Colorado Final Report

for the period

June 1, 2012 - May 31, 2015

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Contents

1 Executive Summary

DoE funds were used as bridge funds for the faculty position for the PI at the University of Colorado. The total funds for the Years 3-5 of the JET Topical Colaboration amounted to about 50 percent of the academic year salary of the PI.

The PI contributed to the JET Topical Collaboration by developing, testing and applying algorithms for a realistic simulation of the bulk medium created in relativistic ion collisions.

Specifically, two approaches were studied, one based on a new Lattice-Boltzmann (LB) framework, and one on a more traditional viscous hydrodynamics framework. Both approaches were found to be viable in principle, with the LB approach being more elegant but needing still more time to develop. The traditional approach led to the super-hybrid model of ion collisions dubbed 'superSONIC', and has been successfully used for phenomenology of relativistic heavy-ion and light-on-heavy-ion collisions.

In the time-frame of the JET Topical Collaboration, the Colorado group has published 15 articles in peer-reviewed journals, three of which were published in Physical Review Letters. The group graduated one Master student during this time-frame and two more PhD students are expected to graduate in the next few years. The PI has given more than 28 talks and presentations during this period.

2 Activities of the Colorado group, years 3-5

2.1 Theory of Lattice Boltzmann Solvers

To achieve the goal of a realistic and fast 3+1d viscous hydrodynamic alogrithm, the PI studied so-called Lattice Boltzmann (LB) Hydrodynamic Solvers for relativistic fluids, based on Ref. [1]. The LB scheme is based on the Boltzmann equation for the single massless particle species distribution function f with the relativistic analogue of the Bhatnagar-Gross-Krook collision term:

$$\left[p^\mu \nabla_\mu - \Gamma^\lambda_{\mu\nu} p^\mu p^\nu \partial^{(p)}_\lambda\right] f = -\frac{p^\mu u_\mu}{\tau_R}\left(f - f^{\rm eq}\right) ,$$

where τ_R is the relaxation time, ∇_μ denotes the geometric covariant derivative and $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols that are given by the derivatives of the underlying metric tensor, and $f^{\rm eq}$ denotes the equilibrium distribution function.

2.2 Challenge 1: Numerical Instabilities

While LB solvers are known to have considerable advantages over more traditional computational fluid dynamics methods, they also have a few disadvantages such as instabilities manifesting themselves in the low density regime. The PI devoted considerable time in the past funding period to identifying the nature of these instabilities and investigating possible ways to overcome them. In particular, the PI studied the one of the simplest relativistic Lattice Boltzmann available, namely

$$p^\mu \partial_\mu f(p^\mu, x^\mu) = -(p\cdot u)\frac{f - f_{\rm eq}}{\tau} . \quad (1)$$

In the LB scheme, such equations are solved on a sparse momentum lattice p^μ. The minimum number of lattice points is determined by requiring the scheme to accurately calculate the hydrodynamic quantities which are compactly given by the energy momentum tensor $T^{\mu\nu}$:

$$T^{\alpha\beta}(x^\mu) = \int \frac{d^3p}{p^0} f(p^\mu, x^\mu) p^\alpha p^\beta . \quad (2)$$

The minimum number of lattice points necessary to accurately describe hydrodynamics had been calculated in Ref. [1] and turns out to be 18 points for three spatial dimensions (D3R18). Selection of the lattice points also dictates the functional basis for the expansion of the equilibrium distribution function $f_{\rm eq}$. Specificially, for the D3R18 lattice with speeds $\mathbf{v} = \mathbf{p}/|\mathbf{p}|$, the first 9 orthogonal polynomials follow directly from the requirement that Eq. (2) be represented exactly:

$$\begin{aligned}
F_0 &= 1\,, \\
F_1 &= v^x\,, \\
F_2 &= v^y\,, \\
F_3 &= v^z\,, \\
F_4 &= (v^x)^2 - \frac{1}{3}\,, \\
F_5 &= v^x v^y\,, \\
F_6 &= v^x v^z\,, \\
F_7 &= (v^y)^2 - \frac{1}{3} + \frac{1}{2}F_4\,, \\
F_8 &= v^y v^z\,.
\end{aligned} \tag{3}$$

Hence these are the *hydrodynamic* basis functions and as a consequence their coefficients correspond to hydrodynamic degrees of freedom.

As outlined above, the equilibrium distribution function has to be expanded in the orthogonal basis function set corresponding to the momentum lattice, e.g.

$$f_{\mathrm{eq}} = \exp\left[-\frac{p^0}{T}\left(u^0 - \mathbf{v}\cdot\mathbf{u}\right)\right] = \exp\left[-\frac{p^0}{T_{\mathrm{ref}}}\right]\sum_{n=0}^{8} F_n a_n^{\mathrm{eq}}(T, u^\mu)\,, \tag{4}$$

for the D3Q18 lattice where T_{ref} is some reference temperature. The coefficients $a_0^{\mathrm{eq}}, \ldots a_8^{\mathrm{eq}}$ correspond to the hydrodynamic degrees of freedom and are given by

$$\begin{aligned}
a_0^{\mathrm{eq}} &= \frac{T^4}{T_{\mathrm{ref}}^4}\left(1 + \frac{4}{3}\mathbf{u}^2\right)\,, \\
a_1^{\mathrm{eq}} &= \frac{T^4}{T_{\mathrm{ref}}^4}4u^x u^0\,, \\
a_2^{\mathrm{eq}} &= \frac{T^4}{T_{\mathrm{ref}}^4}4u^y u^0\,, \\
a_3^{\mathrm{eq}} &= \frac{T^4}{T_{\mathrm{ref}}^4}4u^z u^0\,, \\
a_4^{\mathrm{eq}} &= \frac{T^4}{T_{\mathrm{ref}}^4}15\left((u^x)^2 - \frac{\mathbf{u}^2}{3}\right)\,, \\
a_5^{\mathrm{eq}} &= \frac{T^4}{T_{\mathrm{ref}}^4}20\left(u^x u^y\right)\,,
\end{aligned}$$

$$
\begin{aligned}
a_6^{\rm eq} &= \frac{T^4}{T_{\rm ref}^4} 15\left(u^x u^z\right) , \\
a_7^{\rm eq} &= \frac{T^4}{T_{\rm ref}^4} 20\left((u^y)^2 - \frac{\mathbf{u}^2}{3}\right) + \frac{2}{3} a_4^{\rm eq} , \\
a_8^{\rm eq} &= \frac{T^4}{T_{\rm ref}^4} 20\left(u^y u^z\right) .
\end{aligned} \tag{5}
$$

Indeed, one finds that a LB scheme with such a distribution function works very well for small fluid velocities. However, for large fluid velocities (as usually encountered in the low density regions of say a heavy-ion collision), one encounters instabilities in the evolution that render the scheme unwieldy if not unusable (cf. the results shown in Ref.[1]).

This challenge had not been identified in the orginal proposal, and in the past years, the PI has dedicated considerable effort on curing these instabilities via a variaty of methods, such as those known in the non-relativistic literature as inclusion of non-hydrodynamic degrees of freedom, velocity rescaling and f_0 stabilization. Unfortunately, within the time given, none of these schemes led to a considerable and unequivocal stabilization of the relativistic 3+1 viscous hydrodynamics LB algorithm.

Nevertheless, the LB scheme define above has not proven useless. In fact, it is unconditionally stable even in the case of large gradients in situations where the fluid velocities are mildly relativistic (up to about 50 percent the speed of light). Because of this unconditional stability in the large-gradient regime, the relativistic viscous LB scheme has been successfully used as a 3+1 viscous hydrodynamic comparison to exact quantum field theory results in far-from-equilibrium situations [2].

2.3 Challenge 2: Non-ideal equation of state

The most pressing challenge identified in the original proposal had been the inability of the 'naive' LB scheme to simulate non-ideal gas equations of state, such as those necessary to realistically simulate QCD.

The PI proposed to solve this challenge by implementing temperature-dependent masses. This turned out to be true and has been implemented in a LB scheme outlined in Ref. [3]. Indeed, this line of work has also found applications in so-called 'anisotropic-hydrodynamic' schemes, a separate hydrodynamics algorithm effort, which is also based on the Boltzmann equation.

Furthermore, during the past year the PI has found a second possible solution to this problem inspired by non-relavistic LB schemes. Namely, by suitable deforming the equilibrium distribution function (4), it is possible to simulate non-ideal pressure to energy density relations even for simple (even massless) particles.

The knowledge from the studies in the relativistic LB application has been used in a 'spin-off' study of ultracold Fermi gases with non-ideal equations of state in Ref. [4].

2.4 superSONIC: A successful alternative approach

Faced with the problem of instabilities at high velocities, not initially anticipated in the original proposal, the PI studied alternative approaches towards a realistic viscous hydrodynamics algorithm. In collaboration with the local nuclear physics experimental group, in particular Mike McCumber and Jamie Nagle, it became apparent that the main stumbling block to performing realistic event-by-event heavy-ion medium simulations through 'traditional' hydrodynamics algorithms such as VH2+1 [5] could be removed by performing local smearing of low energy-density regions.

This realization opened up the possibility of performing fully dynamic 2+1d viscous hydrodynamics simulations of heavy-ion and even light-on-heavy ion collisions on an event-by-event basis.

Coupled with first-principles simulations of dynamical equilibration from AdS/CFT and coupled to a late-stage hadronic cascade simulation [6], this approach has led to the creation of a 'super-hybrid' algorithm of hydrodynamic medium simulations in relativistic ion collisions, dubbed 'superSONIC' [7, 8].

The superSONIC package has been used to simulate $^3\mathrm{He} + Au$ collisions and was an important ingredient in motivating the experimental $^3\mathrm{He} + Au$ program at RHIC.

Furthermore, superSONIC (and its predecessor SONIC) have been used to perform super-hybrid simulations of nuclear collisions of C + C, Al + Al, Cu + Cu, Au + Au, and Pb + Pb from $\sqrt{s} = 62.4$ - 2760 GeV [7], storing the simulated space-time medium information in publically available format for use in JET modification simulations of these collision systems.

Moreover, superSONIC has been used to simulate collisions of p+Au d+Au, ^{3}He+Au and p+Pb for various collision energies ranging from $\sqrt{s} = 7$ GeV to $\sqrt{s} = 5.02$ TeV, making predictions for experimental flow signatures, the limits of hydrodynamic applicability in small systems as well as identifying possible experimental signatures of pre-equilibrium QCD dynamics [8].

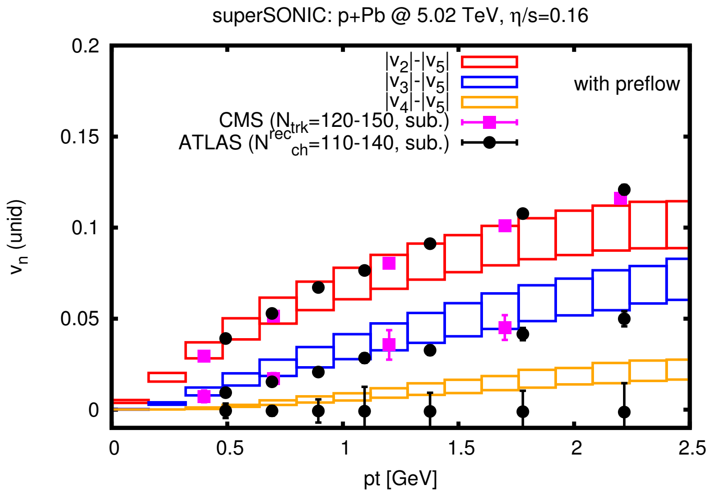

Figure 1: Flow anisotropy coefficients v_n from superSONIC simulations of p+Pb collisions at $\sqrt{s} = 5020$ GeV compared to experimental data from the CMS and ATLAS experiments. Figure from Ref. [8].

2.5 Publications/Preprints/Talks

Since the DOE funds are used entirely to bridge the PI's faculty position salary in the academic year (without any reduction of the PI's teaching load at this institution), the direct costs of the activities below have been carried by other sources, such as the PI's Sloan Award, the PI's DOE Early Career Award and the PI's startup funds at the University of Colorado, Boulder.

	Publications	Preprints/Submitted	Conf. Repts.	Talks
PI (Romatschke)	15	4	0	28
students	5	1	0	0
Total	**15**	**4**	**0**	**28**

2.5.1 Details Publications

1. B. Wu and P. Romatschke, *Shock wave collisions in AdS5: approximate numerical solutions*, Int. J. Mod. Phys. C **22** (2011) 1317.

2. P. Romatschke, *Relativistic (Lattice) Boltzmann Equation with Non-Ideal Equation of State*, Phys. Rev. D **85** (2012) 065012.

3. P. Romatschke and R. E. Young, *Implications of hydrodynamic fluctuations on the minimum shear viscosity of the dilute Fermi gas at*

unitarity, Phys. Rev. A **87** (2013) 053606.

4. P. Romatschke and J. D. Hogg, *Pre-Equilibrium Radial Flow from Central Shock-Wave Collisions in AdS5*, JHEP **1304** (2013) 048.

5. W. van der Schee, P. Romatschke and S. Pratt, *A fully dynamical simulation of central nuclear collisions*, Phys. Rev. Lett. **111** (2013) 222302.

6. A. M. Adare, M. P. McCumber, J. L. Nagle and P. Romatschke, *Tests of the Quark-Gluon Plasma Coupling Strength at Early Times with Heavy Quarks*, Phys.Rev. **C90** (2014) 2, 024911.

7. J. L. Nagle, A. Adare, S. Beckman, T. Koblesky, J. O. Koop, D. McGlinchey, P. Romatschke and J. Carlson *et al.*, *Exploiting Intrinsic Triangular Geometry in Relativistic He3+Au Collisions to Disentangle Medium Properties*, Phys.Rev.Lett. **113** (2014) 11, 112301.

8. M. Habich and P. Romatschke, *Onset of cavitation in the quark-gluon plasma*, JHEP **1412** (2014) 054.

9. P. Kovtun, G. D. Moore and P. Romatschke, *Towards an effective action for relativistic dissipative hydrodynamics*, JHEP **1407** (2014) 123.

10. T. Gorda and P. Romatschke, *Precision studies of v_n fluctuations*, Phys. Rev. C **90** (2014) 5, 054908.

11. P. Arnold, P. Romatschke and W. van der Schee, *Absence of a local rest frame in far from equilibrium quantum matter*, JHEP **1410** (2014) 110.

12. M. Habich, J. L. Nagle and P. Romatschke, *Particle spectra and HBT radii for simulated central nuclear collisions of C + C, Al + Al, Cu + Cu, Au + Au, and Pb + Pb from* $\sqrt{s} = 62.4$ - 2760 *GeV*, Eur. Phys. J. C **75** (2015) 1, 15.

13. H. Bantilan and P. Romatschke, *Simulation of Black Hole Collisions in Asymptotically Antide Sitter Spacetimes*, Phys. Rev. Lett. **114** (2015) 8, 081601.

14. T. Gorda and P. Romatschke, *Equation of state in two-, three-, and four-color QCD at nonzero temperature and density*, Phys. Rev. D **92** (2015) 1, 014019.

15. P. Romatschke, *Light-Heavy Ion Collisions: A window into pre-equilibrium QCD dynamics?*, Eur. Phys. J. C **75** (2015) 7, 305.

2.5.2 Details Preprints

- P. Romatschke and S. Pratt, *Extracting the shear viscosity of a high temperature hadron gas*, arXiv:1409.0010 [nucl-th].
- P. Romatschke, *Collective flow without hydrodynamics: simulation results for relativistic ion collisions*, arXiv:1504.02529 [nucl-th].
- J. Brewer, M. Mendoza, R. E. Young and P. Romatschke, *Lattice Boltzmann simulations of a two-dimensional Fermi gas at unitarity*, arXiv:1507.05975 [cond-mat.quant-gas].
- J. Brewer and P. Romatschke, *Non-hydrodynamic transport in trapped unitary Fermi gases*, arXiv:1508.01199 [hep-th].

2.5.3 Details Talks

1. Paul Romatschke, Johns Hopkins University, March 2012
2. Paul Romatschke, Seminar Los Alamos National Lab, August 2012
3. Paul Romatschke, Seminar Quark Matter 2012
4. Paul Romatschke, Seminar University of Maryland, September 2012
5. Paul Romatschke, Seminar CU Boulder September 2012
6. Paul Romatschke, Seminar Xth Quark Confinement, Munich October 2012
7. Paul Romatschke, Colloqium Denver University, Nov 2012
8. Ryan Edward Young, Seminar at Dynamic Days US 2013, Denver, January 2013
9. Paul Romatschke, JILA Colloqium CU Boulder February 2013
10. Paul Romatschke, Seminar, GHP Group, APS April Meeting, Denver, 2013
11. Paul Romatschke, Colloquium, Colorado School of Mines, April 2013
12. Paul Romatschke, Seminar, JET Collaboration Bulk WG Meeting, June 2013
13. Paul Romatschke, Seminar, Garcia Collin Meeting, Mexico City, September 2013

14. Paul Romatschke, Talk New Frontiers in Dynamical Gravity, Cambridge, UK, 24-28 March 2014
15. Paul Romatschke, Talk The Approach to Equilibrium in Strongly Interacting Matter, BNL April 2014
16. Paul Romatschke, Talk Quark Matter 2014, Darmstadt, May 2014
17. Paul Romatschke, Seminar TU Vienna, May 2014
18. Paul Romatschke, Talk Jet Collaboration Meeting, UC Davies, June 2014
19. Paul Romatschke, Seminar, Perimeter Institute, Nov 2014
20. Paul Romatschke, Seminar, University of Toronto, Nov 2014
21. Paul Romatschke, Talk Initial Stages in Heavy-Ion Collisions, Napa Valley, Dec 2014
22. Paul Romatschke, Talk Numerical Holography, CERN, Dec 2014
23. Paul Romatschke, Talk, Winter Workshop Nuclear Dynamics, Keystone, Jan 2015
24. Paul Romatschke, Talk Holographic Methods for Strongly Coupled Systems, Florence, April 2015
25. Paul Romatschke, Seminar, University of Crete, Jun 2015
26. Paul Romatschke, Seminar, University of Frankfurt, Jun 2015
27. Paul Romatschke, Talk Correlations and Fluctuations in p+A and A+A Collisions, INT, July 2015
28. Paul Romatschke, Talk Equilibration Mechanisms in Weakly and Strongly Coupled Quantum Field Theory INT, Aug 2015

2.5.4 Students

Student	**Date entered Grad School**	**Date Joined Group**	**Program**	**Degree Expected**	**Status**
Tyler Gorda	Aug 2011	Jan 2012	PhD	May 2016	current
Mathis Habich	Aug 2012	Aug 2013	PhD	May 2017	current
Ryan E. Young	Jan 2012	Mar 2012	Master	Aug 2013	graduated

References

[1] P. Romatschke, M. Mendoza and S. Succi, Phys. Rev. C **84** (2011) 034903 [arXiv:1106.1093 [nucl-th]].

[2] H. Bantilan and P. Romatschke, Phys. Rev. Lett. **114** (2015) 8, 081601 [arXiv:1410.4799 [hep-th]].

[3] P. Romatschke, Phys. Rev. D **85** (2012) 065012 [arXiv:1108.5561 [gr-qc]].

[4] J. Brewer, M. Mendoza, R. E. Young and P. Romatschke, arXiv:1507.05975 [cond-mat.quant-gas].

[5] M. Luzum and P. Romatschke, Phys. Rev. C **78** (2008) 034915 [Phys. Rev. C **79** (2009) 039903] [arXiv:0804.4015 [nucl-th]].

[6] W. van der Schee, P. Romatschke and S. Pratt, Phys. Rev. Lett. **111** (2013) 22, 222302 [arXiv:1307.2539].

[7] M. Habich, J. L. Nagle and P. Romatschke, Eur. Phys. J. C **75** (2015) 1, 15 [arXiv:1409.0040 [nucl-th]].

[8] P. Romatschke, Eur. Phys. J. C **75** (2015) 7, 305 [arXiv:1502.04745 [nucl-th]].