Technical Report

for

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1 Summary of Accomplishments

The grant entitled Neoclassical Theory and Its Applications started on January 15 2001 and ended on April 14 2015. The main goal of the project is to develop neoclassical theory to understand tokamak physics, and employ it to model current experimental observations and future thermonuclear fusion reactors. The PI had published more than 50 papers in refereed journals during the funding period.

The PI had devoted his major efforts to develop a comprehensive theory for neoclassical toroidal plasma viscosity for tokamaks with broken symmetry. It began with the first paper on that theory published in 2001, and the last paper appeared in April 2015. The results become a major section, i.e., section 8 in a review paper published online in November 2015 by the PI and coauthors [1]. The theory has been checked by two independent numerical codes [2,3]. There is an agreement between the theoretical and numerical results. Part of the theoretical results is also checked against experiments. There are also agreements between theoretical and experimental results. The PI has published 21 papers on the theory for the neoclassical toroidal plasma viscosity. The PI had worked on other topics that are related to tokamak physics, for example, a physics picture for the origin of the momentum for plasma flow generation without external momentum sources, neoclassical effects on the resistive wall mode and linear tearing mode, the relation between improved neoclassical transport and anomalous transport, transport theory inside transport barrier, direction of the ion ∇B drift and power threshold for the high confinement mode, bootstrap current in the vicinity of the magnetic island and its effects on island evolution, and transport theory in the vicinity of the magnetic axis with finite toroidal slow speed. The summary of these subjects is presented in section 2.

The PI had attended conferences to present research results supported by the grant. The conferences attended are American Physical Society meeting, Sherwood International Conference, Fusion Energy Conference, European Physical Society meeting, and International Tokamak Physics Activity meeting. The oral presentations are listed in section 3.

The list of the published papers is presented in section 4.

2 Neoclassical Theory and Its Applications

Neoclassical theory is a study of plasma transport behaviors when Coulomb collisions are the dissipation mechanism. Because tokamak plasmas are usually plagued by turbulence fluctuations, neoclassical theory is thought not relevant to tokamak plasma confinement. However, the PI has developed a neoclassical quasilinear theory [4], and the results of which indicate that there are physics phenomena observed in tokamak experiments can be understood in terms of neoclassical transport processes. The fundamental reason is that the equilibriu, magnetic field variation is much larger than the turbulent fluctuation level. The research project is devoted to investigate the neoclassical theory that is relevant to tokamak experiments in the US and around the world. In this section, the PI summaries the major high lights of the research results obtained during the funding period. The details are referred to the papers listed in section 4, and the review paper [1].

2.1 Neoclassical Theory inside Transport Barrier

Standard neoclassical theory has been reviewed in Refs. [5,6]. However, for advanced tokamak operation such as high confinement mode (H-mode), the standard theory needs to be modified to accommodate the physics situations that had not been taken into account, such as effects of orbit squeezing, poloidal sonic $\boldsymbol{E}\times\boldsymbol{B}$ speed, … etc. Here, $\boldsymbol{E}$ is the electric field, and $\boldsymbol{B}$ is the magnetic field. The PI had investigated these subjects. The PI had shown that both orbit squeezing, and poloidal sonic flow improve ion thermal transport. Thus, in the pedestal region of H-mode plasmas, not only anomalous losses are reduced as a result of the turbulence suppression [7], but also neoclassical losses. The PI had also shown that when the ion viscous force becomes comparable to electron viscous force, bootstrap current can also be reduced. This can be used to control plasma current density in the pedestal region and thus can control edge localized modes (ELMs) without breaking toroidal symmetry. These results had not been tested in current experiments. However, these predictions should be tested to benefit tokamak operations in the future.

Because the rate of neoclassical poloidal flow damping is affected by the effects of the orbit squeezing and sonic poloidal flow, the anomalous transport is also affected through the influence of the zonal flow on turbulence fluctuation level. This indicates that when neoclassical transport losses are improved, anomalous transport losses should also improve. Thus, it is important to include neoclassical theory in the investigation of turbulence induced transport losses.

The PI had also developed a simple explanation for the dependence of the H-mode power threshold on the direction of ion ∇B drift observed in all tokamaks based on the connection length in the definition of collisionality parameter ν_* [5,6]. This provides one more piece of evidence to support the H-mode theory based on the ion orbit loss and nonlinear plasma viscosity [8].

2.2 Theory for Neoclassical Toroidal Plasma Viscosity

Real tokamaks are not toroidally symmetric. The broken symmetry enhances particle, momentum and energy losses for any value of the Columb collision frequency. The PI had developed a comprehensive theory to describe the enhanced transport fluxes for all collisionality regimes. Starting from the collisional Pfirsch-Schluter regime, plateau regime, $1/\nu$ regime, superbanana plateau regime, superbanana regime, collisional boundary regime, and collisionless detrapping/retrapping regime are all investigated. He also investigated bounce-transit and drift resonance regime using a novel Eulerian approach to solve drift kinetic equation. A summary of these results is presented in [9]. Results of all these regimes are compared in [2] and agreements are reached in all of them in the large aspect ratio limit. The PI has since extended the theory to finite aspect ratio tokamaks to model real tokamaks with broken symmetry. An approximate formula that joins all these results has been constructed [10] using asymptotic limits in all regimes in the large aspect ratio. The construction scheme of the approximate formula can be used for results for finite aspect ratio tokamaks as well. It is believed that implement the results for finite aspect ratio tokamaks in the formula should also produce agreement with numerical codes.

The transport fluxes can be categorized into resonant and nonresonant fluxes. All these fluxes depend on the radial electric field nonlinearly. The resonant fluxes depend on the radial electric field exponentially, and the nonresonant fluxes algebraically. Thus, the momentum equation can have bifurcated solutions similar to the H-mode theory [8]. However, for fusion born alpha particles, because of their energy is much higher than the thermal particles, their transport physics is not affected by the radial electric field. Thus, one cannot rely on the radial electric field to improve energetic alpha particle confinement. The only method control energetic alpha particle confinement is to minimize the symmetry breaking components in the spectrum of the magnetic field strength.

The theory predicts the existence of the steady state toroidal rotation speed without the need of the external momentum sources in all collisionality regimes. The source of the momentum in that case is the non-canceling particle momenta from neighboring particle orbits. A schematic figure to demonstrate the process is shown in Fig. 1. This is similar to the origin of the momentum for the standard diamagnetic flow. Thus, the magnitude of the steady state flow is first order in gyro-radius ordering. The physics picture shown here is based on the insight gleaned in the process of studying the time dependent plasma viscosity. It is discovered that when the time operator dominates the collisional dissipation, the viscous forces only depend on the poloidal flow and not on the poloidal heat flow. This points to the fact that standard neoclassical poloidal flow is a result of the dependence of the collision frequency on particle energy. If there were no such dependence, there would be no neoclassical poloidal flow; in that case, particle momenta from neighboring orbits cancel out exactly. In general, however, step size, dissipation mechanism, and decorrelation time depend on particle energy. Thus, the cancellation is usually incomplete. This leads to the generation of the steady state plasma flow without external momentum sources. Such argument is valid for turbulence generated toroidal flow as well.

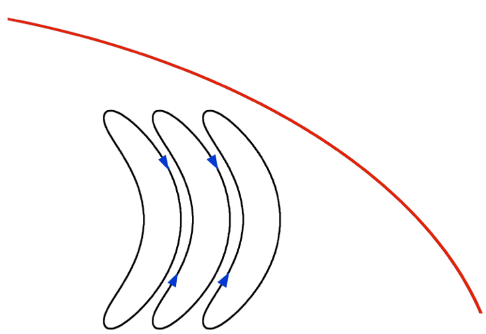

Figure 1.The origin of the momentum for flow generation without external momentum sources is illustrated here. The red line indicates the equilibrium temperature profile. The neighboring banana orbits are shown to indicate the cancelation of the local particles' momenta.

It has been shown that fusion born alpha particles are sensitive to toroidal symmetry breaking due to their high energy as result of the energy dependence of the drift speed. Theory for neoclassical toroidal plasma viscosity had been extended to investigate energetic alpha particle transport losses. The results indicate that for $\delta B/B$ of the order of 10^{-4} or lower, the fusion energy gain factor Q probably will not be significantly impacted by the broken symmetry. Here, δB is the typical value of the perturbed magnetic field strength. However, this also implies that stellarators might not be viable candidates for thermonuclear fusion reactors because poor energetic alpha particle energy confinement. This is true even for optimized stellarators because the magnitude of the $\delta B/B$ in those devices is still of the order of a few percent which is much larger than 10^{-4}. Thus, the energy of fusion born alpha particles are poorly confined in those devices.

To model plasma rotation, a kinetic derivation of the flux-force relation had been developed. The purpose of the kinetic derivation is to make the connection between the transport fluxes, and the viscous forces in the momentum equation more transparent. Thus, the results of the transport fluxes can be used for the viscous forces in the momentum equation without the need to perform new calculations. The flow damping rate and steady state intrinsic flow can be obtained by solving momentum equation. This will help the modeling of plasma rotation in current tokamak experiment and in future tokamak experiments.

Neoclassical toroidal plasma viscosity depends on collision frequency is a complicated way. A schematic dependence of the neoclassical toroidal plasma viscosity on the collision frequency is shown in Fig. 2. Analytic expressions for these asymptotic limits had been derived. The mathematical method employed to obtain these limits can be served as a guide to solve drift kinetic equation in complicated magnetic geometry.

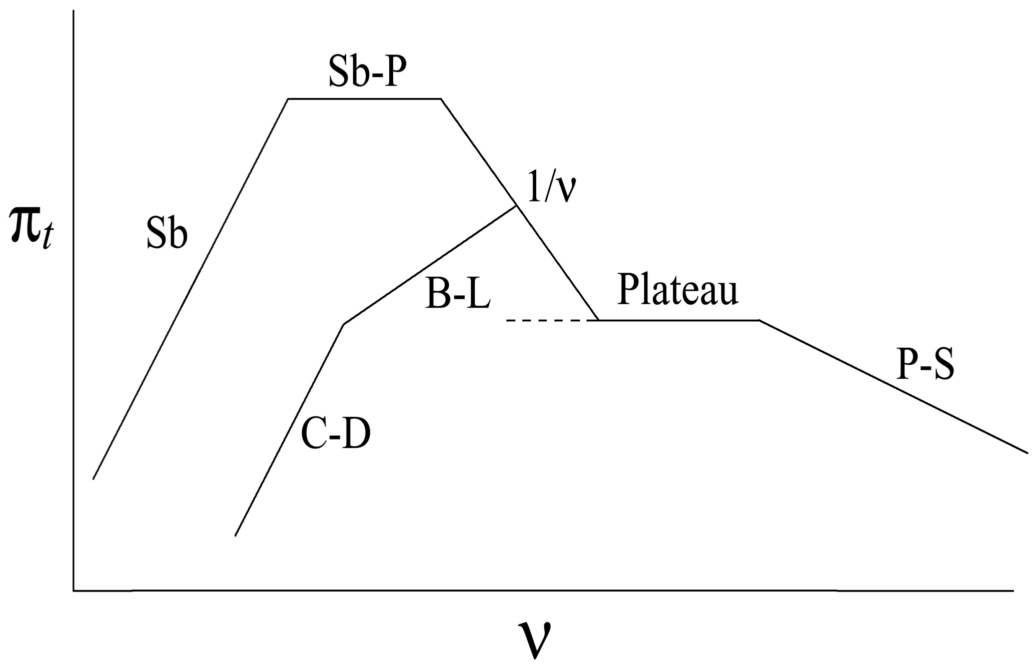

Figure 2 Neoclassical toroidal plasma viscosity π_t versus collision frequency ν in a log-log plot. The superbanana (Sb), superbanana plateau (Sb-P), $1/\nu$, collisional boundary layer (B-L), collisionless detrapping (C-D), plateau, and Pfirsch-Schluter (P-S) regimes are shown. The dotted line is to indicate the bounce-transit and drift resonance.

2.3 Transport Processes in the Vicinity of Magnetic Island and Their Consequences

The presence of a magnetic island also breaks toroidal symmetry in tokamaks, and enhances particle, momentum, and energy transport losses. In this case, transport fluxes are calculated on the helical island magnetic surface. For a large aspect ratio tokamak, the magnetic field strength *B* on the island magnetic surface is

$$\frac{B}{B_0} = 1 - \left[\frac{r_s}{R} \pm \frac{r_w}{R}\left(\overline{\Psi} + \cos m\xi_I\right)^{1/2}\right]\cos\theta, \qquad (1)$$

where B_0 is *B* on the magnetic axis, *R* is the major radius, r_s is the minor radius at χ_s , χ_s is the poloidal flux χ at the resonant magnetic surface with the safety factor $q = q_s$ a rational number, $\xi_I = \left(\theta - \zeta/q_s\right)$ for a static magnetic island, θ is the poloidal angle, ζ is the toroidal angle, m is the poloidal model number, normalized helical flux function $\overline{\Psi} = -\Psi/\tilde{\chi}$, $\tilde{\chi}$ is the amplitude of the perturbed poloidal flux, $r_w = \left[2q_s^2\tilde{\chi}/\left(q_s' B_0 r_s\right)\right]^{1/2}$ is proportional to the island width, and $q_s' = dq/d\chi\big|_{\chi_s}$. The $\pm$ sign in Eq.(1) is chosen to be '+' for $\chi > \chi_s$ and '-' for $\chi < \chi_s$.

The most obvious effects caused by the symmetry breaking are the modifications on the bootstrap current and enhanced plasma viscosity in the vicinity of the magnetic island. Their effects on the island evolution and island rotation frequency are investigated. The predictions are not tested in the experiments.

Transport fluxes in the $1/\nu$ regime, and the collisional boundary layer regime are calculated. These results can also used in the modeling of the toroidal rotation in the vicinity of the magnetic island. The PI had shown that momentum equation can have bifurcated solutions similar to the H-mode theory developed by the PI [8].

In the vicinity of the magnetic island, the enhanced plasma viscosity also determines a radial electric field. Because the width of the island is narrow, the gradient scale length of the radial electric field can be comparable to the width of the trapped ions. Thus, effects of orbit squeezing can be important to suppress the turbulence fluctuations to improve plasma confinement in the vicinity of the magnetic island. This theory can explain the superb confinement inside and in the vicinity of the magnetic island observed in experiments.

The PI also investigated on the controlling of the magnetic island using pellet injection at the island O point. The main physics mechanism for the stablization of the island is the bootstrap current generated by the peaked density profile after the deposition of the pellet inside the island to replenish the missing bootstrap current caused by flattening of the density profile. This stablization scheme can be carried out without the need of electron current drive. It can also fuel plasmas at the same time. This prediction is not tested in the experiments.

2.4 Physics Picture for the Origin of Momentum for Intrinsic Flow in Toroidal Plasmas

To determine radial electric completely using radial force balance equation, toroidal rotation speed is needed. It is known that standard theory for the toroidal viscosity is too small to account for the experimentally observed rate of toroidal rotation damping in axisymmetric tokamaks. Thus, it is likely turbulence fluctuations play a role in the observed damping rate. The PI had developed a neoclassical quasilinear theory for the toroidal viscosity using the methodology of neoclassical theory. This unifies quasilinear theory and neoclassical theory as expected. The important discovery is that the toroidal momentum flux consists of three components. Besides the usual diffusion flux, it has a convective flux and a residual flux [11]. The convective flux is not proportional to the particle flux in the theory. This characteristic is in agreement with the experimental observation [12]. In this theory, there is a toroidal rotation even without external momentum sources. However, it is not clear where the momentum comes from to drive such a flow. The PI had developed a physics picture for the origin of the momentum in this research project. The PI found that the momentum is from non-canceling particle momenta from neighboring drift orbits in analogous to the diamagnetic flow. Because the toroidal flow is diamagnetic flow in nature, the magnitude of the toroidal flow speed is first order in gyro-radius ordering, which is the same as the steady state intrinsic toroidal flow derived form the theory for neoclassical toroidal plasma viscosity. Thus, it will be difficult to distinguish these two experimentally.

2.5 Neoclassical Effects on Linear Magnetiohydrodynamic Modes

Neoclassical effects are usually ignored in studying magnetohydrodynamic (MHD) modes. The PI included such effects in the resistive wall mode and linear tearing mode. When neoclassical effects are included in a resistive wall mode model, the required toroidal rotation speed to stabilize resistive wall mode is much reduced, which is consistent with the experimentally observed trends. Also the growth rate of the linear tearing mode becomes depending on the poloidal magnetic field strength when neoclassical effects are included. Thus, if one uses linear tearing mode growth rate to estimate anomalous diffusion coefficient, it will only depend on the poloidal magnetic field strength. This dependence on the poloidal magnetic field strength might be relevant to the observed scaling of anomalous electron transport losses in tokamaks.

2.6 Physics of Plateau Regime

It is known that plateau regime is caused by the resonant particles that have vanishing speed along the magnetic field line. However, an incorrect theory had been advanced claiming that it is the very circulating particles that contribute to the transport fluxes in the plateau regime. The PI had revisited this old issue and clarified that it is the resonant particles that caused the transport and restored the old, accepted understanding.

2.7 Transport Theory in the Vicinity of Magnetic Axis with Finite Toroidal Ration Speed

In neutral beam heated tokamaks, it is possible to have a near sonic toroidal rotation in the vicinity of the magnetic axis. The PI had developed a neoclassical transport theory for that kind of plasmas. In the vicinity of the magnetic axis, trapped particles are potato orbits. Thus, when solving drift kinetic equation in the potato regime, the potato particle dynamics in the presence of the poloidal density variation has to be taken into account. As expected, unless the toroidal rotation speed is supersonic, its effects of the finite toroidal rotation speed on transport fluxes are not significant. However, once the toroidal rotation speed passes supersonic speed, transport losses increase dramatically.

2.8 Summary

The PI has published a review paper entitled ‘Neoclassical plasma viscosity and transport processes in non-axisymmetric tori’ with Dr. K. Ida at National Institute for Fusion Science, Japan, and Dr. S. A. Sabbagh at Columbia University, USA [1] after the grant was terminated. He is responsible for the theory part. He summarized most of all his works published in the last 35 years. The works supported by the grant reported here are also discussed in great mathematical and physics details there. Readers who are interested in the details of the works described here are suggested to get a copy of that paper.

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3 Oral Presentations of Research Results in Major Conferences

Fusion Energy Conference, International Atomic Energy Agency: 2002
International Tokamak Physics Activity (ITPA) Workshop on Momentum Confinement in Tokamaks: 2006
International Atomic Energy Agency (IAEA) 4th Technical Meeting on Spherical Tori: 2008
Asia Plasma and Fusion Association (APFA) / Asia-Pacific Plasma Theory Conference (APPTC): 2009
480 Wilhelm and Else Heraeus Seminar 2011
European Physical Society/International Conference on Plasma Physics 2012
International Tokamak Physics Activity (ITPA) Workshop on Energetic Particles: 2014
European Physical Society 2014 (Non-invited Oral,)
International Congress on Plasma Physics 2014

4 Grant Supported Publications

The papers that are supported by the research project and are published in the refereed journals are listed as follows.

[1] K. C. Shaing, "Superbanana and superbanana plateau transport in finite aspect ratio tokamaks with broken symmetry," J. Plasma Phys. **81** 905810203 (2015)
[2] K. C. Shaing, and C. T. Hsu, "Transport theory for energetic alpha particles and tolerable magnitude of error fields in tokamaks with broken symmetry," Nucl. Fusion **54**, 033012 (2014).
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