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Executive Summary

The research reported here involves studies of radial particle transport in a cylindrical, low-density Malmberg-Penning non-neutral plasma trap. The research is primarily experimental but involves careful comparisons to analytical theory and includes the results of a single-particle computer code. The transport is produced by applied electric fields that break the cylindrical symmetry of the trap, hence the term "asymmetry-induced transport." Our computer studies have revealed the importance of a previously ignored class of particles that become trapped in the asymmetry potential. In many common situations these particles exhibit large radial excursions and dominate the radial transport. On the experimental side, we have developed new data analysis techniques that allowed us to determine the magnetic field dependence of the transport and to place empirical constraints on the form on the transport equation. Experiments designed to test the computer code results gave varying degrees of agreement with further work being necessary to understand the results. This work expands our knowledge of the varied mechanisms of cross-magnetic-field transport and should be of use to other workers studying plasma confinement.

I. INTRODUCTION

This document is the Final Technical/Scientific Report for Department of Energy grant DE-FG02-06ER54882 to Occidental College, Los Angeles, California. The project title is "Resonance Overlap, Axial Trapping, and Magnetic Field Scaling in Asymmetry-Induced Transport" and the Principal Investigator is Dennis L. Eggleston, Professor of Physics at Occidental College. The report covers the period 7/15/2006 to 7/14/2014. This period includes the original 2006 three-year grant under the above title, a one-year no-cost extension, a 2010 three-year renewal under the title "Studies of Particle Transport in a Non-Neutral Plasma Trap," and a final one-year no-cost extension.

The research reported here involves studies of neoclassical radial particle transport in a cylindrical, low-density Malmberg-Penning non-neutral plasma trap shown in Fig. **??**. The research is primarily experimental but involves careful comparisons to analytical theory and includes the results of a single-particle computer code. The transport is produced by applied electric fields that break the cylindrical symmetry of the trap, hence the term "asymmetry-induced transport." The research builds on our previous studies of asymmetry-induced transport where careful comparisons of the experiments to the predictions of resonant particle theory found significant discrepancies. It thus seemed clear that resonant particle theory is not a complete model for this transport. Our previous work also provided the outlines of a new model where the transport is dominated by low velocity particles that are axially trapped by the asymmetry and scattered by a chaotic, collisionless process. This research was focused on developing and testing this model.

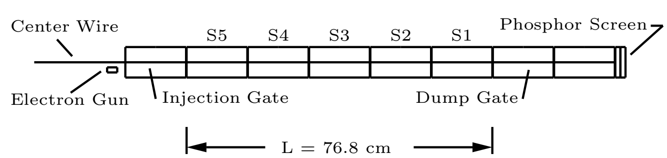

FIG. 1: Schematic of the Occidental College Trap. The usual plasma column is replaced by a biased wire to produce the basic dynamical motions in low density electrons injected from an off-axis gun.

II. ORIGINAL STATEMENTS OF WORK

We list here the original statements of work from the 2006 and 2010 proposals. These were all accomplished during the period of this report and are referred to by number in the Results section.

Statement of Work for 2006 grant:

1. Experimentally determine the magnetic field dependence of asymmetry-induced transport using a new technique that enables one to isolate the various contributions to the radial particle flux.

2. Develop new experimental techniques to verify the existence of axially trapped particles.

3. Determine experimental signatures that allow one to distinguish stochastic transport from other regimes and look for these experimentally.

4. Modify our single particle computer simulation to include a more realistic treatment of the end reflections.

Statement of Work for 2010 grant:

5. Look for evidence of axially trapped particles using two-step dump and central "squeeze" potential.

6. Experimentally study low-frequency transport as a function of available parameters.

7. Extend particle simulations to study possible stochastic dynamics of axially trapped particles.

III. RESULTS

This section is organized topically. Reference is made to the items in the Statement of Work lists thus providing a comparison of the actual accomplishments with the goals and objectives of the project. In most cases the statement of results is brief with details given in the published papers that are included as appendices.

A. Particle dynamics in asymmetry-induced transport

As noted in the Introduction, our previous work found serious discrepancies between resonant particle transport theory and our experimental result, thus making it clear that some physics was missing from the theory. To investigate this, we wrote a simple single-particle computer code to follow the motions of particles in prescribed fields like those in our experiment. The results showed that resonant particle theory missed the existence of a class of low-velocity particles that could be axially trapped in the asymmetry near radii where the asymmetry frequency ω was close to the azimuthal $E \times B$ rotation frequency ω_R. These particles undergo large radial excursions making it likely that they play an important role in radial transport. We also found that these orbits were embedded in a larger region of chaotic orbits. These results were largely qualitative and we were not able at this point to determine the transport coefficients for axially trapped particles. Details of this work are given in the paper: D.L. Eggleston, "Particle dynamics in asymmetry-induced transport", Phys. Plasmas **14**, 012302 (2007).

The code used for the work in the previous paragraph used simple boundary conditions at the end of the trap, either specular reflection or periodic boundary conditions. To insure that we weren't missing important physics, we modified our code to include realistic potentials at the ends of the trap. In order to do this, we first needed to find an analytic expression for those potentials by solving Laplace's equation. While this seems like a textbook problem, the solution for boundary conditions matching our experiment had never been published before, probably because it involved evaluating an integral that was not found in any of the standard integral tables. In the end, we found an obscure theorem that allowed use to evaluate the integral and solve the problem. Details of this work are given in the paper: D.L. Eggleston and Darrell F. Schroeter, "Solution of Laplace's Equation for the Confining End Potentials of a Coaxial Malmberg-Penning Trap", Am. J. Phys. **78**, 287, 2010. When we altered our code to include these realistic potentials, we found several small changes in the particle dynamics. The effective length of the plasma and the azimuthal rotation frequency both had a weak dependence on the particle velocity and radial position. These changes were not significant enough to effect our model of the particle dynamics. This work addressed the Statement of Work item 4.

The next step in this work was to add collisions to the code so that transport would be produced and transport coefficients could be calculated. Collisions were included according to the Langevin prescription. This prescription has two parts. First, a drag term $-\nu v_z$ is included in the equation of motion for v_z. Secondly, after each time step, a velocity Δv is added to v_z. The value of Δv is generated with a random number routine that gives values uniformly distributed over a range determined by the electron temperature and the collision frequency. The diffusion coefficient and the radial drift velocity are determined by following the variations of the particles radial position. For very simple asymmetries, we were able to show good agreement between the values produced by the code and the analytic expressions from resonant particle transport theory. More importantly, for asymmetries of the type found in most experiments and near radii where $\omega = \omega_R$, we found the transport to be dominated by axially trapped particles, in agreement with our earlier speculations. Details of this work are given in the paper: D.L. Eggleston, "Two sources of asymmetry-induced transport", Phys. Plasmas, **19**, 042307 (2012)..

Finally, we examined how this enhanced axially-trapped-particle transport varied with collision frequency. Details of this work are given in the paper: D.L. Eggleston, "Dependence of enhanced asymmetry-induced transport on collision frequency", Phys. Plasmas **21**, 072318 (2014), but two important results can be summarized here. First, the enhanced transport only occurred for low collision frequencies; at higher frequencies, the transport was given by resonant particle theory. We were able to explain this result with a simple model. Secondly, we found no evidence of non-collisional transport (i.e., chaotic transport). We had speculated about this possibility and hoped that at the lowest values of collision frequency the transport would deviate from banana-regime behavior, but this was not observed. This work addressed the Statement of Work item 7.

B. Magnetic field dependence of asymmetry-induced transport

Given the noted discrepancies between theory and experiment, we turned to developing an empirical model of the transport with an eye toward providing guidance for further theoretical developments. Our first target was to find the magnetic field dependence of asymmetry-induced transport. This turned out to be complicated because the magnetic field enters the basic physics in at least two ways: in the zeroth order azimuthal drift frequency and in the first order radial $E \times B$ drift that produces radial transport. To deal with this, we developed a new data analysis technique that allowed us to isolate the magnetic field dependence of the latter by focusing on radial positions where $\omega = \omega_R$ and varying ω. This allowed us to find a scaling law for the diffusion coefficient $D \propto B^{-1.33}$. Details of this work are given in the papers: D.L. Eggleston and J.M. Williams, "Magnetic field dependence of asymmetry-induced transport: a new approach", Phys. Plasmas **15**, 032305 (2008), and D.L. Eggleston, "Using variable frequency asymmetries to probe the magnetic field dependence of radial transport in a Malmberg-Penning trap", in Non-Neutral Plasma Physics VII, edited by J.R. Danielson and T.S. Pedersen, American Institute of Physics, 2009.

Generalizing the above result, we showed that the radial particle flux is empirically constrained to be of the form $\Gamma(\epsilon) = -(B_0/B)^{1.33} D(\epsilon)[\nabla n_0 + f(\epsilon)]$, where $\epsilon = \omega - l\omega_R$, ∇n_0 is the radial density gradient, B is the magnetic field, B_0 is an empirical constant, and $D(\epsilon)$ and $f(\epsilon)$ are unknown functions. We next extended our data analysis technique and showed that further constraints could be placed upon $D(\epsilon)$ and $f(\epsilon)$ by comparing data *near* the $\epsilon = 0$ points to a first order expansion of $\Gamma(\epsilon)$. We showed that $dD/d\epsilon(0) \neq 0$, in contradiction to resonant particle theory, and that $f(\epsilon)$ can only be a fraction of the size predicted by that theory. Finally, we found that $dD/d\epsilon(0)$ exhibits a power-law scaling with radius, magnetic field, and the bias of the center conductor of the trap. Details of this work are given in the paper: D.L. Eggleston, "Constraints on an empirical equation for asymmetry-induced transport", Phys. Plasmas, **17**, 042304 (2010). The above work addressed the Statement of Work item 1.

C. Experimental search for evidence of axially-trapped particles

As noted above, our work with our single-particle computer code produced a new model for the transport in which axially-trapped particles play a major role. We have performed a number of experiments in an attempt to verify this model and to address Statement of Work item 2. One direct approach involved a two-step dump process. Our experiments run in cycles with the plasma dumped out of the trap at the end of each cycle. Usually, our applied asymmetry is turned on for a period of time but turned off prior to the dump. In this new experiment, we first dumped the plasma with the asymmetry still on, expecting that the particles axially trapped by the asymmetry would remain in the machine. We then turned off the asymmetry and dumped a second time to measure these trapped particles.

However, despite repeated attempts with varying parameters, we were not able to observe or measure trapped particles. We do not yet understand this result but are investigating various possible explanations (beyond the obvious that there are no trapped particles).

A second series of experiments involved applying a so-called "squeeze" voltage to the center ring electrode of our trap (electrode S3 in Fig. **??**). The first type of squeeze was a negative voltage applied to the entire ring at the same type as the asymmetry was applied. We found that as the squeeze voltage V_{sq} was made more negative, the flux produced by the asymmetry was reduced by a factor $e^{(V_{sq}/V_0)}$ where $V_0 \approx 1.2$ V. Evidently, particles need to be able to transit the entire machine to produce transport. This is consistent with our transport model (axially trapped particles producing transport when they become untrapped) but the scale factor V_0 is much larger than expected (~ 0.1 V).

A second type of squeeze experiment applies symmetric plus/minus voltages to the two azimuthal halves of the center ring electrode. In this case, we find that a DC squeeze increases the transport produced by the simultaneously applied asymmetry. Kabantsev and Driscoll at UCSD have observed similar behavior and attributed the enhanced transport to chaotic particle dynamics induced by the presence of the squeeze voltage. We continue to investigate this interesting result and will present a poster on the work at the 2014 APS/DPP meeting. The abstract for this presentation is found in D.L. Eggleston, "Effect of a central squeeze potential on asymmetry-induced transport", Bull. Am. Phys. Soc. **59**, 57 (2014). The above experiments address Statement of Work items 3 and 5.

An interesting and consistent feature of our transport experiments is the relative weakness of the transport at low asymmetry frequencies. Our current hypothesis is that this is due to the fact that, at low frequencies, there is no $\omega = \omega_R$ point within the plasma and thus no axially trapped particles. Since such particles normally dominate the transport, the measured flux is small compared to cases where axially trapped particles are present. We have experimentally investigated the radial particle flux Γ produced by frequencies below $f_R = \omega_R/2\pi$. The flux produced by these frequencies is typically largest at the outer edge of the plasma, $r/R \geq 0.75$, where R is the wall radius. The data support an empirical model $\Gamma(r) \propto \exp\left[-(f_0 - f)/f_*\right]$. Both of the parameters f_0 and f_* are proportional to ϕ_{cw}/B, where ϕ_{cw} is the bias of our central wire electrode and B is the axial magnetic field. This scaling suggests a relation with f_R or its derivatives. If we assume the former, then $f_0 \approx 1.5 f_R$ and $f_* \approx f_R/3$. This model is consistent with empirical constraints obtained near the $f = f_R$ points (as discussed above), but the physical basis for this model remains to be found. These results were presented in a poster at the 2013 APS/DPP meeting. The abstract for this presentation is found in D.L. Eggleston, "Empirical model for low-frequency asymmetry-induced transport", Bull. Am. Phys. Soc. **58**, 313 (2013). The above experiments address Statement of Work item 6.

IV. LIST OF PUBLICATIONS

The following is a chronological list of publications produced during the grant periods. Copies of these papers are available for download on our website http://faculty.oxy.edu/dleggles/eggleston/recentpr.htm.

1. D.L. Eggleston, "Particle dynamics in asymmetry-induced transport", Phys. Plasmas **14**, 012302 (2007).

2. D.L. Eggleston and J.M. Williams, "Magnetic field dependence of asymmetry-induced transport: a new approach", Phys. Plasmas **15**, 032305 (2008).

3. D.L. Eggleston, "Using variable frequency asymmetries to probe the magnetic field dependence of radial transport in a Malmberg-Penning trap", in Non-Neutral Plasma Physics VII, edited by J.R. Danielson and T.S. Pedersen, American Institute of Physics, 2009.

4. D.L. Eggleston and Darrell F. Schroeter, "Solution of Laplace's Equation for the Confining End Potentials of a Coaxial Malmberg-Penning Trap", Am. J. Phys. **78**, 287, 2010.

5. D.L. Eggleston, "Constraints on an empirical equation for asymmetry-induced transport", Phys. Plasmas, **17**, 042304 (2010).

6. D.L. Eggleston, "Two sources of asymmetry-induced transport", Phys. Plasmas, **19**, 042307 (2012).

7. D.L. Eggleston, "Dependence of enhanced asymmetry-induced transport on collision frequency", Phys. Plasmas **21**, 072318 (2014).

The following is a list of published abstacts for the annual American Physical Society/Division of Plasma Physics meeting. The last two represent material not yet published in complete papers.

1. D.L. Eggleston, "Effect of Realistic End Boundaries on Particle Dynamics in Asymmetry-Induced Transport", Bull. Am. Phys. Soc. **51**, 246 (2006).

2. D.L. Eggleston and J.M. Williams, "Magnetic Field Dependence of the Diffusion Coefficient in Asymmetry-Induced Transport", Bull. Am. Phys. Soc. **52**, 75 (2007).

3. D.L. Eggleston and C.T. Smith, "Progress towards the determination of an empirical flux equation for asymmetry-induced transport", Bull. Am. Phys. Soc. **53**, 64 (2008).

4. D.L. Eggleston, "Constraints on an empirical flux equation for asymmetry-induced transport", Bull. Am. Phys. Soc. **54**, 48 (2009).

5. D.L. Eggleston, "Two sources of asymmetry-induced transport", Bull. Am. Phys. Soc. **55**, 74 (2010).

6. D.L. Eggleston, "Radial drift to diffusion ratio in asymmetry-induced transport", Bull. Am. Phys. Soc. **56**, 303 (2011).

7. D.L. Eggleston, "Dependence of enhanced asymmetry-induced transport on collision frequency", Bull. Am. Phys. Soc. **57**, 40 (2012).

8. D.L. Eggleston, "Empirical model for low-frequency asymmetry-induced transport", Bull. Am. Phys. Soc. **58**, 313 (2013).

9. D.L. Eggleston, "Effect of a central squeeze potential on asymmetry-induced transport", Bull. Am. Phys. Soc. **59**, 57 (2014).