# DETERMINATION OF CRITICAL EXPERIMENT CORRELATIONS USING THE SAMPLER SEQUENCE WITHIN SCALE 6.2

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**ABSTRACT**

The validation of neutron transport methods used in nuclear criticality safety analyses is required by consensus American National Standards Institute/American Nuclear Society (ANSI/ANS) standards. In the last decade, there has been an increased interest in correlations among critical experiments used in validation that have shared physical attributes and which impact the independence of each measurement. The statistical methods included in many of the frequently cited guidance documents on performing validation calculations incorporate the assumption that all individual measurements are independent, so little guidance is available to practitioners on the topic. Typical guidance includes recommendations to select experiments from multiple facilities and experiment series in an attempt to minimize the impact of correlations or common-cause errors in experiments. Recent efforts have been made both to determine the magnitude of such correlations between experiments and to develop and apply methods for adjusting the bias and bias uncertainty to account for the correlations.

This paper describes recent work performed at Oak Ridge National Laboratory using the Sampler sequence from the SCALE code system to develop experimental correlations using a Monte Carlo sampling technique. Sampler will be available for the first time with the release of SCALE 6.2, and a brief introduction to the methods used to calculate experiment correlations within this new sequence is presented in this paper. Techniques to utilize these correlations in the establishment of upper subcritical limits are the subject of a companion paper and will not be discussed here.

Example experimental uncertainties and correlation coefficients are presented for a variety of low-enriched uranium water-moderated lattice experiments selected for use in a benchmark exercise by the Working Party on Nuclear Criticality Safety Subgroup on Uncertainty Analysis in Criticality Safety Analyses. The results include studies on the effect of fuel rod pitch on the correlations, and some observations are also made regarding difficulties in determining experimental correlations using the Monte Carlo sampling technique.

**KEYWORDS**
validation, experimental correlations

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## 1. INTRODUCTION

The validation of neutron transport methods used in nuclear criticality safety analyses is required by consensus American National Standards Institute/American Nuclear Society (ANSI/ANS) standards [1]. In the last decade, there has been an increased interest in correlations among critical experiments used in validation that have shared physical attributes and which impact the independence of each individual measurement [2]. Many of the frequently cited guidance documents on performing validation calculations incorporate statistical methods which rely on the assumption that all individual measurements are independent [3,4], so little guidance is available to practitioners on the topic. The effects of correlations are potentially mitigated by selecting experiments from multiple facilities and series. Recent efforts have been made both to determine the magnitude of such correlations between experiments [5] and to develop and apply methods for adjusting the bias and bias uncertainty to account for them [6]. A new method proposed for implementation in the USLSTATS trending program distributed with the SCALE code package is presented in a companion paper at this conference [7].

This paper describes recent work performed at Oak Ridge National Laboratory (ORNL) to use the new Sampler sequence [8] within the next release of the SCALE code package [9] to develop experimental correlations among two sets of critical experiments. The first set includes the seven cases within the LEU-COMP-THERM-042 (LCT-042) evaluation in the *International Handbook of Evaluated Criticality Safety Benchmark Experiments* (ICSBEP Handbook) [10]. The second set of experiments is the set defined for a Working Party on Nuclear Criticality Safety Expert Group on Uncertainty Analysis in Criticality Safety Assessments (UACSA) benchmark involving three cases of LCT-007 and all 17 cases of LCT-039. Sampler will be available for the first time with the release of SCALE 6.2, and a brief introduction to the methods used to calculate experimental correlations within this new sequence is presented in this paper. Example correlation coefficients are presented for LCT-042, followed by the results of limited studies investigating the effect of fuel rod pitch on the correlations. Correlation coefficients are also presented for the LCT-007 and LCT-039 cases. Further investigation of the effects of fuel rod pitch on these correlations is also presented. Some observations are also made regarding difficulties in determining experimental correlations using the Monte Carlo sampling technique [11].

## 2. SAMPLER DESCRIPTION AND METHODS

The Sampler sequence will be released for the first time in SCALE 6.2, but it has already been available for beta testing in the beta releases of SCALE. Several previous papers have discussed Sampler in general [8,12], so only a brief summary of the applicable composition and geometry sampling techniques will be described here. The implementation of the Monte Carlo sampling technique for determining experimental correlations [11] will also be discussed.

The Sampler sequence within SCALE allows random sampling and perturbation of a wide range of parameters within virtually any sequence in the SCALE code package. For critical experiment correlation determinations, input compositions and geometry descriptions are varied based on uncertainty information gathered by the user. The perturbed values are sampled randomly according to user-specified distributions for particular input parameters within the appropriate cases. In Sampler, each unique base case input is called a case and a complete input containing a set of perturbed input parameters is called a realization. The requested number of realizations is generated for each case.

The distributions available within Sampler include a uniform distribution, a normal distribution, and a beta distribution [13]. The specification for the uniform distribution includes the maximum and minimum values in addition to a nominal value to be used in an initial nominal calculation. The normal distribution is specified with a nominal mean value, the standard deviation, and optionally with maximum and minimum values to truncate the distribution. The beta distribution is specified with alpha and beta parameters in addition to maximum, nominal, and minimum values. The selection of the alpha and beta parameters control whether the probability density function is symmetric or skewed, how skewed it is, and in which direction it is skewed. Variables within Sampler can also be determined from expressions involving other variables and/or constants.

Each experiment is modeled as a case in Sampler and, as mentioned above, the requested number of realizations is generated for each case. Identical perturbed values are used in each realization across cases with shared features as specified for each variable by the user. As an example, in some analyses the same enrichment is used in all cases because the same fuel material is used in all the experiments. Different values are used for cases that are not specified to use the same values. An example of an independent parameter would be absorber panels used in different experiments within a series. The composition and dimensions are sampled for all cases, but independent values are applied since the experiments used different panels.

Experimental correlations are generated in a post-processing calculation by Sampler. The use of random sampling to determine experimental correlations has been investigated over the last five years based on theoretical developments presented in Ref. [11]. The correlation coefficient is calculated by dividing the covariance or shared uncertainty between two cases by the product of the standard deviations of each case, as shown in Eq (1). It is essential that both the shared and unique contributions to uncertainty are specified for each case in the random sampling process. Including only some uncertainties or uncertainties that are quantified on different bases will impact the final correlation coefficient and reduce its accuracy.

$$c_{ij} = \frac{\mathrm{cov}(i, j)}{\sigma_i \sigma_j}, \tag{1}$$

where:

$c_{ij}$ is the correlation coefficient between cases i and j,
cov(i,j) is the covariance between cases i and j,
$\sigma_i$ is the standard deviation in case i, and
$\sigma_j$ is the standard deviation in case j.

## 3. ANALYSIS OF LCT-042

The experiments evaluated in LCT-042 were performed in the Critical Mass Laboratory at what is now Pacific Northwest National Laboratory. The experiments consisted of three arrays of low-enriched uranium (LEU) fuel rods clad in aluminum. Different poison panels were inserted on two of the outside faces of the central array facing the lateral arrays; criticality was controlled by array separation. The poison material and array separations are provided in Table I. A thick steel reflecting wall was located approximately 1.3 cm from the faces of the fuel arrays. A top view rendering of the KENO model of one of the cases is shown in Fig. 1.

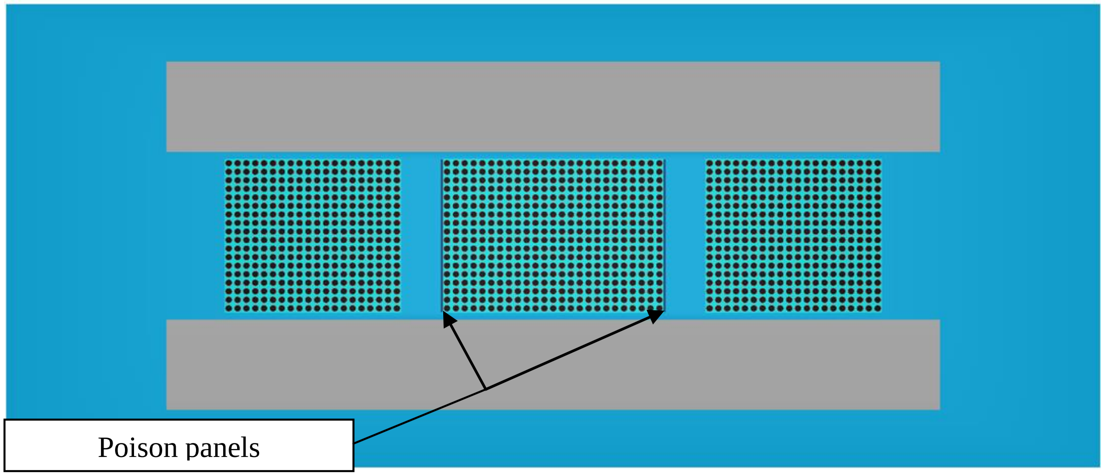



**Figure 1. Top View of LCT-042 Experimental Layout.**

### 3.1 Generation and Application of Uncertainties

The uncertainties associated with materials and dimensions are analyzed in Section 2 of the LCT-042 evaluation in the ICSBEP Handbook [10]. Variables are defined in Sampler for the majority of the composition and dimension inputs in the KENO model of this experiment series based on the information presented in the evaluation. Assessments are also made on the uncertainties shared by all seven cases and on those which are unique. Uncertainties affecting the neutron absorber panels separating the fuel arrays are unique for each case since different panels were used in each experiment. The same fuel rods are used for all experiments, so the composition uncertainties related to fuel and cladding materials are shared among all cases. For many other parameters, however, it is unclear whether or not the parameters are correlated. For example, it is not clear whether the fuel arrays contain the same rods in the same locations or whether the reflecting walls were removed and replaced between any of the experiments. These parameters are assumed to be shared among all seven cases, but this is by no means certain. Tolerances are provided for many composition uncertainties, but the potential distribution of the composition within these bounds is not known. Most of the distributions are assumed to be uniform, except for some parameters like fuel enrichment and fuel rod pitch that specifically mention a confidence interval or standard deviation. These distributions are assumed to be normal. The impact of assuming particular distributions has not been investigated, but it should not be significant since most of the tolerance bands are also small.

Limited studies have been conducted to examine the impact of the stochastic uncertainty in each individual calculation, as well as the impact of the number of realizations on the total uncertainty determined by Sampler [8]. Generally, Sampler uncertainty estimates are not reduced by either large numbers of realizations (more than ~100) or stochastic uncertainties in individual calculations less than about 0.1% $\Delta k$. The calculations reported here used 275 samples with stochastic calculation uncertainties $\leq$ 0.1% $\Delta k$ for each KENO calculation.

The uncertainty in fuel rod pitch is a controlling parameter with a significant reactivity impact and hence a large influence on calculated uncertainty. The pitch uncertainty is provided in Table 13 of the evaluation, along with a footnote explaining the derivation of the uncertainty used [10]. Four references are provided for justification of that value, though none of them is relevant to the grid plates used in this experiment. The uncertainty used ultimately comes from a triangular-pitch grid plate used in related experiments in the same time period, although LCT-042 includes only square-pitch arrays. It is therefore difficult to defend any specific distribution from which to sample the fuel rod pitch. Ultimately, a normal distribution is used, applying the specified uncertainty as a single standard deviation; the sampled distribution is truncated at ±3 standard deviations. The range of ±3 standard deviations is largely arbitrary, and a further examination of this parameter is presented below. It should also be noted that the spacing of all fuel rods is assumed to be equal throughout the experiment. In other words, a single pitch value is used in constructing the model, and it is used uniformly throughout all three fuel rod arrays.

### 3.2 Initial Results

KENO inputs are created with Sampler for 275 realizations of each of the seven cases included in LCT-042. Sampler was not used to manage the execution of the resulting 1,925 KENO calculations, though some parallel job management capability has been included in the code. The output files were post-processed by Sampler to generate uncertainties and experimental correlations among the individual cases. The uncertainty in the average $k_{eff}$ calculated with Sampler is compared with the estimate from the evaluation [10] in Table I. The experimental correlations are shown in Table II.

The Sampler estimated experimental $k_{eff}$ uncertainties are generally somewhat larger than those estimated in the evaluation. The Sampler estimates are between 1.5 and 1.8 times higher except for Case 5. The difference for Case 5 is a result of the uncertainty assessment performed in the evaluation for the mounting and positioning of the cadmium foils used in the experiment. The apparent overestimate of the experimental uncertainties in Sampler is likely an indication that the sampling approach in the model creates too much uncertainty. Likely sources of additional uncertainty include the magnitude of the pitch sampling and the fully correlated nature of the random pitch variations. Since the fuel rod pitch is a shared parameter, this would also indicate that the experimental correlations may be too large.

The correlations are similar among all pairs of cases and range from 0.784 to 0.854. The stochastic uncertainty of the correlation coefficients is not estimated by Sampler at this time. Regardless, the results indicate that the seven cases are highly correlated to each other, as expected for experiments containing the same fuel rods in identical pitch arrays. There are no clear trends for any particular experiment having lower correlations than other cases, nor do there appear to be trends related to array separation or any other experimental parameter.

**Table I. Evaluation and sampler experimental uncertainty estimates**

| Case | Poison material | Fuel array spacing (cm)* | Evaluated uncertainty | Sampler uncertainty | Ratio (Sampler/evaluation) |
|---|---|---|---|---|---|
| 1 | 304L steel | 7.866 | 0.0016 | 0.0027 | 1.68 |
| 2 | 304L steel with 1.1% B | 4.386 | 0.0016 | 0.0029 | 1.78 |
| 3 | Boral B | 2.276 | 0.0016 | 0.0028 | 1.74 |
| 4 | Boraflex | 2.566 | 0.0017 | 0.0029 | 1.69 |
| 5 | cadmium | 3.446 | 0.0033 | 0.0028 | 0.84 |
| 6 | copper | 7.376 | 0.0016 | 0.0027 | 1.66 |
| 7 | Cu-Cd | 5.016 | 0.0018 | 0.0028 | 1.55 |

* Distance between outer cell boundaries of adjacent arrays.

**Table II. Calculated correlations among cases**

| | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 | Case 7 |
|---|---|---|---|---|---|---|---|
| Case 1 | 1 | 0.832 | 0.830 | 0.826 | 0.838 | 0.803 | 0.814 |
| Case 2 | | 1 | 0.831 | 0.831 | 0.854 | 0.810 | 0.829 |
| Case 3 | | | 1 | 0.831 | 0.820 | 0.784 | 0.823 |
| Case 4 | | | | 1 | 0.837 | 0.791 | 0.806 |
| Case 5 | | | | | 1 | 0.823 | 0.796 |
| Case 6 | | | | | | 1 | 0.803 |
| Case 7 | | | | | | | 1 |

### 3.3 Sensitivity to Sampling Range of Fuel Rod Pitch

A sensitivity study on the effect of the range of fuel rod pitch sampling is included for several reasons. There is large uncertainty in the distribution to be used for sampling this parameter, and the values used to describe the distribution are also highly uncertain. It is also not clear whether each fuel rod pitch should be assumed to be unique and thus sampled independently for each rod in each case, or whether it should be assumed to be shared and therefore sampled identically for all rods in all cases. It is likely that manufacturing differences in the fuel rods and grid plates could create both systematic variation and some randomness in the distribution of rod pitches, but this cannot be proven at this time. The sensitivity study consisted of varying the range over which the fuel rod pitch is sampled, but the same sampled rod spacing is used between all pairs of rods in all seven cases. A model allowing each pitch to be sampled separately has not been developed at this time because of the effort required to assemble such a model and to sample such a large number of variables.

As discussed previously, the initial determination of experimental correlations sampled the fuel rod pitch over a range of ±3 standard deviations from the mean. This parametric study includes sampling ranges of ±1.5 and ±0.75 standard deviations, with experimental uncertainties and correlation coefficients determined for each assumed range. An additional set of uncertainties and correlations was generated with a fixed fuel rod pitch, accounting for only the other uncertain parameters (e.g., fissile material composition, fuel rod dimensions, reflector dimensions).

The resulting Sampler estimated experimental uncertainty values are provided in Table III. It is evident that the overall experimental uncertainties are reduced in the cases sampling the pitch uncertainty over smaller ranges. Sampling over a range of about ±0.75 σ yields overall uncertainties similar to the estimates provided in the evaluation. It is clearly not realistic that all pin pitches would vary by less than one standard deviation, so this may be indicative of the additional uncertainty induced by assuming all pin pitches are identical. Further investigation is warranted with models allowing the sampling of each pin position independently.

The experimental correlation results are shown in Fig. 2, where the numbers in the legend represent the individual experiments. For example, the data shown as “1–2” represent the correlation between cases 1 and 2 from LCT-042. The correlation coefficients are all less than 0.2 for the samples with no fuel rod pitch variability, but the correlations increase quickly as the sampled pitch range increases. The effect appears to saturate such that increasing the range from ±1.5 standard deviations to ±3 standard deviations has only a relatively small impact. It is also noteworthy that the correlation coefficients are similar at each fuel rod pitch range, and the variability among the cases appears slightly larger with no rod pitch variation. Thus it appears that the primary factor affecting correlation of LEU fuel array experiments might be the fuel rod pitch and not the fuel material itself.

The experimental correlations assuming a ±0.75 σ range for pitch sampling vary from about 0.45 to 0.56. This set of correlation coefficients may be more representative of the true correlations than those presented in Table II given the agreement in Sampler estimated uncertainties with those in the evaluation, as discussed in the previous paragraph. The uncertainty evaluations in the ICSBEP Handbook [10] are not necessarily performed uniformly or rigorously, so relying on them as an accurate prediction may not be possible. Without further investigation of the random pin pitch effects, it is impossible to draw firm conclusions on the true experimental correlations among the seven cases in LCT-042.

**Table III. Sampler experimental uncertainty estimates for different pitch sampling ranges**

| Case | ±3 σ | ±1.5 σ | ±0.75 σ | Fixed rod positions |
|---|---|---|---|---|
| 1 | 0.00269 | 0.00222 | 0.00166 | 0.00135 |
| 2 | 0.00285 | 0.00220 | 0.00164 | 0.00125 |
| 3 | 0.00278 | 0.00218 | 0.00158 | 0.00125 |
| 4 | 0.00288 | 0.00229 | 0.00166 | 0.00139 |
| 5 | 0.00277 | 0.00219 | 0.00150 | 0.00116 |
| 6 | 0.00266 | 0.00218 | 0.00163 | 0.00121 |
| 7 | 0.00279 | 0.00232 | 0.00179 | 0.00126 |

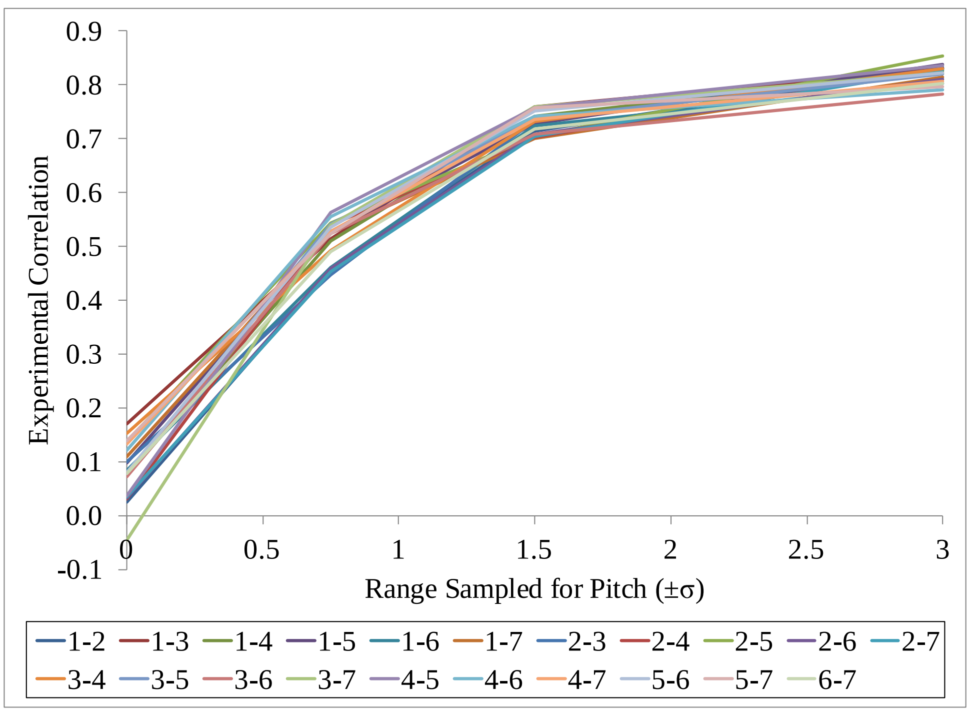
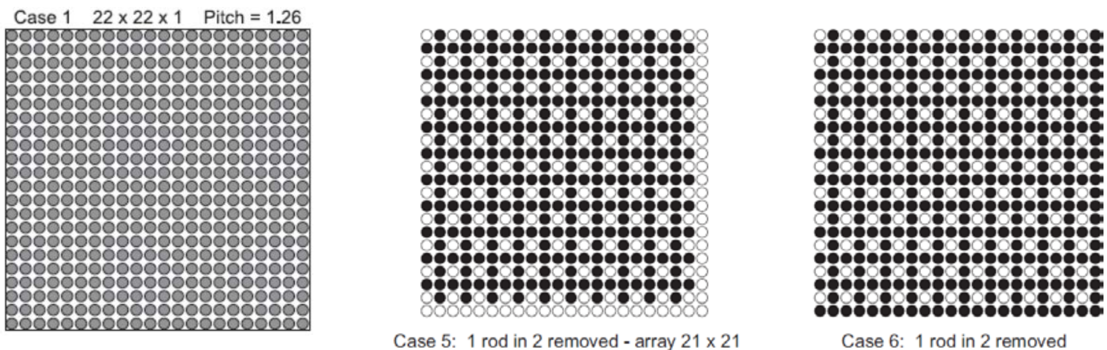



**Figure 2. Sensitivity of Experimental Correlations to Sampled Range for Fuel Rod Pitch.**

## 4. ANALYSIS OF LCT-007 AND LCT-039

The analysis of LCT-007 and LCT-039 has been suggested as part of a UACSA benchmark exercise related to experimental correlations [14]. Both series of experiments were performed at the Criticality Laboratory in Valduc, France, on “Apparatus B” in 1978 [10] using the same set of fuel rods and thus have the potential for significant correlations. Both sets of experiments involve square pitch arrays of

LEU fuel rods moderated with light water; criticality was controlled with variable water height. Each of the three LCT-007 cases uses a different fuel rod pitch within the uniform fuel rod array, but all 17 cases in LCT-039 use the same pitch which is the same pitch used in LCT-007 Case 1. Example fuel rod patterns for LCT-007 Case 1 and some LCT-039 cases are provided in Fig. 3. This combination of experiments provides insight into the importance of shared fissile material and shared fuel rod pitch in LEU fuel lattice experiments.




Filled circle represents fuel rod; empty circle represents empty lattice location.

**Figure 3. Fuel Rod Patterns for LCT-007 Case 1 (left), LCT-039 Case 5 (center), and LCT-039 Case 6 (right).**

### 4.1 Correlated Pitch Cases

As with LCT-042, the initial calculations to determine experimental correlations within the LCT-007 and LCT-039 cases assume that all fuel rod pitches within a model were equal. Furthermore, cases with the same pitch used the same random samples of fuel rod pitch as provided in the benchmark specification [14]. Other shared parameters were varied, including enrichment, cladding thickness, and fuel rod outer diameter. The only unique component of uncertainty in each case was the water height. The parameters that are varied in the benchmark and the distribution and parameters for the sampling are shown in Table IV. The benchmark specification eliminated some sources of uncertainty, such as cladding composition uncertainties, but the neglected components are expected to be minor and are largely shared sources of uncertainty.

KENO inputs are created with Sampler for 250 realizations of each of the 20 cases in the benchmark. As with the LCT-042 calculations, Sampler was not used to manage the execution of the resulting 5,000 KENO calculations. The output files were post-processed by Sampler to generate uncertainties and experimental correlations among the individual cases. The uncertainty in the average $k_{\mathrm{eff}}$ calculated with Sampler is compared with estimate from the evaluation [10] in Table V. A sample of the experimental correlations is shown in Table VI.

The experimental uncertainties generated by Sampler are more than 10 times greater than those reported in the evaluation [10]. This difference has at least two major components: (1) the correlated pitch assumption and (2) a discrepancy between the pitch uncertainty report in the evaluations and that provided in the benchmark specification. The result of this error is that the pitch is sampled from a normal distribution with a standard deviation more than three times as large as that provided in the evaluation. The value provided in the specification was used primarily because an independent comparison of the input specifications in the benchmark with the values in the ICSBEP Handbook was not conducted when the models were built.

The experimental correlations shown in Table VI are extremely high for cases that share a fuel rod pitch. These results agree with those of other participants in the benchmark [5], so they are not indicative of a problem with Sampler, the models, or the implementation of the benchmark specification. The fuel rod pitch uncertainty is the overwhelming source of uncertainty in these calculations, and it is shared in all cases with the same pitch. These results provide additional proof that correlated pitch modeling is likely to result in large correlation coefficients. It is worth noting that there is an indication in these results that the use of the same fissile material is not the primary source of high correlation coefficients, but this could be a result of the error in the fuel rod pitch uncertainty.

**Table IV. Sampled parameters in UACSA benchmark**

| Parameter | Distribution | Nominal value | Standard deviation or range |
|---|---|---|---|
| $^{234}$U at. % | Normal | 0.0307 | 0.0005 |
| $^{235}$U at. % | Normal | 4.79525 | 0.0020 |
| $^{236}$U at. % | Normal | 0.1373 | 0.0005 |
| $^{238}$U at. % | Normal | 95.03675 | 0.0100 |
| Fuel density | Normal | 10.38 | 0.0133 |
| $^{10}$B impurity | Normal | 6.9037e-8 | 8.0e-9 |
| Clad ID | Uniform | 0.82 | 0.81 – 0.83 |
| Clad thickness | Uniform | 0.06 | 0.055 – 0.065 |
| Fuel diameter | Normal | 0.7892 | 0.0017 |
| Fissile height | Normal | 89.7 | 0.3 |
| Water height | Normal | Variable | Variable (0.06 – 0.1) |

**Table V. Evaluation and sampler experimental uncertainty estimates**

| Case | Evaluated uncertainty | Sampler uncertainty | Ratio (Sampler/evaluation) |
|---|---|---|---|
| LCT-007-001 | 0.0014 | 0.0255 | 18.22 |
| LCT-007-002 | 0.0008 | 0.0083 | 10.39 |
| LCT-007-003 | 0.0007 | 0.0020 | 2.91 |
| LCT-039-001 | 0.0014 | 0.0233 | 16.66 |
| LCT-039-002 | 0.0014 | 0.0229 | 16.35 |
| LCT-039-003 | 0.0014 | 0.0211 | 15.09 |
| LCT-039-004 | 0.0014 | 0.0210 | 14.98 |
| LCT-039-005 | 0.0009 | 0.0159 | 17.61 |
| LCT-039-006 | 0.0009 | 0.0154 | 17.15 |
| LCT-039-007 | 0.0012 | 0.0223 | 18.56 |
| LCT-039-008 | 0.0012 | 0.0214 | 17.86 |
| LCT-039-009 | 0.0012 | 0.0202 | 16.81 |
| LCT-039-010 | 0.0012 | 0.0176 | 14.70 |
| LCT-039-011 | 0.0013 | 0.0237 | 18.23 |
| LCT-039-012 | 0.0013 | 0.0230 | 17.67 |
| LCT-039-013 | 0.0013 | 0.0226 | 17.36 |
| LCT-039-014 | 0.0013 | 0.0222 | 17.03 |
| LCT-039-015 | 0.0013 | 0.0217 | 16.72 |
| LCT-039-016 | 0.0013 | 0.0211 | 16.22 |
| LCT-039-017 | 0.0013 | 0.0217 | 16.67 |

**Table VI. Calculated correlations among cases**

| | 7-1 | 7-2 | 7-3 | 39-1 | 39-2 | 39-3 | 39-4 | 39-5 | 39-6 |
|---|---|---|---|---|---|---|---|---|---|
| 7-1 | 1 | 0.124 | 0.080 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 |
| 7-2 | | 1 | 0.095 | 0.120 | 0.122 | 0.122 | 0.124 | 0.127 | 0.124 |
| 7-3 | | | 1 | 0.081 | 0.082 | 0.083 | 0.083 | 0.095 | 0.094 |
| 39-1 | | | | 1 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 |
| 39-2 | | | | | 1 | 0.999 | 0.999 | 0.999 | 0.998 |
| 39-3 | | | | | | 1 | 0.999 | 0.999 | 0.998 |
| 39-4 | | | | | | | 1 | 0.999 | 0.998 |
| 39-5 | | | | | | | | 1 | 0.998 |
| 36-6 | | | | | | | | | 1 |

## 4.2 Random Pitch Cases

After preliminary results were generated like those shown in Table VI, the UACSA benchmark was modified to examine the effect of random factors influencing fuel rod position. This required significant model development within the KENO models being used, so an intermediate Sampler model was implemented which positioned the origin of each fuel rod randomly. The x- and y-coordinate for the origin is sampled randomly from a normal distribution, though the same incorrect standard deviation discussed in Section 4.1 is used in the random placements. This is potentially useful, however, as it provides an equal basis for comparison with the uniform pitch results.

A subset of the 20 cases in the benchmark has been reanalyzed with the random pin placement models to examine the effects of this approach. Only 150 realizations of each case were created, and the resulting uncertainties are provided in Table VII. The $k_{\mathrm{eff}}$ values from the perturbed cases with fully correlated pitches and random pitches are shown in Fig. 4. It is clear that the uncertainty has been significantly reduced between the two modeling approaches.

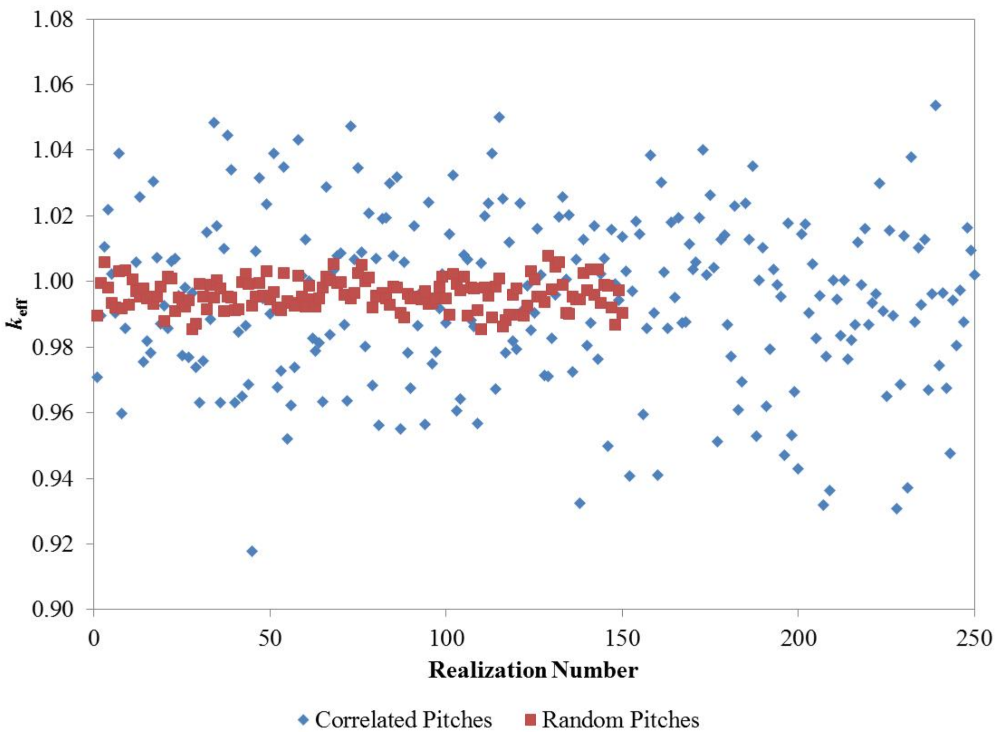
Sampler's random number generation requires these random pin location cases to be run from within the same input, which is challenging for models of this size. For this reason, only two correlation coefficients are calculated: the correlation of LCT-007 Case 1 with LCT-039 Case 1, and the correlation of LCT-039 Cases 1 and 5. All three of these cases have the same pitch. LCT-007 Case 1 is shown in Fig. 3 and is a simple 22 × 22 array. LCT-039 Case 1 is a 22 × 22 array with every fifth rod removed, so it is a very similar experiment which would be expected to have a high correlation coefficient with LCT-007 Case 1. As shown in Fig. 3, LCT-039 Case 5 is a 21 × 21 array with every other rod removed, so this experiment is likely to be less similar to LCT-039 Case 1. The resulting correlation coefficients are shown in Table VIII. Also included in Table VIII are correlation coefficients determined for the same two pairs of cases with fixed nominal rod positions. These correlation coefficients indicate the correlation that exists without any uncertainty contributed by rod position. It is evident from these results that the correlation coefficients change insignificantly between the random rod location case and the fixed rod location cases, indicating that the fuel rod location sampling has no effect on the correlation coefficients when treated independently. Further study is required to confirm this result and to understand the impact of different sampling approaches for the fuel rod positions. It is also clear that the correlation coefficient remains high in the random pitch cases even though the overall magnitude of the variations is significantly reduced.

**Table VII. Evaluation and sampler experimental uncertainty estimates**

| Case | Evaluated uncertainty | Sampler uncertainty | Ratio (Sampler/evaluation) |
|---|---|---|---|
| LCT-007-001 | 0.0014 | 0.0046 | 3.26 |
| LCT-007-002 | 0.0008 | 0.0019 | 2.39 |
| LCT-007-003 | 0.0007 | 0.0010 | 1.45 |
| LCT-039-001 | 0.0014 | 0.0043 | 3.06 |
| LCT-039-002 | 0.0014 | 0.0040 | 2.84 |
| LCT-039-003 | 0.0014 | 0.0036 | 2.60 |
| LCT-039-004 | 0.0014 | 0.0035 | 2.53 |
| LCT-039-005 | 0.0009 | 0.0026 | 2.88 |
| LCT-039-006 | 0.0009 | 0.0029 | 3.18 |
| LCT-039-007 | 0.0012 | 0.0040 | 3.30 |
| LCT-039-008 | 0.0012 | 0.0039 | 3.21 |
| LCT-039-009 | 0.0012 | 0.0037 | 3.06 |
| LCT-039-010 | 0.0012 | 0.0032 | 2.68 |

**Table VIII. Experimental correlations for random rod location cases**

| Case | Random rod locations | Fixed rod locations |
|---|---|---|
| LCT-007-001/LCT-039-001 | 0.974 | 0.968 |
| LCT-039-001/LCT-039-005 | 0.955 | 0.954 |




Note: The magnitude of the stochastic uncertainty in the KENO calculations is smaller than the data markers.

**Figure 4. $k_{eff}$ Values for Realizations with Different Pitch Assumptions.**

## 5. CONCLUSIONS AND FUTURE WORK

The detailed investigation of critical experiment correlations is a fairly recent area of study, and as such, there are few conclusions to offer and significant future work remaining. Both the LCT-042 and LCT-007/LCT-039 work indicate that for LEU lattice experiments, fuel rod pitch is an extremely important parameter that must be well characterized and sampled appropriately. In LCT-042, there are sources of unique uncertainty related to the poison panels and the associated changes in the fuel array separations that act to reduce the correlation caused by the use of the same fissile material. The LCT-007 and LCT-039 cases lack this same degree of uniqueness because water height is the primary unique component of uncertainty, and the reported uncertainties are quite small. Such components with small uncertainties contribute little to the total system uncertainty. The large correlation coefficients presented in Tables VI and VIII are plausible given the experiment designs involved. It is more difficult to draw a firm conclusion on the appropriate correlation coefficients among the LCT-042 cases given the range shown in Fig. 2.

Additional work is needed in both sets of experiments to further examine the effect of individual fuel rod modeling. No models have yet been constructed with fuel rod parameters (clad thickness and/or outer diameter) varied uniquely, and no random pitch models have been constructed for any of the LCT-042 models. More work is planned to implement uncertainty propagation within Sampler to determine the uncertainties in the correlation coefficients resulting from the stochastic uncertainties in the individual KENO calculations, though this is expected to be a relatively small uncertainty and should not significantly impact the results presented in this paper.

In a broader sense, there are many important aspects of experimental correlations that need to be examined. Little work has been performed to date within Sampler to calculate correlations for other types of experiments, such as solution or metal systems. These systems are geometrically simpler and are likely to provide insights on the experimental correlation methods without the overriding impact of the fuel rod pitch modeling uncertainty. The impact of implementing critical experiment correlations will also require more investigation, potentially via the methods described in [7]. The application of rigorously generated correlation coefficients to data assimilation tools such as TSURFER module of SCALE is straightforward, but has not yet been performed.

The determination of experimental correlations via stochastic sampling presents many challenges. The evaluations [10] generally do not contain detailed data about many of the uncertainties or their distributions, and it is often unknown if the uncertainties are shared for all cases, some cases, or none of the cases. Additional investigations are needed to determine the impact of sampling from specific distributions. For some experiments, details of the experimental configuration have been lost, and this information cannot be accounted for through stochastic sampling. An example from LCT-042 relates to the mounting and orientation of the cadmium foil used in Case 5. The input needed to calculate correlations across several evaluations will be difficult to assemble and will require a huge computational effort. Many of the challenges have already been encountered in the generation of correlation coefficients among the seven cases in LCT-042 and the 20 cases in LCT-007 and LCT-039.

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