LA-UR-15-26444
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Title:	Calculation of Latitude and Longitude for Points on Perimeter of a Circle on a Sphere
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Intended for:	Report
Issued:	2015-08-14

Calculation of Latitude and Longitude for Points on Perimeter of a Circle on a Sphere

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6 August 2015

LANL, XCP-6

This document describes the calculation of the Earth-Centered Earth Fixed (ECEF) coordinates for points lying on the perimeter of a circle. Here, the perimeter of the circle lies on the surface of the sphere and the center of the planar circle is below the surface. These coordinates are converted to latitude and longitude for mapping fields on the surface of the earth.

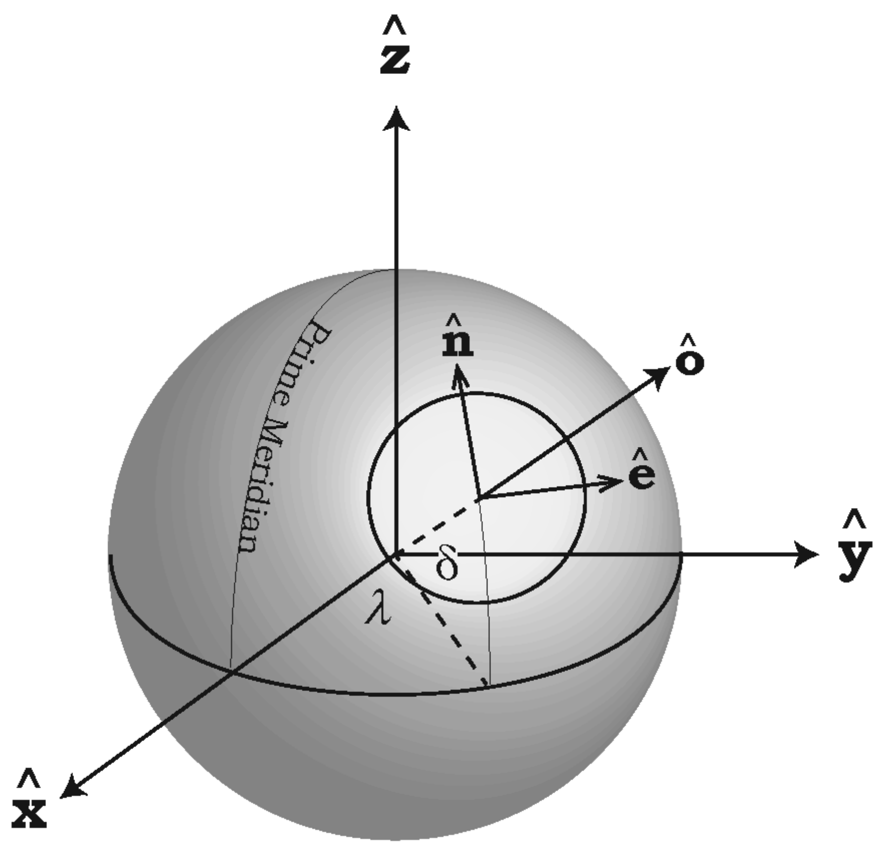

Figure 1. Diagram of geometry by H. Morris

Figure 1 shows a diagram of the problem geometry in ECEF coordinates. For any location on the surface of the sphere the coordinates can be expressed using latitude, δ, longitude, λ, and the radius of the earth, R_e. The ECEF (x,y,z) coordinates of this point are expressed in equation set 1.

$$x = R_e \cos\delta \cdot \cos\lambda$$

$$y = R_e \cos\delta \cdot \sin\lambda \qquad 1$$

$$z = R_e \sin\delta$$

This point shares the same radially-outward unit vector, $\hat{\mathbf{o}}$, with the planar circle centered at this point and shown in Figure 1. The orthogonal unit vectors in the plane of this circle are $\hat{\mathbf{n}}$ and $\hat{\mathbf{e}}$. These unit vectors are the same whether the circle center is on the surface of the sphere or depressed and define the orientation of the plane of the circle. The vector $\hat{\mathbf{e}}$ can be expressed in ECEF coordinates, and is $\hat{\mathbf{z}} \times \hat{\mathbf{o}}$. This cross product gives a unit vector perpendicular to both $\hat{\mathbf{z}}$ and $\hat{\mathbf{o}}$, which points in the eastward direction. The vector $\hat{\mathbf{n}}$ in the plane of the circle and pointing northward is $\hat{\mathbf{o}} \times \hat{\mathbf{e}}$. The unit vector $\hat{\mathbf{o}}$ has components $(x, y, z)/R_e$ since for any point on the surface of the spherical earth $\sqrt{x^2 + y^2 + z^2} = R_e$.

The geometry used in CHAP for calculating the EMP along a line-of-sight path, denoted "range", from a burst at altitude h to an observation point on the earth's surface is shown in Figure 2. The angle ϕ is measured east of magnetic north, and θ is the angle measured from a vector from the burst to the burst nadir to a vector from the burst to the observation point as shown. For a given value of θ, ϕ can be allowed to vary from 0 to 180° at some discrete interval. The electric field values are symmetric about a line defined by ϕ = 0°. For a given value of θ, a circle can be traced out in ϕ along the sphere's surface. For the remaining discussion, the burst nadir corresponds to the coordinates given in equation 1.

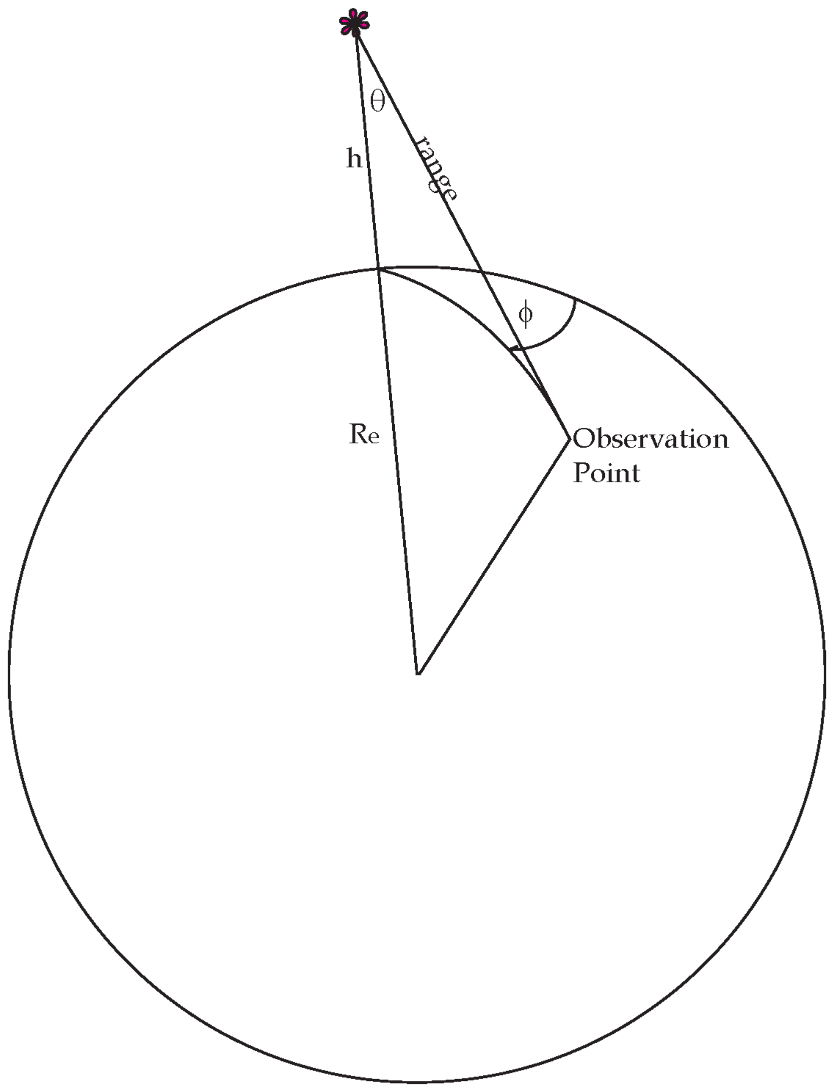

Figure 2. Geometry of CHAP by H. Morris

We wish to calculate the latitude and longitude values defining the perimeter of the circle traced out in ϕ of Figure 2, and also shown in Figure 1. It is clear from Figure 2 that the center of this circle is below the surface of the earth a depth, d, given in equation 2. The depth of this circle increases with θ and range.

$$d = range \times \cos\theta - h \qquad 2$$

The coordinates of the center of the circle are given in equation 3, by recognizing that the trigonometric terms are for the unit vector and the length is now given by $R_e - d$.

$$xc = (R_e - d)\cos\delta\cdot\cos\lambda$$
$$yc = (R_e - d)\cos\delta\cdot\sin\lambda \qquad 3$$
$$zc = (R_e - d)\sin\delta$$

The radius, r, of this circle is $range \times \sin\theta$. The general equation for a circle centered at an origin of a plane defined by $\hat{\mathbf{n}}$ and $\hat{\mathbf{e}}$ is

$$\vec{\boldsymbol{r}} = r\cos\omega\,\hat{\boldsymbol{e}} + r\sin\omega\,\hat{\boldsymbol{n}}. \qquad 4$$

However, for ϕ (also φ) defined as degrees east of north, this is written

$$\vec{\boldsymbol{r}} = r\sin\varphi\,\hat{\boldsymbol{e}} + r\cos\varphi\,\hat{\boldsymbol{n}}. \qquad 5$$

Since our circle is not centered at zero, the ECEF coordinates of points on the perimeter of the circle are given by equation 6. The *x,y,z* components of the unit vectors are also required, and used, here.

$$xp = xc + r\sin\varphi\cdot e_x + r\cos\varphi\cdot n_x$$
$$yp = yc + r\sin\varphi\cdot e_y + r\cos\varphi\cdot n_y \qquad 6$$
$$zp = zc + r\sin\varphi\cdot e_z + r\cos\varphi\cdot n_z$$

Given these components, the latitude and longitude values for the points are readily obtained by back substitution into equation 1. Finally, it is often the case that the magnetic declination, *dec*, is not zero. This requires a phase correction to equation 6 as given in equation 7.

$$xp = xc + r\sin(\varphi + dec)\cdot e_x + r\cos(\varphi + dec)\cdot n_x$$
$$yp = yc + r\sin(\varphi + dec)\cdot e_y + r\cos(\varphi + dec)\cdot n_y \qquad 7$$
$$zp = zc + r\sin(\varphi + dec)\cdot e_z + r\cos(\varphi + dec)\cdot n_z$$

Like ϕ, magnetic declination is also conventionally given in degrees east of north. Thus, if ϕ of zero is treated as magnetic north in a CHAP calculation, using equation 7, the fields for the point (*xp*, *yp*, *zp*) would be mapped to a small positive value of declination.

This calculation removes any planar geometry assumptions for mapping fields that have significant magnitude at ranges of thousands of kilometers from the burst nadir. It does not, however, take the oblate earth or terrain into consideration.