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# NetMOD Version 2.0 Mathematical Framework

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**Abstract**

NetMOD (Network Monitoring for Optimal Detection) is a Java-based software package for conducting simulation of seismic, hydroacoustic and infrasonic networks. Network simulations have long been used to study network resilience to station outages and to determine where additional stations are needed to reduce monitoring thresholds. NetMOD makes use of geophysical models to determine the source characteristics, signal attenuation along the path between the source and station, and the performance and noise properties of the station. These geophysical models are combined to simulate the relative amplitudes of signal and noise that are observed at each of the stations. From these signal-to-noise ratios (SNR), the probabilities of signal detection at each station and event detection across the network of stations can be computed given a detection threshold.

The purpose of this document is to clearly and comprehensively present the mathematical framework used by NetMOD, the software package developed by Sandia National Laboratories to assess the monitoring capability of ground-based sensor networks. Many of the NetMOD equations used for simulations are inherited from the NetSim network capability assessment package developed in the late 1980s by SAIC (Sereno et al., 1990).

## ACKNOWLEDGMENTS

We would like to thank the reviewers of this document for their contributions.

# CONTENTS

## FIGURES



## TABLES

# NOMENCLATURE

| | |
|---|---|
| dB | Decibel |
| CDF | Cumulative Distribution Function |
| GUI | Graphical User Interface |
| JVM | Java Virtual Machine |
| NetMOD | Network Monitoring for Optimal Detection |
| PDF | Probability Density Function |
| PSD | Power Spectral Density |
| RMS | Root Mean Squared |
| SNR | Signal to Noise Ratio |

# 1 INTRODUCTION

The purpose of this document is to clearly and comprehensively present the mathematical framework used by NetMOD, the seismic, hydroacoustic, and infrasonic network capability assessment software package developed by Sandia National Laboratories. Many of the NetMOD equations are inherited from the NetSim seismic network capability assessment package developed in the late 1980s by SAIC (Sereno et al., 1990), which has been the tool of choice for high-quality seismic network assessments prior to NetMOD.

The initial goal for NetMOD was to develop a more user-friendly, modernized version of NetSim (which was written in Fortran and had no Graphical User Interface), but since then the scope of the project has expanded to include improving the fidelity of the NetSim-style simulations as well as adding new types of simulations.

In the following sections, we describe all of the calculations performed by NetMOD for simulations, beginning with signal amplitude calculations, then proceed to the calculation of probabilities for signal and event detection. The complete set of equations includes both those inherited from NetSim for detection, as well as new equations needed to increase fidelity and to extend the capability assessments to new sensor technologies (hydroacoustic and infrasonic).

The desired products for network capability simulation are maps showing how well a given network can detect, locate, or identify events. These maps can be used for a variety of purposes: to see if current networks meet monitoring requirements, to identify which stations are most critical, to decide whether a proposed new station installation will significantly improve capability, etc. The maps typically either show capability for a specified event size (e.g. probability of detecting a 1 kt explosion), or show the capability for the threshold event that can be detected at a specified probability. The maps are made by calculating capabilities for a discrete set of source positions (generally on an evenly spaced grid) and then interpolating the results.

## 1.1 Logarithmic Quantities

Many of the quantities being simulated represent amplitudes that are best expressed as a logarithmic value. For clarification, these logarithmic quantities are generally computed as a base 10 logarithm. It is the convention used in this document that unless otherwise stated, a logarithm is inferred to mean a base 10 logarithm. This applies for all descriptions and equations:

$$log \equiv log_{10}$$

If a logarithm with a base of $x$, other than 10, is used then it will be represented as:

$$log_x$$

A natural logarithm will be represented as:

$$ln$$

## 1.2 Random Variables

One of the key differences between how NetMOD and NetSim implement the equations for computing amplitudes is in their handling of random variables. NetSim generally assumes that variables are normally distributed with respect to the mean and standard deviation of their log value. The mean of the log value is computed from the established amplitude equations for signal and noise. The standard deviation is then computed separately, generally abiding by the conventions for summing normally distributed random variables.

NetMOD does not make any specific assumption about the probability distribution of each of the variables within the equations. Instead, unless otherwise stated, each variable is represented as a probability function that may take on any of the forms that are implemented within NetMOD (see section 5.1 Probability Distributions). The operations described in the amplitude equations are then performed on the random variables that are consistent with the properties of each probability function.

# 2 SIGNAL AMPLITUDES

Detection, location, and event identification capability estimates all require estimating amplitudes of waves (signals) that have propagated from a specified source through the body of the Earth (seismic), the oceans (hydroacoustic), or the atmosphere (infrasonic), to a specified sensor. These estimates are compared with noise estimates at the sensor to determine signal quality, which is used to estimate detection, location, and/or identification capability.

The general equation that is used for computing signal amplitudes is:

$$\log\left(S_{ijk}(f)\right) = \log\left(C_{ijk}\right) + s_{jk}(f) + B_K\left(Path_{ij}, f\right) + SR_{ik}(f) + stacorr_{ik} \tag{1}$$

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $S_{ijk}(f)$ | = | Signal amplitude for a particular *ijk* combination. |
| $C_{ijk}$ | = | Constant dependent on physical properties of the source and station media for wave *k*. |
| $s_{jk}(f)$ | = | Log source spectrum for a particular source location and wave. |
| $B_K\left(Path_{ij}, f\right)$ | = | Log attenuation of amplitude for wave *k* along the *ij* path. This term accounts for both geometric spreading and attenuation. |
| $SR_{ik}(f)$ | = | Log site response and array gain as a function of frequency at station *i* for wave *k*. |
| $stacorr_{ik}$ | = | Station specific log amplitude correction at station *i* for wave *k*. |

Signal amplitudes may be calculated in a variety of units, depending upon the technology being simulated and the selection of the source spectrum.  In the case of simulations involving explosive yield, NetMOD performs simulations in units of conventional, chemical explosive yield.

## 2.1 Seismic

Seismic signal amplitudes are calculated using one of several variations depending upon the distance between the source and receiver locations and the type of wave being modeled (i.e. body wave versus surface wave).

NetMOD defines a regional to teleseismic cross over distance, typically 20 degrees, that may be specified within a simulation. If the distance from the source to receiver is greater than or equal to the cross over distance, then the signal amplitude is calculated as a teleseismic wave. If less than the cross over distance, then the signal amplitude is calculated as a regional wave.

For seismic signal amplitudes, the following definitions apply to some of the terms from equation (1):

$$s_{jk}(f) = \log\left(\kappa_{jk}\right) + \log\left(M_{0j}(f)\right) \tag{2}$$

$$C_{ijk} = \frac{10^9}{4\pi(\rho_j \rho_i v_{jk}^5 v_{ik})} \tag{3}$$

Where

| | | |
|---|---|---|
| $\kappa_{jk}$ | = | Source excitation factor for the *k*th wave type at source location *j*. |
| $M_{0j}(f)$ | = | Moment spectrum for the source location interpolated from a set of amplitude versus moment and frequency curves. Selection of an explosion source (the typical choice) or an earthquake source is controlled by the choice of $M_{0j}(f)$. |

And

| | | |
|---|---|---|
| $C_{ijk}$ | = | Constant dependent on physical properties of the source and station media. The factor of $10^9$ converts from meters to nanometers. |
| $\rho_i$ | = | Density of the media at the station location in kg/m$^3$. |
| $\rho_j$ | = | Density of the media at the source location in kg/m$^3$. |
| $v_{ik}$ | = | Velocity of the media at station location for wave *k* in m/s. |
| $v_{jk}$ | = | Velocity of the media at source location for wave *k* in m/s. |

### 2.1.1 Teleseismic Body Wave

For seismic signal amplitudes at teleseismic distances, the following additional definitions apply to equation (1):

$$B_K(Path_{ij}, f) = B_k^{ref}(\Delta_{ij}, f) + Scor_{jk} + Rcor_{ik} \tag{4}$$

Where

| | | |
|---|---|---|
| $B_k^{ref}(\Delta_{ij}, f)$ | = | Log attenuation & geometric spreading factor for wave *k* at source to receiver distance $\Delta_{ij}$, interpolated from a set of reference amplitude vs. distance curves per frequency, e.g. Veith-Clawson. |
| $Scor_{jk}$ | = | Log amplitude correction for wave *k* and the source location at teleseismic distances. |
| $Rcor_{ik}$ | = | Log amplitude correction for for wave *k* and the receiver location at teleseismic distances. |

### 2.1.2 Regional Body Wave

For seismic signal amplitudes at regional distances, the total path attenuation is computed as a linear combination of the path attenuation from each path media type that intersects the great circle path between the source and receiver.

$$B_K(Path_{ij}, f) = \sum_p \left[ \frac{\Delta_{ij\,p}}{\Delta_{ij}} B_{K\,p}(\Delta_{ij}, f) \right] \tag{5}$$

Where

| | | |
|---|---|---|
| $\Delta_{ij}$ | = | Distance traversed along the *ij* path. |
| $\Delta_{ij\,p}$ | = | Distance traversed through path media type *p* along the *ij* path. |
| $B_{K\,p}(\Delta_{ij}, f)$ | = | Log attenuation & geometric spreading factor for wave *k* at source to receiver distance $\Delta_{ij}$, interpolated from a set of amplitude vs. distance curves per frequency defined for path media type *p*. |

The weighting given to each path media type is equal to the distance traversed through that path media type, normalized by the total path distance.

A propagation grid is defined within a simulation from which appropriate path attenuations and grid region selections may be made.

### 2.1.3 Regional Surface Wave

NetMOD does not currently calculate surface wave amplitudes.

## 2.2 Hydroacoustic

For hydroacoustic signal amplitudes, the source terms in equation (1) are defined similarly to seismic signal amplitudes with the exception of lack of a media constant:

$$s_{jk}(f) = \log\left(\kappa_{jk}\right) + \log\left(M_{0j}(f)\right) \tag{6}$$

$$C_{ijk} = 1.0 \tag{7}$$

Where

| | | |
|---|---|---|
| $\kappa_{jk}$ | = | Source excitation factor for the *k*th wave type at the source location, typically 1.0 in water and 0.5 on land. |
| $M_{0j}(f)$ | = | Moment spectrum for the source location interpolated from a set of amplitude versus moment and frequency curves. Typically only an explosion source with separate models for either in water or on land is chosen. |

And

| | | |
|---|---|---|
| $C_{ijk}$ | = | Media constant. Hydroacoustic signals do not utilize a media constant in their simulations. |

Hydroacoustic signal amplitudes are calculated using the same method of computing a $B_K(Path_{ij}, f)$ path weighted attenuation as for seismic regional body waves (see 2.1.2 Regional Body Wave) with the exception that if the source to receiver path intersects any section of path media that has been flagged as blocked (i.e. land), then the path will be assigned to be completely attenuated.

In addition, path attenuations are calculated for both the major and minor arcs of the great circle path. The path with the least amount of attenuation is the one that will be used for computing the hydroacoustic signal amplitudes. In practice, the selected major or minor path is the one, if any, that is not blocked.

The justification for this method is that because the oceans are such a very low attenuating medium and land masses are so high attenuating, the observing path ends up being the one for which there is line-of-sight through the ocean between the source and receiver, regardless of whether it is the shortest distance.

The hydroacoustic simulations in NetMOD do not account for any reflection or refraction of the signal around blockages, phenomena that have been well documented. This line-of-sight assumption is appropriate for a first order approximation of signal amplitude, but should always be kept in mind when interpreting NetMOD hydroacoustic simulations.

## 2.3 Infrasonic

For infrasonic amplitude simulation, the distinction from the other technologies is the effect of winds on the path attenuation of the signal. Winds serve to reduce attenuation (and increase amplitude) in the direction of the wind path and increase attenuation (and decrease amplitude) in the opposite direction. Winds have a dramatic effect, both positive and negative, on the observed signal amplitudes at stations.

Stratospheric arrivals have been shown to dominate detections for the typical infrasound stations (Le Pichon et al., 2008), so these are the only infrasonic waves that are modeled in NetMOD.

Due to the state of ongoing research in the infrasound community, NetMOD has the ability to perform infrasonic simulations using several different methods, described in the following sections.

Infrasonic signal amplitudes, using equation (1), do not utilize a media constant in their simulation. Therefore:

$$C_{ijk} = 1.0 \tag{8}$$

Infrasonic simulations are performed in units of chemical explosive yield in kilotons. For infrasonic, as in seismic, nuclear equivalent yields are 2 times greater than chemical yields (Glasstone and Dolan, 1957; American National Standards Institute, 1983).

$$Y_n = 2 * Y_c \tag{9}$$

Where

| | | |
|---|---|---|
| $Y_n$ | = | Nuclear explosive yield in kilotons |
| $Y_c$ | = | Chemical explosive yield in kilotons |

Infrasonic simulations are unique in that the source spectra from explosions are limited to a frequency band around a central frequency that is a function of the explosive yield (Davidson and Whitaker, 1992). Because of this, infrasonic simulations may be performed over a frequency pass-band rather than at discrete frequencies as is done in seismic or hydroacoustic simulations. Infrasonic frequency pass-bands are typically configured to reflect operational systems that have multiple passbands 2 octaves wide and overlapped by an octave (i.e. 0.1- 0.4 Hz, 0.2 – 0.8 Hz, 0.4 – 1.6 Hz). This also ensures that the frequency range fully covers the span of observable yields and maintains the same number of degrees of freedom within each pass band (Green and Bower, 2010). See section 4.1 Frequency Type for a description of how NetMOD handles discrete frequency versus frequency pass-band simulations.

NetMOD obtains wind vectors from the Horizontal Wind Model HWM07 (Drob, 2008). Wind vectors within NetMOD are calculated at an altitude of 50 km and are sampled at one of the following:

- Source location
- Receiver location
- Average of the source and receiver locations
- Average across 50 km increments of the Source-Receiver path

Selection of how the wind vectors are sampled is a trade-off between accuracy and computational speed. Sampling the wind vectors at just one or two locations may be performed relatively rapidly; however, it may not reflect wind conditions along the full path. Calculating the wind vector across the entire source to receiver path is more representative; however, it takes significantly longer to compute.

In addition, infrasonic simulations may be performed without any wind contribution, in which case a wind velocity of zero is used.

### *2.3.1 NetSim Method*

The first method of simulating infrasonic signal amplitudes within NetMOD is based on the capability that was introduced in later versions of NetSim. This method was devised to fit into the existing NetSim framework for representing source amplitude and path attenuation models. The following additional definitions apply to equation (1):

$$s_{jk}(f) = \log\left(\kappa_{jk}\right) + \log\left(M_{0j}(f)\right) \quad (10)$$

$$B_K\left(Path_{ij}, f\right) = B^{ref}_{k}\left(\Delta_{ij}, f\right) + 0.019\ v_{ij} \quad (11)$$

Where

| | | |
|---|---|---|
| $\kappa_{jk}$ | = | Source excitation factor for the *k*th wave type |
| $M_{0j}(f)$ | = | Moment spectrum for the source location interpolated from a set of amplitude versus moment and frequency curves. Typically only an explosion source in the atmosphere is chosen. |

And

| | | |
|---|---|---|
| $B^{ref}_{k}\left(\Delta_{ij}, f\right)$ | = | Log attenuation & geometric spreading factor for wave *k* at source to receiver distance $\Delta_{ij}$, interpolated from a set of reference amplitude vs. distance curves per frequency, e.g. a simple 1/R geometric spreading model. |
| $\Delta_{ij}$ | = | Distance traversed along the *ij* path in degrees. |
| $v_{ij}$ | = | Component of the stratospheric wind velocity (m/s) at an altitude of 50 km in the direction of propagation from the source to the receiver. |

The NetSim infrasonic method treats the entire globe as having a single reference media type composed of the atmosphere so there is no path averaging calculation.

### 2.3.2 LANL / Green & Bowers (2010) Method

Another method of performing infrasonic signal simulations in NetMOD may be based on the empirical relation derived from the Los Alamos National Laboratory high explosive database (Whitaker et al., 2003; Green and Bowers, 2010):

$$P_{no\,wind} = 5.95 \times 10^4 (SR_{ij})^{-1.4072} \tag{12}$$

$$SR_{ij} = \frac{R_{ij}}{\sqrt{2 \times Y_c}} \tag{13}$$

$$P_{obs} = P_{no\,wind} \times 10^{0.018 v_s} \tag{14}$$

Where

| | | |
|---|---|---|
| $P_{no\,wind}$ | = | Pressure in the absence of wind measured in peak-to-peak microbars. |
| $SR_{ij}$ | = | Scaled range of the chemical yield |
| $R_{ij}$ | = | Source to receiver distance in kilometers. |
| $Y_c$ | = | Explosive chemical yield in kilotons |
| $P_{obs}$ | = | Observed signal amplitude, including the contribution of wind, at the receiver in peak-to-peak microbars. |

Equations (**12**), (**13**), and (**14**) define a relationship between the chemical yield and the observed time domain peak amplitude in Pa, including the effects of wind.

The simulated amplitude from equation (**14**) is measured in peak-to-peak microbars. However, NetMOD simulates infrasonic amplitudes in Pascal (Pa). For easy comparison to noise amplitudes the signal amplitude needs to be expressed as a root-mean-squared (RMS) quantity. The conversion from peak-to-peak microbars to RMS Pascal (Green et al, 2010) uses a factor of 10 to convert from microbar to Pa and a factor of 6 to convert from peak-to-peak to RMS:

$$P_{rms\,Pa} = \frac{P_{pp\,microbar}}{6 \times 10} \tag{15}$$

We also make use of a relationship between the explosive yield and the dominant period of the observed signal. This period is due to the time constant of the blast over-pressure rarefication when observed at distances beyond the blast wave. The equation may be expressed in terms of chemical yield as (Green et al, 2010):

$$Y_c = \left(\frac{t_{Y_c}}{5.92}\right)^{3.34} \tag{16}$$

$$f_{Y_c} = \frac{1}{t_{Y_c}} = \frac{1.0}{5.92 \times Y_c^{1/3.34}} \tag{17}$$

Where

$Y_c$ = Explosive chemical yield in kilotons.

$t_{Y_c}$ = Time period in seconds of the dominant signal for yield $Y_c$.

$f_{Y_c}$ = Frequency, one over the period, in Hz of the dominant signal for yield $Y_c$.

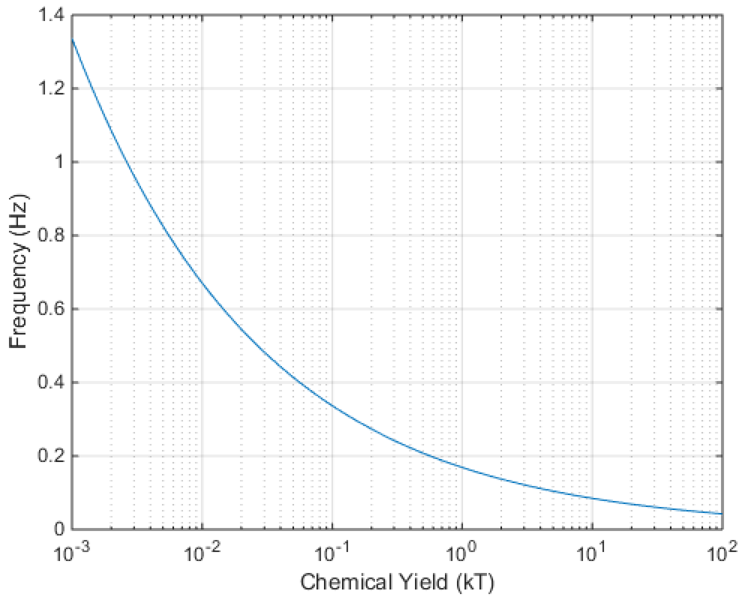


**Figure 1 Dominant Infrasonic Frequency vs Yield**

Estimates from observed explosions (Davidson and Whitaker, 1992) indicate that the 90% of the amplitude is contained within +/- 1 octave of the dominant period.

If we make the assumption that amplitude has a Gaussian distribution with respect to log frequency and that 90% of the amplitude is contained within +/- 1 octave of the dominant frequency, then the amplitude may be represented as a normal distribution:

$$P \sim N(\mu_{Y_c}, \sigma_{Y_c}) \tag{18}$$

Where

$\sigma_{Y_c}$ = Log standard deviation of the pressure distribution frequency for an explosion with yield $Y_c$ where

$$\sigma_{Y_c} = \frac{\log(2)}{1.645}$$

1.645 is the approximate number of standard deviations required for limiting 90% of the distribution to within +/- one octave.

$\mu_{Y_c}$ = Log mean of the pressure distribution frequency for an explosion with yield $Y_c$ where

$$\mu_{Y_c} = \log\left(f_{Y_c}\right)$$

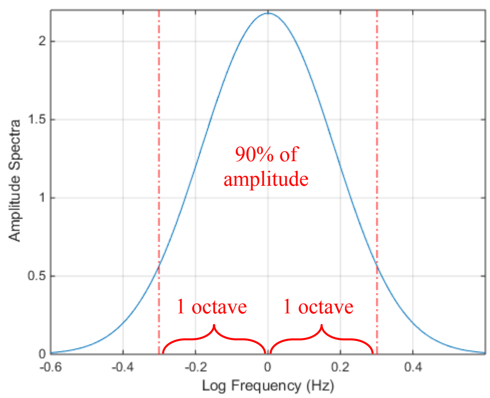


**Figure 2 Infrasonic Amplitude Distribution for a dominant frequency of 1.0 Hz.**

The relationships above are refactored for NetMOD in terms of equation (**1**) in order to attempt to separate out the source and path terms:

$$s_{jk}(f) = \log\left(\frac{5.95\ \times 10^4}{60} \times \left(\frac{1}{\sqrt{2 \times Y_c}}\right)^{-1.4072} \times N\left(\mu_{Y_c}, \sigma_{Y_c}\right)\Big|_{\log(f)}\right) \quad (19)$$

$$B_K\left(Path_{ij}, f\right) = -1.4072\ \times \log\left(R_{ij}\right) + 0.018 \times V_s \quad (20)$$

Where

| | | |
|---|---|---|
| $Y_c$ | = | Explosive chemical yield in kilotons |
| $N\left(\mu_{Y_c}, \sigma_{Y_c}\right)\Big\|_{\log(f)}$ | = | The Normal distribution of amplitudes within +/- 1 octave of the dominant frequency, evaluated at the log frequency *f*. |

And

| | | |
|---|---|---|
| $R_{ij}$ | = | Distance in kilometers along the *ij* path. |
| $V_s$ | = | Stratospheric wind velocity at 50 km elevation along the path from source to receiver. |

A plot of the simulated source spectra for a range of explosive chemical yields is shown below.

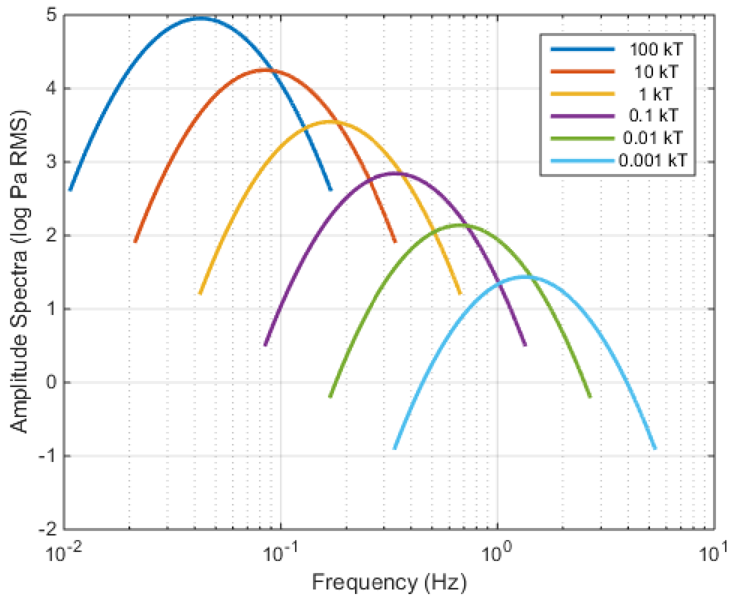


**Figure 3 Infrasonic Amplitude Spectra**

### 2.3.3 LePichon (2012) / Kinney & Graham (1985) Method

Another method of estimating infrasonic signal amplitude has been proposed by LePichon (LePichon, 2012). This method differs from the LANL relationship largely with proposing a parameterized attenuation model that takes into account the relative sound propagation speed between the ground and stratospheric altitudes.

LePichon does not make an attempt in this paper to specify an explosive infrasound source spectra. Instead, the conclusion of the paper mentions that an appropriate explosive source spectrum may be selected (Kinney & Graham, 1985) that would allow for the equivalent yield in units of TNT to be simulated. NetMOD makes use of this source spectra for performing infrasound detection simulations.

The relationships for path attenuation and source spectra in the sections below are refactored for NetMOD in terms of equation (**1**) in order to attempt to separate out the source and path terms.

$$s_{jk}(f) = FT\{p(t)\}(f) \tag{21}$$

$$B_K(Path_{ij}, f) = \log\left(A_p(R, f, V_{eff-ratio})\right) \tag{22}$$

Where

| | | |
|---|---|---|
| $FT$ | = | Fourier Transform to represent the time-series model in the frequency domain |
| $p(t)$ | = | Time-series model simulating the explosive source obtained from section 2.3.3.2. |

And

| | | |
|---|---|---|
| $A_p(f, V_{eff-ratio})$ | = | LePichon attenuation coefficient (non-log) of the pressure wave from section 2.3.3.1 |
| $R$ | = | Source to receiver distance in kilometers. |
| $f$ | = | Frequency in Hz. |
| $V_{eff-ratio}$ | = | Ratio between the effective sound speed at 50 km altitude and the sound speed at ground level. |

### 2.3.3.1 LePichon (2012) Path Attenuation Model

LePichon has devised a parameterized model for geometric spreading and wind effects derived from a numerical simulation of path attenuation through the atmosphere:

$$A_p(R,f,V_{eff-ratio}) = \frac{1}{R}10^{\frac{\alpha(f)R}{20}} + \frac{R^{\beta(f,V_{eff-ratio})}}{1+10^{\frac{\delta-R}{\sigma(f)}}} \tag{23}$$

Where

| | | |
|---|---|---|
| $A_p(f,V_{eff-ratio})$ | = | Attenuation coefficient (non-log) of the pressure wave. |
| $R$ | = | Source to receiver distance in kilometers. |
| $f$ | = | Frequency in Hz. |
| $V_{eff-ratio}$ | = | Ratio between the effective sound speed at 50 km altitude and the sound speed at ground level: |

$$V_{eff-ratio} = \frac{v_{sound} + v_{50km}}{v_{sound} + v_{0km}}$$

Where $v_{sound}$ is the nominal speed of sound at 20 degrees Celsius (343.59 m/s). $v_{50km}$ and $v_{0km}$ are the wind velocities in m/s at 50 km and 0 km, respectively, along the path from the source to the receiver.

$\alpha(f)$ = Dissipation of the direct wave. Mean and standard deviation values are interpolated from the provided table:

| f (Hz) | Mean | Std |
|---|---|---|
| 0.1 | -0.28 | 0.05 |
| 0.2 | -0.33 | 0.04 |
| 0.4 | -0.39 | 0.03 |
| 0.8 | -0.47 | 0.04 |
| 1.6 | -0.59 | 0.06 |
| 3.2 | -0.69 | 0.06 |

$\beta(f,V_{eff-ratio})$ = Geometrical spreading and dissipation of stratospheric and thermospheric waves. Mean and standard deviation values are interpolated from the provided multidimensional table:

| | f (Hz) | | | | | |
|---|---|---|---|---|---|---|
| $V_{eff-ratio}$ | 0.1 Hz | 0.2 Hz | 0.4 Hz | 0.8 Hz | 1.6 Hz | 3.2 Hz |
| 0.85 Mean | -1.00 | -1.20 | -1.35 | -1.70 | -1.95 | -2.40 |
| Std | 0.10 | 0.10 | 0.15 | 0.15 | 0.30 | 0.45 |
| 0.88 Mean | -1.00 | -1.20 | -1.40 | -1.60 | -1.95 | -2.30 |
| Std | 0.10 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 |
| 0.91 Mean | -1.00 | -1.15 | -1.40 | -1.60 | -1.85 | -2.10 |
| Std | 0.10 | 0.10 | 0.15 | 0.15 | 0.20 | 0.25 |
| 0.94 Mean | -1.00 | -1.10 | -1.35 | -1.55 | -1.75 | -1.85 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Std | 0.10 | 0.10 | 0.15 | -0.20 | 0.25 | 0.25 |
| 0.97 Mean | -1.00 | -1.05 | -1.15 | -1.30 | -1.40 | -1.45 |
| Std | 0.15 | 0.10 | 0.15 | 0.15 | 0.20 | 0.25 |
| 1.0 Mean | -0.90 | -0.95 | -1.00 | -1.05 | -1.15 | -1.20 |
| Std | 0.10 | 0.10 | 0.10 | 0.15 | 0.20 | 0.20 |
| 1.03 Mean | -0.90 | -0.90 | -0.90 | -0.90 | -0.95 | -1.00 |
| Std | 0.05 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 1.06 Mean | -0.85 | -0.85 | -0.90 | -0.90 | -0.95 | -1.00 |
| Std | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 1.09 Mean | -0.85 | -0.85 | -0.90 | -0.90 | -1.00 | -1.05 |
| Std | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 1.12 Mean | -0.85 | -0.85 | -0.85 | -0.90 | -1.00 | -1.10 |
| Std | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 1.15 Mean | -0.80 | -0.80 | -0.85 | -0.95 | -1.05 | -1.15 |
| Std | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 1.18 Mean | -0.80 | -0.80 | -0.85 | -0.90 | -1.05 | -1.20 |
| Std | 0.05 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |

$\delta$ = Width of the shadow zone in km with a mean of 180 km and standard deviation of 50 km.

$\sigma(f)$ = Scaling distance controlling the strength of the attenuation in the shadow zone. Mean and standard deviation values are interpolated from the provided table:

| f (Hz) | Mean | Std |
|---|---|---|
| 0.1 | 79 | 22 |
| 0.2 | 55 | 10 |
| 0.4 | 43 | 4 |
| 0.8 | 36 | 4 |
| 1.6 | 27 | 7 |
| 3.2 | 20 | 3 |

From the proposed model, we see in the figures below that the model predicts there to be significant variation in the attenuation with respect to range, frequency, and wind speed that are not captured in earlier models of infrasonic path attenuation.

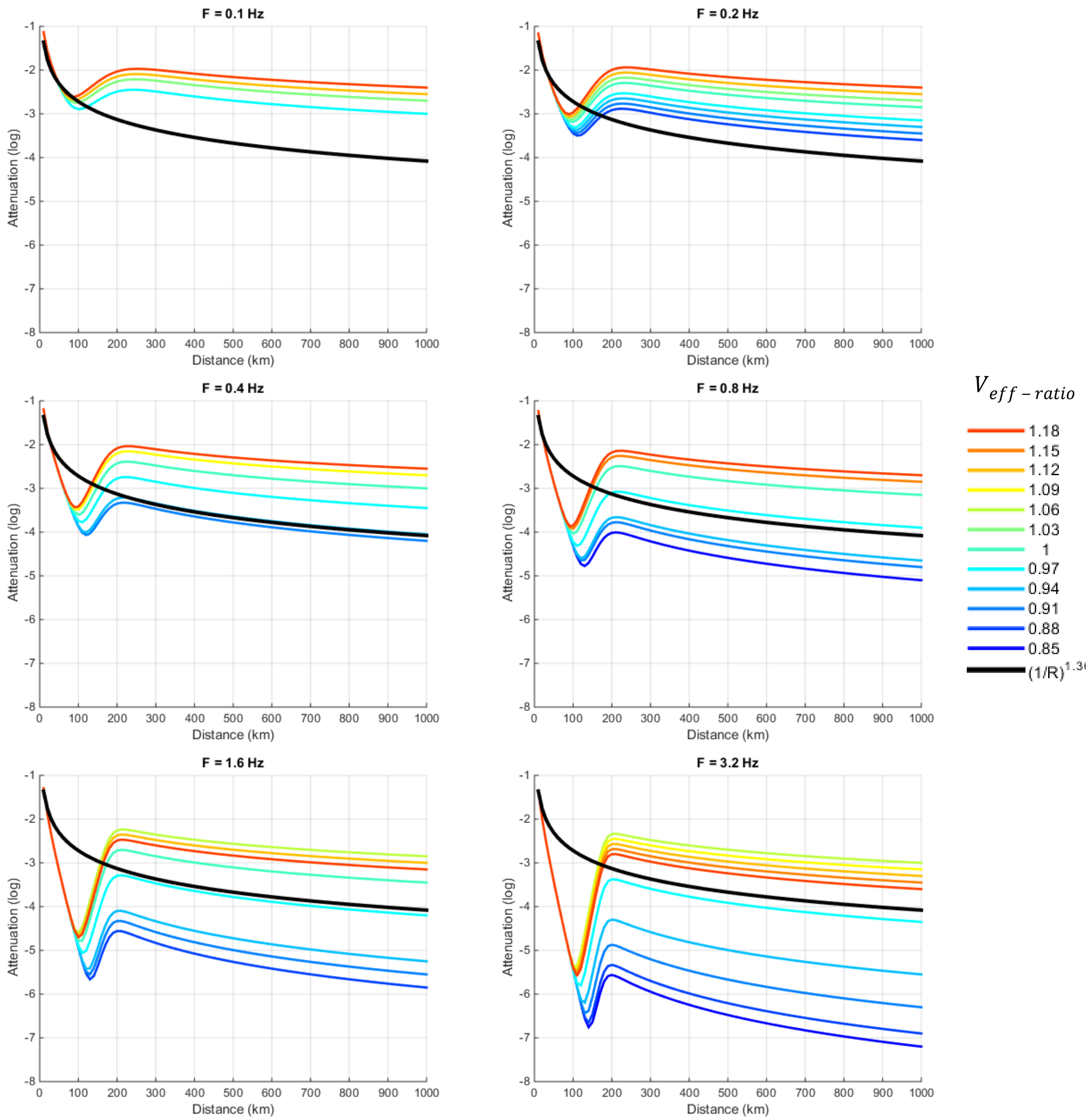


**Figure 4 LePichon attenuation versus distance across frequency and velocity ratio**

NetMOD presumes that the attenuation is a normal random variable with respect to the log of equation (**23**) with mean and standard deviation estimated from the means and standard deviations of the $\alpha, \beta, \delta$, and $\sigma$ terms.

### 2.3.3.2 Kinney & Graham (1985) Source Spectra

Simulating the infrasound source spectra from an explosion may be calculated from the time series of the simulated signal. The time series is then converted to a frequency spectrum using a Fourier-transform and from there converted into RMS amplitude measurements.

NetMOD calculates the time series using different methods for a chemical versus nuclear explosion (Kinney & Graham, 1985).

#### 2.3.3.2.1 Chemical Explosion

The time series of a chemical explosion of a given yield in kilograms is simulated at a nominal distance of 1 kilometer for a standard atmosphere with an ambient pressure of 1013 millibar and temperature of 15 Celsius.

The scaled range may be calculated from the yield:

$$Z = \frac{r}{W^{1/3}} \tag{24}$$

Where

| | | |
|---|---|---|
| $Z$ | = | Scaled Range in m/kg$^{1/3}$. |
| $W$ | = | Explosive yield in kilograms |
| $r$ | = | 1000 (nominal range in meters) |

The peak over pressure of a chemical blast wave (Kinney & Graham, 1985, equation 6-2 on pg. 94):

$$P = 100 \times p_a \frac{808 \times \left(1 + \left(\frac{Z}{4.5}\right)^2\right)}{\sqrt{\left(1 + \left(\frac{Z}{0.048}\right)^2\right)\left(1 + \left(\frac{Z}{0.32}\right)^2\right)\left(1 + \left(\frac{Z}{1.35}\right)^2\right)}} \tag{25}$$

Where

| | | |
|---|---|---|
| $P$ | = | Peak over pressure in Pascal. |
| $Z$ | = | Scaled Range in m/kg$^{1/3}$. |
| $p_a$ | = | 1013 (ambient pressure in millibar), the factor of 100 converts from millibar to Pascal. |

The positive time duration of a chemical blast wave (Kinney & Graham, 1985, equation 6-10 on pg. 97):

$$T_c = \frac{1}{1000} W^{1/3} \frac{980\ \times \left(1 + \left(\frac{Z}{0.54}\right)^{10}\right)}{\left(1 + \left(\frac{Z}{0.02}\right)^{3}\right)\left(1 + \left(\frac{Z}{0.74}\right)^{6}\right)\sqrt{\left(1 + \left(\frac{Z}{6.9}\right)^{2}\right)}} \tag{26}$$

Where

| | | |
|---|---|---|
| $T_c$ | = | Positive time duration of the chemical blast wave in seconds. The factor of 1000 is to convert from millisecond to seconds. |
| $W$ | = | Explosive yield in kilograms. |
| $Z$ | = | Scaled Range in m/kg$^{1/3}$. |

The pressure waveform time series (Kinney & Graham, 1985, equation 6-13 on pg. 100):

$$p(t) = P\left(1 - \frac{t}{T_c}\right) e^{-\frac{\alpha t}{T_c}} \tag{27}$$

Where

| | | |
|---|---|---|
| $p(t)$ | = | Pressure time series in Pascal. |
| $P$ | = | Peak over pressure in Pascal. |
| $t$ | = | Time in seconds. |
| $\alpha$ | = | Wave form parameter adjustable factor. In the absence of knowledge of the blast wave characteristics, an assumed value of 1.0 is reasonable. |
| $T_c$ | = | Positive time duration of the chemical blast wave in seconds. |

Plots of the simulated time series and source spectra at a distance of 1 kilometer for a range of explosive chemical yields are shown below.

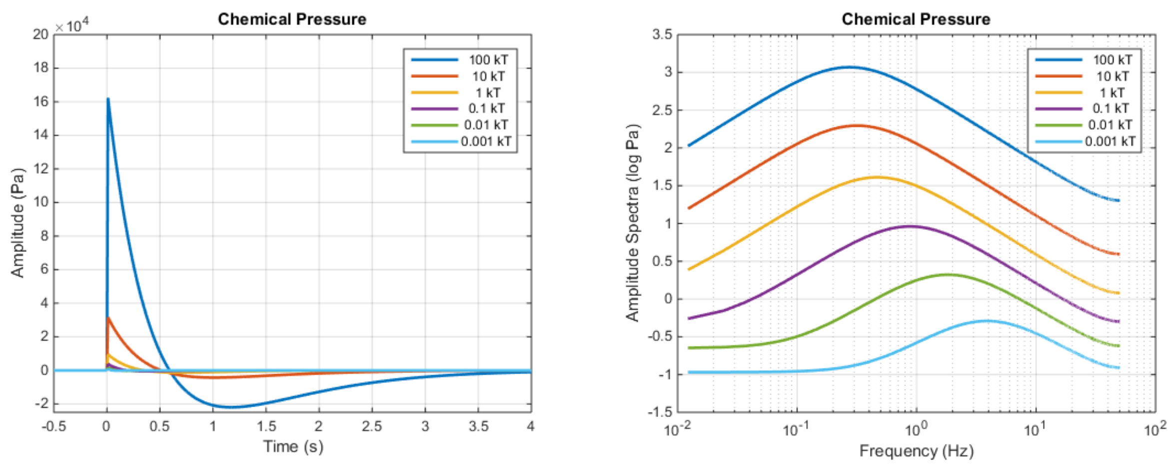


**Figure 5 Kinney & Graham Infrasonic Chemical Time Series and Spectra**

#### 2.3.3.2.2 Nuclear Explosion

The time series of a nuclear explosion for a given yield in kilotons is simulated at a nominal distance of 1 kilometer for a standard atmosphere with an ambient pressure of 1013 millibar and temperature of 15 Celsius.

The scaled range may be calculated from the yield:

$$Z = \frac{r}{W^{1/3}} \tag{28}$$

Where

| | | |
|---|---|---|
| $Z$ | = | Scaled Range in m/kt$^{1/3}$. |
| $W$ | = | Explosive yield in kilotons |
| $r$ | = | 1000 (nominal range in meters) |

The peak over pressure (Kinney & Graham, 1985, equation 6-3 on pg. 94):

$$P = 100 \times p_a \times 3.2 \times 10^6 \times R^{-3} \sqrt{\left(1 + \left(\frac{Z}{87}\right)^2\right)\left(1 + \frac{Z}{800}\right)} \tag{29}$$

Where

| | | |
|---|---|---|
| $P$ | = | Peak over pressure in Pascal. |
| $Z$ | = | Scaled Range in m/kt$^{1/3}$. |
| $p_a$ | = | 1013 (ambient pressure in millibar), the factor of 100 converts from millibar to Pascal. |

The positive time duration of a nuclear blast wave (Kinney & Graham, 1985, equation 6-11 on pg. 98):

$$T_n = \frac{1}{1000} W^{1/3} \frac{180 \times \sqrt{1 + \left(\frac{Z}{100}\right)^3}}{\left(1 + \left(\frac{Z}{40}\right)\right)^{1/2}\left(1 + \left(\frac{Z}{285}\right)^5\right)^{1/6}\left(1 + \frac{Z}{50000}\right)^{1/6}} \tag{30}$$

Where

| | | |
|---|---|---|
| $T_n$ | = | Positive time duration of the chemical blast wave in seconds. The factor of 1000 is to convert from millisecond to seconds. Note that in the reference, the equation states that the result is in seconds without any conversion. However, examination of the results from the equation does not appear to support that. |
| $W$ | = | Explosive yield in kilotons. |
| $Z$ | = | Scaled Range in m/kt$^{1/3}$. |

The pressure waveform time series (Kinney & Graham, 1985, equation 6-13 on pg. 100):

$$p(t) = P\left(1 - \frac{t}{T_n}\right)e^{-\frac{\alpha t}{T_n}} \tag{31}$$

Where

| | | |
|---|---|---|
| $p(t)$ | = | Pressure time series in Pascal. |
| $P$ | = | Peak over pressure in Pascal. |
| $t$ | = | Time in seconds. |
| $\alpha$ | = | Wave form parameter adjustable factor. In the absence of knowledge of the blast wave characteristics, an assumed value of 1.0 is reasonable. |
| $T_n$ | = | Positive time duration of the nuclear blast wave in seconds. |

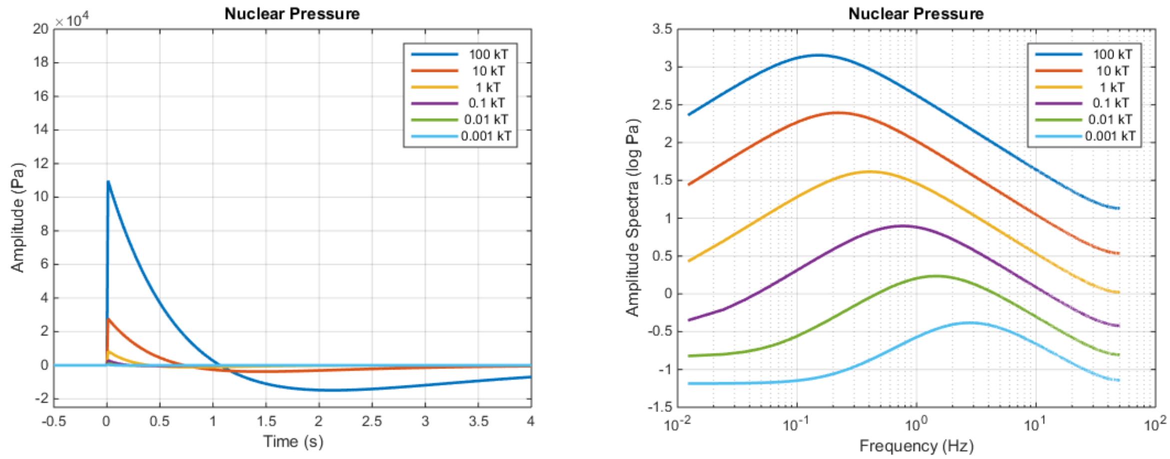


**Figure 6 Kinney & Graham Infrasonic Nuclear Time Series and Spectra**

# 3 NOISE AMPLITUDES

The amplitude of the noise for a given station and phase is expressed as the sum of the ambient site noise and the signal amplitude of the prior phase, if any, scaled by the coda decay. These sums are performed as power spectral densities and then converted to spectral amplitudes.

| Non-Log | Log | |
|---|---|---|
| $N_{ijk}(f) = [\,T_k PSD_{ijk}(f)]^{1/2}$ | $\log\left(N_{ijk}(f)\right) = \frac{1}{2}\log\left(T_k\right) + \frac{1}{2}\log\left(PSD_{ijk}(f)\right)$ | (32) |
| $PSD_{ijk}(f) = PSD_{a_i}(f) + PSD_{S_{ijk}}(f)$ | $\log\left(PSD_{ijk}(f)\right) = \log\left(PSD_{a_i}(f) + PSD_{S_{ijk}}(f)\right)$ | (33) |
| $PSD_{S_{ij(k_p)}}(f) = \dfrac{\left[\gamma_k(\Delta_{ij}) S_{ij(k_p)}(f)\right]^2}{T_{k_p}}$ | $\log\left(PSD_{S_{ijk_p}}(f)\right) = 2\log\left(\gamma_k(\Delta_{ij})\right) - log\left(T_{k_p}\right) + 2\log\left(S_{ijk_p}(f)\right)$ | (34) |

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $N_{ijk}(f)$ | = | Noise amplitude for a particular *ijk* combination. |
| $T_k$ | = | Time window length in seconds used to calculate the signal spectrum for wave *k*. |
| $PSD_{ijk}(f)$ | = | Power Spectral Density of the total noise in $U^2/Hz$ where $U$ is the unit of $N_{ijk}(f)$. |
| $PSD_{a_i}(f)$ | = | Power Spectral Density of the ambient site noise in $U^2/Hz$ where $U$ is the unit of $N_{ijk}(f)$. |
| $PSD_{S_{ijk_p}}(f)$ | = | Power Spectral Density of the signal from the prior phase in $U^2/Hz$ where $U$ is the unit of $N_{ijk}(f)$. |
| $\gamma_k(\Delta_{ij})$ | = | Coda decay rate as a fraction for wave *k* at source to receiver distance $\Delta_{ij}$. |
| $S_{ijk_p}(f)$ | = | Signal amplitude for the prior phase. |
| $T_{k_p}$ | = | Time window length in seconds used to calculate the signal spectrum for the prior phase. |

Note that the quantities above are ***not*** in log units. Conversions from log are performed where appropriate. The equivalent set of equations using log quantities are shown as well.

## 3.1 NetSim Noise Amplitude

The method that NetSim implements for computing total noise amplitude is consistent with equations (32), (33), and (34). NetSim assumes that the site noise and prior phase noise are random variables with a normal distribution with respect to the log of amplitude. The mean noise value is computed with those equations. However, the standard deviation is computed with the following equation:

$$\sigma_{N_{ijk}}(f) = \frac{\left[PSD_{a_i}(f)^2\sigma_{N_{ai}}(f)^2 + PSD_{S_{ijkprev}}(f)^2\sigma_{S_{ijkprev}}(f)^2\right]^{1/2}}{PSD_{a_i}(f) + PSD_{S_{ijkprev}}(f)} \tag{35}$$

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $\sigma_{N_{ijk}}(f)$ | = | Log standard deviation of total noise amplitude for a particular *ijk* combination. |
| $PSD_{a_i}(f)$ | = | Power Spectral Density of the ambient site noise in $U^2/Hz$ where $U$ is the unit of $N_{ijk}(f)$. |
| $\sigma_{N_{ai}}$ | = | Log standard deviation of ambient site noise amplitude for a particular *ijk* combination. |
| $PSD_{S_{ijkprev}}(f)$ | = | Power Spectral Density of the signal from the prior phase in $U^2/Hz$ where $U$ is the unit of $N_{ijk}(f)$. |
| $\sigma_{S_{ijkprev}}(f)$ | = | Standard deviation of prior phase signal amplitude for a particular *ijk* combination. |

Note that the relationship between the log standard deviation of a PSD and the log standard deviation of amplitude, due to equations (32) and (34) and the log of amplitude and PSD is:

$$\sigma_{amplitude} = \frac{\sigma_{PSD}}{2} \tag{36}$$

## 3.2 NetMOD Noise Amplitude

In addition to the classic NetSim implementation of Noise Amplitude, NetMOD employs a method based upon equations (32), (33), and (34) that treats each term within those equations as a unique random variable depending upon its definition. In particular, amplitude and PSD values are treated a log-normal random values and the log of those values are treated a normal random values. The random variables are then transformed and summed by those equations, properly honoring the individual properties of each random variable (see 5.1 Probability Distributions).

The key distinction between the NetSim and NetMOD methods of noise amplitude simulation is the handling of equations (33) and (**35**) in which summing the log-normal PSD values are computed (Fenton, 1960).

## 3.3 Comparison of NetSim and NetMOD Noise Amplitudes

The NetSim method of calculating the mean and standard deviation of the total noise amplitude in equations (32), (33), (34), and (35) would be accurate if the PSD values that are being summed were normally distributed. However, the PSD values are actually log-normally distributed and should be summed according to the method described in Table 3 Properties of Log Normal Random Variables (Fenton, 1960).

As an example, numerical simulations were performed summing two log-normal random variables. These simulations were performed using the NetSim and NetMOD (using the Fenton-Wilkinson solution for summing log-normal random variables) methods described above and compared to the results from a Monte Carlo simulation for reference. Two identically distributed random variables across a range of means and standard deviations were used.

It was found that results were consistent across the range of mean values. However, there was variability in the results across the range of standard deviation values.

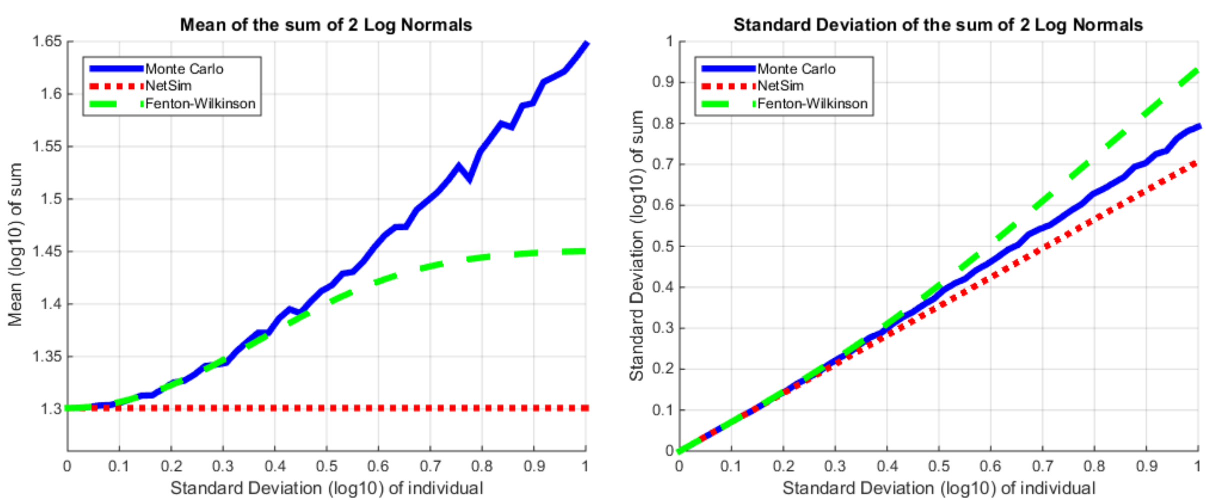


**Figure 7 Mean and Standard Deviation of the Sum of Log Normal Random Variables**

What we find is that there is a considerable amount of error in the estimate of the log10 mean for the NetSim method of summing. The Fenton-Wilkinson method does not have significant error until the standard deviation is above approximately 0.5 log10 units. Both methods do a good job of estimating the standard deviation and have only slight errors above 0.5.

Another issue is that the sum of multiple log-normal random variables is not guaranteed to be a log-normal random variable itself. Examining the kurtosis of the log of the summed variable from the Monte Carlo simulation yields the plot below.

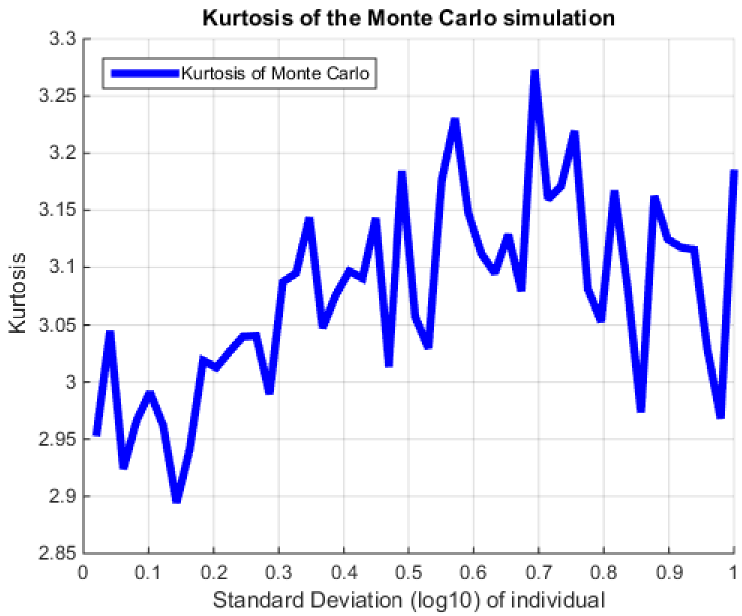


**Figure 8 Kurtosis of the Sum of Log Normal Random Variables**

The kurtosis of the log of the sum is close to 3, which indicates a normally distributed random variable, although with some increases at higher standard deviations. This would suggest that the sum may be reasonably approximated as a log-normal random variable.

Comparing the predicted means and standard deviations of distributions provides for a quick way to summarize the statistics. However, it is important to bear in mind that what we primarily are interested in for the purpose of the network detection simulation problem is the Cumulative Distribution Function (CDF) of the random variable. The CDF is what we are actually using to determine the detection probabilities. So, for the simple case of averaging two identically distributed log-normal random variables with log10 means of 1 and standard deviations ranging from 0.1 to 0.6, the following CDFs were generated for the Monte-Carlo, NetSim, and Fenton-Wilkinson methods:

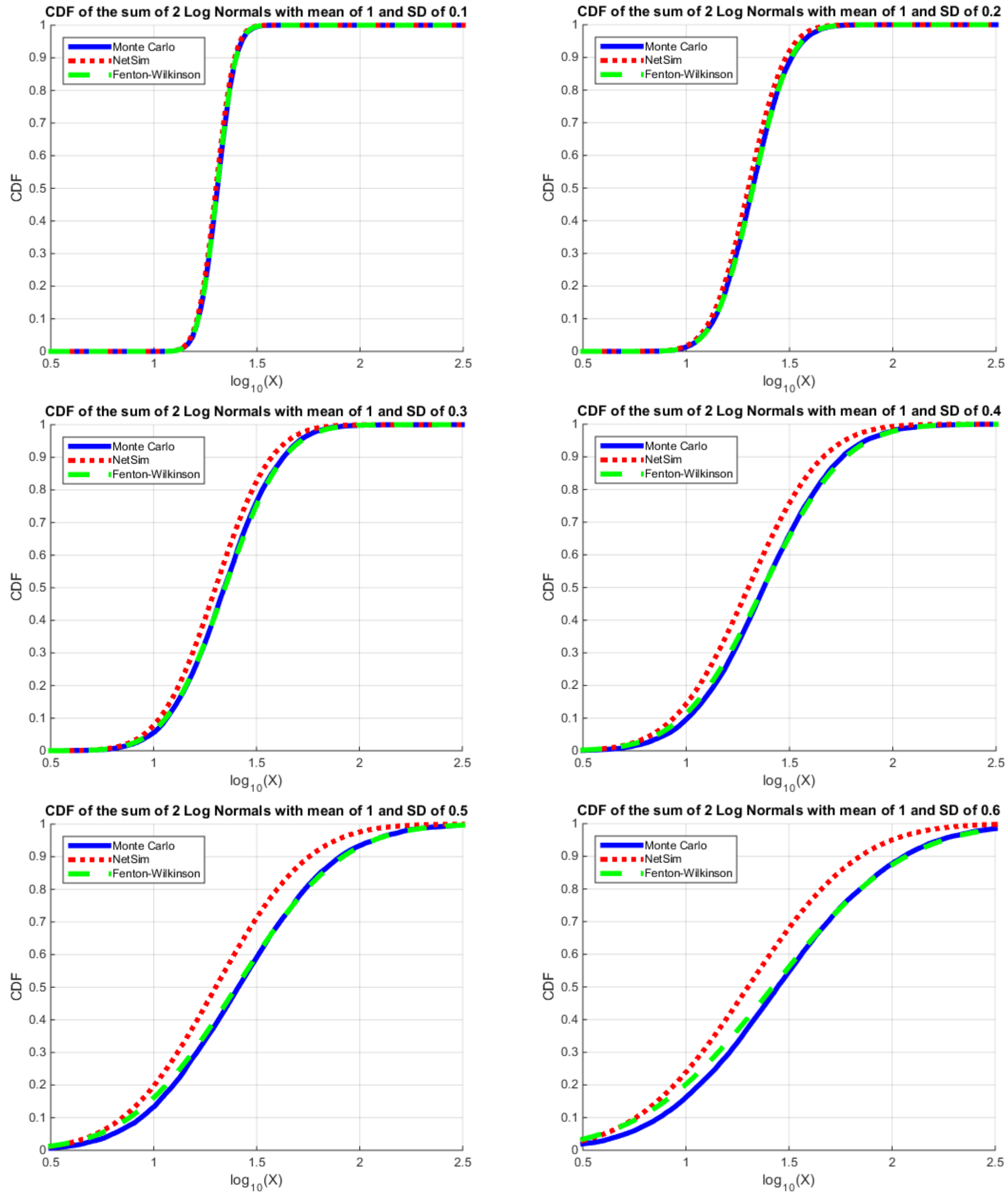


**Figure 9 CDF of the Sum of Log Normal Random Variables**

We observe the NetSim method can be reasonably accurate for very low standard deviation values. However, the Fenton-Wilkinson method of summing log-normal random variables used in NetMOD is a more accurate estimate than the method implemented in NetSim across a wide range of standard deviations.

Based on these results, we strongly recommend using the NetMOD method of calculating noise amplitudes. However, the NetSim method remains available as an option within NetMOD in order to draw comparisons to historical results.

# 4 DETECTION SIMULATION

The NetMOD calculation for probability of detection follows the NetSim formulation wherein the individual probabilities of detecting specified waves at each station are combined to form a probability of detecting the event by the network. The initial step in calculating a probability of detection is to calculate a predicted signal to noise ratio (SNR) value for every combination of source location, station, phase, and frequency.

## 4.1 Frequency Type

NetMOD has the ability to perform detection simulations at multiple discrete frequencies, frequency pass-bands, or any combination of the two.

In the case of a discrete frequency, NetMOD computes the equations for signal and noise amplitude at that frequency. The SNR at that frequency is then computed as the ratio (or log difference) of those two amplitudes.

In the case of a frequency pass-band, NetMOD will internally sample the specified pass-band as a set of discrete frequencies that span the pass-band:

$$f_{min} - f_{max} = \left\{ \bigcup_{f_{min}}^{f_{max}} f \right\} \tag{37}$$

The signal and noise amplitudes are computed for each of the frequencies within the pass-band and then averaged across the pass-band, accounting for the bin-width that each frequency represents:

$$S_{ijk}(f_{min} - f_{max}) = \frac{1}{f_{max} - f_{min}} \sum_{f = f_{min}}^{f_{max}} \frac{(f_{i+1} - f_{i-1})}{2} S_{ijk}(f) \tag{38}$$

$$N_{ijk}(f_{min} - f_{max}) = \frac{1}{f_{max} - f_{min}} \sum_{f = f_{min}}^{f_{max}} \frac{(f_{i+1} - f_{i-1})}{2} N_{ijk}(f) \tag{39}$$

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $S_{ijk}(f_{min} - f_{max})$ | = | Signal amplitude for a particular *ijk* combination at frequency pass-band $f_{min} - f_{max}$. |
| $N_{ijk}(f_{min} - f_{max})$ | = | Noise amplitude for a particular *ijk* combination at frequency pass-band $f_{min} - f_{max}$. |
| $S_{ijk}(f)$ | = | Signal amplitude for a particular *ijk* combination at discrete frequency $f$. |
| $N_{ijk}(f)$ | = | Noise amplitude for a particular *ijk* combination at discrete frequency $f$. |

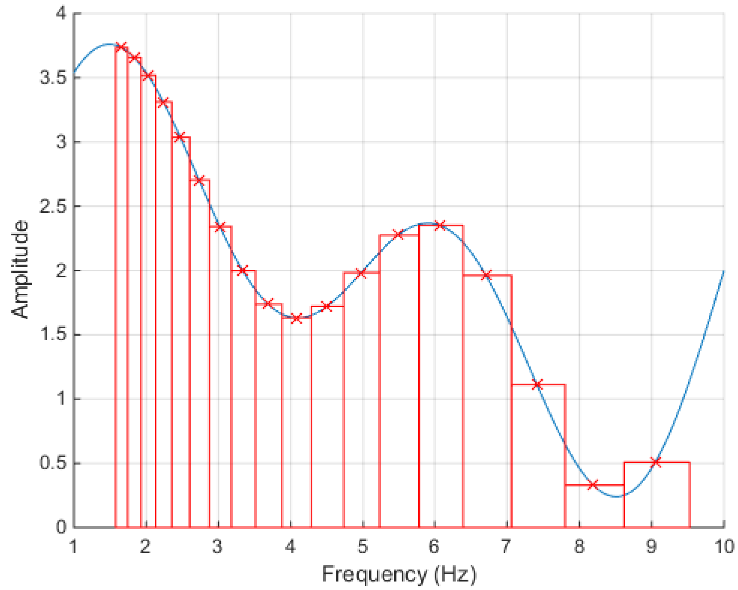


**Figure 10 Frequency pass-band integration**

This performs a rectangular integration of the amplitude across frequency in which each rectangular bin is centered on the frequency sample and the width of each bin is accounted for.

Note that the signal and noise amplitudes are in their non-log representations and will need to be converted to and from log values. Also, the amplitudes are represented as random variables and will need to be scaled and summed accordingly (see 5.1 Probability Distributions).

## 4.2 Signal to Noise Ratio

The SNR is calculated from the signal (see 2 Signal Amplitudes) and noise (see 0

Noise ) amplitudes calculated previously:

| Non-Log | Log | |
|---|---|---|
| $SNR_{ijk}(f) = \dfrac{S_{ijk}(f)}{N_{ijk}(f)}$ | $\log\left(SNR_{ijk}(f)\right) = \log\left(S_{ijk}(f)\right) - \log\left(N_{ijk}(f)\right)$ | (40) |

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station $i$, event $j$, wave (phase) $k$, respectively. |
| $S_{ijk}(f)$ | = | Signal amplitude for a particular $ijk$ combination. |
| $N_{ijk}(f)$ | = | Noise amplitude for a particular $ijk$ combination. |

The SNRs that NetMOD calculates are treated as random variables. They may take any form, however, the only random variable functions that NetMOD supports at this time (see 5.1 Probability Distributions) are Normal, Log-Normal, and Non-Parametric Cumulative Distribution Function (CDF).

The most common scenario is for the SNR to be represented as a Normal distribution random variable with respect to the log of SNR, assuming that the log of both the signal and noise amplitudes is also a normal distribution. In this case the log SNR is parameterized by a mean and standard deviation:

$$\log\left(SNR_{ijk}(f)\right) \sim N\left(\mu_{log\ SNR_{ijk}}, \sigma_{log\ SNR_{ijk}}\right) \quad (41)$$

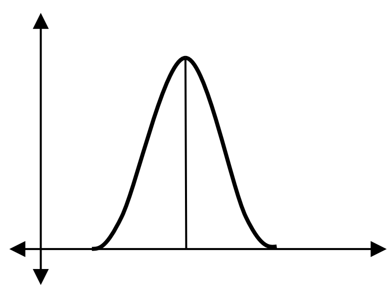

**Figure 11 Normal SNR Distribution**

## 4.3 Probability of Detection

Computing the probability that a network of sensors can detect an event may be performed by two different methods: Probabilistic and Monte Carlo.

In network detection, solving for the probability of detection for a given source location is broken into two parts. First, the probabilities of the individual station detections are solved for each station and phase. Next, the individual station probabilities are combined using a network detection rule that specifies the minimum number of phase detections that are required.

Note that combining the individual station probabilities into a network probability makes the assumption that the station probabilities are statistically independent of one another.

### 4.3.1 Probabilistic

#### 4.3.1.1 Station Detection

The probability that an individual station will detect a phase of an event for a given source location may be represented as the probability that the log SNR is greater than some threshold:

$$\begin{aligned} P_{ijk}(f) &= P\left(\log\ SNR_{ijk}(f) > \log\ SNR_{threshik}\right) \\ &= 1.0 - F_{log\ SNR_{ijk}(f)}\left(\log\ SNR_{threshik}\right) \end{aligned} \quad (42)$$

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $P_{ijk}(f)$ | = | Probability of detection for a particular *ijk* combination at frequency *f*. |
| $P(X > x)$ | = | Probability of the observed random variable X being greater than some threshold *x*. |
| $\log\ SNR_{ijk}(f)$ | = | Observed log SNR for a particular *ijk* combination. |
| $\log\ SNR_{threshik}$ | = | Minimum detectability log threshold for station *i* and phase *k*. |
| $F_X(x)$ | = | Cumulative Distribution Function of the observed random variable such that $F_X(x) = P(X \le x)$ |

The CDF of the log SNR random variable (see 5.1 Probability Distributions) may then be evaluated at the log threshold level to determine the probability of detection at the station.

In the case of a normally distributed log SNR value, this may be thought of as integrating the portion of the probability density function above the threshold:

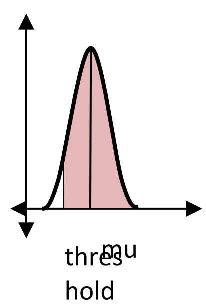


**Figure 12 SNR Distribution**

The SNR for a given station, event, and phase may be determined at multiple frequencies. If this is the case, then it is necessary to determine a representative probability that takes into account the SNR at each of these frequencies. The two methods that NetMOD supports for combining detection frequencies are High and Average.

#### 4.3.1.1.1 High Frequency

The High Frequency method of determining the individual station probability examines the SNR values for each frequency and selects the highest probability across those SNR values.

$$P_{ijk} = MAX_f\left(P_{ijk}(f)\right) \tag{43}$$

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $P_{ijk}$ | = | Probability of detection for a particular *ijk* combination. |
| $P_{ijk}(f)$ | = | Probability of detection for a particular *ijk* combination at frequency *f*. |

Note that the SNR with the highest probability is not necessarily the SNR with the largest mean value.

#### 4.3.1.1.2 Average Frequency

The Average Frequency method of determining the individual station probability computes the average log SNR across frequency. The probability of detection is then estimated from this average log SNR value.

$$\log\ SNR_{ijk} = \frac{1}{N_f}\sum_f \log\ SNR_{ijk}(f) \tag{44}$$

$$\begin{aligned} P_{ijk} &= P\left(\log\ SNR_{ijk} > \log\ SNR_{threshik}\right) \\ &= 1.0 - F_{\log SNR_{ijk}}\left(\log\ SNR_{threshik}\right) \end{aligned} \tag{45}$$

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $\log\ SNR_{ijk}$ | = | Average log SNR for a particular *ijk* combination. |
| $\log\ SNR_{ijk}(f)$ | = | Log SNR for a particular *ijk* combination at frequency *f*. |
| $\log\ SNR_{threshik}$ | = | Minimum detectability log threshold for station *i* and phase *k*. |
| $P_{ijk}(f)$ | = | Probability of detection for a particular *ijk* combination at frequency *f*. |
| $P(X > x)$ | = | Probability of the observed random variable X being greater than some threshold *x*. |
| $F_X(x)$ | = | Cumulative Distribution Function of the observed random variable such that $F_X(x) = P(X \le x)$ |

### 4.3.1.2 Network Detection

Once the probabilities for each station/phase have been determined, these can be combined to come up with a probability of event detection, according to the user-specified criteria for detecting an event. The two basic types of detection criteria are "*form an event detection if this list of phases are detected at n or more stations*" (as in requiring 3 P arrivals) and "*form an event detection if at least one of these phases is detected at n or more stations*" (as in requiring 4 arrivals that are either P or S). Basic criteria can be combined using union and intersection operations to create compound criteria.

NetMOD assumes independence among phase detections at individual stations and also between detections at different stations. Intersections are therefore computed using simple multiplication. The probability of detecting both Pg and Sn at a station $i$ for event $j$ is: $P_{ij}(Pg, Sn) = P_{ij}(Pg) * P_{ij}(Sn)$. Union calculations use this definition of intersection as well, so, for example, $P_{ij}(Pg) \cup P_{ij}(Sn) = P_{ij}(Pg) + P_{ij}(Sn) - \left(P_{ij}(Pg) * P_{ij}(Sn)\right)$. Extensions of these formulas to more phases allow us to compute probabilities of meeting station detection criteria involving arbitrary numbers of phases.

Finding network probabilities from these station probabilities requires basic combinatorics to enumerate all of the ways certain numbers of detections can occur across a network and then uses these two formulas to find the probabilities of each detection scenario. For detection criteria of the form "*detect these phases at $n$ or more stations*," the probability of meeting this type of criteria is the same as one minus the probability of meeting the criteria at fewer than $n$ stations: $P(N \geq n) = 1.0 - P(N < n)$. Using this equivalent form can result in simpler combinatorics for the various ways a certain number of stations can detect an event as $n$ is small in typical NetMOD simulations, while the number of stations can be large. The independence assumption among stations means the intersection composing each of these probability calculations proceeds in the same manner as they did for station detection criteria with multiple phases.

As an example, suppose there are $m$ stations with probabilities of meeting phase detection criteria $P_i$. $n$ station detections are required to form a network detection, meaning that at least $n$ stations must meet the phase detection criteria. This is the sum of all the ways $n, n+1, n+2, \ldots m$ stations can meet the criteria:

$$P(N \geq n) = P(N = n) + P(N = n+1) + P(N = n+2) + \ldots + P(N = m)$$
$$= 1.0 - P(N < n)$$
$$= 1.0 - P(N = n-1) - P(N = n-2) - \ldots - P(N = 0)$$

Each of these probabilities requires finding all of the ways the indicated number of detections can occur, which is why it is computationally more efficient to perform the second calculation when *n* is small. There is a simple, iterative calculation that may be followed to identify these probabilities (Luft and Brown, 1993).

Calculating the probability that an exact number of stations detected may be performed by following an iterative procedure in which the probability of the individual station detections are accounted for:

$$q_i = 1.0 - p_i \quad (46)$$
$$p_{i,j} = p_{i-1,j} q_i + p_{i-1,j-1} p_i \quad (47)$$

Where

| | | |
|---|---|---|
| $i, j$ | = | Station and iteration *i, j* stations detecting. |
| $p_i$ | = | Probability that station *i* detected. |
| $q_i$ | = | Probability that station *i* did not detect. |
| $p_{i,j}$ | = | Probability that exactly *j* stations detect at iteration *i*. |

| Station | | | # of events | | | | |
|---|---|---|---|---|---|---|---|
| $i$ | $p_i$ | $q_i$ | 0 | 1 | 2 | … | k-1 |
| 0 | | | $p_{0,0} = 1.0$ | $p_{0,1} = 0.0$ | $p_{0,2} = 0.0$ | … | $p_{0,k-1} = 0.0$ |
| 1 | $p_1$ | $q_1$ | $p_{1,0} = q_1$ | $p_{1,1} = p_1$ | $p_{1,2} = 0.0$ | … | … |
| 2 | $p_2$ | $q_2$ | $p_{2,0} = q_1q_2$ | $p_{2,1} = q_1p_2 + q_2p_1$ | $p_{2,2} = p_1p_2$ | … | … |
| 3 | $p_3$ | $q_3$ | $p_{3,0} = q_1q_2q_3$ | $p_{3,1} = q_1q_2q_3$ | $p_{3,2} = p_1p_2q_3 + (q_1p_2$ | … | … |
| … | … | … | … | … | … | … | … |
| n | $p_n$ | $q_n$ | $p_{n,0} = p_{n-1,0}q_i$ | $p_{n,1} = p_{n-1,1}q_i + p_{n-1,}$ | $p_{n,2} = p_{n-1,2}q_i + p_{n-1}$ | … | $p_{n,k-1} = p_{n-1,k-1}q_i + p_n$ |

At the end of iterating through all of the stations, the resulting probabilities of detecting *0…k-1* may be summed and summed and subtracted from 1.0 to find the probability of *k* or more detections.

#### 4.3.1.2.1 Detection Criteria

NetMOD defines a language for specifying the detection criteria. The detection criteria enumerates the set of phases and the rules under which each phase must be observed at individual stations in order to declare that the network has detected a given event. A detection criteria string is parsed using the rules below and then then station probabilities are evaluated against the hierarchical criteria tree.

Italicized names represent defined quantities. String literals are enclosed within quotes (“”).

**Table 1 Detection Criteria Rule**

*Criteria*:

*SubCriteria* “/” *n*
*Criteria op Criteria*
“(“ *Criteria* “)”

*SubCriteria*:

*phase*
*SubCriteria op SubCriteria*
“(“ Sub*Criteria* “)”

*n*: An integer value greater than 0 defining the minimum number of detections for the *SubCriteria*. This indicates that the probability of *n* or more detections will be calculated:

$P(N \geq n)$

*op*: A probabilities operator.

“*”: An “and” operator indicating an intersection of probabilities

$$P(A\ and\ B) = P(A) * P(B)$$

“+”: An “or” operator indicating a union of probabilities

$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

*phase*: A string representing a detecting phase.

So, for example, the detection rule:

$((P + Pg)/3 \ * \ S/1)$

Would be parsed into:

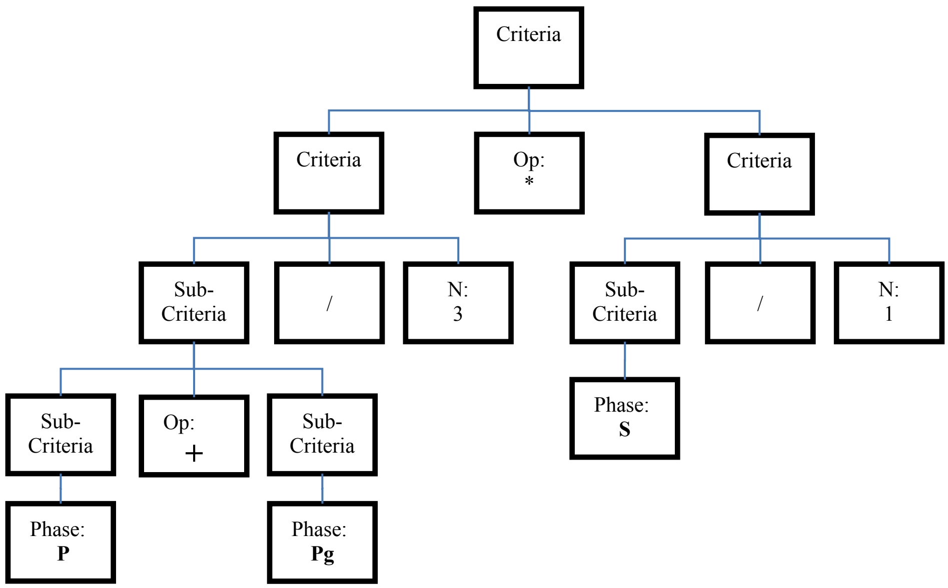


**Figure 13 Detection Rule Example Hierarchy**

The evaluation of the detection rule may then be evaluated using basic rules for evaluating intersections, unions, and combinatorics of statistically independent probabilities.

#### *4.3.2 Monte Carlo*

In Monte Carlo detection, the approach is somewhat different than for probabilistic detection. Rather than computing probability distributions for each of the signal and noise amplitudes, and thus the SNR, a random value is sampled from the probability distribution for each variable that makes up the equations for computing amplitudes (see sections 2 Signal Amplitudes and 0

Noise ). This results in a random sample of the predicted SNR for each station, phase, and frequency. Monte Carlo simulations make no assumptions about the individual probability distributions or how probability distributions are to be combined. Multiple iterations of the Monte Carlo simulations are performed.

The station and phase results are then combined across frequency similar to sections 4.3.1.1.1 High Frequency and 4.3.1.1.2 Average Frequency:

- High: The iterations of SNR samples are used that correspond to the frequency with the most sample SNR values above the threshold.
- Average: For each iteration the sample SNR values are average across frequency.

In a Monte Carlo simulation, the individual station and phase probabilities are assigned a simple binomial quantity: 1 if the sample SNR is above the threshold and 0 if the sample SNR is less than or equal to the threshold. So, either a detection was made for the event or not without any middle ground.

$$P_{ijkn}(f) = \begin{cases} 1, & \log\ SNR_{ijkn} > \log\ SNR_{threshik} \\ 0, & \log\ SNR_{ijkn} \leq \log\ SNR_{threshik} \end{cases} \tag{48}$$

Where

| | | |
|---|---|---|
| $i, j, k, n$ | = | Station *i*, event *j*, wave (phase) *k*, iteration n, respectively. |
| $P_{ijkn}$ | = | Probability of detecting combination *ijkn*. |
| $\log\ SNR_{ijkn}$ | = | Sample log SNR value for combination *ijkn*. |
| $\log\ SNR_{threshik}$ | = | Log SNR threshold for station *i* and phase *k*. |

Network detection is performed on the probabilities of the station and phase detections as described in section 4.3.1.2 Network Detection for each of the individual iterations. Because the station probabilities can only be 1 (the station and phase detected) or 0 (the station and phase did not detect), the network probability will also be either 1 (the network detected) or 0 (the network did not detect).

The probability of the network detecting an event is then calculated as the fraction of the Monte Carlo iterations that resulted in a network detection.

$$P_j = \frac{1}{N_{mc}} \sum_n P_{jn} \tag{49}$$

Where

| | | |
|---|---|---|
| $i, j, k$ | = | Station *i*, event *j*, wave (phase) *k*, respectively. |
| $P_j$ | = | Network detection probability of event *j*. |
| $N_{mc}$ | = | Number of Monte Carlo iterations. |
| $P_{jn}$ | = | Network detection probability of event j for iteration *n*. This value may only be 1 or 0. |

The number of Monte Carlo iterations should be large enough, on the order of 1000, in order to allow sufficient stability and resolution in the results.

## 4.4 Detection Type

There are two basic types of detection simulations that NetMOD may perform: Probability and Threshold.

In a probability detection simulation, NetMOD performs a straight forward simulation for an event of a fixed magnitude to determine the probability that the network is able to detect such an event.

A threshold detection simulation solves the opposite problem of a probability detection simulation. It poses the question of what magnitude threshold is the network able to detect at a given probability, typically 90%.

There is no simple method of inverting the simulation computation to answer this question. Instead, NetMOD is able to search across a specified range of magnitudes. NetMOD iteratively bisects a range of magnitudes to simulate until it has narrowed in on a magnitude whose simulated network detection probability sufficiently matches the desired probability.

# 5 APPENDIX

## 5.1 Probability Distributions

### 5.1.1 Normal Distribution

A random value X with a normal distribution may be entire defined by its mean and standard deviation:

$X \sim N(\mu, \sigma)$

The probability density function of X is:

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{50}$$

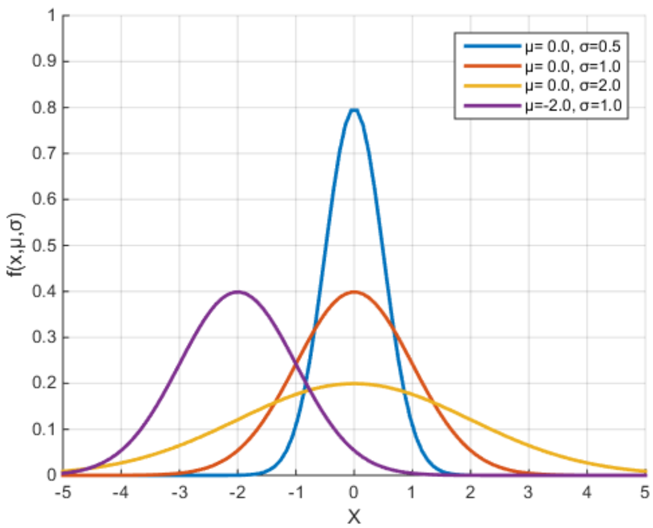


**Figure 14 Normal Distribution Probability Density Function**

The cumulative distribution function (CDF) of X is:

$$F(x,\mu,\sigma) = P(X \le x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \tag{51}$$

$$= \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

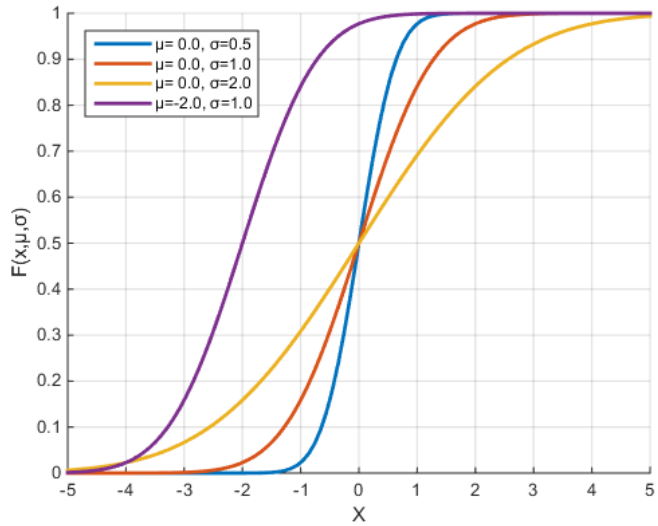


**Figure 15 Normal Distribution Cumulative Distribution Function**

#### 5.1.1.1 Properties

For two Normal random variables X and Y such that:

$X \sim N(\mu_x, \sigma_x)$

$Y \sim N(\mu_y, \sigma_y)$

The following properties hold:

**Table 2 Properties of Normal Random Variables**

| Function of X and Y | Mean | Standard Deviation |
|---|---|---|
| $-X$ | $-\mu_x$ | $\sigma_x$ |
| $aX$ | $a\mu_x$ | $\lvert a\rvert\sigma_x$ |
| $X+b$ | $\mu_x+b$ | $\sigma_x$ |
| $aX+b$ | $a\mu_x+b$ | $\lvert a\rvert\sigma_x$ |
| $X+Y$ | $\mu_x+\mu_y$ | $\sqrt{\sigma_x^2+\sigma_y^2}$ |

### 5.1.2 Log-Normal Distribution

A log-normal distribution is a probability distribution of a random variable whose natural logarithm is normally distributed.

$X \sim lnN(\mu_x, \sigma_x)$
$ln(X) \sim N(\mu_x, \sigma_x)$

By definition, the log-normal distribution is parameterized by the mean and standard deviation of the natural-log of the variable.

The probability density function of X is:

$$f(x, \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln(x) - \mu)^2}{2\sigma^2}} \tag{52}$$

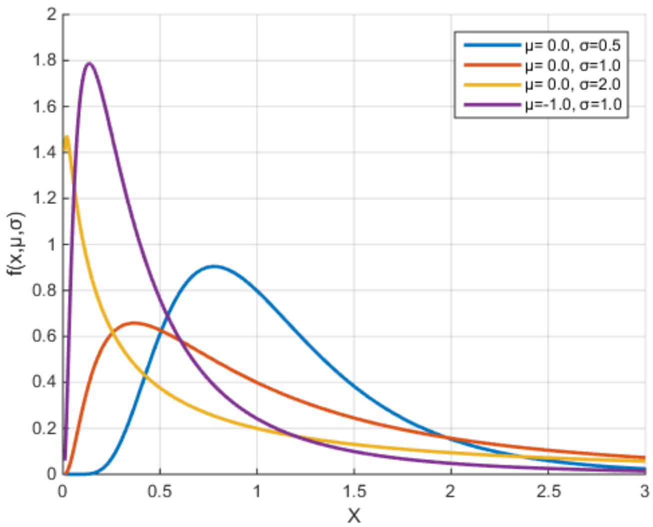


**Figure 16 Log-Normal Probability Density Function**

The cumulative distribution function (CDF) of X is:

$$F(x, \mu, \sigma) = P(X \leq x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(t - \mu)^2}{2\sigma^2}} dt \tag{53}$$

$$= \frac{1}{2}\left[1 + \text{erf}\left(\frac{\ln(x) - \mu}{\sigma\sqrt{2}}\right)\right]$$

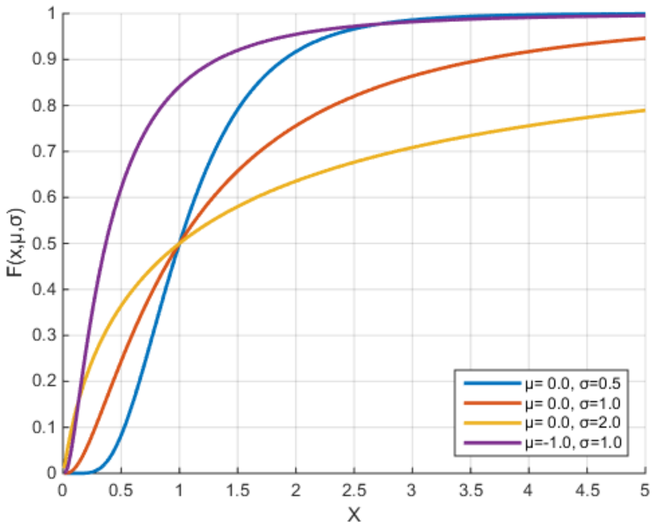


**Figure 17 Figure 18 Log-Normal Cumulative Distribution Function**

#### 5.1.2.1 Properties

For a Log-Normal random variable X such that:

$X \sim lnN(\mu_x, \sigma_x)$

The following properties hold:

**Table 3 Properties of Log Normal Random Variables**

| Function of X | $\mu_Z$ | $\sigma_Z$ |
|---|---|---|
| $Z = aX$ | $\mu_x + \ln(a)$ | $\sigma_x$ |
| $Z = X^a,\ a \neq 0$ | $a\mu_x$ | $\lvert a\rvert\sigma_x$ |
| $Z = X_1 * X_2 * \dots * X_n$ | $\sum_j \mu_j$ | $\left[\sum_j \sigma_j^2\right]^{1/2}$ |
| $Z = X_1 + X_2 + \dots + X_n$ * | $\ln\left[\sum_j e^{\mu_j + \sigma_j^2/2}\right] - \frac{\sigma_Z^2}{2}$ | $\left[\ln\left(\frac{\sum_j \left(e^{2\mu_j + \sigma_j^2}\right)\left(e^{\sigma_j^2} - 1\right)}{\left(\sum_j e^{\mu_j + \sigma_j^2/2}\right)^2} + 1\right)\right]^{1/2}$ |

*Note that the summation of log normal random variables (Fenton, 1960) is expressed in units of natural log and not log base 10, which is commonly used within network modeling. Conversion of the mean and standard deviations between natural log and log10 is easily performed:

$\mu_{ln} = ln(10) \times \mu_{log10}$
$\sigma_{ln} = ln(10) \times \sigma_{log10}$

The means and standard deviations of the individual log-normal random variables must be converted from log10 to a natural log base before they are summed. Once the summed mean and standard deviations are computed, the summed moments will then be converted back to log10.

### *5.1.3 Non-Parametric Cumulative Distribution Function*

NetMOD provides support for random variables that are defined with an arbitrary Cumulative Distribution Function (CDF). CDFs are primarily used in the definition of station site noise where a typical Normal or Log-Normal function may not be a good fit for the empirical data.

A CDF is typically defined by the user as a mean, standard deviation, a series of data values $x$, and the corresponding CDF $F_X(x)$. The only constraint on the definition of the CDF is that the values for $F_X(x)$ are bound between 0 and 1 and the values for both $x$ and $F_X(x)$ must be monotonically increasing. The CDF is assumed to be continuous across the range of defined points.

#### 5.1.3.1 Properties

For a Non-Parametric CDF random variable X:

$$X \sim F_X(x)$$

The following properties hold:

**Table 4 Properties of CDF Random Variables**

| Function of X | $\mu_Z$ | $\sigma_Z$ | $F_Z(z)$ |
|---|---|---|---|
| $Z = aX$ | $a\mu_x$ | $\lvert a\rvert\sigma_x$ | $F_x(z/a)$ |
| $Z = X + b$ | $\mu_x + b$ | $\sigma_x$ | $F_x(z - b)$ |
| $Z = X_1 + X_2$ | $\mu_{x_1} + \mu_{x_2}$ | $\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}$ | $\int F_{xX_1}(z - x) f_{X_2}(x) dx$ or $\int F_{X_2}(z - x) f_{X_1}(x) dx$ Where $f_X(x) = \frac{d}{dx} F_X(x)$ |

Note that because the CDF is defined by a discrete set of points, integration and differentiation of the CDF must be performed numerically.

## 5.2 Error Function

The error function is a special function in probability and statistics representing the partial integral of a symmetric exponential.

$$\mathrm{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{54}$$

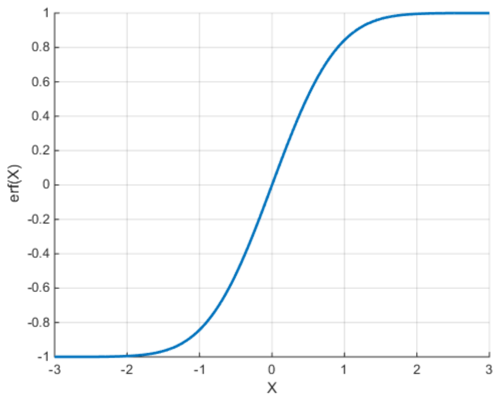


**Figure 19 Error Function**

The error function is used within NetMOD for evaluating the CDF of normal and log-normal distributions.

Numerically solving for the CDF of a normal distribution is difficult because the error function is defined to be the integral of a continuous variable. However, a reasonable approximation of the error function may be made with a maximum error of 1.2 x $10^{-7}$ (Numerical Recipes, 1992):

$$
\begin{aligned}
a_0 &= -1.26551223 \\
a_1 &= 1.00002368 \\
a_2 &= 0.37409196 \\
a_3 &= 0.09678418 \\
a_4 &= -0.18628806 \\
a_5 &= 0.27886807 \\
a_6 &= -1.13520398 \\
a_7 &= 1.48851587 \\
a_8 &= -0.82215223 \\
a_9 &= 0.17087277
\end{aligned} \tag{55}
$$

$$t = \frac{1}{1 + 0.5|x|}$$

$$\tau = t \times e^{\left(-x^2 + a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 + a_8 t^8 + a_9 t^9\right)}$$

$$erf(x) = \begin{cases} 1 - \tau & for\ x \geq 0 \\ \tau - 1 & for\ x < 0 \end{cases}$$

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