

**LA-UR-15-26222**

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**Title:** Modeling of non-linear interaction of waves in rock

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**Intended for:** 20th International Symposium of Non-Linear Acoustics, 2015-06-29  
(Lyon, France)

**Issued:** 2015-08-05

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# Modeling of non-linear interaction of waves in rocks

C. Larmat, R.A. Guyer, Pierre-Yves Le Bas, Paul A. Johnson, Jim TenCate

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# Outline

- Non-linearity in rocks and Earth
- Spectral Element Method
- Validation:
  - Simple test with monochromatic sources
  - Mixing of two non-collinear beams

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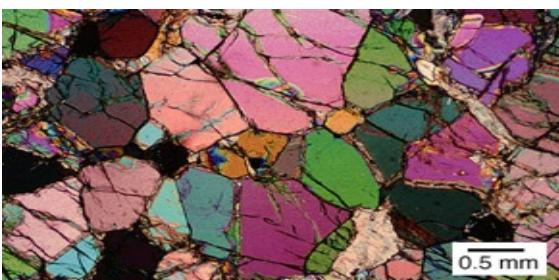
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# Non-linearity acoustics for Earth applications

*The brick and mortar model (e.g. Guyer and Johnson, 2009)*

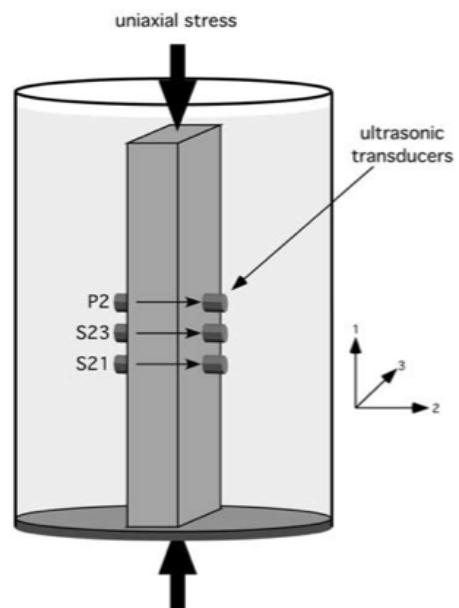
- Change of velocities of porous rocks documented by Birch and Bandcroft in 1938.
- Non-linear models applied to rocks: Johnson and Shankland, 1989; Meegan et al., 1993; Winkler and Liu, 1996.
- Memory effects known as slow-dynamic.
- Hysteresis : Mc Call and Guyer, 1994.



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# Non-linearity measurements

*From the laboratory to in-situ Non-Destructive Evaluation methods*



*Measurement by Winkler and Mc Gowan, 2004*

	Portland sandstone	Indiana limestone	Berea 1 sandstone	Berea 2 sandstone	Lucite
$\beta$ Dry	124 $\pm$ 19	849 $\pm$ 50	1193 $\pm$ 28	717 $\pm$ 115	9 $\pm$ 2
$\beta$ Saturated	332 $\pm$ 28	1201 $\pm$ 43	1155 $\pm$ 93	2568 $\pm$ 63	

*Adaption to in-situ imaging will be through innovative exploration methods and imaging techniques.*

*Renaud et al., 2014.*



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# Numerical Methods

*Theory for nonlinear waves in **fluids** solved by some of the famous names in Science, Poisson, Euler, Lagrange, Rayleigh and others.*

*Elastic and anisotropic media involve a lot of parameters*

*Different numerical methods:*

- Finite Difference: Kosik, 1993; Zhang, Wang, and Harris, 1999; problem of dispersion
- Pseudospectral method: Xu, Day, and Minster, 1999; stringent constrains on time step, complex geometries and heterogeneous media challenging
- Finite Element Method: Zumpano and Meo, 2007; global mass matrix
- Discontinuous Galerkin: Bou Matar et al., 2012; expensive

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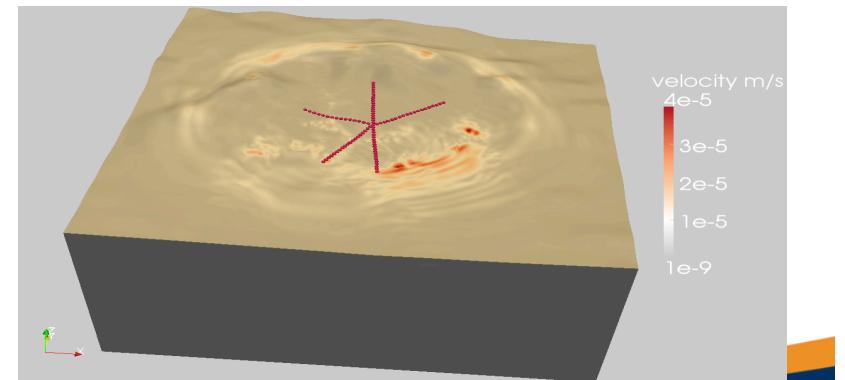
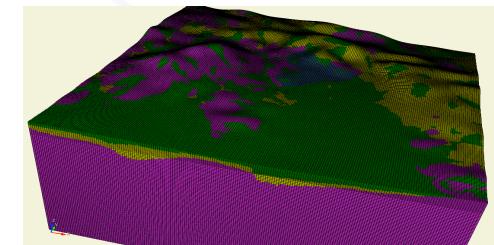
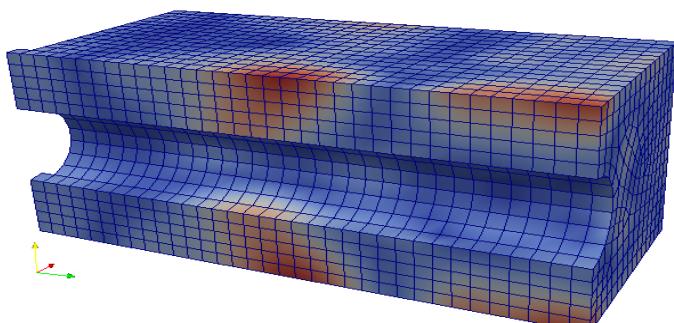
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# The Spectral Element Method

## *Verify Physics Interpretation – Extrapolate to field applications*

- Open-source SPECFEM3D
  - <https://geodynamics.org/cig/software/specfem3d/>
- Laboratory and seismic scale
- From linear to non-linear, homogeneous to complex, arbitrary geometries



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# Implementation of the non-linearity

Newmark scheme:

$$d_{n+1}^0 = d_n + \Delta t v_n + \Delta t^2 \left( \frac{1}{2} - \beta \right) a_n$$

(P)

$$v_{n+1}^0 = v_n + \Delta t (1 - \gamma) a_n$$

$$a_{n+1}^0 = 0$$

(R)

$$\Delta \ddot{a} = M^{-1} \left[ F_{n+1} - K d_{n+1}^{(i)} - B^t \lambda_{n+1}^{(i+1)} - M a_{n+1}^{(i)} \right]$$

(C)

$$a_{n+1}^{i+1} = a_{n+1}^i + \Delta a$$

$$v_{n+1}^{i+1} = v_{n+1}^0 + \Delta t \gamma a_{n+1}^{i+1}$$

$$d_{n+1}^{i+1} = d_{n+1}^0 + \Delta t^2 \beta a_{n+1}^{i+1}$$

*Internal forces with classical NL:*

$$K(d_{n+1}^i) = - \int_{\Omega} \nabla w : T dV$$

With  $T = c: \Delta d + c_{NL} \Delta d \Delta d$

Following Johnson, Kostek and Norris (1994)

Still small deformation approximation

**Modification is local to each element!**

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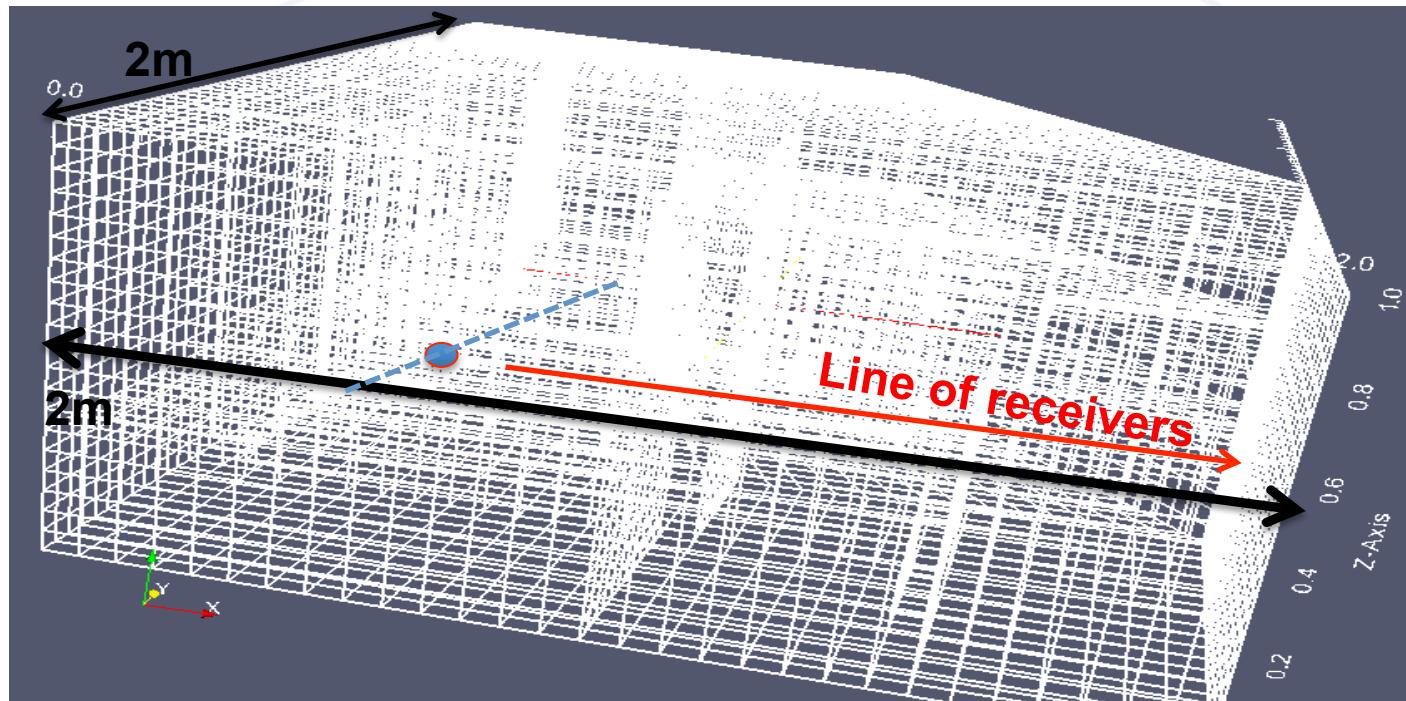
- Non-linearity in rocks and Earth
- Spectral Element Method
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# Test on monochromatic sources



Block Berea

$\rho=2100 \text{ kg/m}^3$

$Vp=2.1 \text{ km/s}$

$Vs=1.344 \text{ km/s}$

$A=-451 \text{ GPa}$

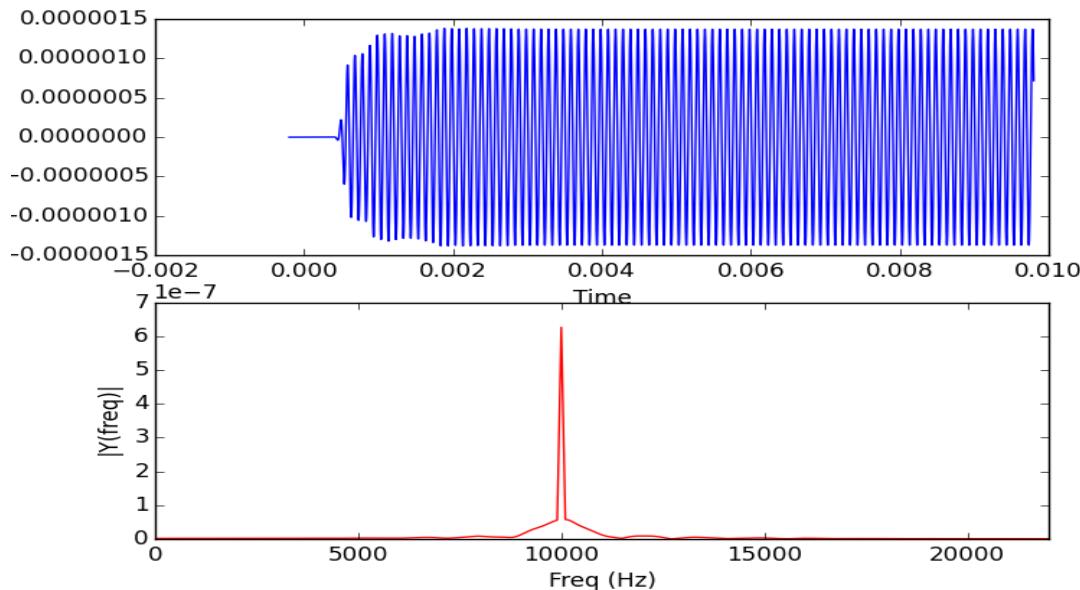
$B=-3216 \text{ GPa}$

$C=1580 \text{ GPa}$

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# Monochromatic Source – 10kHz

*Radial component generated 1m away from the source –  
Linear modeling*



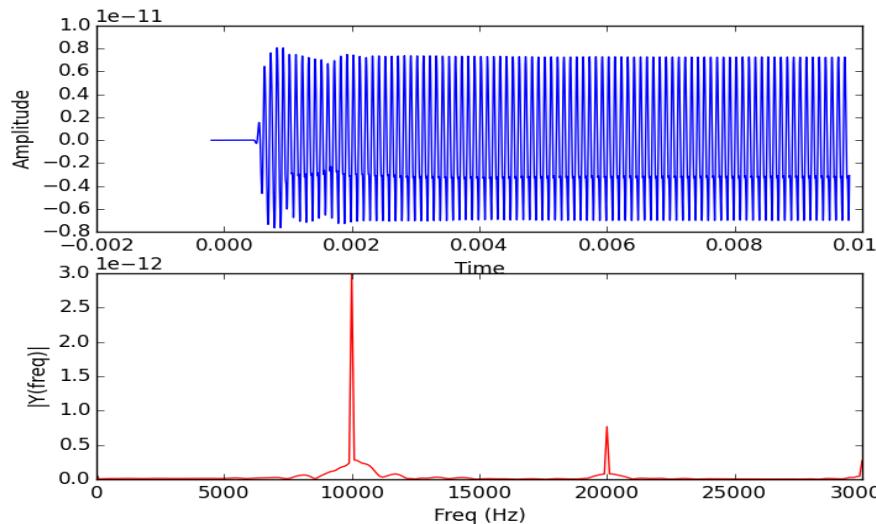
Displacement  $10^{-6}$  m

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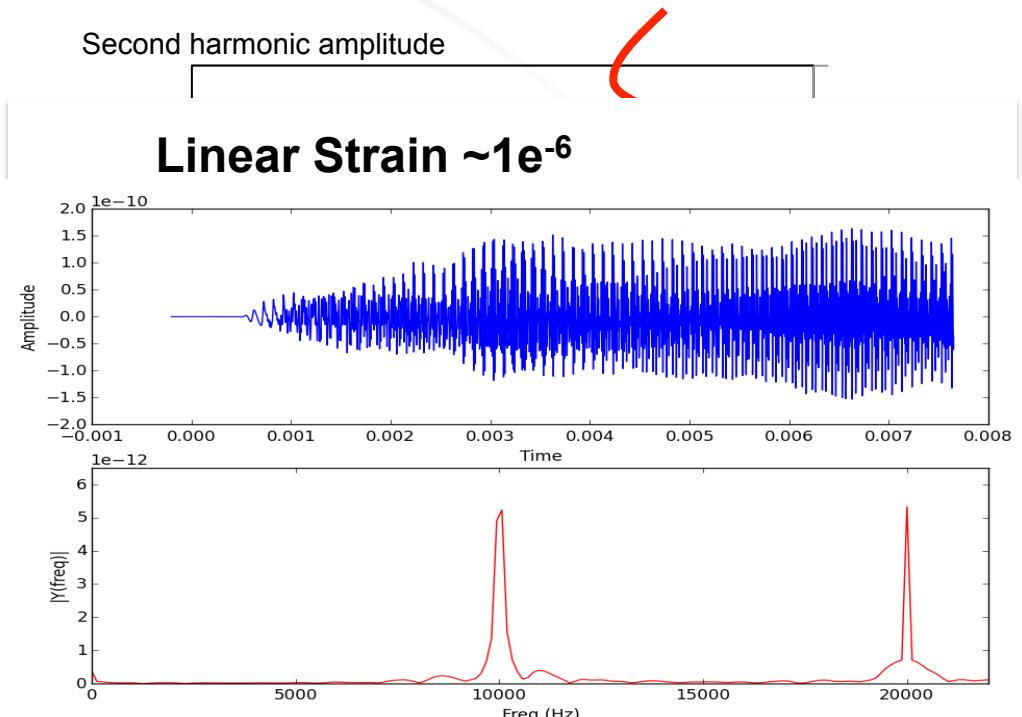
# Monochromatic Source – 10kHz

*Radial component displacement generated 1m away from the source – Non-linear modeling*

Linear Strain  $\sim 1e^{-8}$

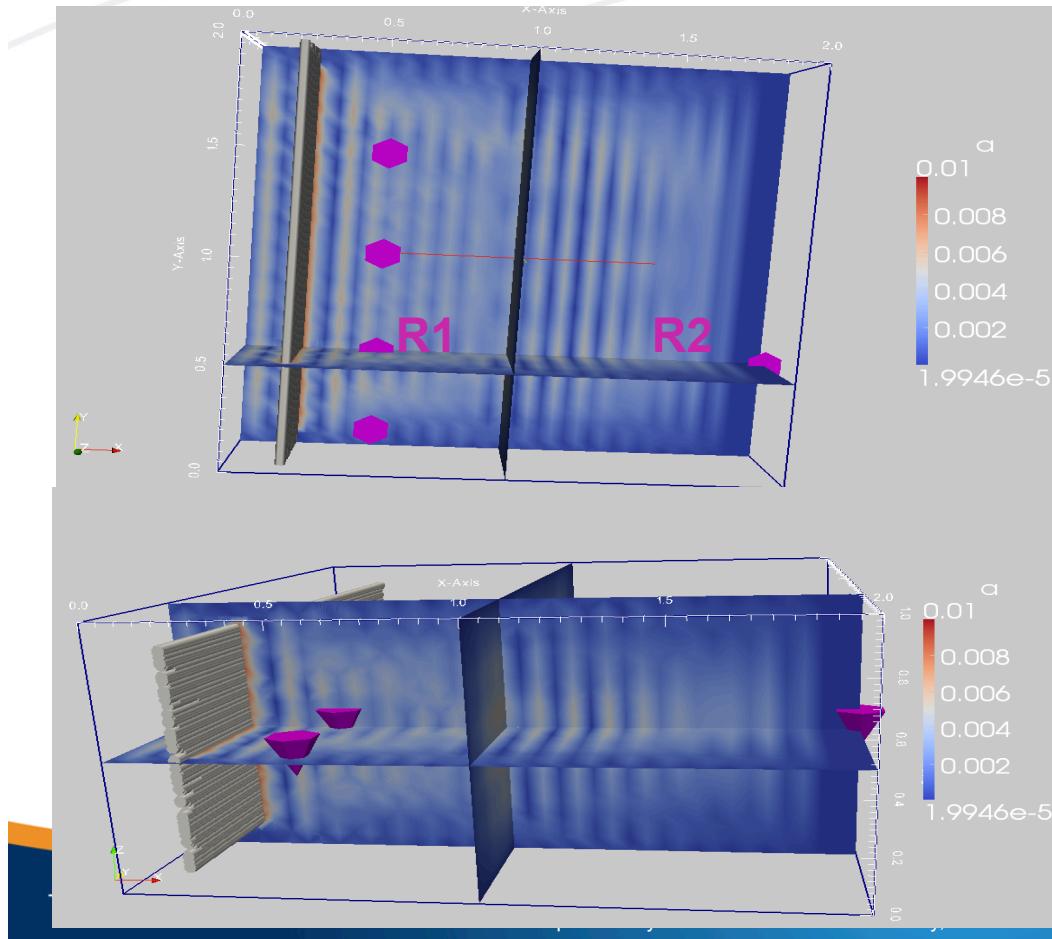


Linear Strain  $\sim 1e^{-6}$



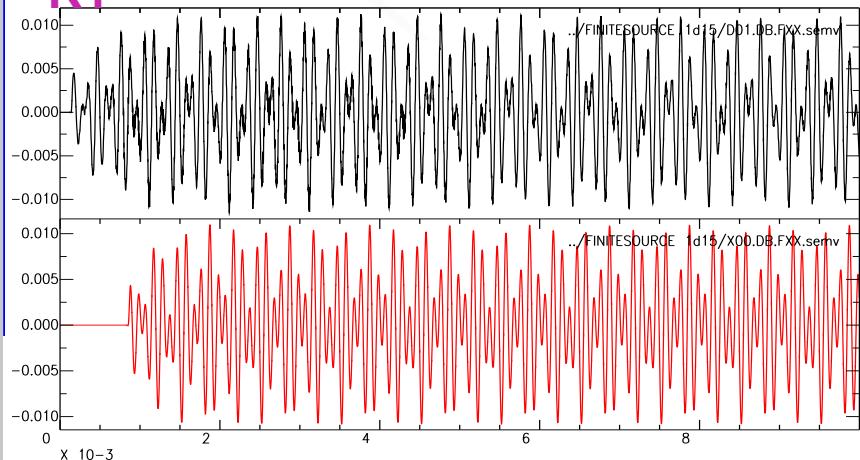
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# Planar Waves 10kHz + 7kHz



Radial component velocity

R1



R2



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- **Validation:**
  - Simple test with monochromatic sources
  - Benchmarking with other modeling tools
  - By experiment

## Mixing of two non-collinear beams

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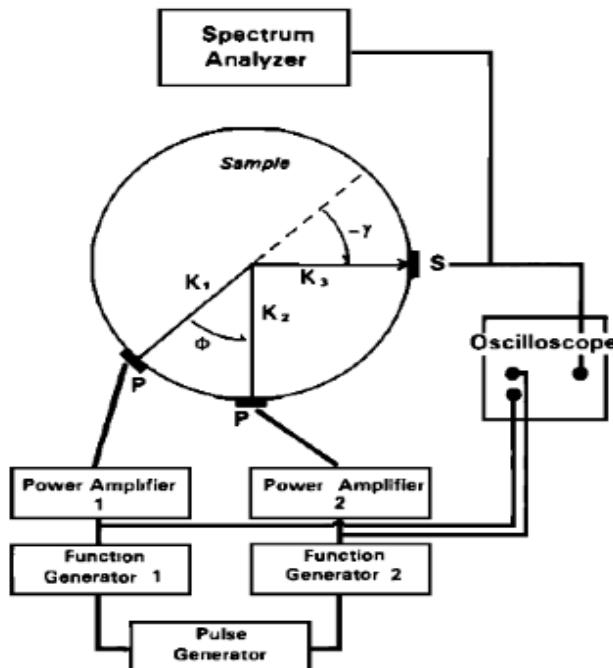
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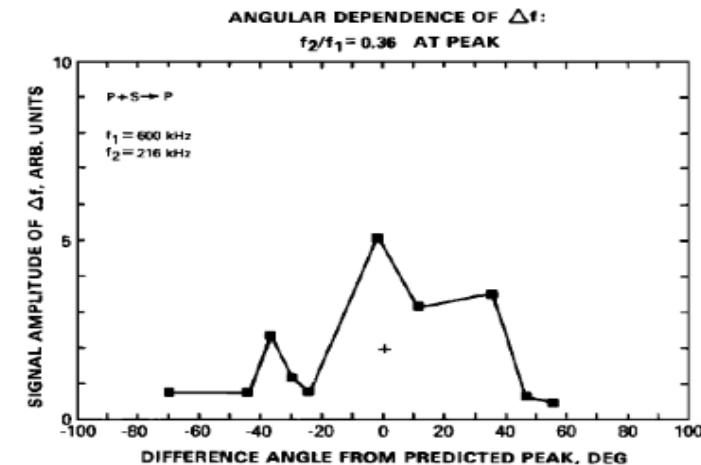
# Non-linear mixing of non-colinear beams



Mixing happens following selection rules:

$$\mathbf{k}_{f3} = \mathbf{k}_{f2} - \mathbf{k}_{f1}$$

Selection of frequencies and angles.



Johnson, Shankland, O'Connell, Albright, 1987.

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# Conclusions

*Non-linearity implemented in Spectral Element method 3D with minor modification of the scheme and CPU cost.*

*Validation being done following the few analytical solutions available: Xu, Day, & Minster, 2000; Bou Matar, 2012.*

*Modeling will be applied to the non-collinear beams experiment of Johnson et al., 1987.*

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*Condition of stability?*

*CFL modified – compute  $dt$  at each step*  
*hyperbolicity of the equation?*



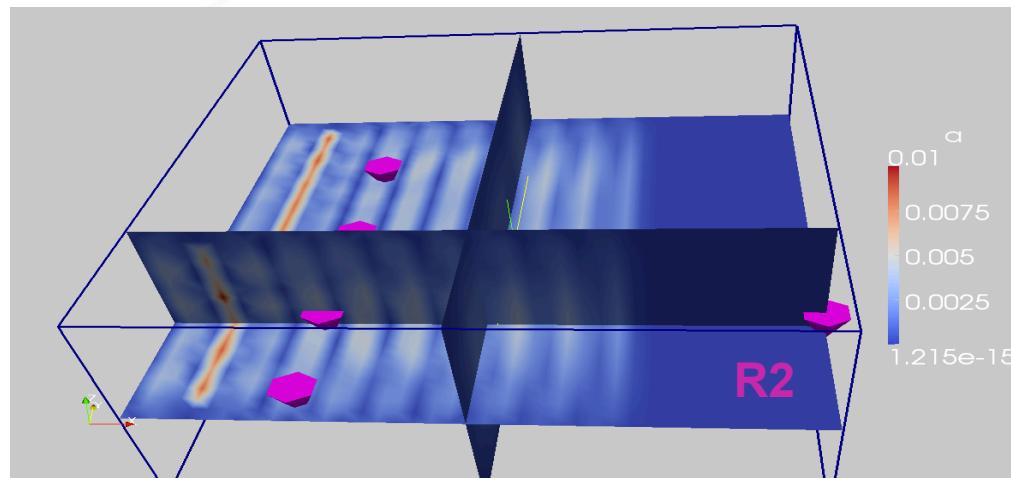
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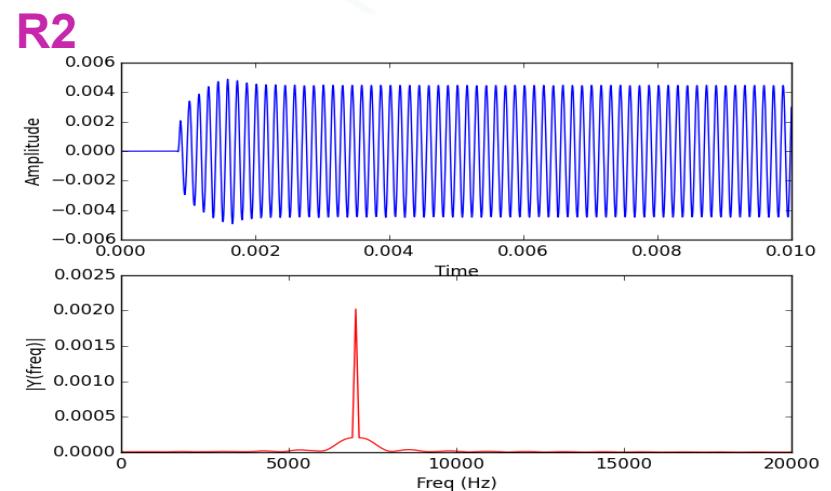
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# monochromatic 7000Hz



## Radial component



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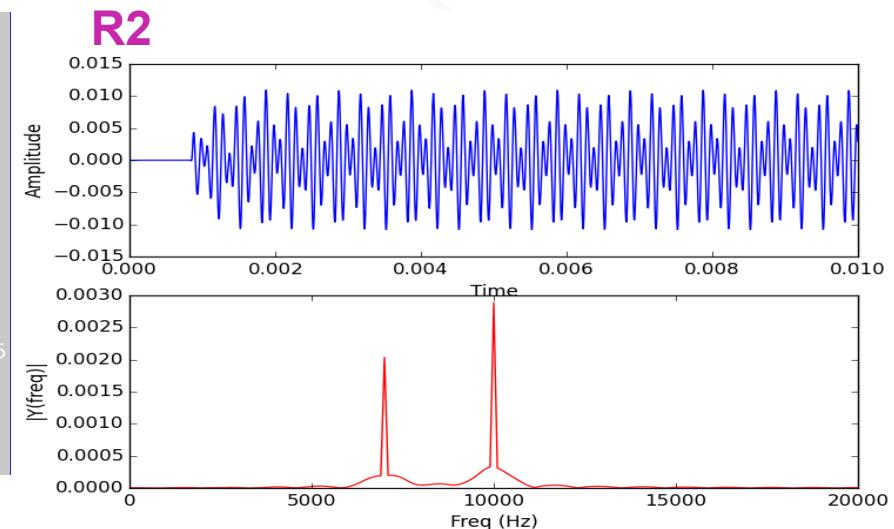
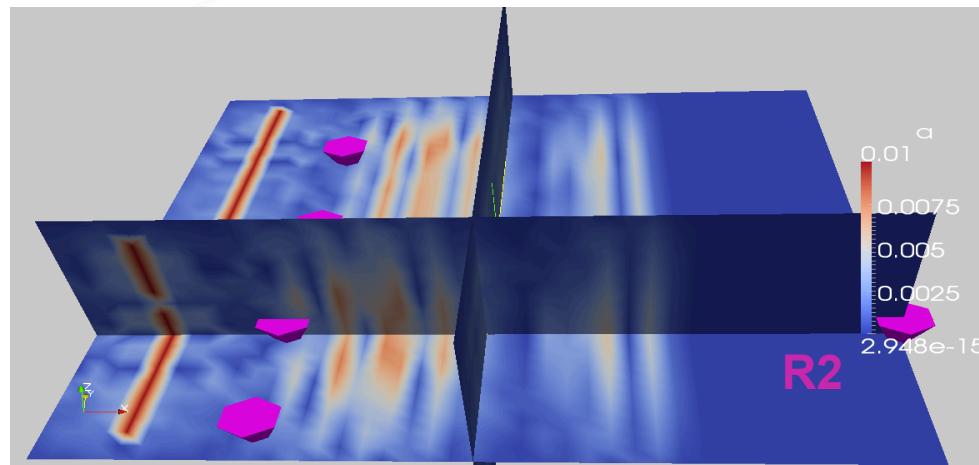


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# monochromatic 7000Hz + 10000Hz linear

## Radial component



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