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Title: Modeling of non-linear interaction of waves in rock

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Modeling of non-linear interaction of waves in rocks

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Outline

- Non-linearity in rocks and Earth
- Spectral Element Method
- Validation:
  - Simple test with monochromatic sources
  - Mixing of two non-collinear beams

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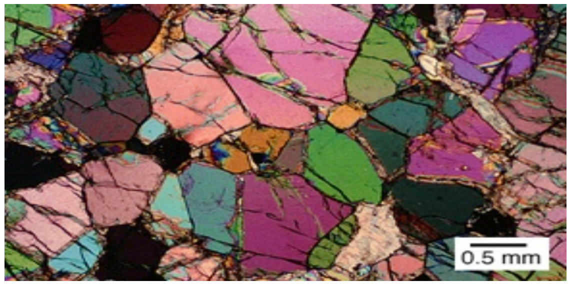
Non-linearity acoustics for Earth applications

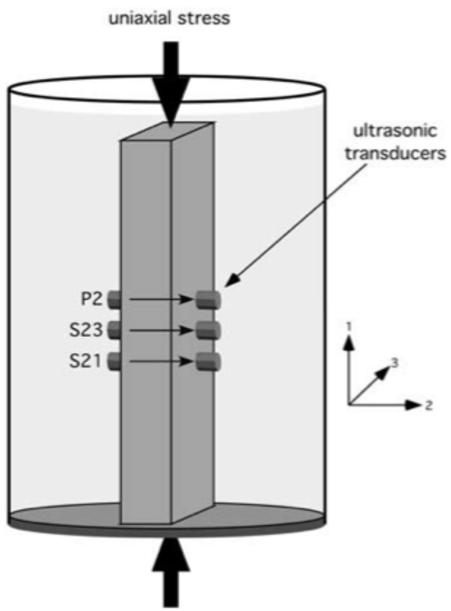
The brick and mortar model (e.g. Guyer and Johnson, 2009)

- Change of velocities of porous rocks documented by Birch and Bandcroft in 1938.
- Non-linear models applied to rocks: Johnson and Shankland, 1989; Meegan et al., 1993; Winkler and Liu, 1996.
- Memory effects known as slow-dynamic.
- Hysteresis : Mc Call and Guyer, 1994.



Non-linearity measurements

From the laboratory to in-situ Non-Destructive Evaluation methods



Measurement by Winkler and Mc Gowan, 2004

	Portland sandstone	Indiana limestone	Berea 1 sandstone	Berea 2 sandstone	Lucite
β Dry	124 ±19	849 ±50	1193 ±28	717 ±115	9 ±2
β Saturated	332 ±28	1201 ±43	1155 ±93	2568 ±63	

Adaption to in-situ imaging will be through innovative exploration methods and imaging techniques.

Renaud et al., 2014.

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Numerical Methods

Theory for nonlinear waves in fluids solved by some of the famous names in Science, Poisson, Euler, Lagrange, Rayleigh and others.

Elastic and anisotropic media involve a lot of parameters

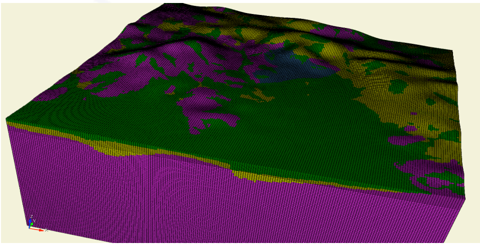
Different numerical methods:

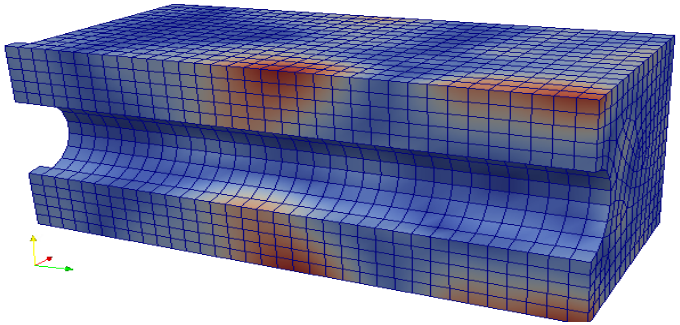
- Finite Difference: Kosik, 1993; Zhang, Wang, and Harris, 1999; problem of dispersion
- Pseudospectral method: Xu, Day, and Minster, 1999; stringent constrains on time step, complex geometries and heterogeneous media challenging
- Finite Element Method: Zumpano and Meo, 2007; global mass matrix
- Discontinuous Galerkin: Bou Matar et al., 2012; expensive

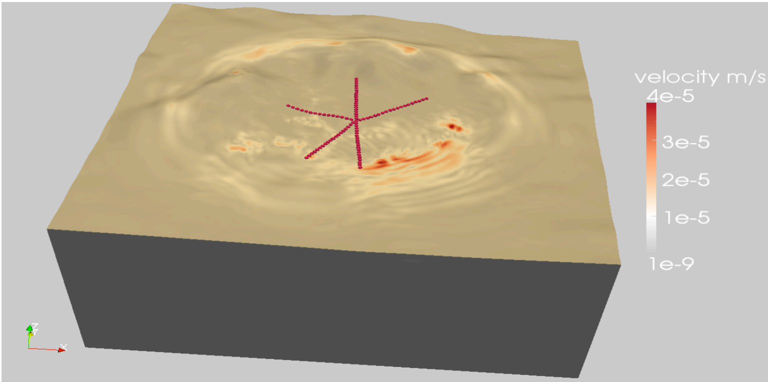
The Spectral Element Method

Verify Physics Interpretation – Extrapolate to field applications

- Open-source SPECFEM3D
  - https://geodynamics.org/cig/software/specfem3d/
- Laboratory and seismic scale
- From linear to non-linear, homogeneous to complex, arbitrary geometries







Implementation of the non-linearity

Newmark scheme:

(P)

$$d_{n+1}^{0} = d_n + \Delta t v_n + \Delta t^2 \left(\frac{1}{2} - \beta\right) a_n$$

$$v_{n+1}^{0} = v_n + \Delta t (1 - \gamma) a_n$$

$$a_{n+1}^{0} = 0$$

(R)

$$\Delta \mathrm{a} = \mathrm{M}^{-1}\left[\mathrm{F}_{n+1} - \mathrm{Kd}_{n+1}^{(i)} - \mathrm{B}^t \lambda_{n+1}^{(i+1)} - \mathrm{Ma}_{n+1}^{(i)}\right]$$

(C)

$$a_{n+1}^{i+1} = a_{n+1}^{i} + \Delta a$$

$$v_{n+1}^{i+1} = v_{n+1}^{0} + \Delta t \gamma a_{n+1}^{i+1}$$

$$d_{n+1}^{i+1} = d_{n+1}^{0} + \Delta t^2 \beta a_{n+1}^{i+1}$$

Internal forces with classical NL:

$K(d^{i}_{n+1}) = -\int_\Omega \nabla \mathbf{w} : \mathbf{T}\, dV$

With $T = c{:}\Delta d + c_{NL} \Delta d \Delta d$

Following Johnson, Kostek and Norris (1994)

Still small deformation approximation

Modification is local to each element!

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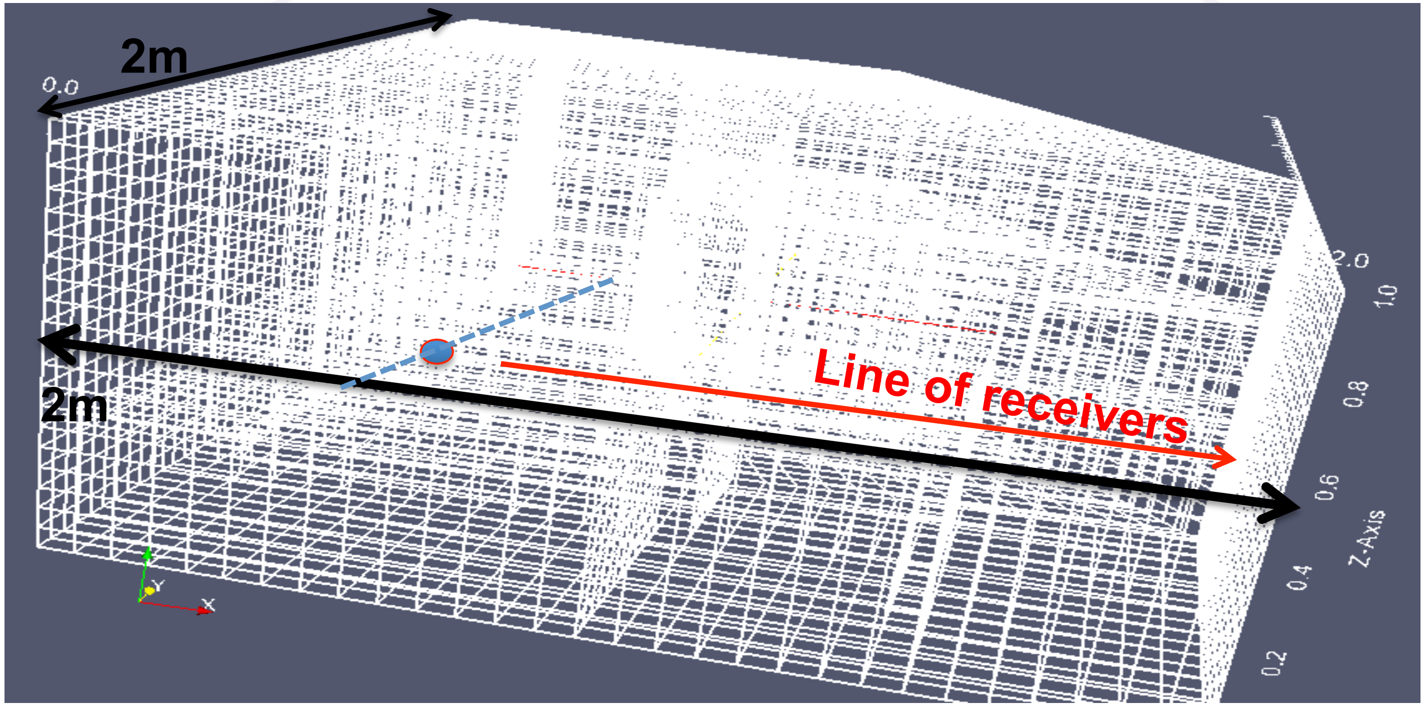
Test on monochromatic sources



Block Berea

ρ=2100 kg/m^3

Vp=2.1 km/s

Vs=1.344 km/s

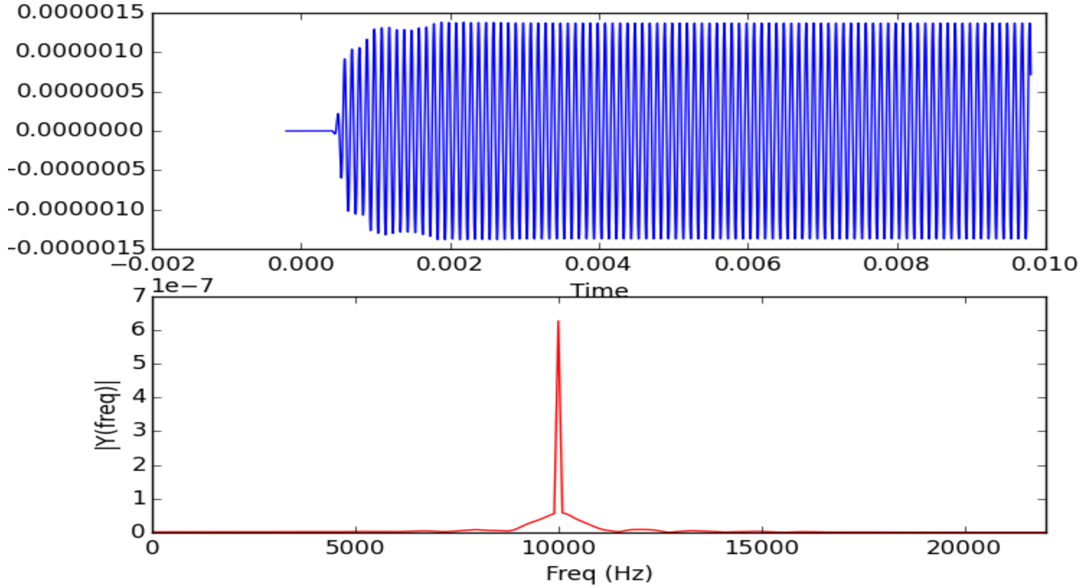
A=-451 GPa

B=-3216 GPa

C=1580 GPa

Monochromatic Source – 10kHz

Radial component generated 1m away from the source – Linear modeling



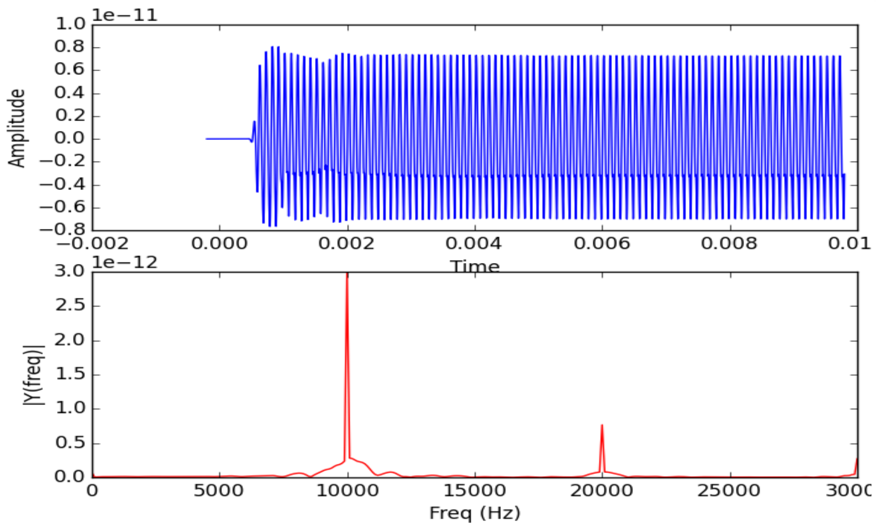
Displacement 10^{-6} m

Monochromatic Source – 10kHz

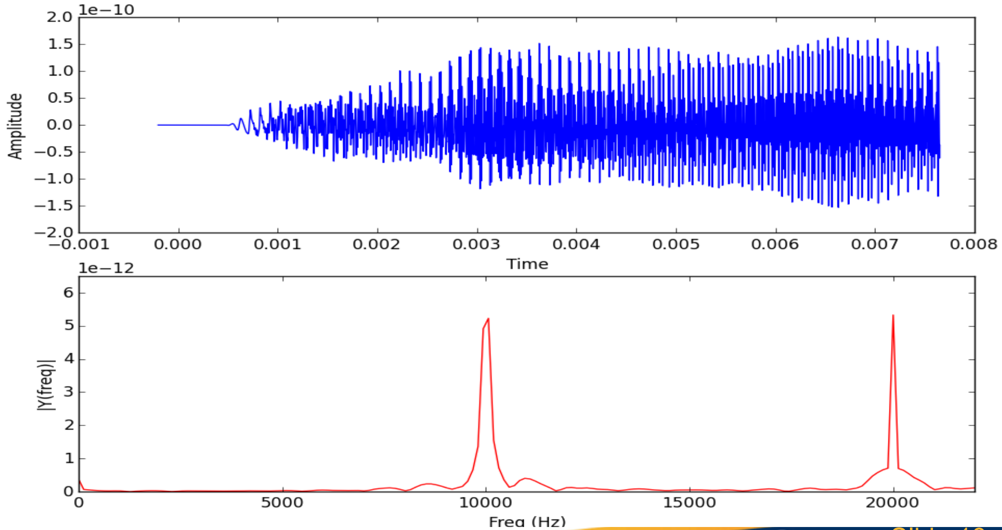
Radial component displacement generated 1m away from the source – Non-linear modeling

Second harmonic amplitude

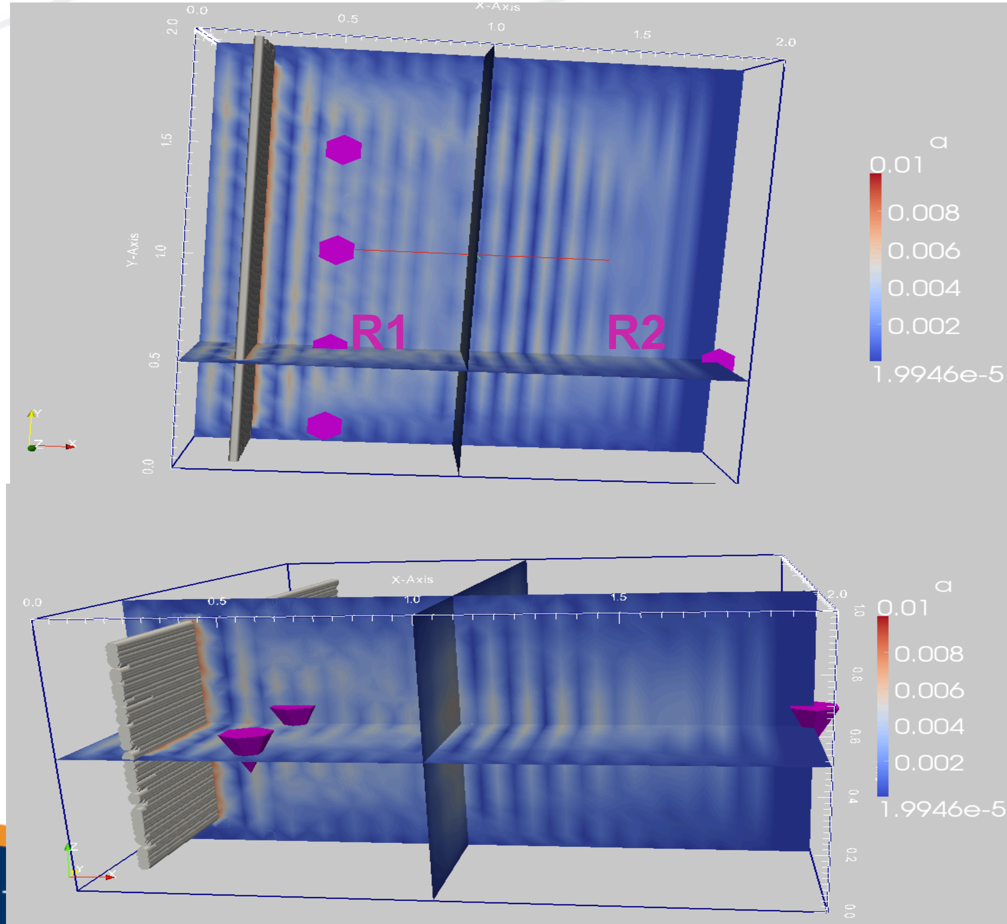
Linear Strain ~1e^{-8}



Linear Strain ~1e^{-6}

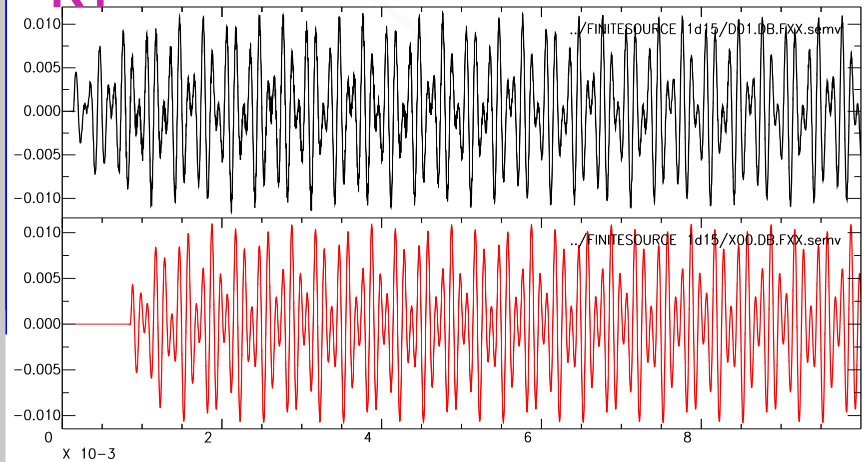


Planar Waves 10kHz + 7kHz



Radial component velocity

R1

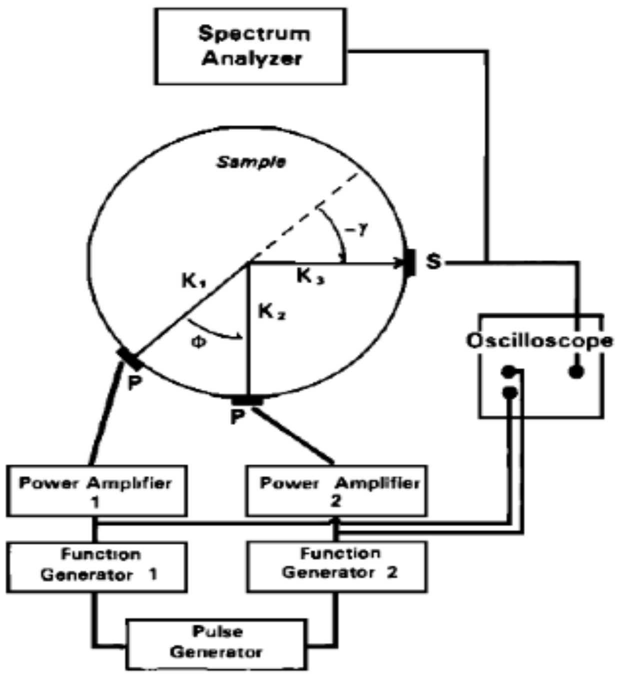


R2

- **Validation:**
  - Simple test with monochromatic sources
  - Benchmarking with other modeling tools
  - By experiment

Mixing of two non-collinear beams

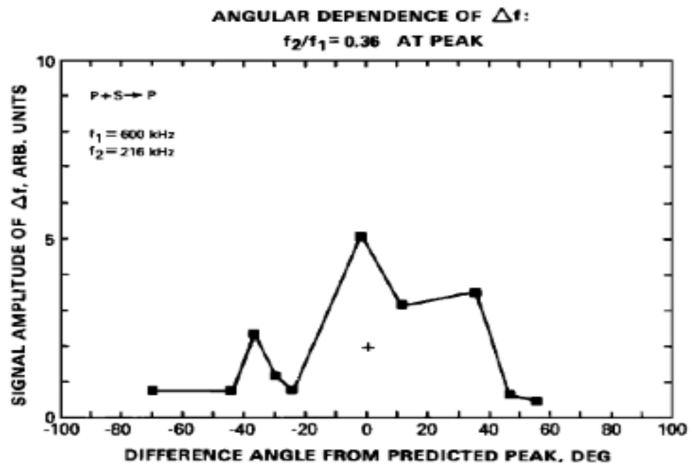
Non-linear mixing of non-colinear beams



Mixing happens following selection rules:

$\mathbf{k}_{f3} = \mathbf{k}_{f2} - \mathbf{k}_{f1}$

Selection of frequencies and angles.



Johnson, Shankland, O' Connell, Albright, 1987.

Conclusions

Non-linearity implemented in Spectral Element method 3D with minor modification of the scheme and CPU cost.

Validation being done following the few analytical solutions available: Xu, Day, & Minster, 2000; Bou Matar, 2012.

Modeling will be applied to the non-collinear beams experiment of Johnson et al., 1987.

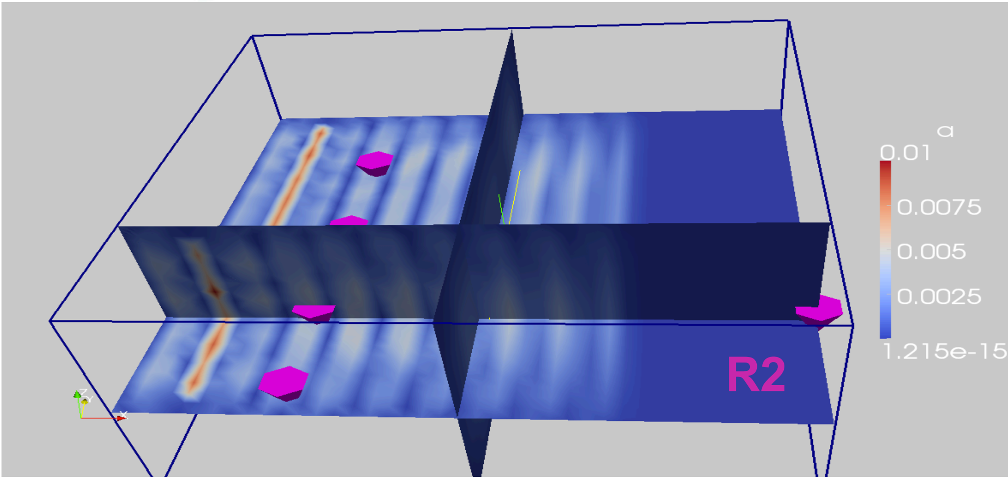
Condition of stability?

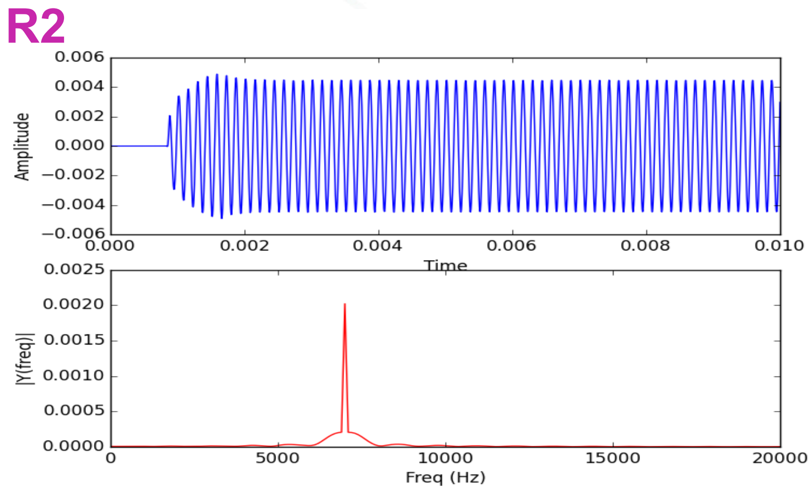
CFL modified – compute dt at each step

hyperbolicity of the equation?

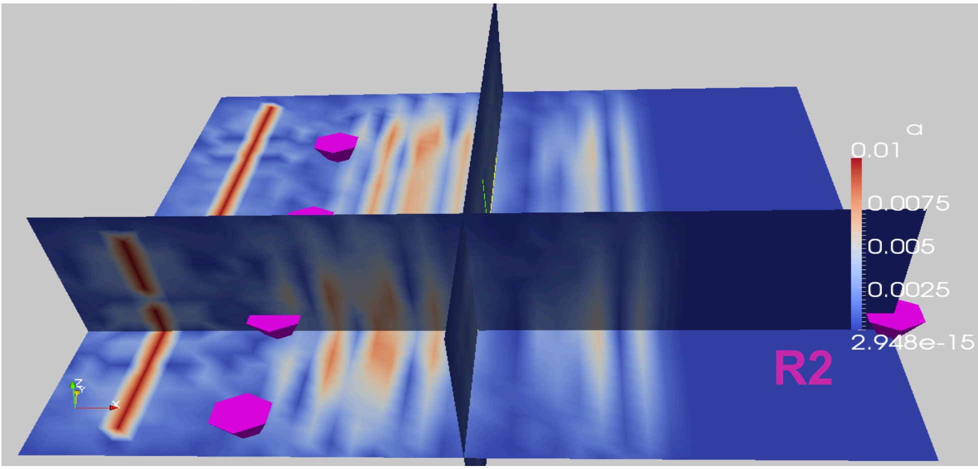
monochromatic 7000Hz

Radial component





monochromatic 7000Hz + 10000Hz linear



Radial component