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SAND2014-1071
Unlimited Release
Printed February 2014

Radar Channel Balancing with Commutation

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Radar Channel Balancing with Commutation

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Abstract

When multiple channels are employed in a pulse-Doppler radar, achieving and maintaining balance between the channels is problematic. In some circumstances the channels may be commutated to achieve adequate balance. Commutation is the switching, trading, toggling, or multiplexing of the channels between signal paths. Commutation allows modulating the imbalance energy away from the balanced energy in Doppler, where it can be mitigated with filtering.

Acknowledgements

This report is the result of an unfunded Research and Development effort.

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Contents

Foreword

This report details the results of an academic study. It does not presently exemplify any modes, methodologies, or techniques employed by any operational system known to the author.

Classification

The specific mathematics and algorithms presented herein do not bear any release restrictions or distribution limitations.

This distribution limitations of this report are in accordance with the classification guidance detailed in the memorandum “Classification Guidance Recommendations for Sandia Radar Testbed Research and Development”, DRAFT memorandum from Brett Remund (Deputy Director, RF Remote Sensing Systems, Electronic Systems Center) to Randy Bell (US Department of Energy, NA-22), February 23, 2004. Sandia has adopted this guidance where otherwise none has been given.

This report formalizes preexisting informal notes and other documentation on the subject matter herein.

1 Introduction

Often in radar applications a single transmitted signal will need to be received by more than one receiver, or more than one receiver channel. These channels might correspond to different polarizations, antenna phase centers, time/frequency/phase offsets, or other characteristic desired or required for a particular application. System applications might include Synthetic Aperture Radar (SAR), Interferometric SAR (IFSAR or InSAR), Ground Moving Target Indicator (GMTI) radar, Dismount Moving Target Indicator (DMTI) radar, and/or Direction of Arrival (DOA) measurements.

It is generally desired that the different channels are identical, or at least adequately similar, such that the difference in output signal characteristics are dependent solely on input signal differences, and not on channel characteristic differences. However, when analog signal paths are involved, even with the most careful of designs and fabrications the normal variation in analog component characteristics and circuit construction naturally render differences in channel characteristics, sometime exceeding differences that are tolerable for the application at hand. Furthermore, channel imbalances are often frequency dependent, and manifest in both amplitude and phase. This can be quite a problem, generally requiring some mitigation.[1,2]

The obvious solution is to build in adjustments to allow ‘tweaking’ or calibrating the channels to match each other, sometimes even in an automated fashion. Indeed this is the most common approach. However analog matching networks can become quite complex and still only exhibit marginal effectiveness. Digital Signal Processing (DSP) techniques can also become quite computation-intensive, just to get close. Furthermore, should balance change with time, temperature, or other factor, a re-calibration becomes necessary to preserve balance. Consequently the ‘obvious’ solution may not always be the ‘best’ solution.

We discuss herein the idea of “commutation” between different channels on a pulse-to-pulse basis, for the purpose of manifesting effective balance in the channel characteristics in a pulse-Doppler radar. Commutation means switching, trading, toggling, or multiplexing physical channels between signal inputs/outputs, so that all distinct signals see the same channel transmission characteristics, at least statistically. We show that imbalance energy may thereby be modulated away from the balanced energy in Doppler frequency space, and then filtered to effectively provide balanced channels.

"The truth is balance. However the opposite of truth, which is unbalance,
may not be a lie."
-- Susan Sontag

2 Basic Channel Commutation

Consider two pulse-Doppler radar channels, each with separate input and output, and with separate gain and phase characteristics. This is illustrated in Figure 1. We identify them with the transfer function equations

$$Y_1(n) = H_1(n)X_1(n) \text{ , and}$$
$$Y_2(n) = H_2(n)X_2(n), \tag{1}$$

where

n = pulse index number,
$X_1(n)$ = input spectrum of channel 1 for pulse *n*,
$X_2(n)$ = input spectrum of channel 2 for pulse *n*,
$Y_1(n)$ = output spectrum of channel 1 for pulse *n*,
$Y_2(n)$ = output spectrum of channel 2 for pulse *n*,
$H_1(n) = A_1 e^{j\Theta_1}$ = transfer function (frequency response) of channel 1, and
$H_2(n) = A_2 e^{j\Theta_2}$ = transfer function (frequency response) of channel 2, (2)

where

A_1 = amplitude response of channel 1 transfer function,
Θ_1 = phase response of channel 1 transfer function,
A_2 = amplitude response of channel 2 transfer function, and
Θ_2 = phase response of channel 2 transfer function. (3)

In general, we will allow that all parameters are frequency-dependent within a pulse. We emphasize that index *n* is a 'pulse' index, and not a frequency index, or intra-pulse time index. We are omitting any overt frequency variable to keep the notation simplified so as to not obscure the larger points made in this report.

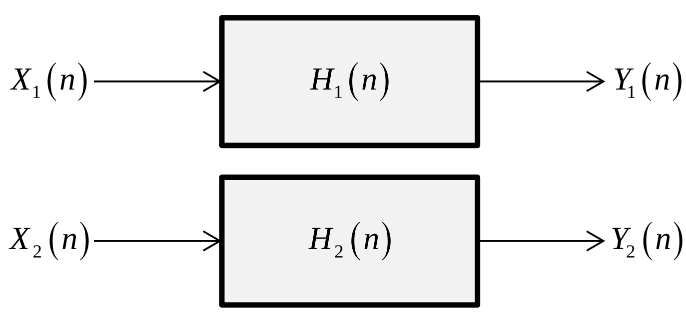

Figure 1. Two channels without commutation.

2.1 Perfectly Balanced Channels

In a balanced-channel system, we have

$$A_2 = A_1 \text{, and}$$
$$\Theta_2 = \Theta_1 . \quad (4)$$

For such a balanced-channel system, equal inputs will yield equal outputs. That is

$$\text{if } X_2(n) = X_1(n),$$
$$\text{then } Y_2(n) = Y_1(n). \quad (5)$$

We note that Eq. (4) is a different set of requirements than for a 'distortionless' system, which requires a constant amplitude, and linear (with frequency) phase response. That is, we can in fact have two entirely distortionless channels that are nevertheless not balanced. Similarly, we can have perfectly balanced channels that are not distortionless.

In practice, especially with analog signal paths, perfect balance is an ideal that can be approached but never achieved. Consequently we are faced with the following questions.

1. How much balance is good enough?

2. How can we make balance better?

2.2 Unbalanced Channels

We now consider the case where channels are not balanced. That is we allow

$$A_2 \neq A_1 \text{, and/or}$$
$$\Theta_2 \neq \Theta_1 . \quad (6)$$

With some forethought, we define the channel parameters in terms of an ideal value and an error. That is, we define

$$A_1 = A_0 (1 + a_\varepsilon),$$
$$A_2 = A_0 (1 - a_\varepsilon),$$
$$\Theta_1 = \Theta_0 + \theta_\varepsilon \text{, and}$$
$$\Theta_2 = \Theta_0 - \theta_\varepsilon , \quad (7)$$

where

a_ε = amplitude error parameter, and
θ_ε = phase error parameter. (8)

We note that the errors are related to the individual channel parameters as

$$a_\varepsilon = \frac{A_1 - A_2}{A_1 + A_2}, \text{ and}$$
$$\theta_\varepsilon = \frac{\Theta_1 - \Theta_2}{2}. \tag{9}$$

In general, we expect all new terms to be frequency-dependent as well, although as a practical matter the error terms can typically be expected to be relatively low-frequency compared to other parameters.

We note that $a_\varepsilon = 0$ and $\theta_\varepsilon = 0$ yield balanced channels, where

$$H_2(n) = H_1(n) = A_0 e^{j\Theta_0} \text{ for balanced channels.} \tag{10}$$

More generally, channel transfer functions may be written as

$$H_i(n) = A_i e^{j\Theta_i} = A_0(1 \pm a_\varepsilon) e^{j(\Theta_0 \pm \theta_\varepsilon)}, \tag{11}$$

where the plus sign holds for channel 1, i.e. $i = 1$, and the minus sign holds for channel 2, i.e. $i = 2$.

With a little algebra and trigonometry we may expand the channel transfer functions to

$$H_i(n) = A_0 e^{j\Theta_0} \left(\cos\theta_\varepsilon \pm j\sin\theta_\varepsilon \pm a_\varepsilon \cos\theta_\varepsilon + ja_\varepsilon \sin\theta_\varepsilon\right), \tag{12}$$

and then to

$$H_i(n) = A_0 e^{j\Theta_0} \left(\left(\cos\theta_\varepsilon + ja_\varepsilon \sin\theta_\varepsilon\right) \pm \left(a_\varepsilon \cos\theta_\varepsilon + j\sin\theta_\varepsilon\right)\right), \tag{13}$$

For small errors, we may approximate this as

$$H_i(n) \approx \left[A_0 e^{j\Theta_0}\right] - e^{j\pi i}\left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right]. \tag{14}$$

We make several observations.

- The first square bracket is the desired ‘perfectly balanced’ channel term.
- The second square bracket represents the departure from perfect balance, which is a copy of the ‘true’ term but attenuated and shifted in phase.
- The imbalance term sits on top of (in frequency), and corrupts the true term.
- Attenuating the second term will improve balance.

If the errors are known, well characterized, and stable, then calibration schemes might be employed to 'correct' the analog channel and/or the data sampled from it. In practice such calibration schemes can help, perhaps even sufficiently for some applications. However this is not always the case, and begs for alternate or additional measures to achieve channel balance. One such measure is "channel commutation."

2.3 Simple Channel Commutation

We begin with the need to process two input signals simultaneously through respective signal channels, which we are not able to guarantee are sufficiently balanced for our purposes. Now consider the prospect of achieving balance by operating in a manner so as to put both signals through both channels, so that any error between channels affects both signals equally.

We achieve this with "commutation." That is, we swap channels for each signal as a function of pulse index number. This is written in a general sense as

$$Y_1(n) = H_{f_1(n)}(n) X_1(n) \text{ , and}$$
$$Y_2(n) = H_{f_2(n)}(n) X_2(n), \tag{15}$$

where

$$f_1(n) \in \{1, 2\} \text{, based on some function of } n \text{, and}$$
$$f_2(n) = \begin{cases} 1 & if \quad f_1(n) = 2 \\ 2 & if \quad f_1(n) = 1 \end{cases} \tag{16}$$

Here we will assume that channels are swapped every other pulse. This is illustrated in Figure 2. We may write this as

$$f_i(n) = \text{mod}(i + n - 1, 2) + 1. \tag{17}$$

This means that the channel error is added or subtracted on alternating pulses, with each channel doing the opposite of the other. This allows us to write the channel transfer function as

$$H_{f_i(n)}(n) \approx \left[A_0 e^{j\Theta_0} \right] - e^{j\pi f_i(n)} \left[A_0 e^{j\Theta_0} (a_\varepsilon + j\theta_\varepsilon) \right]. \tag{18}$$

The previous equations can be combined, manipulated, and simplified to

$$H_{f_i(n)}(n) \approx \left[A_0 e^{j\Theta_0} \right] - e^{j\pi n} e^{j\pi i} \left[A_0 e^{j\Theta_0} (a_\varepsilon + j\theta_\varepsilon) \right]. \tag{19}$$

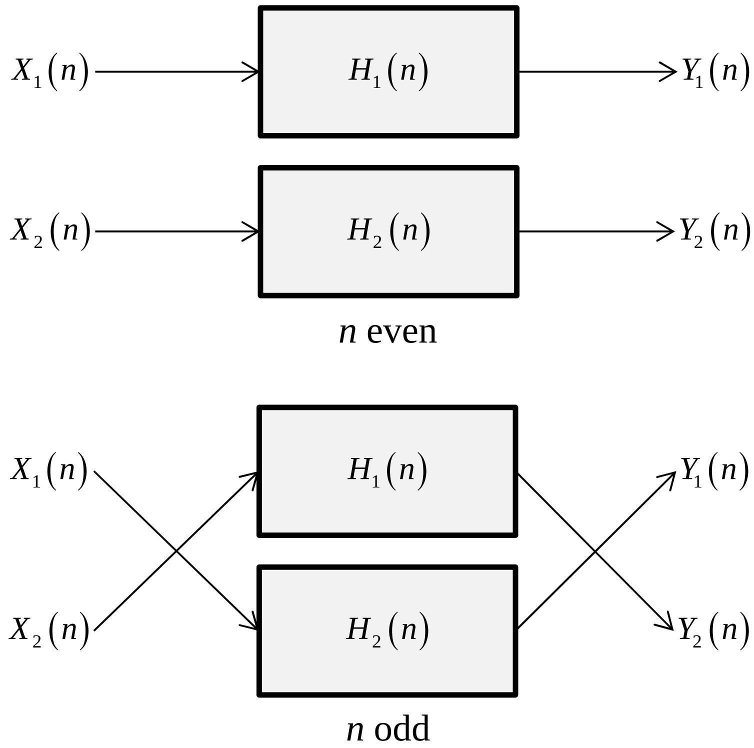

Figure 2. Two channels with commutation on alternate pulses.

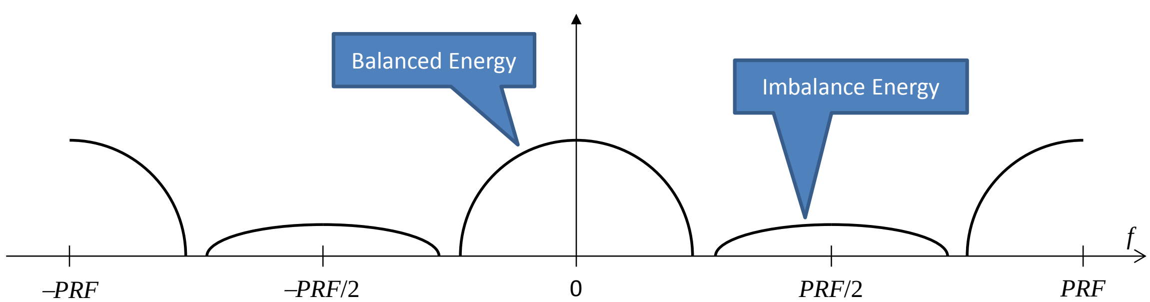

Figure 3. Doppler spectrum of commutated signal energy with adequate PRF.

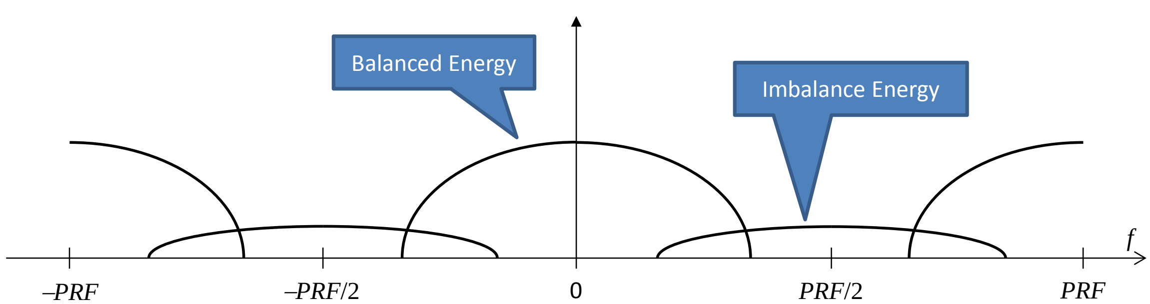

Figure 4. Doppler spectrum of commutated signal energy with inadequate PRF.

From this, we make the following observations

- Sometimes the un-doing of the commutation is called "decommutation."
- Comparing the commutated expression in Eq. (19) with the non-commutated expression in Eq. (14) shows that the big difference is that the imbalance energy is modulated by $e^{j\pi n}$. That is, the imbalance energy is in a neighborhood centered around a different Doppler frequency than the balanced energy. Commutation attempts to separate balanced energy from imbalance energy in Doppler.
- In a radar, if the commutation is done on a pulse-to-pulse basis, then the term $e^{j\pi n}$ represents half the Pulse Repetition Frequency (PRF), or the frequency *PRF*/2.
- The relative magnitude of the imbalance signal is $\sqrt{a_\varepsilon^{\,2}+\theta_\varepsilon^{\,2}}$.
- Commutation works to balance both amplitude and phase. It does not prefer one over the other.
- If the ideal balanced output spectrum of $Y_i(n)$ is band-limited to something less than *PRF*/2, then the imbalance energy can be completely separated from the balanced energy, in frequency. This is illustrated in Figure 3. This is good.
- If the ideal balanced output spectrum of $Y_i(n)$ is not band-limited to something less than *PRF*/2, then the imbalance energy cannot be completely separated from the balanced energy, and will overlap in frequency. This is illustrated in Figure 4. Commutation will not help much in this case. This is bad.
- The degree to which commutation aids balancing is the degree to which the imbalance energy can be suppressed, i.e. filtered. Filtering is generally easier the more the undesired spectral components are separated from the desired spectral components.

2.4 Some Radar Design Considerations

We now pose and then answer some questions with respect to using commutation in a radar design.

<u>What kinds of channels might be balanced?</u>

Multiple channels are employed in a radar for a variety of reasons. These might include

1. Multi-polarization radars,
2. Elevation IFSAR systems,
3. Endo-clutter GMTI systems,
4. Tracking radars,
5. Other DOA measurements,
6. Baseband quadrature demodulator architectures. (More on this later.)

<u>What constraints are there on radar operation?</u>

The radar PRF should be at least twice the Doppler bandwidth, but the more the better. The farther apart the balanced energy is from the imbalance energy in Doppler, the easier will be the filtering. In particular, if presumming is used, it is advantageous to employ commutation before the presummer since the filtering action of the presummer will attenuate the imbalance energy in the normal course of operation.

<u>Where should commutation/decommutation be applied?</u>

Commutation will only help balance those circuits that are in fact switched. Consequently, it is advantageous to apply commutation as near to the signal source as possible, that is very early in the receiver signal path. Since imbalance is typically a function of analog components in the signal path, it is advantageous to decommutate after the signals have been digitized.

<u>How much channel balance is needed?</u>

The degree of balance needed is a function of the application of the radar. We opine the following typical limits.

For simple SAR images, imbalance energy should be −35 dBc or better.

For DMTI range-Doppler maps, imbalance energy should be −55 dBc or better.

Of course 'better' is better.

"The trick to balance is to not make sacrificing important things become the norm."
-- *Simon Sinek*

3 Commutation of Sum and Difference Channels

Often, instead of transmitting separate inputs, the channels will transmit sum and difference signals. This is the case with monopulse radars. In such a case, the channel inputs are algebraic sums of other signals. Accordingly we define

$U_a(n)$ = input spectrum of first signal for pulse *n*,

$U_b(n)$ = input spectrum of second signal for pulse *n*,

$V_a(n)$ = output spectrum of first signal for pulse *n*, and

$V_b(n)$ = output spectrum of second signal for pulse *n*. (20)

Channel inputs now become

$$X_1(n) = U_a(n) + U_b(n) \text{ , and}$$
$$X_2(n) = U_a(n) - U_b(n), \tag{21}$$

Corresponding outputs can be reconstructed as

$$V_a(n) = \frac{Y_1(n) + Y_2(n)}{2} \text{ , and}$$
$$V_b(n) = \frac{Y_1(n) - Y_2(n)}{2}. \tag{22}$$

In such situations, channel 1 is often referred to as the "sum" channel, or Σ-channel, and channel 2 is often referred to as the "difference" channel, or Δ-channel.

We can expand the output using commutated channels as

$$V_a(n) = \frac{1}{2}\Big(H_{f_1(n)}(n)\big(U_a(n) + U_b(n)\big) + H_{f_2(n)}(n)\big(U_a(n) - U_b(n)\big)\Big) \text{ , and}$$
$$V_b(n) = \frac{1}{2}\Big(H_{f_1(n)}(n)\big(U_a(n) + U_b(n)\big) - H_{f_2(n)}(n)\big(U_a(n) - U_b(n)\big)\Big). \tag{23}$$

This may be simplified to

$$V_a(n) = \frac{1}{2}\Big(H_{f_1(n)}(n) + H_{f_2(n)}(n)\Big)U_a(n) + \frac{1}{2}\Big(H_{f_1(n)}(n) - H_{f_2(n)}(n)\Big)U_b(n) \text{ , and}$$
$$V_b(n) = \frac{1}{2}\Big(H_{f_1(n)}(n) - H_{f_2(n)}(n)\Big)U_a(n) + \frac{1}{2}\Big(H_{f_1(n)}(n) + H_{f_2(n)}(n)\Big)U_b(n). \tag{24}$$

Note that if channels are balanced, then signals are properly passed. However, if channels are not balanced, then each output is corrupted by the non-corresponding input.

To facilitate the subsequent discussion we define a virtual channel as

$$H_p(n) = \frac{1}{2}\left(H_{f_1(n)}(n) + H_{f_2(n)}(n)\right) \text{ , and}$$
$$H_m(n) = \frac{1}{2}\left(H_{f_1(n)}(n) - H_{f_2(n)}(n)\right). \tag{25}$$

We may then write

$$V_a(n) = H_p(n)U_a(n) + H_m(n)U_b(n) \text{ , and}$$
$$V_b(n) = H_m(n)U_a(n) + H_p(n)U_b(n). \tag{26}$$

In this formulation, $H_p(n)$ passes the desired component, and $H_m(n)$ is ideally zero. Now let us presume simple channel commutation, that is pulse-to-pulse channel switching. This is illustrated in Figure 5. In this case, recall that

$$H_{f_i(n)}(n) \approx \left[A_0 e^{j\Theta_0}\right] - e^{j\pi n} e^{j\pi i}\left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right]. \tag{27}$$

Accordingly, we expand and simplify to

$$H_p(n) = \left[A_0 e^{j\Theta_0}\right] - \frac{1}{2} e^{j\pi n}\left(e^{j\pi 1} + e^{j\pi 2}\right)\left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right] \text{ , and}$$
$$H_m(n) = \left[\quad 0 \quad\right] - \frac{1}{2} e^{j\pi n}\left(e^{j\pi 1} - e^{j\pi 2}\right)\left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right]. \tag{28}$$

Of course this simplifies to

$$H_p(n) = \left[A_0 e^{j\Theta_0}\right] \text{ , and}$$
$$H_m(n) = e^{j\pi n}\left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right]. \tag{29}$$

Clearly, the undesired signal is a function of the imbalance, but is modulated to *PRF*/2. We like this. Consequently commutation ***does*** work for sum and difference signals.

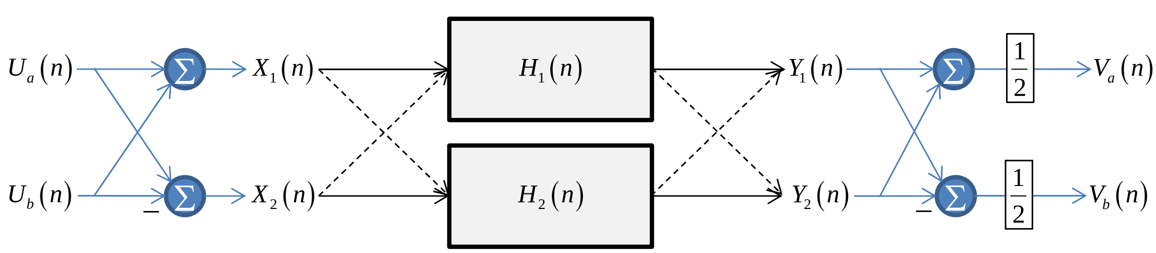

Figure 5. Commutation of Sum and Difference Signals.

4 Random Commutation

The commutation discussed in previous sections simply moves the imbalance energy, to a Doppler center frequency of *PRF*/2 in the case of two channels. This works just fine if the Doppler spectrum to which the imbalance energy is moved is not otherwise needed by the radar. For many systems this might indeed be true, e.g. SAR under the right circumstances. However, for other radar systems the analysis of the entire Doppler spectrum is desired. In such cases there is no convenient spectral region to which imbalance energy might be moved.

Whereas "Plan A" was to move the offending energy out of the way, we now consider a "Plan B" whereby the offending energy is smeared to hopefully reduce its effects on the radar data.

Since our choice on any one pulse is to either "switch" channels or to "not switch" channels, an obvious mechanism to smear the imbalance energy is to do the switching in some zero-mean random manner.

Our model continues to be written in a general sense as

$$Y_1(n) = H_{f_1(n)}(n) X_1(n) \text{ , and}$$
$$Y_2(n) = H_{f_2(n)}(n) X_2(n), \tag{30}$$

where

$f_1(n) \in \{1,2\}$, but now randomly selected with equal for each pulse *n*, and

$$f_2(n) = \begin{cases} 1 & if \quad f_1(n) = 2 \\ 2 & if \quad f_1(n) = 1 \end{cases}. \tag{31}$$

We may write this as

$$f_i(n) = \mathrm{mod}\big(i + f_1(n) - 2, 2\big) + 1. \tag{32}$$

This again allows us to write the channel transfer function as

$$H_{f_i(n)}(n) \approx \left[A_0 e^{j\Theta_0} \right] - e^{j\pi f_i(n)} \left[A_0 e^{j\Theta_0} \left(a_\varepsilon + j\theta_\varepsilon \right) \right]. \tag{33}$$

The previous equations can be combined, manipulated, and simplified to

$$H_{f_i(n)}(n) \approx \left[A_0 e^{j\Theta_0} \right] - e^{j\pi\left(\mathrm{mod}\left(i + f_1(n) - 2, 2\right) + 1\right)} \left[A_0 e^{j\Theta_0} \left(a_\varepsilon + j\theta_\varepsilon \right) \right]. \tag{34}$$

This can be further simplified to

$$H_{f_i(n)}(n) \approx \left[A_0 e^{j\Theta_0}\right] + e^{j\pi i} e^{j\pi f_1(n)} \left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right]. \tag{35}$$

This can otherwise be written as

$$H_{f_i(n)}(n) \approx \left[A_0 e^{j\Theta_0}\right] + \xi(n) e^{j\pi i} \left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right], \tag{36}$$

where we equate

$$\xi(n) = 2 f_1(n) - 3. \tag{37}$$

This makes $\xi(n)$ a random variable with equally likely values from the set $\{-1, +1\}$. Consequently, the imbalance energy is modulated with $\xi(n)$. This random modulation will smear the imbalance energy in Doppler. This is illustrated in Figure 6.

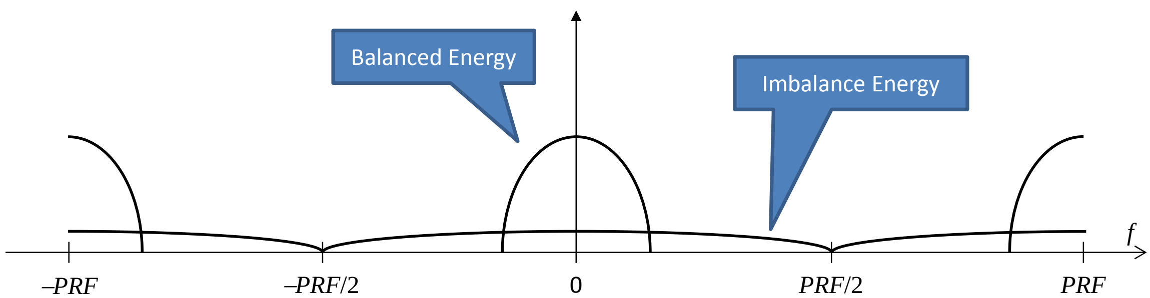

Figure 6. Doppler spectrum of randomly commutated signal energy.

Several points are worth observing include the following.

- As the imbalance energy is spread in Doppler, its maximum peak value can be expected to be reduced. This is an example of "Squish Theory."†
- The reduction in Doppler spectral density of the imbalance energy will be greater for larger ratios of PRF to balanced energy Doppler bandwidth.
- No amount of filtering will completely remove 'in-band' imbalance energy. However, it will be diminished.

† "If it squishes in here, it squishes out there."

5 Quadrature Demodulation

Heretofore we have considered channel switching, or modulation schemes that included alternating on every pulse, and randomly alternating for each pulse. Sometimes we can effectively employ other modulation/switching schemes. We now examine a particular application to exemplify this, namely Quadrature Demodulation.

Quadrature Demodulation is also sometimes called “I/Q Demodulation,” “Complex Demodulation,” and/or “Baseband Demodulation.”

Essentially, Quadrature Demodulation recognizes that real signals have both positive and negative frequency components that mirror each other in spectrum, and contain identical information. Consequently only one (either positive or negative) frequency component needs to be processed. This processing begins essentially by first shifting (demodulating) only one frequency component to baseband, i.e. centered at DC. This baseband signal is now complex, with the real component being called the “In-phase”, or “I” component, and the imaginary component being called the “Quadrature”, or “Q” component. This complex baseband signal creation is done by employing two channels to implement the Euler formula for a complex exponential multiplication. It is implemented in practice as in Figure 7.

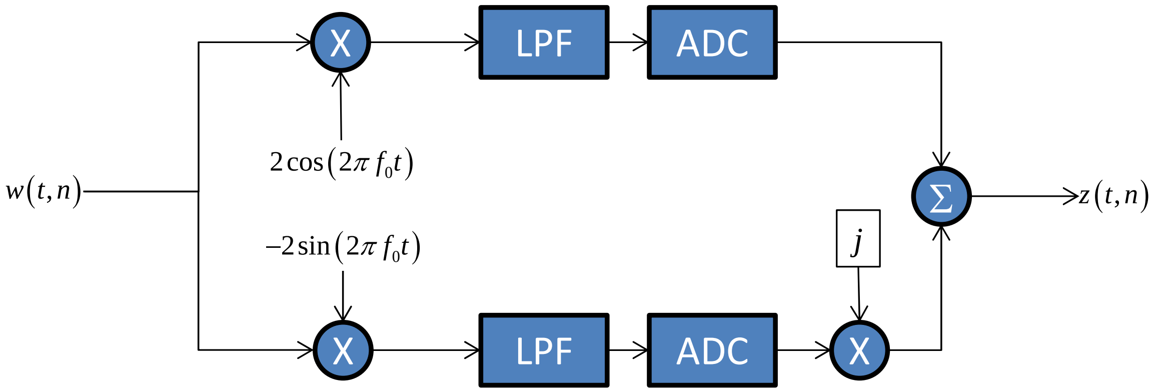

Figure 7. Conventional Quadrature Demodulator.

Here we identify the following.

$w(t,n)$ = input signal (real-valued) to Quadrature Demodulator,

$z(t,n)$ = output signal from Quadrature Demodulator, and

f_0 = carrier signal of input to Quadrature Demodulator. (38)

In general, inputs and outputs may be time (t) varying, and/or pulse (n) varying. For our model, each Lowpass Filter (LPF) is ideal and identical, and each Analog-to-Digital Converter (ADC) is ideal and identical. Consequently, if we define an input signal as

$$w(t,n) = b(t)\cos\left(2\pi f_0 t + \beta(t)\right), \tag{39}$$

then the output signal becomes

$$z(t,n) = b(t)\exp\left(j\beta(t)\right). \tag{40}$$

This effectively takes the positive frequency component centered at $+f_0$ and translates it to baseband, centered at DC. This is what we want.

However, to achieve perfect results requires identical signal paths in the two branches, with the exception of a precise $\pi/2$ phase difference in the modulations at the first mixers (multipliers). This balance is problematic.

Consequently, we rewrite Figure 7 with overt acknowledgement of channel differences by incorporating different channel characteristics in Figure 8, where we also have identified parameters in terms of frequency-domain definitions. Furthermore, we have removed distinction between analog and digital data from the signal paths, and thereby removed the ADCs.

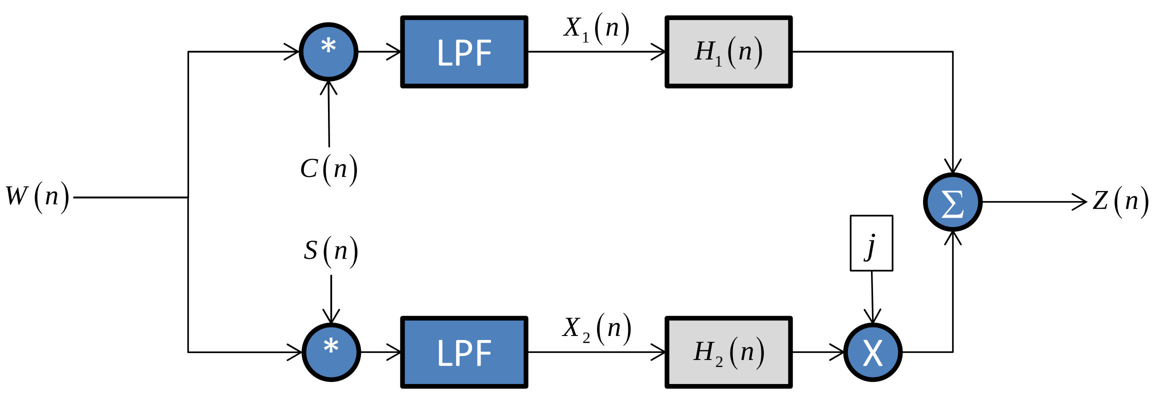

Figure 8. Quadrature Demodulator in frequency-domain with independent channel characteristics.

Specifically, we identify

$W(n) = \mathbb{F}\{w(t,n)\}$ = input signal spectrum,

$Z(n) = \mathbb{F}\{z(t,n)\}$ = output signal spectrum,

$\delta(f)$ = Dirac delta function, and

f = frequency variable, (41)

where

$\mathbb{F}\{g(t)\}$ = Fourier Transform of $g(t)$ with respect to time variable *t*. (42)

Once again, for later convenience we are omitting the overt frequency dependence of the function descriptions. In addition, note that the first two time-domain multiplications have been replaced with convolutions with functions

$$C(n) = \mathbb{F}\left\{2\cos\left(2\pi f_0 t\right)\right\} = \delta\left(f - f_0\right) + \delta\left(f + f_0\right), \text{ and}$$
$$S(n) = \mathbb{F}\left\{-2\sin\left(2\pi f_0 t\right)\right\} = j\delta\left(f - f_0\right) - j\delta\left(f + f_0\right). \quad (43)$$

By expanding the input signal into constituents, and following the processing, we may identify

$$X_1(n) = \mathbb{F}\left\{b(t)\cos\left(\beta(t)\right)\right\}, \text{ and}$$
$$X_2(n) = \mathbb{F}\left\{b(t)\sin\left(\beta(t)\right)\right\}. \quad (44)$$

Recall that for small errors, we may approximate the filter functions as

$$H_i(n) \approx \left[A_0 e^{j\Theta_0}\right] - e^{j\pi i}\left[A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right]. \quad (45)$$

Combining some things lets us write

$$Z(n) \approx \left(X_1(n) + jX_2(n)\right)A_0 e^{j\Theta_0} + \left(X_1(n) - jX_2(n)\right)A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right). \quad (46)$$

This may be simplified to

$$Z(n) \approx \left[\mathbb{F}\left\{b(t)e^{j\beta(t)}\right\}A_0 e^{j\Theta_0}\right] + \left[\mathbb{F}\left\{b(t)e^{-j\beta(t)}\right\}A_0 e^{j\Theta_0}\left(a_\varepsilon + j\theta_\varepsilon\right)\right]. \quad (47)$$

The first bracketed term is the desired response, and the second bracketed term is the undesired response due to channel imbalance. From this we observe that the effect of a channel imbalance is to perturb the output with the addition of the complex conjugate of the desired output signal. In a range-Doppler map this manifests as a ghost signal with symmetry through the DC point.

While commutation as previously described would indeed help balance the channels to provide the desired output, we offer a similar but slightly different solution. We propose to add a pulse-to-pulse phase shift to the channel input, and to remove it in the output data, as illustrated in Figure 9.

In this formulation, we allow phase-shifted quadrature beat signals given by

$$C(n) = \mathbb{F}\left\{2\cos\left(2\pi f_0 t - \phi(n)\right)\right\}, \text{ and}$$
$$S(n) = \mathbb{F}\left\{-2\sin\left(2\pi f_0 t - \phi(n)\right)\right\}. \quad (48)$$

This will generate signals with the same phase shift, namely

$$X_1(n) = \mathbb{F}\left\{b(t)\cos\left(\beta(t) + \phi(n)\right)\right\}, \text{ and}$$
$$X_2(n) = \mathbb{F}\left\{b(t)\sin\left(\beta(t) + \phi(n)\right)\right\}. \quad (49)$$

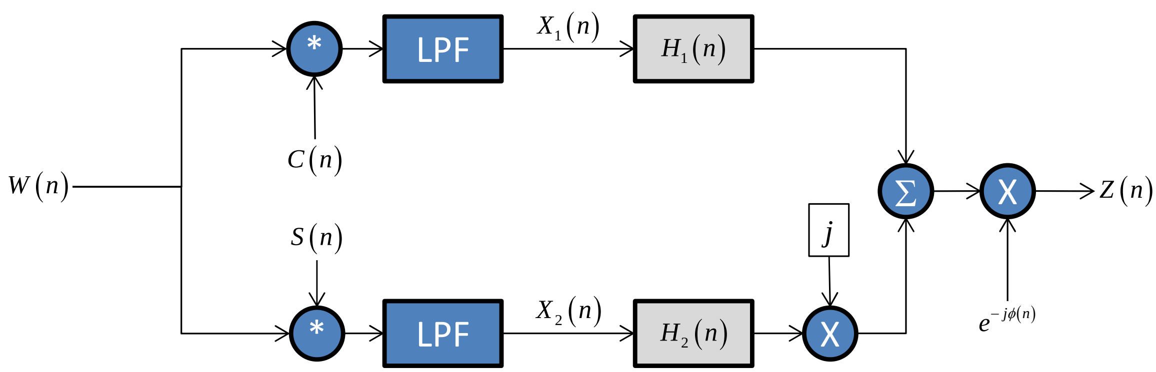

Figure 9. Quadrature Demodulator with support for channel balancing with phase shifts.

With some algebra, the final output is then calculated to be

$$Z(n) \approx \left[\mathbb{F}\left\{ b(t) e^{j\beta(t)} \right\} A_0 e^{j\Theta_0} \right] + \left[\mathbb{F}\left\{ b(t) e^{-j\beta(t)} \right\} A_0 e^{j\Theta_0} \left(a_\varepsilon + j\theta_\varepsilon \right) e^{-j2\phi(n)} \right]. \tag{50}$$

We observe that the imbalance term is shifted in Doppler with respect to the desired energy. Consequently, by selecting a rolling phase shift defined as

$$\phi(n) = \frac{n\pi}{4}, \tag{51}$$

we may then shift the undesired imbalance energy to be centered at a Doppler frequency of *PRF*/2.[3] We manifest all the attendant advantages of channel switching. We have effectively moved the ghost signal to be symmetric through a different reference point, namely –PRF/4.

One might wonder "Why would we want to implement this rather than a more conventional channel switching architecture?" We answer this with the following points.

- If incorporated into the waveforms with a phase shift, no overt switching hardware is required with its attendant circuitry.
- In modern radars, signals $C(n)$ and $S(n)$ are often implemented with Digital Waveform Synthesizers, making precise phase shifts trivial to generate.
- Alternatively, instead of incorporating the phase shift into signals $C(n)$ and $S(n)$, the pulse-to-pulse phase shift may be incorporated directly into the generated signal $W(n)$. This might even be the transmitted radar signal.

6 Commutation Between More Than Two Channels

We now consider the case of switching between more than two channels on a pulse-to-pulse basis, as illustrated in Figure 10. Accordingly, we define channel characteristics as follows.

$$H_i(n) = A_i e^{j\Theta_i} = A_0\left(1 + a_{\varepsilon,\mathrm{i}}\right)e^{j\left(\Theta_0 + \theta_{\varepsilon,i}\right)}. \tag{52}$$

With a little algebra and trigonometry we may expand the channel transfer functions to

$$H_i(n) = A_0 e^{j\Theta_0}\left(\cos\theta_{\varepsilon,\mathrm{i}} + j\sin\theta_{\varepsilon,i} + a_{\varepsilon,i}\cos\theta_{\varepsilon,i} + ja_{\varepsilon,i}\sin\theta_{\varepsilon,i}\right), \tag{53}$$

which for small errors can be approximated with

$$H_i(n) \approx \left[A_0 e^{j\Theta_0}\right] + \left[A_0 e^{j\Theta_0}\left(a_{\varepsilon,i} + j\theta_{\varepsilon,i}\right)\right]. \tag{54}$$

We stipulate that for N channels,

$$\frac{1}{N}\sum_{i}^{N} A_i = A_0 \text{, and}$$
$$\frac{1}{N}\sum_{i}^{N} \Theta_i = \Theta_0 . \tag{55}$$

The implication is that the mean error is zero, both for amplitude and phase.

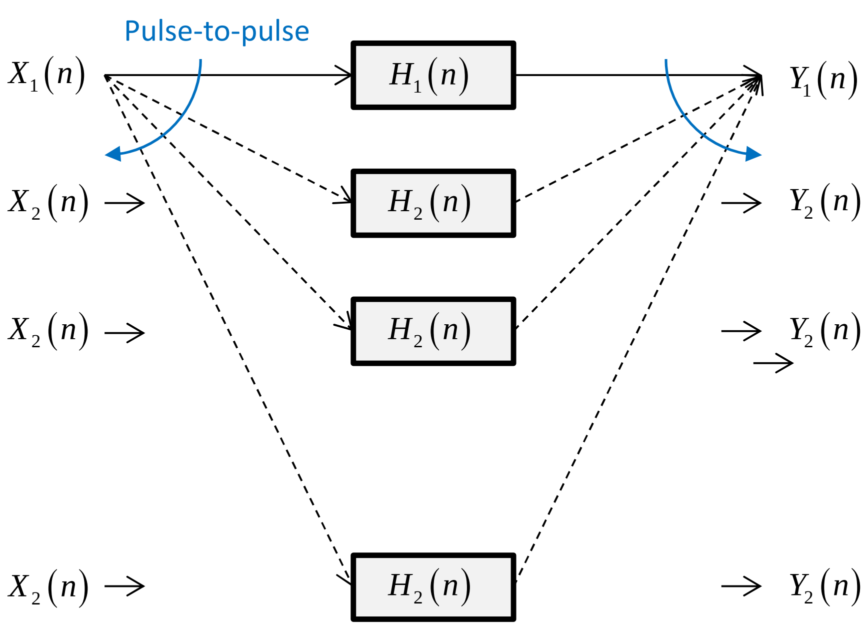

Figure 10. Signals may be commutated between *N* channels.

Now consider that from pulse to pulse we cycle through all filters with indices *i*, from 1 to *N*, and then continuously repeating the same order of the sequence.

In this case, an input X_1 will yield an output Y_1 that contains a steady term due to the first bracket of Eq. (54), and a pulse-to-pulse changing (albeit periodic) error term due to the second bracket of Eq. (54). The period of the error sequence is in fact *N* pulses. The frequency of repetition for the error sequence is *PRF*/*N*. Consequently, imbalance will modulate energy to centers that are multiples of *PRF*/*N*, but not to centers of multiples of *PRF* itself due to the zero mean of the error sequence. Furthermore, this is all true for all input/output pairs X_i and Y_i. Nevertheless, all channels will effectively be balanced with each other.

Several points are worth observing.

- For a repetition period of *N* samples, with a corresponding repetition frequency of *PRF*/*N*, the signal's Doppler bandwidth needs to be limited to something less than *PRF*/*N* in order to separate the balanced energy entirely from the imbalance energy. In other words, the PRF needs to be at least *N* times the Doppler bandwidth of the signal to allow filtering of the imbalance energy.

- Not all sub-multiples of the PRF will exhibit the same imbalance energy.

- Other commutation sequences may be chosen such that the repetition period is something other than the number of available channels. For an arbitrary repetition period of *M* samples, with a corresponding repetition frequency of *PRF*/*M*, the signal's Doppler bandwidth needs to be limited to something less than *PRF*/*M* in order to separate the balanced energy entirely from the imbalance energy.

- For a random sequence that has effectively infinite period, the imbalance energy will be smeared across all Doppler frequencies.

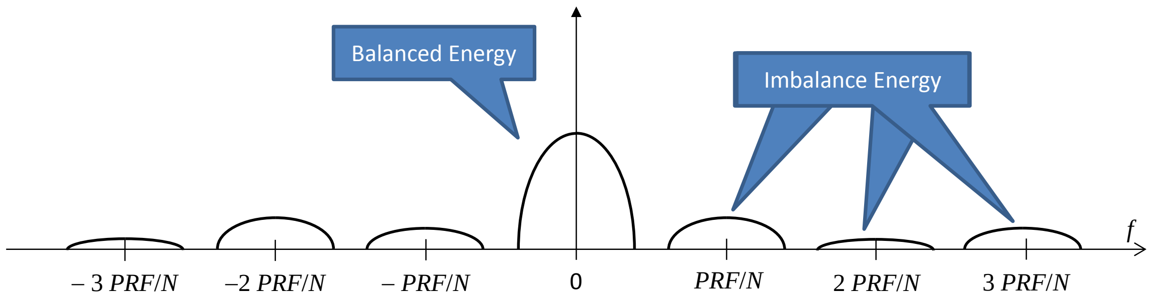

Figure 11. Imbalance energy will be centered at multiples of the sequence repetition frequency.

7 Conclusions

The following points are worth repeating.

- Commutation is the trading, switching, toggling, or multiplexing channels.
- Commutation allows all channels to see an average of channel characteristics, with any imbalance modulated.
- Imbalance energy may be modulated to different Doppler spectral regions than balanced energy, and then mitigated by filtering.
- Commutation will work on two or more channels.
- Commutation will work on sum and difference channels.
- In Quadrature Demodulators, commutation may be applied with a rolling phase shift in the input signal, or the quadrature local oscillator signals.

“It's hard to balance everything. It's always challenging.”
-- Vera Wang

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