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STATISTICAL ANALYSES OF PLUME COMPOSITION AND DEPOSITED RADIONUCLIDE MIXTURE RATIOS

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Statistical Analyses of Plume Composition and Deposited Radionuclide Mixture Ratios

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ABSTRACT

A proposed method is considered to classify the regions in the close neighborhood of selected measurements according to the ratio of two radionuclides measured from either a radioactive plume or a deposited radionuclide mixture. The subsequent associated locations are then considered in the area of interest with a representative ratio class. This method allows for a more comprehensive and meaningful understanding of the data sampled following a radiological incident.

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TABLE OF CONTENTS

LIST OF FIGURES.....	6
LIST OF TABLES.....	7
LIST OF ACRONYMS.....	8
1. INTRODUCTION.....	10
1.1 Background.....	10
2. PROBLEM DESCRIPTION.....	14
2.1 Presentation of a Simple Problem.....	14
2.2 Description of Enhanced Complexity	14
3. REPRESENTATION OF THE AVAILABLE DATA	15
3.1 Matrix Representation	15
3.2 Graphical Representation	15
4. ANALYSIS OF DATA TO DEFINE RATIO GROUPS	17
4.1 Simple Estimate of Influence and Grouping	17
4.2 Selection of Correct Metric Comparison	19
4.3 Treatment of Multiple Ratios	19
4.4 Propagation of Uncertainty.....	20
5. ORGANIZING MULTIPLE OBSERVATIONS INTO RATIO GROUPS	21
5.1 Simple Quantitative Comparison.....	21
5.2 Grouping Method: The Neighborhood Method.....	22
5.3 Selection of a Representative Ratio Value.....	24
5.4 Analysis Incorporating Multiple Radionuclide Ratios.....	25
6. USE OF RESULTS ON SPATIAL MAP	26
6.1 Nearest Neighbor Approach.....	26
6.2 Determination of Spatial Boundaries.....	27
7. EXAMPLE USING NARAC RESULTS.....	28
7.1 Extraction of Data.....	28
7.2 Matrix Creation and Results	32
7.3 Grouping of Data	34
7.4 Creation of Ratio Map	35
8. CONCLUSIONS.....	36
9. REFERENCES.....	37
APPENDIX A.....	38
APPENDIX B.....	40

LIST OF FIGURES

Figure 1:	Dose Rate DRL Contours for Hypothetical Fukushima Daiichi Data	13
Figure 2:	Graphical Representation of the Ratio Distributions amongst the Observations	15
Figure 3:	Distribution Function Representation of the Ratios Observed	16
Figure 4:	Overlapping Groups for the Application of Proposed Method	22
Figure 5:	Graphical Representation of Step 1 of the Neighborhood Method	23
Figure 6:	Graphical Representation of Step 2 of the Neighborhood Method	24
Figure 7:	Graphical Representation of Step 3 of the Neighborhood Method	24
Figure 8:	Spatial Map Showing Groups of Activity Ratios across the Sampling Area.....	26
Figure 9:	Accuracy of the Nearest Neighbor Technique	27
Figure 10:	Cs-137 Simulated Plume	29
Figure 11:	I-131 Simulated Plume with Ten-Point Sample Plan	30
Figure 12:	I-131 Ten-Point Sample Plan.....	31
Figure 13:	Cs-137 Simulated Plume with Ten-Point Sample Plan.....	31
Figure 14:	Cs-137 Ten-Point Sample Plan	32
Figure 15:	Spatial Locations of Sampling Points in NARAC Model	33
Figure 16:	Spatial Map of Activity Ratio Groups	35
Figure 17:	Cs-137 Ground Deposition at 2 mph	42
Figure 18:	Cs-137 Ground Deposition at 10 mph	42
Figure 19:	I-131 Ground Deposition at 1 mph.....	43
Figure 20:	I-131 Ground Deposition at 2 mph.....	43
Figure 21:	Sampling Scheme for HotSpot Model.....	44
Figure 22:	Spatial Representation of Grouping Created for HotSpot Data	46

LIST OF TABLES

Table 1:	Activity Ratio Differences from Fukushima Daiichi Environmental Releases.....	11
Table 2:	Dose Rate 2-Pathway DRL Comparison	12
Table 3:	Dose Rate DRL Contour Effects	13
Table 3:	Matrix Representation of Ratio Differences	15
Table 4:	Example of Ratios Calculated across Sampling Area.....	17
Table 5:	Example of Representative Matrix Populated with Simple Difference between Ratios	18
Table 6:	Example of <i>log</i> of One plus the Absolute Value of Simple Difference	18
Table 7:	Example of the Difference Divided by the Mean	18
Table 8:	Example Matrix of Comparative Determination	22
Table 9:	Cs-137 and I-131 Location and Material Released.....	29
Table 10:	I-131 Deposition Values	29
Table 11:	Cs-137 Deposition Values	30
Table 12:	Cs-137 and I-131 Concentration and Calculated Activity Ratio Values	33
Table 13:	Matrix of Simple Difference between Ratios	34
Table 14:	Matrix of Comparative Determination following Matrix Representation of Difference	34
Table 15:	Chosen Groups Based on NARAC Activity Ratio Comparison	34
Table 16:	Parameters for HotSpot Analysis of Cs-137 and I-131	40
Table 17:	Highest and Lowest Ground Deposition for Cs-137.....	41
Table 18:	Highest and Lowest Ground Deposition for I-131	41
Table 19:	Data Collected from HotSpot Model and Calculated Activity Ratios	45
Table 20:	Simple Difference Calculated among All Pairs of Points Modeled in HotSpot.....	45
Table 21:	Comparison of Ratios across Sampling Area using a Fixed Strength of Relation	45

LIST OF ACRONYMS

AMS	Aerial Measuring System
CDF	Cumulative Distribution Function
CM	Consequence Management
DRL	Derived Response Level
FRMAC	Federal Radiological Monitoring and Assessment Center
MARSSIM	Multi-Agency Radiation Survey and Site Investigation Manual
NARAC	National Atmospheric Release Advisory Center

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1. INTRODUCTION

The management and analysis of field sample data following a radiological incident is an important and time sensitive problem. In particular, the characterization of radionuclide activity ratios across a sampling area can be very complex, especially when the large number of possible radionuclide activity ratio combinations and the variability of sampling capabilities and results are considered. Up to this point, it was assumed that activity ratios of radionuclides determined based on an initial sampling could be considered to be the same across the sampling area. This assumption in turn allowed analysts to apply the same Derived Response Levels (DRLs) across the sampling region to identify where protective actions (e.g., sheltering, evacuation, relocation) may be warranted. While the application of the same radionuclide activity ratio across a sampling area simplifies the problem, this simplification may mask important patterns in the distribution of radionuclides that a more in-depth analysis of the sampling data might reveal.

This report proposes a method to classify the regions in the close neighborhood of selected measurements according to the activity ratio of two radionuclides measures (Cs-137 and I-131), and then associate each location in the area of interest with a representative ratio class. This method will allow for a more comprehensive and meaningful understanding of the data sampled following a nuclear incident. More precisely, this report will focus on answering the following questions:

1. How many different reference ratios should be considered for a given sampling area?
2. Which ratio should be used for any specific location?
3. What recommendations can be given regarding the location of future measurements in order to reduce the overall uncertainty over the sampling region?

Section 2 gives a more complete representation of the problem considered. Section 3 details the representation of data in a matrix or graphical form. Section 4 proposes a matrix analysis of the data as a step toward organizing all observations into ratio groups to analyze the data. This method is developed further in Section 5 for situations with large numbers of observations. Section 6 concludes the discussion of the proposed methodology by creating a spatial mapping of the determined ratio groups. An example of the entire approach for a simple model is presented in Section 7.

1.1 Background

During the Fukushima Daiichi nuclear power station meltdowns and subsequent environmental radiological releases, the Department of Energy's National Nuclear Security Agency activated Consequence Management (CM) assets. The CM assets provided technical analysis, advice to the U.S. and Japanese Government officials to support immediate decision-making and longer-term stabilization planning, and conducted technical analyses in support of potential mitigation and recovery strategies. As part of this response, CM assets established DRLs based on the radionuclide activity ratios observed at a single location, the Aerial Measuring System (AMS) Test Line. As the accident response progressed, it became apparent to CM Assessment Scientists that using a single ratio of radionuclides for the entire deposition footprint may not be appropriate. Table 1 provides the Cs-137 to I-131 ratio at specified distances and directions from the Fukushima Daiichi site. The samples used to create Table 1 were collected between

March 27, 2011 and April 24, 2011 by CM assets, and are decay adjusted to noon March 14, 2011.

Table 1: Activity Ratio Differences from Fukushima Daiichi Environmental Releases

Distance (miles)	Direction	Cs-137 to I-131 Activity Ratio
12.2	S	0.018
18.9	NW	0.059
21.3	NW	0.080
24.2	NW	0.062
35.1	SSW	0.020
35.2	NNW	0.058
36.2	WNW	0.161
38.9	SSW	0.017
41.3	NW	0.237
41.8	WNW	0.086
45.3	WSW	0.049
48.5	NNW	0.044
48.8	NNW	0.044
49.7	N	0.053

For the CM DRL analyses, a Cs-137 to I-131 activity ratio of 0.378 was chosen based on the AMS Test Line radionuclide mixture on March 14, 2011. The calculated DRLs depend on the radionuclide activity ratios. The large range of radionuclide activity ratios in Table 1 result in an order of magnitude (i.e., the CM ratio to the lowest ratio from Table 1 is $0.378/0.017 \approx 22$) difference. Because different DRL values can identify significantly different areas where protective actions may be warranted, research is needed to determine at what level does the difference in the radionuclide activity ratios become statistically significant such that a new radionuclide activity ratio should be considered for the DRL calculation and public protection decisions.

To better understand the dependence of the calculated DRL on the radionuclide activity ratios, consider the Cs-137 to I-131 activity ratios in Table 1. The default assessment methods specified in the Federal Radiological Monitoring and Assessment Center (FRMAC) Assessment Manual and the TurboFRMAC 2013 software package were used to calculate the Early Phase (96 hours) and 1st year (365 days) DRLs for the Cs-137 to I-131 activity ratios in Table 1. Table 2 presents these results. Although the AMS Test Line DRL for the Early Phase (listed as 'AMS' in Table 2) is essentially the same when compared with all other activity ratios (~10% difference), marked differences (~25-85%) are noted when the 1st year AMS Test Line DRL is compared to the 1st year DRLs calculated from the other radionuclide activity ratios.

Table 2: Dose Rate 2-Pathway DRL Comparison

Direction	Distance (miles)	Early Phase (mrem/hr)	First Year (mrem/hr)	Early Phase % Difference	First Year % Difference
N	49.7	11.4	2.3	9%	72%
NNW	35.2	11.4	2.2	9%	71%
NNW	48.5	11.5	2.6	9%	75%
NNW	48.8	11.5	2.6	9%	75%
NW	18.9	11.4	2.1	9%	70%
NW	21.3	11.3	1.7	8%	64%
NW	24.2	11.4	2.1	8%	69%
NW	41.3	10.7	0.84	3%	25%
S	12.2	11.6	4.0	10%	84%
SSW	35.1	11.6	3.8	10%	84%
SSW	38.9	11.6	4.1	10%	85%
WNW	36.2	11.0	1.1	5%	42%
WNW	41.8	11.3	1.7	8%	62%
WSW	45.3	11.4	2.4	9%	74%
WSW (AMS)	45.0	10.4	0.63	--	--

If relocation decisions for the entire contaminated area were based solely on the 1st year dose rate DRL for the AMS Test Line ratio, inappropriate public protection actions may have been implemented over hundreds or thousands of square miles of area. To better illustrate this, Figure 1 and Table 3 provide the 1st year dose rate DRL contours and effects from Table 2 for the AMS Test line ratio (0.63 mrem/hr), the 18.9 mile northwest result (2.1 mrem/hr), and the 38.9 mile south-southwest results (4.1 mrem/hr) with hypothetical Fukushima Daiichi contamination data (assuming a two day release time period). As seen in Figure 1 and Table 3, the AMS Test Line ratio would have a significant impact for regional relocation decisions (~2200 square miles and ~1,000,000 people effected between the 0.63 - 2.1 mrem/hr contours).

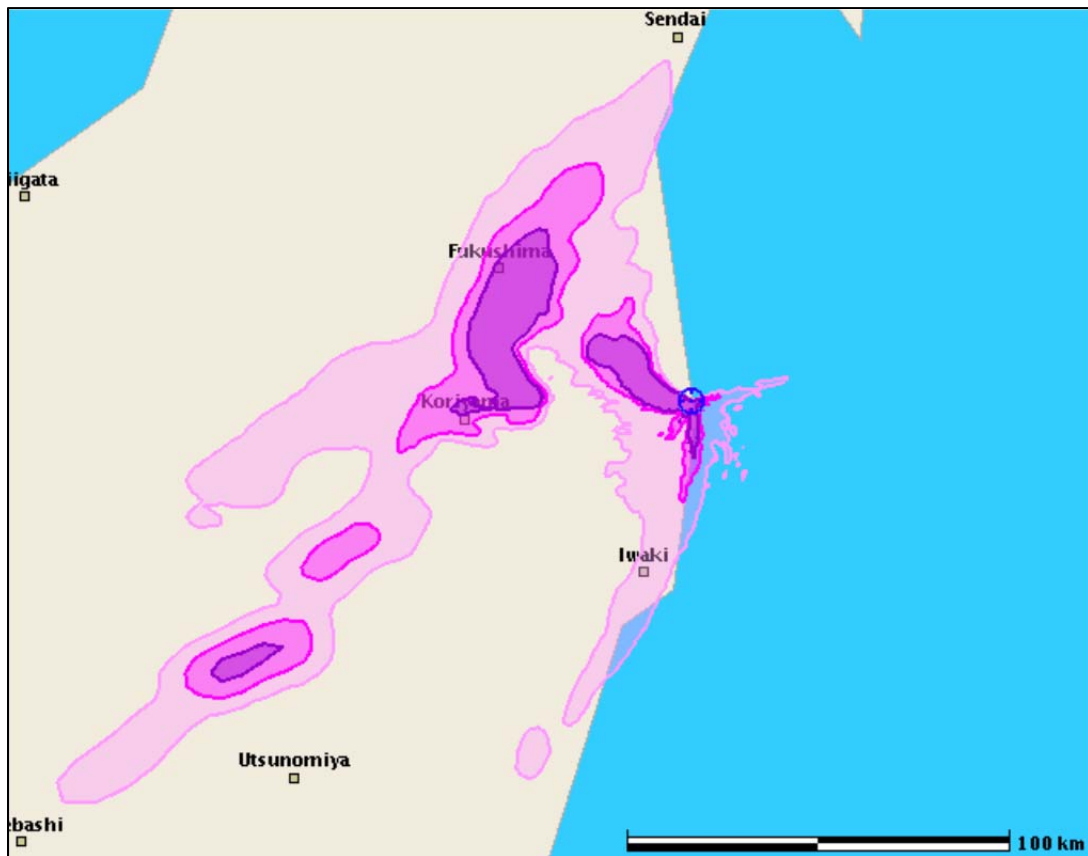


Figure 1: Dose Rate DRL Contours for Hypothetical Fukushima Daiichi Data

Table 3: Dose Rate DRL Contour Effects

Contour	Dose Level (mrem/hr)	Extent (miles)	Area (mile ²)	Effected Population
	>4.1	89	373	346,000
	>2.1	92	940	840,000
	>0.6	120	3135	1,880,000

2. PROBLEM DESCRIPTION

It is recognized that the radionuclide mixture deposited on the ground (footprint) may have different radionuclide activity ratios at different directions and distances from the release point. Inappropriate protective action decisions may be recommended if those decisions are based upon an incorrect DRL. A defensible, statistically-based method is needed to determine when radionuclide activity ratios are sufficiently different to warrant a different DRL. This section provides a discussion on this problem which is analyzed within rest of this report.

2.1 Presentation of a Simple Problem

The simplest problem that can be considered is the measurement of two radionuclides concentrations (e.g., Cs-137 and I-131) with no uncertainty present in their measurements. Under the prescribed initial sampling scheme that is usually applied following a radiological release, described in the FRMAC Monitoring Manual Volume 1, Section 3.3.1 [1], the following discussion considers the application for the proposed method to a case in which ten initial values of the radionuclide activity ratio are known and is further discussed in Section 4.

2.2 Description of Enhanced Complexity

There are several additional elements present in the real-world consideration of this problem that may add complexity to the proposed method of analysis:

- As was discussed previously, the concurrent comparison of many different radionuclide activity ratios in order to create a spatial representation of similarity across the sampling area strongly increases the complexity of the problem under consideration.
- Additionally, the values for parameters measured in the field may be presented with a certain amount of uncertainty that must also be considered and will further complicate the determination of similarity.
- Finally, the addition of hundreds or thousands of more data points and the number of groups that these points might be split into would necessarily complicate the consideration of the problem.

3. REPRESENTATION OF THE AVAILABLE DATA

To begin the analysis, the radionuclide activity ratio values at each sample point must be compared to one another. There are several ways that the available data might be represented as the first step in analysis. The choice for the representation of available data depends upon the ultimate method that will be used to group the data.

3.1 Matrix Representation

In order to mathematically compare the ratio values across the sampling area, an “ $n \times n$ ” matrix, in which ‘ n ’ represents the number of observations, is constructed (see Table 4). Each cell (i, j) is populated with the comparative difference between observations ‘ i ’ and ‘ j ’. This comparative difference value could be either the absolute difference between the ratio values under a fixed value approach or could be the difference normalized by the mean ratio under a statistical approach. Additional applications of the proposed method could also include the calculation of comparative difference values among sampling points regarding measured uncertainty or confidence intervals that might be presented in the original data. The diagonal value represents the relation of the observation ‘ i ’ to itself and is therefore equal to zero.

Table 4: Matrix Representation of Ratio Differences

	Point 0	Point 1	Point 2	..	Point i	Point i+1	..	Point n
Point 0	0	0.433833	0.42414	..	-0.61294	-0.95475	..	-0.407
Point 1	-0.43383	0	-0.00969	..	-1.04678	-1.38858	..	-0.84083
Point 2	-0.42414	0.009693	0	..	-1.03708	-1.37889	..	-0.83114
..	..	..	..	..	..	..	..	..
Point i	0.612944	1.046777	1.037084	..	0	-0.3418	..	0.205949
Point i+1	0.954747	1.388579	1.378886	..	0.341803	0	..	0.547751
..	..	..	..	..	..	..	..	..
Point 10	0.406996	0.840828	0.831135	..	-0.20595	-0.54775	..	0

3.2 Graphical Representation

A graphical representation of the data points without any spatial information included might also help to inform the selection of groups of similar ratios. Ratio values indexed by their data point number could be shown on one line such that clustering patterns and points that are not close to any others might be identified (see Figure 2). A cumulative distribution function (CDF) can then be constructed in order to present a more classical representation of the ratios variation (see Figure 3).



Figure 2: Graphical Representation of the Ratio Distributions amongst the Observations

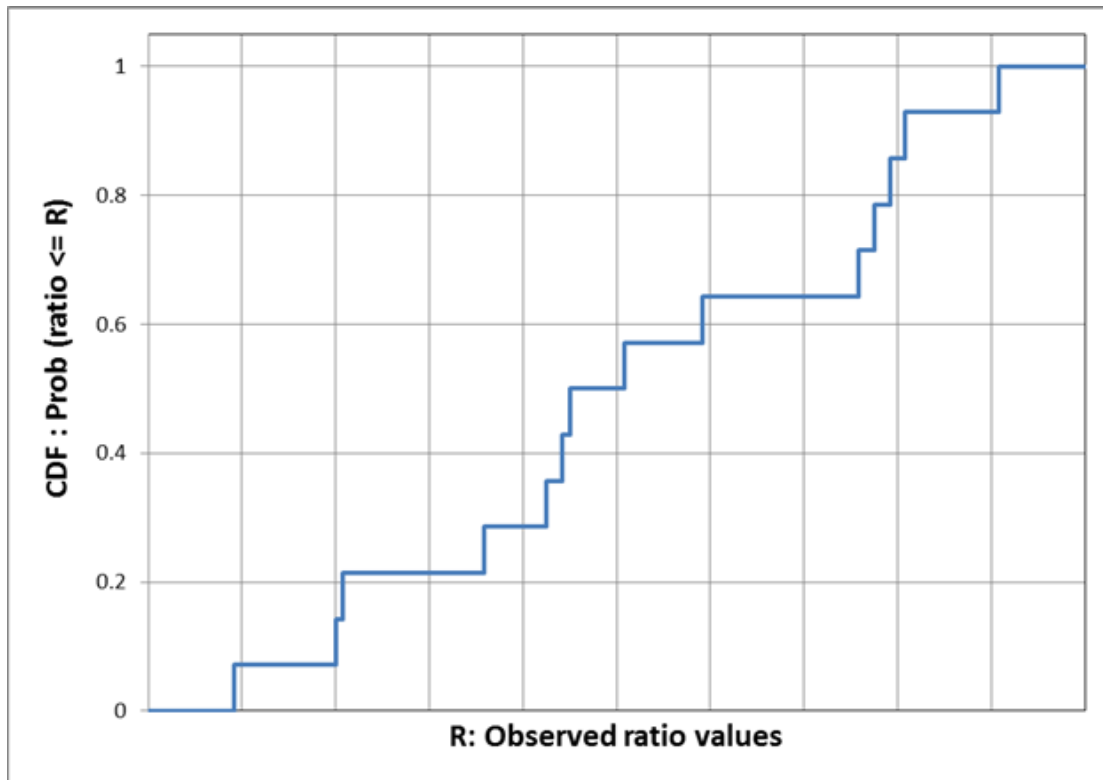


Figure 3: Distribution Function Representation of the Ratios Observed

4. ANALYSIS OF DATA TO DEFINE RATIO GROUPS

This section provides discussion on which selection methodology was considered for this work, how the Cs-137 to I-137 ratio is treated, and the propagation of uncertainty.

4.1 Simple Estimate of Influence and Grouping

There are several methods that might be used to calculate the comparative difference between ratio values among all possible combinations of sampling locations.

1. The simplest way to calculate a value that could be used to make such a comparison is to find the absolute difference between the ratios at any of the two selected locations. These calculated differences populate the representative matrix discussed in Section 3.1 and can be used to determine groups of similar ratios using a fixed reference value, as discussed in Section 5.1. An example of a matrix populated using this approach is shown in Table 6. The original ratios sampled in this example are given in Table 5. This approach would be an appropriate choice if the ratios values are close to one another. This calculation of a comparative difference value could be extended by calculating the absolute value of the simple difference between ratios.
2. In situations in which the ratios across the sampling area may span orders of magnitude, it may be more appropriate to calculate the *log* of the absolute value of the difference between the ratios. It is recommend the calculation of the *log* of 1 plus the absolute difference in order to avoid an infinite result for cells on the diagonal and keep all numbers positive. An example of a matrix populated using this approach is shown in Table 7.
3. A more statistically based approach to calculating a value for the comparative difference between the ratios at any two sampling locations involves the calculation of the mean ratio and the standard deviation. Under this approach, the difference between the ratios at any two sampling points is divided by the mean ratio. These calculated values are used to populate the representative matrix which, in turn, is used to determine ratio groupings based on a statistically calculated reference value. This is further discussed in Section 5.1. An example of a matrix populated using this approach is shown in Table 8. This approach would be an appropriate choice if a statistically based method is preferred.

Table 5: Example of Ratios Calculated across Sampling Area

	Cs-137 to I-131 Activity Ratio
Point 0	1.00E+00
Point 1	5.66E-01
Point 2	5.76E-01
Point 3	1.49E+00
Point 4	1.61E+00
Point 5	1.95E+00
Point 6	1.72E+00
Point 7	1.34E+00
Point 8	9.38E-01
Point 9	5.98E-01
Point 10	1.41E+00

Table 6: Example of Representative Matrix Populated with Simple Difference between Ratios

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	0	0.433833	0.42414	-0.48771	-0.61294	-0.95475	-0.71669	-0.33695	0.061836	0.40208	-0.407
Point 1	-0.43383	0	-0.00969	-0.92154	-1.04678	-1.38858	-1.15052	-0.77079	-0.372	-0.03175	-0.84083
Point 2	-0.42414	0.009693	0	-0.91185	-1.03708	-1.37889	-1.14083	-0.76109	-0.3623	-0.02206	-0.83114
Point 3	0.487707	0.92154	0.911847	0	-0.12524	-0.46704	-0.22898	0.150754	0.549544	0.889787	0.080712
Point 4	0.612944	1.046777	1.037084	0.125237	0	-0.3418	-0.10375	0.275991	0.674781	1.015024	0.205949
Point 5	0.954747	1.388579	1.378886	0.467039	0.341803	0	0.238057	0.617793	1.016583	1.356826	0.547751
Point 6	0.71669	1.150523	1.14083	0.228983	0.103746	-0.23806	0	0.379736	0.778526	1.11877	0.309694
Point 7	0.336954	0.770786	0.761093	-0.15075	-0.27599	-0.61779	-0.37974	0	0.39879	0.739033	-0.07004
Point 8	-0.06184	0.371996	0.362303	-0.54954	-0.67478	-1.01658	-0.77853	-0.39879	0	0.340243	-0.46883
Point 9	-0.40208	0.031753	0.02206	-0.88979	-1.01502	-1.35683	-1.11877	-0.73903	-0.34024	0	-0.80908
Point 10	0.406996	0.840828	0.831135	-0.08071	-0.20595	-0.54775	-0.30969	0.070042	0.468832	0.809075	0

Table 7: Example of $\log$ of One plus the Absolute Value of Simple Difference

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	0	0.308725	0.302772	0.341178	0.41278	0.585548	0.468444	0.247578	0.050268	0.289092	0.292157
Point 1	0.308725	0	0.008049	0.570013	0.627382	0.769026	0.672533	0.496285	0.27013	0.026128	0.531217
Point 2	0.302772	0.008049	0	0.565433	0.623057	0.765273	0.6684	0.491353	0.263942	0.018224	0.526455
Point 3	0.341178	0.570013	0.565433	0	0.099312	0.328852	0.174712	0.118391	0.377173	0.554929	0.065122
Point 4	0.41278	0.627382	0.623057	0.099312	0	0.250729	0.082955	0.207091	0.44633	0.613145	0.158456
Point 5	0.585548	0.769026	0.765273	0.328852	0.250729	0	0.181045	0.415452	0.613849	0.756681	0.376147
Point 6	0.468444	0.672533	0.6684	0.174712	0.082955	0.181045	0	0.275043	0.500205	0.658929	0.229677
Point 7	0.247578	0.496285	0.491353	0.118391	0.207091	0.415452	0.275043	0	0.287036	0.480037	0.056753
Point 8	0.050268	0.27013	0.263942	0.377173	0.44633	0.613849	0.500205	0.287036	0	0.249717	0.329927
Point 9	0.289092	0.026128	0.018224	0.554929	0.613145	0.756681	0.658929	0.480037	0.249717	0	0.515532
Point 10	0.292157	0.531217	0.526455	0.065122	0.158456	0.376147	0.229677	0.056753	0.329927	0.515532	0

Table 8: Example of the Difference Divided by the Mean

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	0	0.361688	0.353606	-0.4066	-0.51101	-0.79598	-0.59751	-0.28092	0.051553	0.335215	-0.33931
Point 1	-0.36169	0	-0.00808	-0.76829	-0.8727	-1.15766	-0.95919	-0.64261	-0.31013	-0.02647	-0.701
Point 2	-0.35361	0.008081	0	-0.76021	-0.86462	-1.14958	-0.95111	-0.63453	-0.30205	-0.01839	-0.69292
Point 3	0.406603	0.768291	0.760209	0	-0.10441	-0.38937	-0.1909	0.125684	0.458156	0.741818	0.06729
Point 4	0.511013	0.872701	0.86462	0.10441	0	-0.28496	-0.08649	0.230094	0.562567	0.846228	0.1717
Point 5	0.795975	1.157663	1.149581	0.389372	0.284962	0	0.198469	0.515056	0.847528	1.13119	0.456662
Point 6	0.597507	0.959194	0.951113	0.190904	0.086493	-0.19847	0	0.316587	0.64906	0.932722	0.258193
Point 7	0.280919	0.642607	0.634526	-0.12568	-0.23009	-0.51506	-0.31659	0	0.332472	0.616134	-0.05839
Point 8	-0.05155	0.310134	0.302053	-0.45816	-0.56257	-0.84753	-0.64906	-0.33247	0	0.283662	-0.39087
Point 9	-0.33522	0.026473	0.018391	-0.74182	-0.84623	-1.13119	-0.93272	-0.61613	-0.28366	0	-0.67453
Point 10	0.339313	0.701001	0.69292	-0.06729	-0.1717	-0.45666	-0.25819	0.058394	0.390867	0.674528	0

4.2 Selection of Correct Metric Comparison

The selection of the correct metric for comparing the values calculated in Section 4.1 is determined by the desired approach for analysis. The metric selected is related to the manner in which the representative matrix is populated. The metric for comparison is referred to as the strength of relation within this work.

A fixed value for the strength of relation can be used to compare the simple differences populating the representative matrix like that shown in Table 6. This fixed value is compared to the calculated difference between ratios at two points. If the difference between the ratios is less than the fixed value of the strength of relation, the ratios are assumed to be the same.

Under the comparison approach motivated by statistics, the coefficient of variation is first calculated by dividing the mean ratio by the standard deviation. The chosen value of the strength of relation is then multiplied by the coefficient of variation to determine a reference value that will be used to compare the ratios among sampling points based on the difference of ratios divided by the mean that populate the representative matrix like that shown in Table 8. If the difference between ratios divided by the mean ratio is less than the strength of relation multiplied by the coefficient of variation, the ratios are assumed to be the same. Additional variations of this method could use the strength of relation selected statistically from a t-distribution perspective or from a standard error perspective. If the t-distribution perspective is used, the selected degree of freedom would be $n - 1$ where 'n' represents the number of observations and the corresponding t-distribution could be found based on a certain confidence level. Under the standard error perspective, the strength of relation would be equal to $1/\sqrt{n}$ where 'n' represents the number of observations.

It is important to consider the span of the ratios under consideration and the physical meaning of the value of this span when the strength of relation is chosen under either approach. In either case, the smaller the strength of relation, the less likely that any two points will be grouped together, while a larger strength of relation will lead to a greater likelihood of point being grouped together.

4.3 Treatment of Multiple Ratios

The case in which ratios of multiple radionuclide combinations need to be considered in order to make a comparative determination of ratios across sampling points adds complexity to the problem. Analysis of such a problem begins with the calculation of differences for each ratio of interest across the sampling area as was done with one pair of radionuclides. After these differences are calculated, the importance of one ratio can be weighted based on some type of chosen metric. This weighting might be based on what is more important for analysis (e.g., dose conversion factors), which measurements are more accurate, knowledge that one ratio is more representative than the other, etc. After each ratio is assigned a weight, the convoluted distance can be calculated according to any kind of normalization. Two possibilities could be the simple L^1 and L^2 norms, that is to say:

- Take the absolute value for each ratio measure and apply the weight. For each location, sum the weighted measures of the different ratios and work with the resulting sum.

- Take the square value for each ratio measure and apply the weight. For each location, sum the weighted measures and work with the square root of the resulting sum (note that the weight is applied after squaring the results to avoid dilution).

L^1 and L^2 norms are from the family of mathematical L^i norms (called vector norms), which are defined as $|x|_i = (\sum (x_k)^i)^{1/i}$. One important point is that such norms conserve the unit as it is defined in the same space as each individual element of the vector. It is fairly common to define a distance as a norm, since a norm is always positive (i.e., independent of the direction), and brings back a multi-dimensional space into a mono-dimensional space which simplifies the comparison.

L^2 norm is the most commonly used norm in geometry. In geometry it is used to estimate the distance between two points in a 2D or 3D space. However, for this work the interest is the estimated distance in ratios, which is a different dimension; so L^2 , while still valid, may not be the norm of preference. What is happening when one looks at the norms is that the higher 'i' is, the biggest emphasize is given to the largest numbers. L^1 does not distort the space while L^∞ distorts it so much that it is equivalent to consider only the maximum value in the vector.

4.4 Propagation of Uncertainty

The measurement of some radionuclide concentrations may be presented with a certain amount of confidence or uncertainty. It is important to track the propagation of this uncertainty to the calculation of a ratio between two measured radionuclides at a sampling point. Depending on the distribution type used to represent uncertainty in the concentration measurement, the ratio distribution will have different characteristics. In a simple case, the uncertainty can be defined analytically [2]-[4] (also see http://en.wikipedia.org/wiki/Ratio_distribution (January 28, 2014) for a more complete discussion). In the case of a more complex relation, a Monte Carlo technique can be used to estimate the resulting distribution.

The uncertainty on a ratio can also be used as an indicator for the weighting process when multiple ratios are under consideration as was discussed in Section 4.3. The larger the uncertainty in the ratio, the lower the weight associated with this ratio should be because the associated weight may be more variable.

5. ORGANIZING MULTIPLE OBSERVATIONS INTO RATIO GROUPS

The determination of groups of ratios that could be considered to be the same directly follows the determination of the strength of relation and subsequent treatment of multiple ratios and uncertainty. There are several methods that can be applied in order to select appropriate groups of ratios and appropriate representative ratios for those groups.

5.1 Simple Quantitative Comparison

As was discussed in Section 4.2, the chosen metric for comparison can be used as a simple reference value in order to determine what ratios can be grouped with one another. If a fixed value is chosen for the strength of relation, this value will be used as a reference to compare the calculated differences between ratios. If the difference between the ratios calculated at two points is less than the strength of relation, the ratios will be assumed to be the same. If a statistically based approach is chosen, the reference value is calculated by multiplying the chosen strength of relation by the coefficient of variation. This reference value is then compared to the populated matrix of the difference between ratios divided by the mean. Again, if the difference between ratios divided by the mean ratio is less than the calculated reference value, the ratios will be assumed to be the same.

Following the comparison of ratio differences using the selected metric, a matrix showing the comparative results can be created. An example of a matrix showing the comparative determination made for the values shown in Table 5 based on the use of a fixed value of 0.5 for the strength of relation is shown in Table 9. Pairs of points at which the chosen radionuclide ratios have been determined to be close enough to be considered to be the same are marked with a green “yes.” In this specific example, the simple difference of ratios between pairs of points marked with a “yes” was determined to be smaller than the fixed value of the strength of relation. Pairs of points whose ratio difference was larger than this value are marked with a red “no.” Groups are determined by clusters of points that are all determined to be close enough to one another.

One weakness in utilizing this grouping method is the risk of creating overlapping groups. A point that is in a certain group may be determined to have the same ratio as a point that is not the same as any other of the points in the group. An example of this drawback is shown in Table 9 in which two groups intersect. Point 3 is shown to be the same as points 0, 1, and 2 but is also grouped with point 4, 5, 6, 7, and 10, which is not comparatively the same as the previous group. A graphical representation of this overlapping grouping can be seen in Figure 4. This risk of overlapping might be avoided by applying another method for determining groups of similar points.

Table 9: Example Matrix of Comparative Determination

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	yes	yes	yes	yes	no	no	no	yes	yes	yes	yes
Point 1	yes	yes	yes	no	no	no	no	no	yes	yes	no
Point 2	yes	yes	yes	no	no	no	no	no	yes	yes	no
Point 3	yes	no	no	yes	yes	yes	yes	yes	no	no	yes
Point 4	no	no	no	yes	yes	yes	yes	yes	no	no	yes
Point 5	no	no	no	yes	yes	yes	yes	no	no	no	no
Point 6	no	no	no	yes	yes	yes	yes	yes	no	no	yes
Point 7	yes	no	no	yes	yes	no	yes	yes	yes	no	yes
Point 8	yes	yes	yes	no	no	no	no	yes	yes	yes	yes
Point 9	yes	yes	yes	no	no	no	no	no	yes	yes	no
Point 10	yes	no	no	yes	yes	no	yes	yes	yes	no	yes

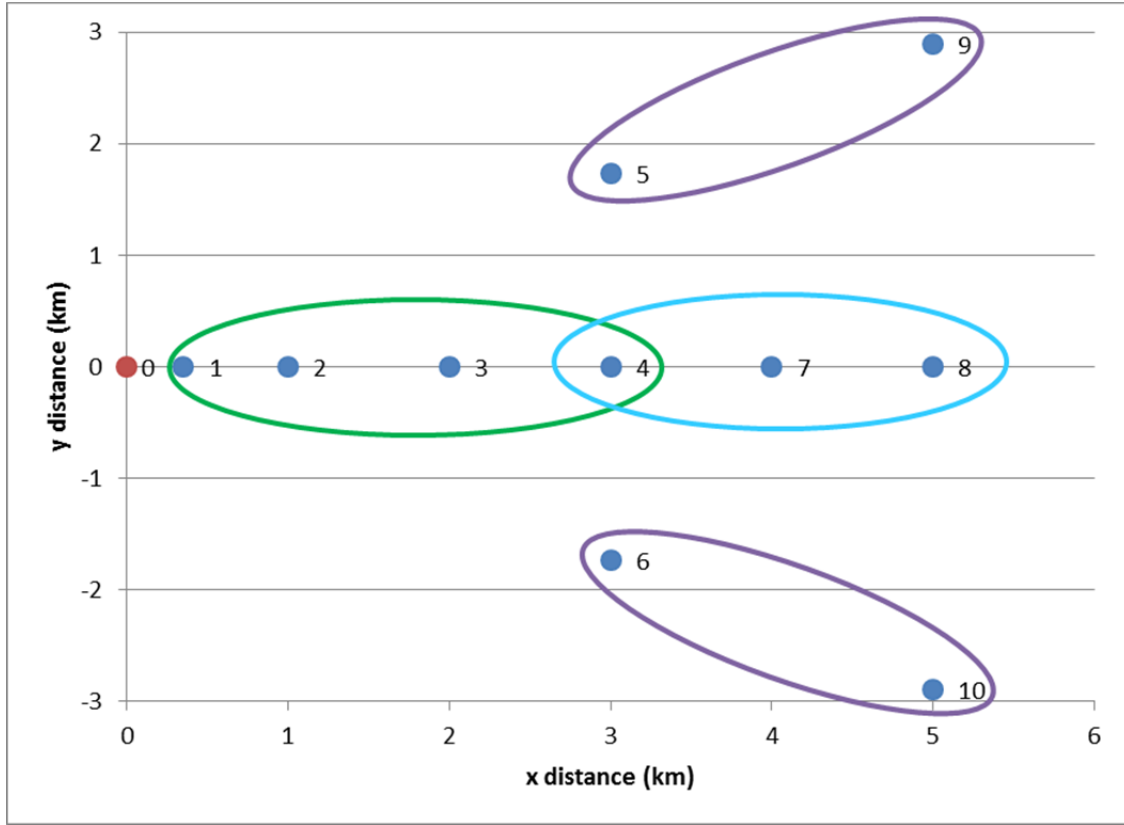


Figure 4: Overlapping Groups for the Application of Proposed Method

5.2 Grouping Method: The Neighborhood Method

While the matrix method works well when few observations are involved, it becomes impractical when the number of observations increases from tens to several hundreds or thousands. In such a case, it is likely that many overlapping groups will be created and the grouping will become too difficult.

In such a situation, the graphical representation shown in Figure 2 can be used so that clusters of points that are close to one another can be identified. Under this grouping approach, the zone of influence of each point is estimated by progressively increasing the ratio neighborhood and comparing how many other points each successive neighborhood includes. Those points with the biggest neighborhoods could be used as central references for a ratio group.

There are various methods by which appropriate reference neighborhood distances might be determined. If the ratios observed are uniformly distributed, the reference distance might be calculated by dividing the number of observations by the number of groups, which would result in an estimate of the approximate number of observations in each group. However, this situation is unlikely as ratios are likely to be clustered and a larger number of observations will need to be defined in some groups, meaning that this approach will probably underestimate the number of points that the biggest group should have.

In a classical distribution of ratios, one can expect to have about fifty to seventy percent of the distribution to occur within plus or minus one standard deviation. Under another approach, this number could be used to estimate the number of points that will be included between the mean or median plus or minus one standard deviation. However, as a result, this number will tend to overestimate the number of points that should be included in this first group.

The average of these two values could be taken as the first neighborhood value in order to mitigate the underestimation or overestimation of the number of observations in the first chosen group caused by the respective approaches proposed above. This reference distance might be calculated by using plus and minus one standard deviation about the mean or median. Another estimate might be found by dividing the number of observations by the number of groups. Under the methodology that is considered, the average of the two previously described numbers was used as the neighborhood size.

After the initial neighborhood size is determined, this neighborhood will be applied to each observation and groups will be created according to the largest neighborhood (see representation in Figure 5). The groups are then removed one by one and the neighborhood size will be estimated and applied again until all groups are created (see representation in Figure 6). Following the creation of all groups, some points that are relatively far from any others may be left without a group. In this case, such a point might be placed in the group with the closest defined neighborhood (see Figure 7).

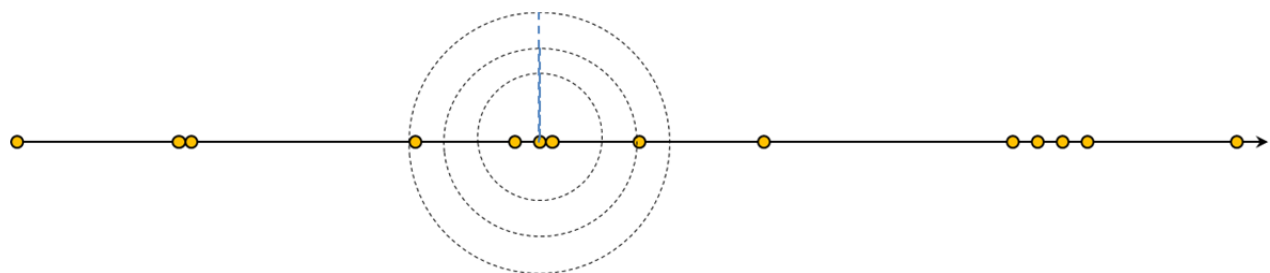


Figure 5: Graphical Representation of Step 1 of the Neighborhood Method

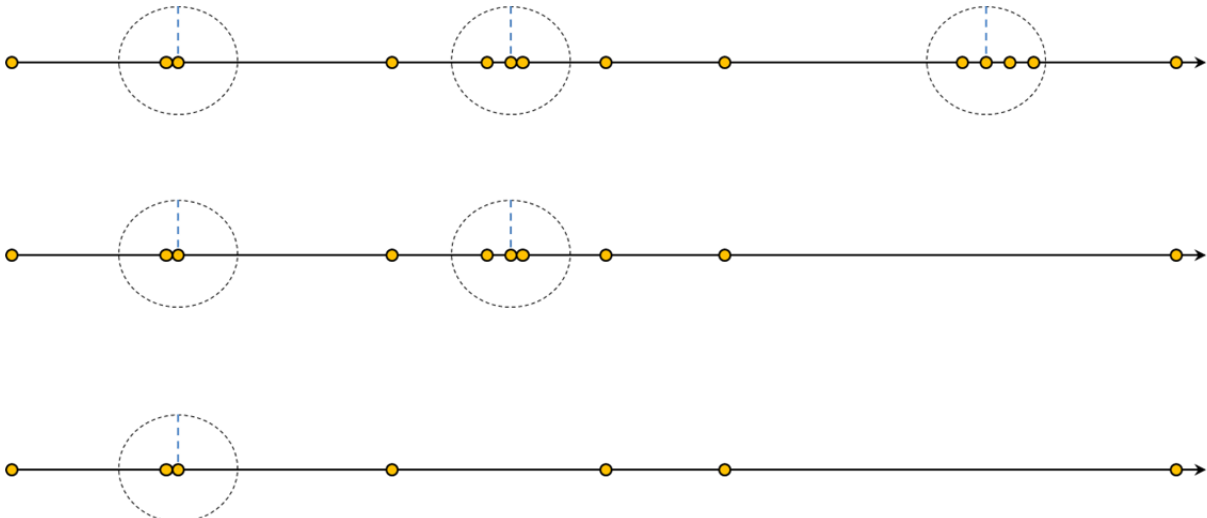


Figure 6: Graphical Representation of Step 2 of the Neighborhood Method

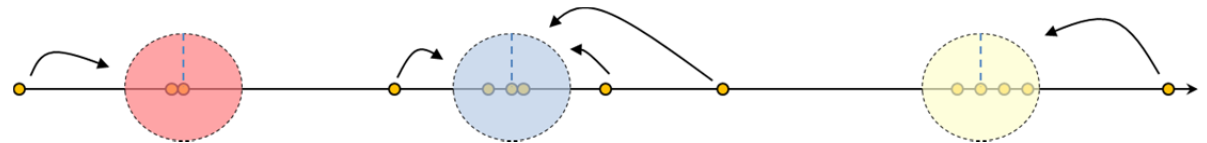


Figure 7: Graphical Representation of Step 3 of the Neighborhood Method

The method described above is one of many methods that could be used to regroup the ratios into zones of influence.

5.3 Selection of a Representative Ratio Value

A representative ratio value must be selected following the determination of ratio groups. There are several ways that this representative ratio can be estimated for each ratio group. Three methods are proposed below:

1. The simplest method is to take the average of all of the ratios that have been grouped together and to use this as the representative ratio for the group.
2. Another approach could be to take the value of the ratio at the most representative point of the group as the representative ratio for the entire group. This method logically follows from the application of the neighborhood method discussed in Section 5.2. The representative ratio chosen in this case would be the ratio at the central reference point around which the chosen influence neighborhood has been determined.
3. Finally, if ratio groups vary by orders of magnitude, the geometric mean could be used as a representative value as opposed to the arithmetic mean.

5.4 Analysis Incorporating Multiple Radionuclide Ratios

There may be a need for multiple radionuclide ratios to be taken into consideration as groups of sampling points with similar characteristics are created. If multiple ratios are used to determine groups of points, one method might be more suitable than others. It is of course perfectly valid to use a different representative value for each ratio. If for instance some ratios vary by orders of magnitude, a geometric mean would be more appropriate. For some other ratios, the arithmetic mean may be the best representation for each group. This choice is completely independent and therefore does not affect the weighting scheme described in Section 4.3.

6. USE OF RESULTS ON SPATIAL MAP

This section provides discussion on the application of the statistical analysis to a field map and discusses how statistical transitions can be used for further field monitoring.

6.1 Nearest Neighbor Approach

In order to implement the proposed analysis in a field situation, it is important that the results be presented spatially so that any decisions that must be made can be applied. One method for creating such a spatial map uses nearest neighbor polygons [5] (also see http://en.wikipedia.org/wiki/Nearest-neighbor_interpolation (January 28, 2014) for a more complete discussion).

This method follows directly from the selection of groups and the determination of a representative ratio value for each group. Under this approach, polygons dividing the space under consideration are constructed around sampling points where the ratio value is set to the representative value that is chosen for each point's group. Every point inside of the polygon constructed around a sample point is closer to that sample point than to any other sample point. This creates a spatial mapping like that shown in Figure 8. In this figure, sampling points are marked at their locations within their respective polygons.

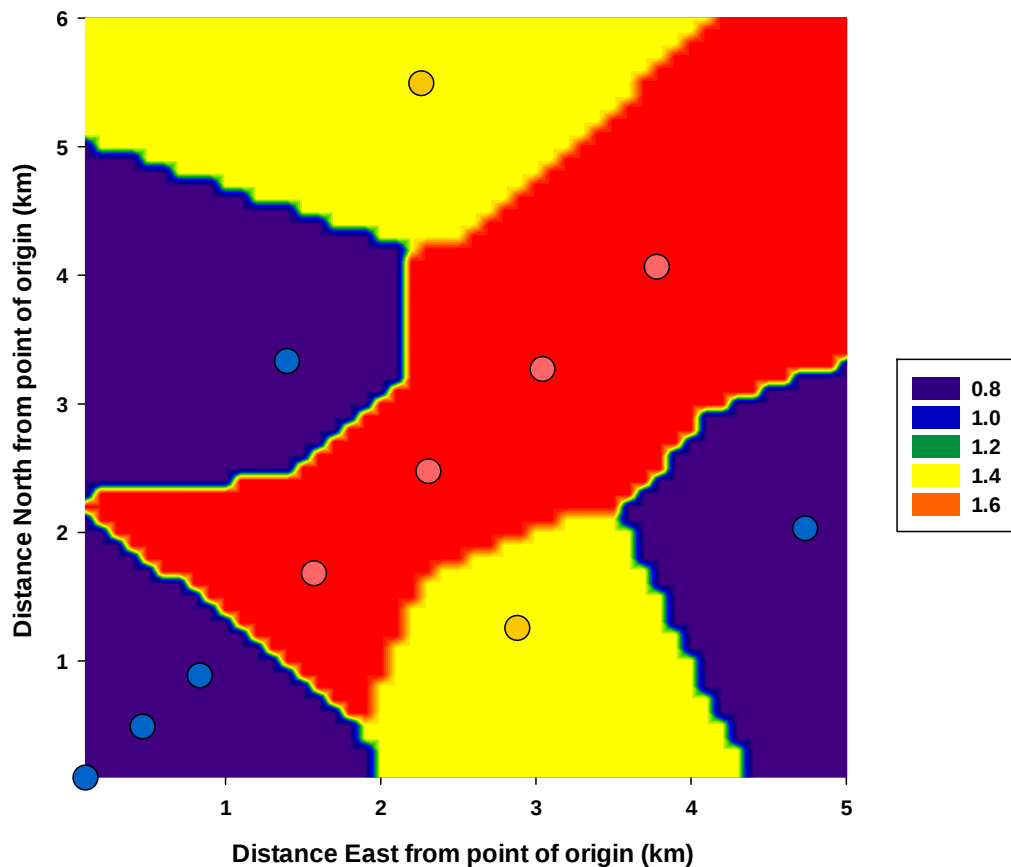


Figure 8: Spatial Map Showing Groups of Activity Ratios across the Sampling Area

6.2 Determination of Spatial Boundaries

An additional advantage of the spatial representation of ratio groups across the sampling area under consideration is the identification of spatial boundaries or zones of low knowledge that may be present in the data. These zones are shown in Figure 8 as the boundaries between ratio groups. A representation of the accuracy for the technique can be considered by plotting the minimum distance from a measurement at each location. Figure 9 displays such a contour map, showing the uncertainty amongst the areas in terms of ratio representativeness. The assumption is that there should be a smooth transition between ratios and, consequently, there is a larger uncertainty in areas further from any observation (if Gaussian is assumed).

In addition to showing the accuracy of the nearest neighbor technique, this representation could help to identify areas where the most meaningful subsequent observations (sampling points) should be made. This will help to ensure that additional sampling is conducted in a manner that will generate the most knowledge about the total area under examination.

Appendix A provides a review on how this proposed methodology is consistent with guidance provided in the U.S. Environmental Protection Agency's Multi-Agency Radiation Survey and Site Investigation Manual (MARSSIM).

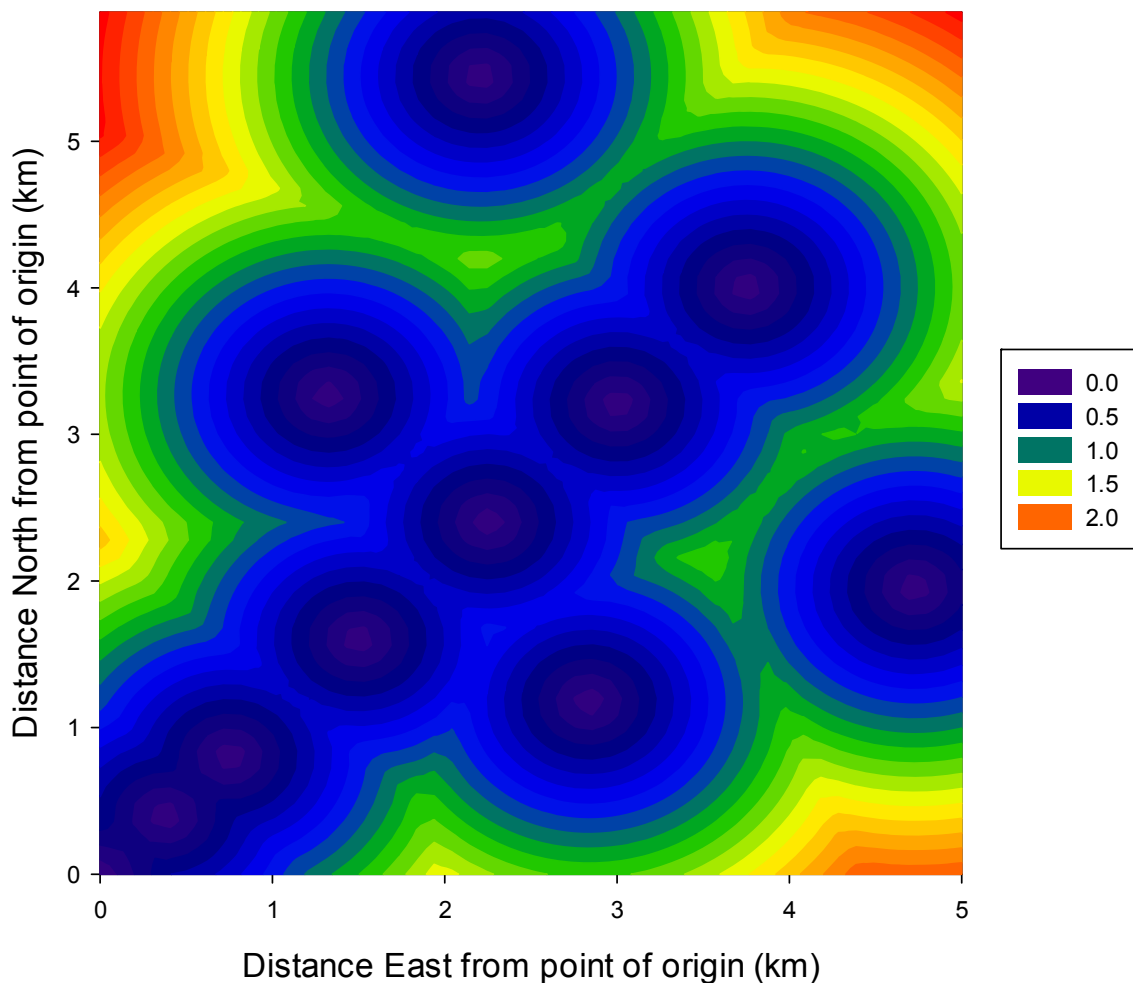


Figure 9: Accuracy of the Nearest Neighbor Technique

7. EXAMPLE USING NARAC RESULTS

This section discusses the methods used to create real world radionuclide deposition data, statistical analyses, and subsequent mapping of plume deposition of a mixture of cesium and iodine. A more simplistic plume model (HotSpot [6]) is also provided in Appendix B.

7.1 Extraction of Data

The National Atmospheric Release Advisory Center (NARAC) provides computer predictions for atmospheric transport of radionuclides downwind of the release using real-time weather data. Located at the Lawrence Livermore National Laboratory, NARAC is a national support and resource center for planning, real-time assessment, emergency response, and detailed studies of incidents involving a wide variety of hazards, including nuclear, radiological, chemical, biological, and natural emissions. NARAC provides tools and services to the FRMAC that map the probable spread of radiological material accidentally or intentionally released into the atmosphere.

NARAC was used to create plumes with as many points from a ten point sample plan as possible. The theoretical plumes were used for the release of Cs-137 and I-131 using real-time weather data, and each radionuclide had a varied deposition velocity of 0.3 cm/sec and 0.1 cm/sec, respectively. Figure 10 shows the theoretical plume for the Cs-137 release. The ten-point sample plan as determined in Section 3.3.1 of the FRMAC Monitoring Manual was implemented for the data points selected for this analysis [1].

For the simulated plume release, the ten-point sample plan was applied after a 15 minute release. Table 10 provides the location, NARAC model used, and material released. Based on this information, the ground deposition was evaluated at 12 hours. Since wind speeds vary downwind between the two NARAC models, the coordinates (X/Y) for the ten point sample plan are different in the plume for I-131 as compared to Cs-137 and are shown in Table 11 and Table 12, respectively.

Figure 11 and Figure 12 illustrate the I-131 sample points within the plume using the ten-point sample plan shown in Table 11. Additionally, Figure 13 and Figure 14 illustrate the Cs-137 sample points within the plume using the ten-point sample plan shown in Table 12.



Figure 10: Cs-137 Simulated Plume

Table 10: Cs-137 and I-131 Location and Material Released

Parameters	
Release Location	35.0072 N, 106.4369 W
Model	ADAPT/LODI
Material at Risk	1.0E08

Table 11: I-131 Deposition Values

Probe name	X distance from release (km)	Y distance from release (km)	Value ($\mu\text{Ci}/\text{m}^2$)
Probe00	0.393185261	0.376061242	6.65E+06
Probe01	0.786259326	0.752233679	1.08E+06
Probe02	1.572518653	1.504356163	3.04E+05
Probe03	2.358777979	2.256589841	2.89E+05
Probe04	3.145037305	3.008712325	1.89E+05
Probe05	3.931296632	3.760946004	1.18E+05
Probe06	2.889622559	0.987410949	3.23E+04
Probe07	4.816185858	1.64535133	3.99E+04
Probe08	1.468662591	3.181842826	1.46E+06
Probe09	2.447845115	5.303219636	1.24E+05

Table 12: Cs-137 Deposition Values

Probe name	X distance from release (km)	Y distance from release (km)	Value ($\mu\text{Ci}/\text{m}^2$)
Probe00	0.357602884	0.012026017	3.77E+06
Probe01	0.715205768	0.024052033	6.23E+05
Probe02	1.430411536	0.048107208	4.53E+05
Probe03	2.145617305	0.072159242	4.67E+05
Probe04	2.860823073	0.096214417	3.69E+05
Probe05	3.576028841	0.12026645	2.03E+05
Probe06	2.785099328	0.038446811	4.32E+04
Probe07	4.641943408	0.064069641	3.74E+04
Probe08	1.177554273	0.094813266	8.70E+05
Probe09	1.962590455	0.158028394	1.74E+05

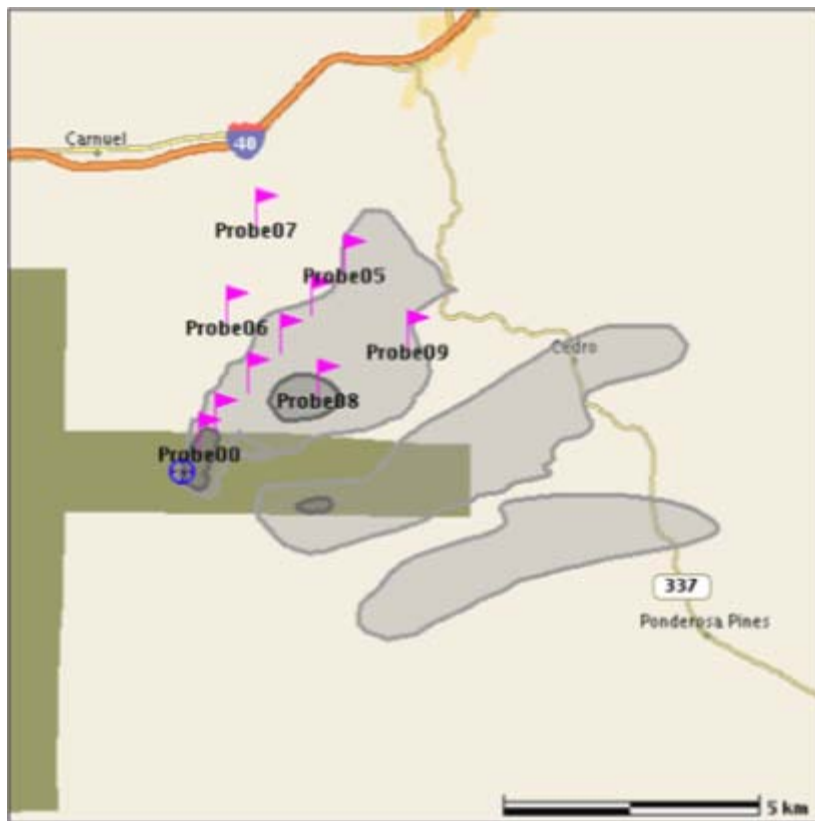


Figure 11: I-131 Simulated Plume with Ten-Point Sample Plan

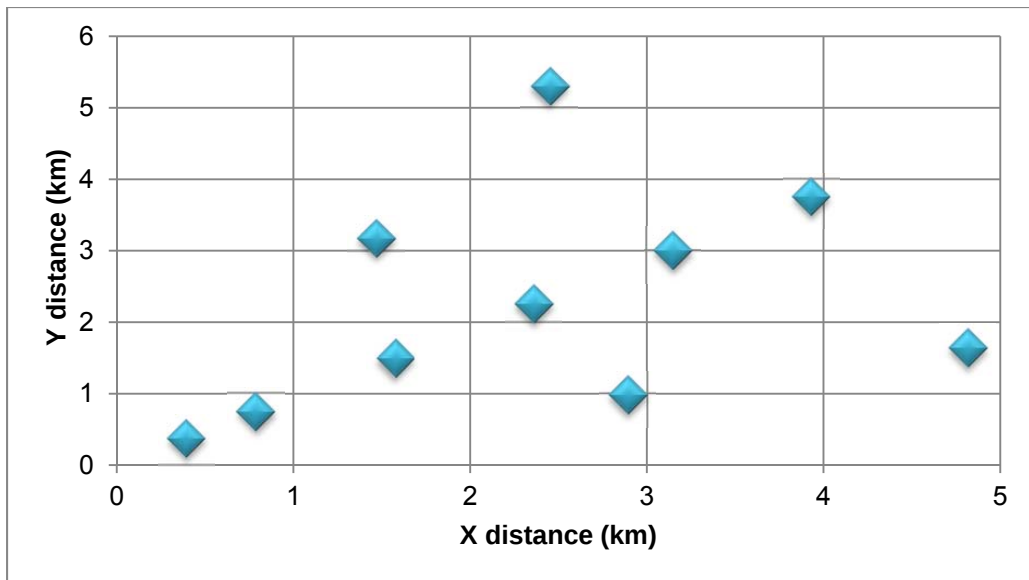


Figure 12: I-131 Ten-Point Sample Plan

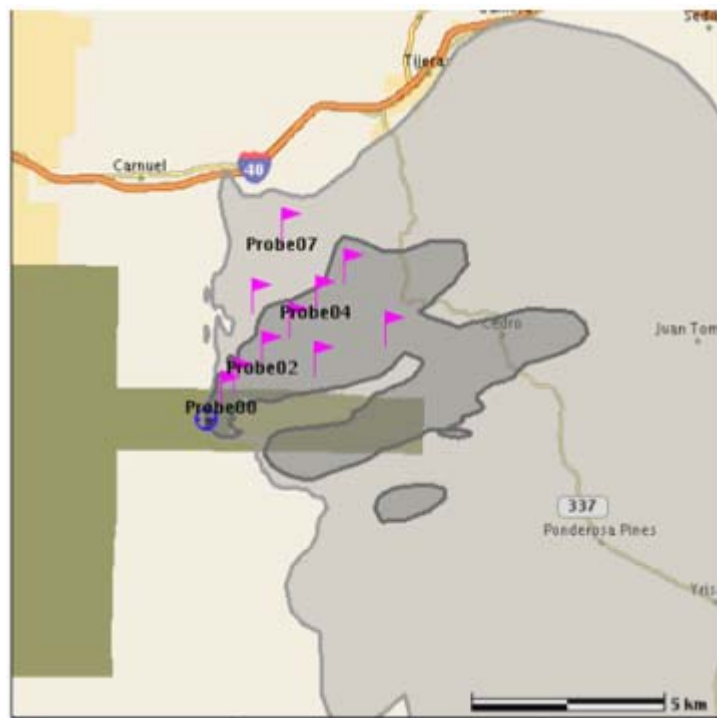


Figure 13: Cs-137 Simulated Plume with Ten-Point Sample Plan

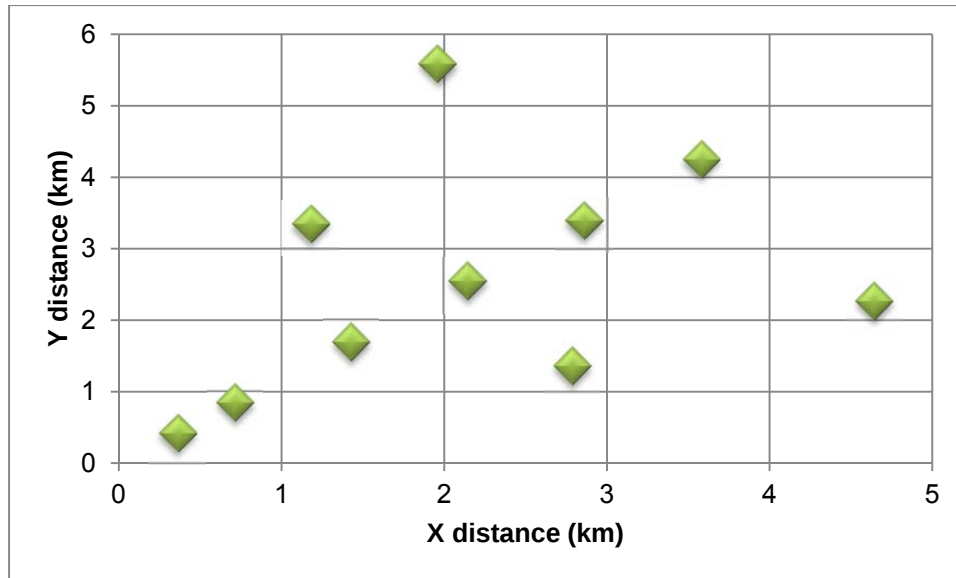


Figure 14: Cs-137 Ten-Point Sample Plan

7.2 Matrix Creation and Results

Due to the nature of the NARAC model, samples for Cs-137 and I-131 that were taken from the model had to be sampled from the dispersion plume at separate times, and could not be taken at exactly the same location. The map of the sampling scheme for these values is shown in Figure 15. In order to simplify the application of the proposed methods to this modeled data, the sampling locations were averaged, the ratios of Cs-137 to I-131 were calculated, and the ratios were assumed to occur at these averaged locations. The Cs-137 and I-131 concentration values and their calculated ratios are shown in Table 13.

Following the determination of sampling locations and the calculation of ratio values at these locations, a matrix of comparative difference like those discussed in Section 4.1 can be created. For this example, a simple difference calculation was chosen as the value for comparison. The matrix was populated with the simple difference between all pairs of ratios across the sampling area and is shown in Table 14. Following the creation of this matrix, these values were compared using a fixed value of 0.5 for the strength of relation. Pairs of points were determined to have the same ratio value when the difference between their respective ratio values was less than 0.5. A matrix of these comparative decisions was created in which points that were determined to be the same under the chosen method of analysis were marked with a green “yes” and points that were determined to be different were marked with a red “no.” This matrix is shown in Table 15.

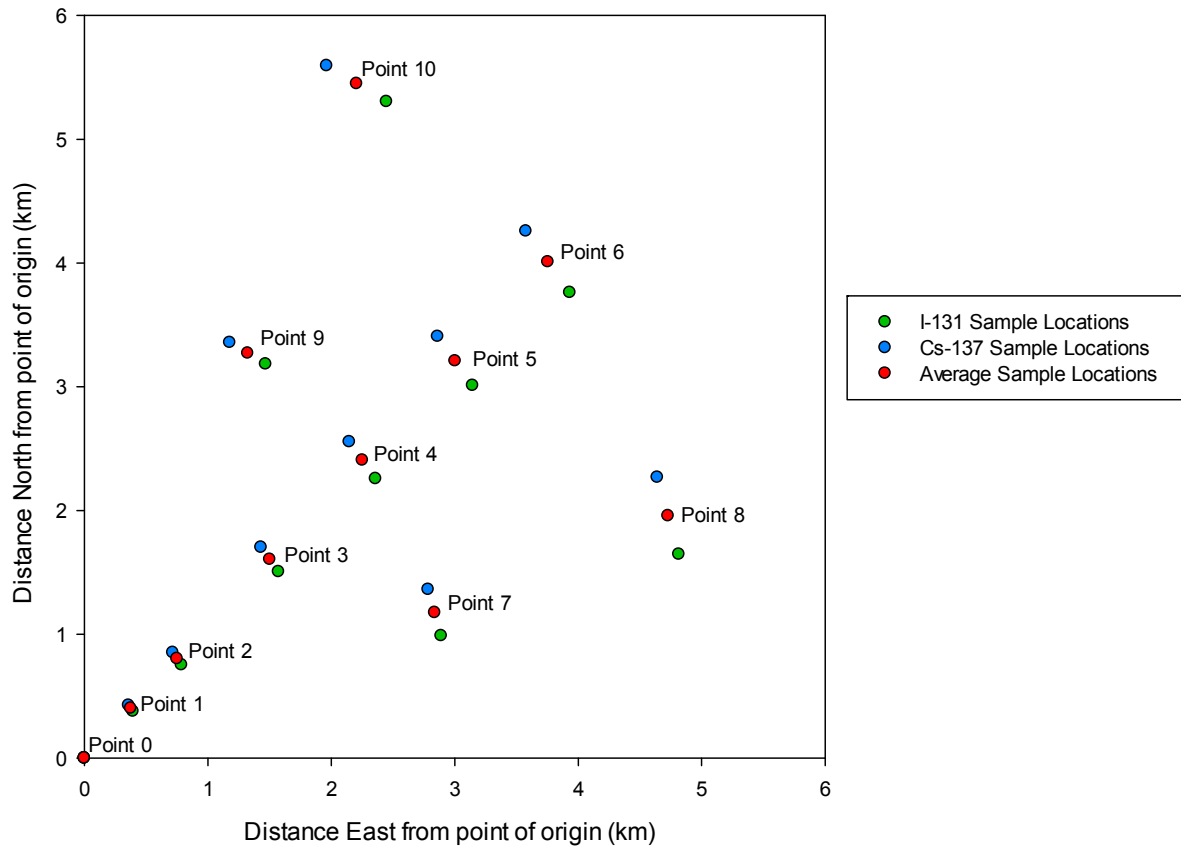


Figure 15: Spatial Locations of Sampling Points in NARAC Model

Table 13: Cs-137 and I-131 Concentration and Calculated Activity Ratio Values

Location Index	Cs-137 ($\mu\text{Ci}/\text{m}^2$)	I-131 ($\mu\text{Ci}/\text{m}^2$)	Cs-137/I-131 Activity Ratio
Point 0 (Release)	---	---	1.00E+00
Point 1	3.77E+06	6.65E+06	5.66E-01
Point 2	6.23E+05	1.08E+06	5.76E-01
Point 3	4.53E+05	3.04E+05	1.49E+00
Point 4	4.67E+05	2.89E+05	1.61E+00
Point 5	3.69E+05	1.89E+05	1.95E+00
Point 6	2.03E+05	1.18E+05	1.72E+00
Point 7	4.32E+04	3.23E+04	1.34E+00
Point 8	3.74E+04	3.99E+04	9.38E-01
Point 9	8.70E+05	1.46E+06	5.98E-01
Point 10	1.74E+05	1.24E+05	1.41E+00

Table 14: Matrix of Simple Difference between Ratios

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	0	0.433833	0.42414	-0.48771	-0.61294	-0.95475	-0.71669	-0.33695	0.061836	0.40208	-0.407
Point 1	-0.43383	0	-0.00969	-0.92154	-1.04678	-1.38858	-1.15052	-0.77079	-0.372	-0.03175	-0.84083
Point 2	-0.42414	0.009693	0	-0.91185	-1.03708	-1.37889	-1.14083	-0.76109	-0.3623	-0.02206	-0.83114
Point 3	0.487707	0.92154	0.911847	0	-0.12524	-0.46704	-0.22898	0.150754	0.549544	0.889787	0.080712
Point 4	0.612944	1.046777	1.037084	0.125237	0	-0.3418	-0.10375	0.275991	0.674781	1.015024	0.205949
Point 5	0.954747	1.388579	1.378886	0.467039	0.341803	0	0.238057	0.617793	1.016583	1.356826	0.547751
Point 6	0.71669	1.150523	1.14083	0.228983	0.103746	-0.23806	0	0.379736	0.778526	1.11877	0.309694
Point 7	0.336954	0.770786	0.761093	-0.15075	-0.27599	-0.61779	-0.37974	0	0.39879	0.739033	-0.07004
Point 8	-0.06184	0.371996	0.362303	-0.54954	-0.67478	-1.01658	-0.77853	-0.39879	0	0.340243	-0.46883
Point 9	-0.40208	0.031753	0.02206	-0.88979	-1.01502	-1.35683	-1.11877	-0.73903	-0.34024	0	-0.80908
Point 10	0.406996	0.840828	0.831135	-0.08071	-0.20595	-0.54775	-0.30969	0.070042	0.468832	0.809075	0

Table 15: Matrix of Comparative Determination following Matrix Representation of Difference

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	yes	yes	yes	yes	no	no	no	yes	yes	yes	yes
Point 1	yes	yes	yes	no	no	no	no	no	yes	yes	no
Point 2	yes	yes	yes	no	no	no	no	no	yes	yes	no
Point 3	yes	no	no	yes	yes	yes	yes	yes	no	no	yes
Point 4	no	no	no	yes	yes	yes	yes	yes	no	no	yes
Point 5	no	no	no	yes	yes	yes	yes	no	no	no	no
Point 6	no	no	no	yes	yes	yes	yes	yes	no	no	yes
Point 7	yes	no	no	yes	yes	no	yes	yes	yes	no	yes
Point 8	yes	yes	yes	no	no	no	no	yes	yes	yes	yes
Point 9	yes	yes	yes	no	no	no	no	no	yes	yes	no
Point 10	yes	no	no	yes	yes	no	yes	yes	yes	no	yes

7.3 Grouping of Data

Three groups of ratios were created following the comparative determinations made using the fixed strength of relation. These groups were chosen based on dominant blocks of points that were determined to be the same as one another with outlying points grouped in a manner similar to the neighborhood method. The average of the ratios of points grouped together was chosen as the representative value for each group. These designated groups, their members, and calculated representative values are shown in Table 16.

Table 16: Chosen Groups Based on NARAC Activity Ratio Comparison

Group Number	Group 1	Group 2	Group 3
Group Members	Points 0, 1, 2, 8, and 9	Points 3, 4, 5, and 6	Points 7 and 10
Representative Activity Ratio	0.735622	1.693022	1.371975

7.4 Creation of Ratio Map

A spatial map of these groups across the sampling area was created using the nearest neighbor polygon method. This map is shown in Figure 16. As was discussed in Section 6.2, the boundaries between the polygons shown on the spatial mapping represent areas where knowledge regarding the application of ratio values is the lowest. Further sampling of this area should be conducted in these zones of low knowledge in order to gain the most meaningful information about the patterns present in the spatial variability of radionuclide activity ratios across the sampling area.

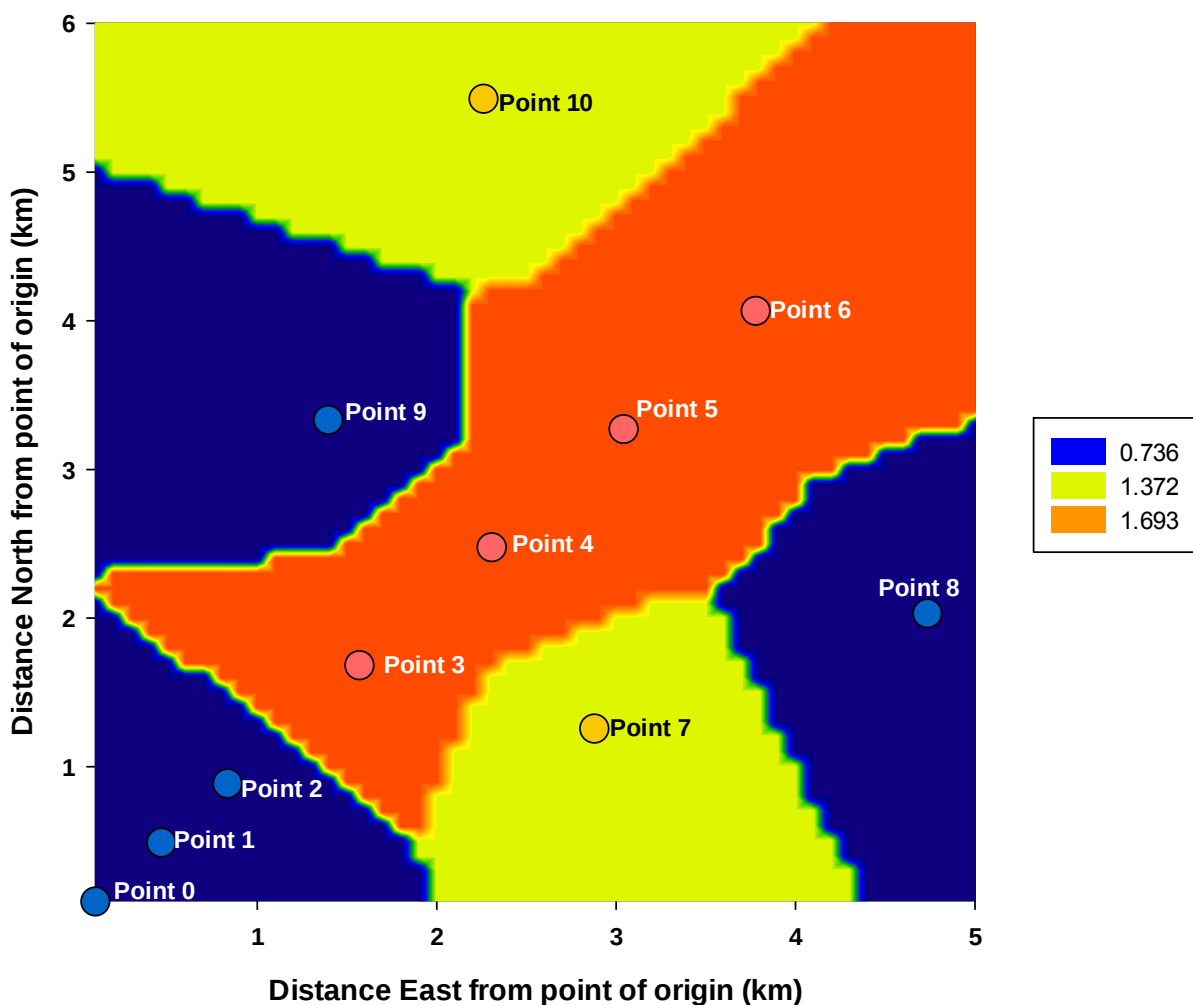


Figure 16: Spatial Map of Activity Ratio Groups

8. CONCLUSIONS

While initially this study was driven by a very simple question, “Can we use the same representative ratio for all observations,” it was discovered that the problem was more complex than expected and required a more sophisticated technique in order to report meaningful results. The problem has to be split into two parts.

The first part considers all observations regardless of their locations and determines how many different groups should be considered. This determination needs to be done with both physical and statistical interpretations in mind, meaning that the range and variation of values need to be considered (i.e., statistical interpretation) as well as the consequence of changing the ratio by a certain amount (i.e., physical interpretation). The methodology presented in this report gives the tools to apply such considerations, but selecting the correct representative split can only be done along with consultation of experts and through deeper understanding of the problem and the statistics involved.

The second part considers the geo-location of the observations in order to:

1. Create a map that could be used to follow the plume/deposition and apply the correct ratio group to each location, and
2. Determine where the biggest uncertainty is located so that the next set of samples can be considered.

This method has been tested on synthetic and realistic (NARAC) data and has been presented using examples from these tests. Further work will be necessary to fully develop and V&V (i.e., verify and validate) this approach so that it can be applied to real data taken following a radiological release.

9. REFERENCES

- [1] FRMAC, "Monitoring Manual – Volume 1," Federal Radiological Monitoring and Assessment Center, Washington D.C., 2012.
- [2] Marsaglia, G., "Ratios of Normal Variables and Ratios of Sums of Uniform Variables," Defense Technical Information Center, 1964.
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APPENDIX A

Independent Review of Proposed Region Classification Method for Radionuclide Mixtures

Reviewer: Robert G. Knowlton, Ph.D., P.E., Sandia National Laboratories

Date: January 27, 2014

Purpose: The purpose of this review is to determine if a proposed methodology is consistent with guidance provided in the Multi-Agency Radiation Survey and Site Investigation Manual (MARSSIM). A method is proposed method to classify the regions in the close neighborhood of selected measurements according to the ratio of two radionuclides measured from either a radioactive plume or a deposited radionuclide mixture.

Observations: MARSSIM provides guidance for conducting radiation surveys and performing investigations at potentially contaminated sites. MARSSIM follows a step-wise process for planning, implementation, assessment, and decision making. The guidance strongly recommends the application of the Data Quality Objectives (DQO) process in order to assure adequate quality and quantity of data and information. MARSSIM provides specific statistical methods for designing sampling strategies that account for decision errors, uncertainty, and derived concentration guidelines. The Data Quality Assessment (DQA) phase utilizes statistical methods to analyze data collected during implementation. Again, MARSSIM has specific statistical methods and plotting guidelines to aid in the interpretation of the data to assess whether project objectives have been met. In addition, MARSSIM recognizes that other methods may exist that can also aid in the DQA phase.

Typically, the distribution of contamination is assumed to be uniformly distributed unless proven otherwise. The ability to evaluate the potential uncertainty in the spatial distribution of radionuclide mixtures is the purpose of the proposed method for evaluating regions for radionuclide mixtures. The proposed method evaluates the results of a 10-spot sampling plan within a potentially contaminated area by performing analyses of comparative differences between cesium-137 to iodine-131 activity ratios. Matrix tables and spatial plots are used to assess regions of similar activity ratios. The method is intended to explicitly address the uncertainty in the data and how it relates to the decision making process. Therefore, this method is part of the DQA phase.

MARSSIM states in Section 2.3: "Usually a decision maker will make a correct decision after evaluating the data. However, since uncertainty in the survey results is unavoidable, the possibility of errors in decisions supported by survey results is unavoidable. For this reason, positive actions must be taken to manage the uncertainty in the survey results so that sound, defensible decisions may be made." In addition, consider the following from MARSSIM Section 2.6 Flexibility in Applying MARSSIM Guidance: "The plan should also demonstrate that the extrapolation from measurements performed at specific locations to the entire site or survey unit is performed in a technically defensible manner." The proposed method meets these criteria. Also, in Section 2.6.1 Alternate Statistical Methods: "MARSSIM encourages the use of statistics to provide a quantitative estimate of the probability that the release criterion is not exceeded at a site." MARSSIM does not specifically recommend any statistical methods to address the spatial variability of the activity ratios and how that relates to the assignment of different regions of concern. But obviously from the statements listed above, MARSSIM allows

for the use of alternate methods to address this uncertainty in order to provide decision makers with the best available data.

Therefore, the proposed method is deemed consistent with the MARSSIM guidance.

Conclusion: The proposed method is consistent with the MARSSIM guidance.

APPENDIX B

HOTSPOT Analysis

For the purpose of verification, another simple plume model was created to test the proposed method. The HotSpot Health Physics Code [6] provides a first order approximation of atmospheric release of radioactive materials and radiation effects in a short-term and short-range (i.e., within a few hours and less than 10 km) predictions for one direction with a single meteorological input (e.g., wind speed and atmospheric stability class). HotSpot does not take into account varying geological and weather conditions.

Simulated plumes for Cs-137 and I-131 were considered using HotSpot with the ten-point sample plan [1]. For the simulated plume releases, the ten-point sample plan was applied after a 15 minute release. The radionuclide dispersion values for each of the ten points were considered at different constant wind speeds of 1 to 10 mph in one direction. The ground surface deposition values are based on sampled (i.e., wind speed and deposition velocity) and default values as seen in Table 17. The Material-at-risk (MAR) and respirable source term values are sample values. The parameters do not include weather conditions or geological variations.

Table 17: Parameters for HotSpot Analysis of Cs-137 and I-131

Parameters	Cs-137	I-131
Material-at-risk (MAR)	1.0000E+10 Curies	1.0000E+10 Curies
Damage Ratio (DR)	1	1
Airborne Fraction (ARF)	1	1
Respirable Fraction (RF)	1	1
Leak path Factor (LPF)	1	1
Respirable Source Term	1.00E+10 Curies	1.00E+10 Curies
Non-respirable Source Term	0.00E+00 Curies	0.00E+00 Curies
Heat Emission	1.00E+05 cal/s	1.00E+05 cal/s
Air Temperature	68.0 °F	68.0 °F
Release Radius	3.3 feet	3.3 feet
Physical Height of Fire	0 feet	0 feet
Effective Release Height	94 feet	94 feet
Wind Direction	270.0 degrees (wind from the West)	270.0 degrees (wind from the West)
Wind Speed (h=33 feet)	1.00 to 10.00 mph	1.00 to 10.00 mph
Average Wind Speed (h=H _{eff})	1.41 to 10.28 mph	1.41 to 10.28 mph
Stability Class	D	D
Respirable Deposition Velocity	0.30 cm/sec	1.00E-01 cm/s
Non-respirable Deposition Velocity	8.00 cm/sec	8.00 cm/s
Receptor Height	4.9 feet	4.9 feet
Inversion Layer Height	1001 feet	1001 feet
Sample Time	15.000 min	15.000 min
Breathing Rate	4.17E-04 m ³ /sec	4.17E-04 m ³ /sec
Location Coordinates	Plume Centerline = x-axis, Crosswind Direction = y-axis	Plume Centerline = x-axis, Crosswind Direction = y-axis
Maximum Dose Distance	0.19 miles	0.19 miles
Maximum TED	5.06E+07 rem	2.35E+06 rem
Inner Contour Dose	1.00E+02 rem	1.00E+02 rem
Middle Contour Dose	10 rem	10 rem
Outer Contour Dose	1.0 rem	1.0 rem
Exceeds Inner Dose Out To:	> 120 miles	> 120 miles
Exceeds Middle Dose Out To:	> 120 miles	> 120 miles
Exceeds Outer Dose Out To:	> 120 miles	> 120 miles

The HotSpot resulting values in Table 18 and Table 19 for Cs-137 and I-131, respectively, are based on the parameters in Table 17 and the ten data points are the resulting (X/Y) coordinates measured in kilometers based on the ten-point sample plan. The lowest deposition value for Cs-137 is with constant wind speed at 1 mph, and the highest deposition value is with constant wind speed at 2 mph. The lowest deposition value for I-131 is with constant wind speed at 2 mph and, the highest deposition value with constant wind speed at 10 mph.

Table 18: Highest and Lowest Ground Deposition for Cs-137

Wind Speed	X distance from release (km)	Y distance from release (km)	Cs-137 ($\mu\text{Ci}/\text{m}^2$)	I-131 ($\mu\text{Ci}/\text{m}^2$)
1 mph (lowest)	3.50E-01	0.00E+00	1.20E+08	4.00E+07
	1.00E+00	0.00E+00	4.20E+08	1.40E+08
	2.00E+00	0.00E+00	3.40E+08	1.10E+08
	3.00E+00	0.00E+00	2.40E+08	8.30E+07
	3.00E+00	1.73E+00	5.50E-08	7.20E+07
	3.00E+00	-1.73E+00	5.50E-08	7.20E+07
	4.00E+00	0.00E+00	1.80E+08	6.30E+07
	5.00E+00	0.00E+00	1.40E+08	5.00E+07
	5.00E+00	2.89E+00	3.30E-12	4.30E+07
	5.00E+00	-2.89E+00	3.30E-13	4.30E+07
2 mph (highest)	3.50E-01	0.00E+00	9.90E+08	3.30E+08
	1.00E+00	0.00E+00	6.60E+08	2.20E+08
	2.00E+00	0.00E+00	3.10E+08	1.10E+08
	3.00E+00	0.00E+00	1.80E+08	6.50E+07
	3.00E+00	1.73E+00	1.30E-08	5.40E+07
	3.00E+00	-1.73E+00	1.30E-08	5.40E+07
	4.00E+00	0.00E+00	1.30E+08	4.50E+07
	5.00E+00	0.00E+00	9.40E+07	3.40E+07
	5.00E+00	2.89E+00	9.80E-13	2.90E+07
	5.00E+00	-2.89E+00	9.80E-13	2.90E+07

Table 19: Highest and Lowest Ground Deposition for I-131

Wind Speed	X distance from release (km)	Y distance from release (km)	Cs-137 ($\mu\text{Ci}/\text{m}^2$)	I-131 ($\mu\text{Ci}/\text{m}^2$)
2 mph (lowest)	3.50E-01	0.00E+00	9.90E+08	3.30E+08
	1.00E+00	0.00E+00	6.60E+08	2.20E+08
	2.00E+00	0.00E+00	3.10E+08	1.10E+08
	3.00E+00	0.00E+00	1.80E+08	6.50E+07
	3.00E+00	1.73E+00	1.30E-08	5.40E+07
	3.00E+00	-1.73E+00	1.30E-08	5.40E+07
	4.00E+00	0.00E+00	1.30E+08	4.50E+07
	5.00E+00	0.00E+00	9.40E+07	3.40E+07
	5.00E+00	2.89E+00	9.80E-13	2.90E+07
	5.00E+00	-2.89E+00	9.80E-13	2.90E+07
10 mph (highest)	3.50E-01	0.00E+00	1.40E+09	4.80E+08
	1.00E+00	0.00E+00	2.80E+08	9.60E+07
	2.00E+00	0.00E+00	1.00E+08	3.40E+07
	3.00E+00	0.00E+00	5.60E+07	1.90E+07
	3.00E+00	1.73E+00	1.30E-09	1.60E+07
	3.00E+00	-1.73E+00	1.30E-09	1.60E+07
	4.00E+00	0.00E+00	3.80E+07	1.30E+07
	5.00E+00	0.00E+00	2.80E+07	9.70E+06
	5.00E+00	2.89E+00	1.30E-13	8.00E+06
	5.00E+00	-2.89E+00	1.30E-13	8.00E+06

Figure 17 through Figure 20 were created using HotSpot to illustrate the simulated distance the radionuclides would disperse with constant wind speeds for Cs-137 and I-131. The figures are for the highest and lowest ground surface deposition values from the same data used to produce Table 18 and Table 19. From the figures it can be concluded that Cs-137 has the farthest dispersion (note both Cs-137 plumes in Figure 17 and Figure 18 extend to the outer most distance of 120 miles).

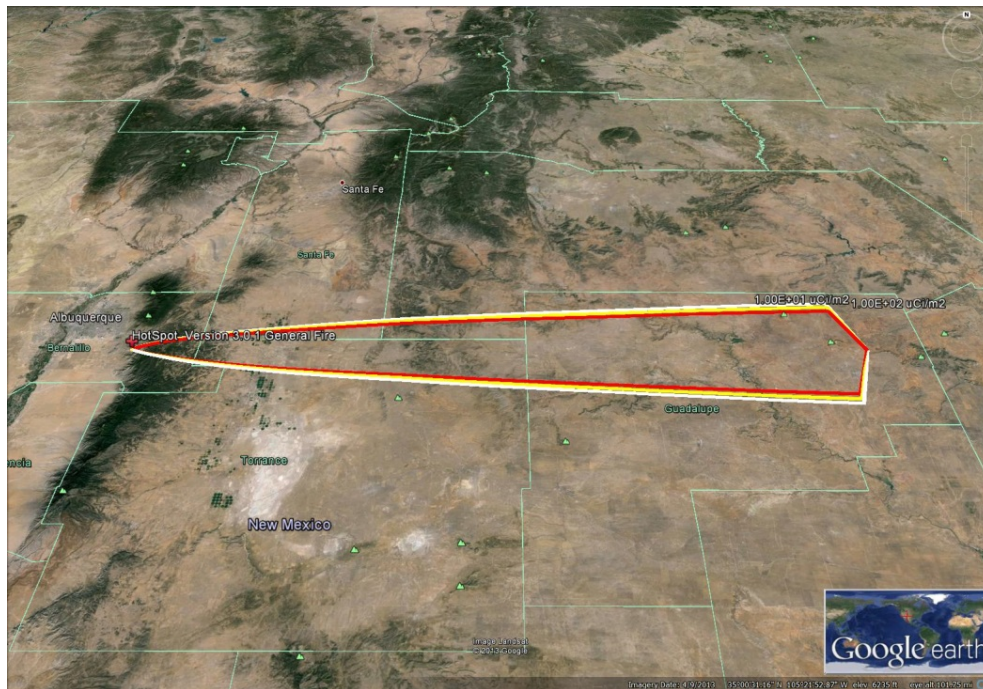


Figure 17: Cs-137 Ground Deposition at 2 mph

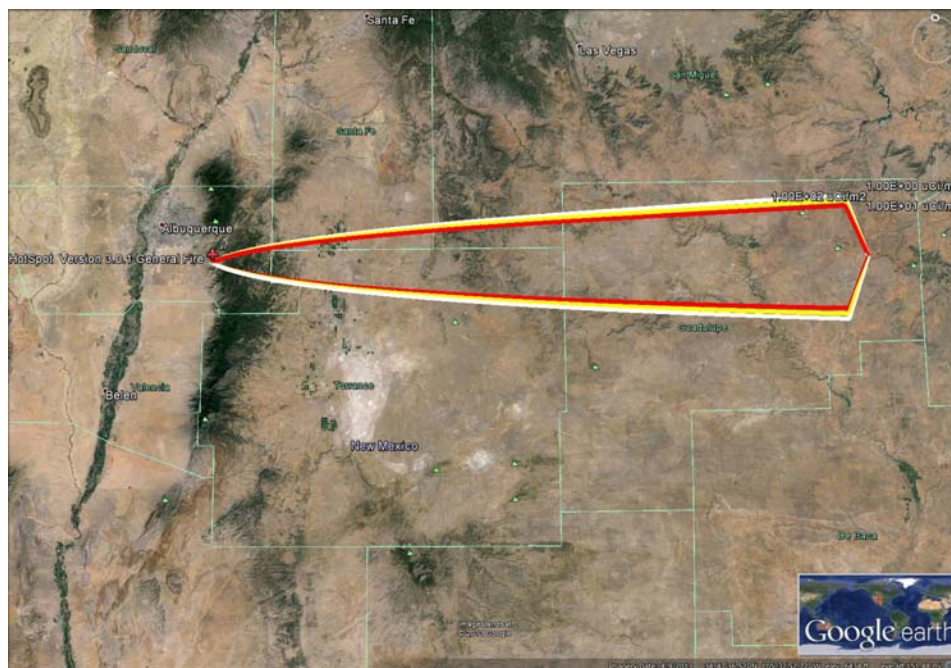


Figure 18: Cs-137 Ground Deposition at 10 mph

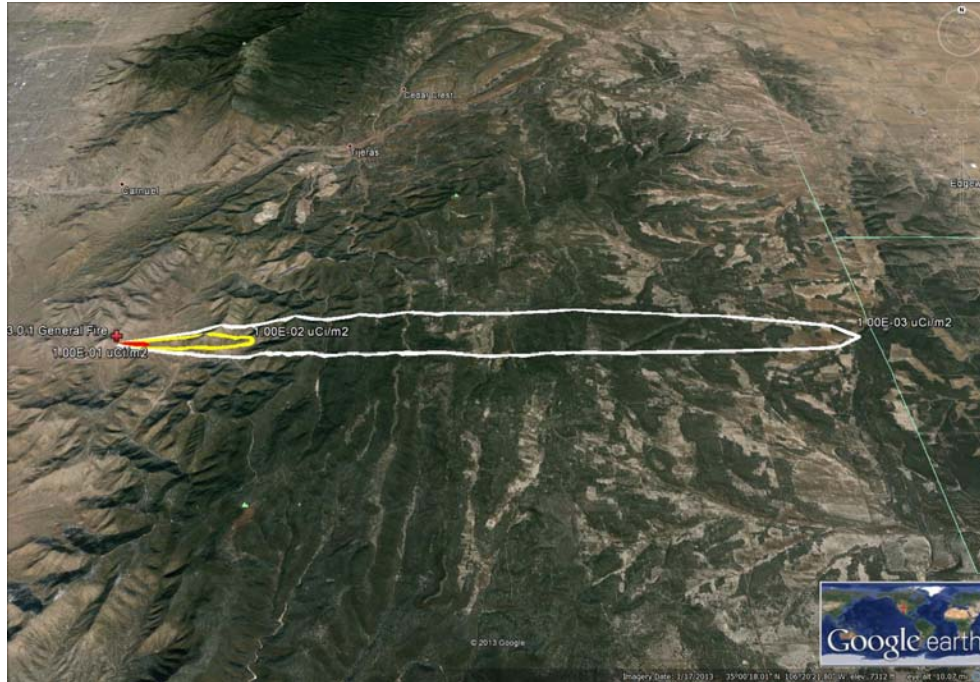


Figure 19: I-131 Ground Deposition at 1 mph

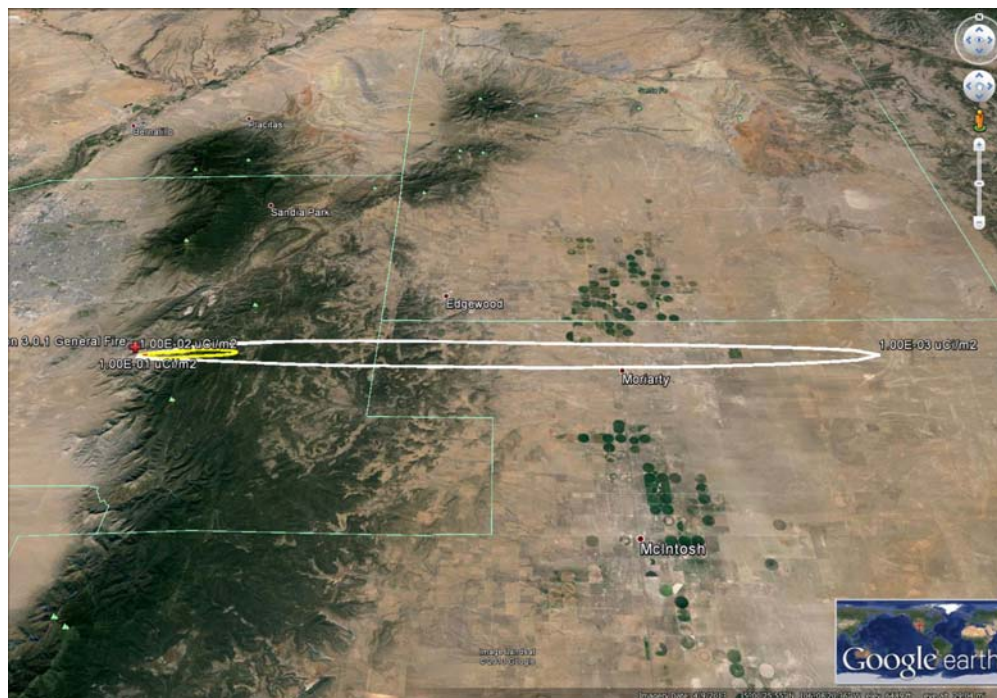


Figure 20: I-131 Ground Deposition at 2 mph

Using HotSpot, a theoretical release plume was modeled with a constant wind blowing in the same direction. The plume was sampled using the same ten-point sampling scheme [1] described in the previous example. Similar results were found across all of the wind speeds modeled. The grouping of sampling points with points that are considered to have the same

Cs-137 to I-131 ratio remains fairly constant as the wind speed and strength of relation are varied to a certain degree. However, these groupings show some sensitivity to very small relative values of the strength of relation. As the strength of relation becomes smaller and smaller, the grouping becomes more sensitive and the points are broken into a greater number of smaller groups. This sensitivity is expected because the bounds for each group become smaller as the strength of relation becomes smaller and, thus, fewer points are found to have the same ratio as one another.

The ten-point sampling scheme for this model is shown in Figure 21. The original data from the HotSpot model [6] for a 10 mph wind speed is shown in Table 20. The simple difference was used to populate a matrix in order to compare the ratios across all samples. This matrix representation of difference is shown in Table 21. Following this calculation of difference, a fixed value of 0.7 was used as the strength of relation to compare the ratios across the sampling area. A matrix showing the results of this comparison is shown in Table 22.

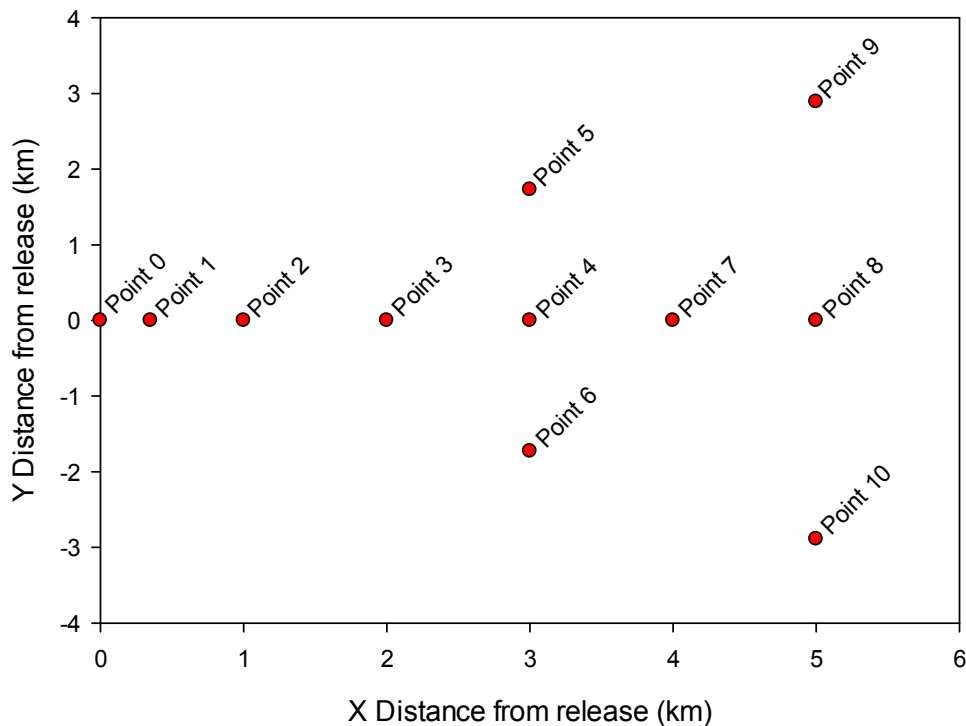


Figure 21: Sampling Scheme for HotSpot Model

Table 20: Data Collected from HotSpot Model and Calculated Activity Ratios

X distance from release (km)	Y distance from release (km)	Cs-137 ($\mu\text{Ci}/\text{m}^2$)	I-131 ($\mu\text{Ci}/\text{m}^2$)	Cs-137/I-131 Activity Ratio
0.00E+00	0.00E+00	---	---	1.00E+00
3.50E-01	0.00E+00	1.40E+09	4.80E+08	2.92E+00
1.00E+00	0.00E+00	2.80E+08	9.60E+07	2.92E+00
2.00E+00	0.00E+00	1.00E+08	3.40E+07	2.94E+00
3.00E+00	0.00E+00	5.60E+07	1.90E+07	2.95E+00
3.00E+00	1.73E+00	1.30E-09	1.60E+07	8.13E-17
3.00E+00	-1.73E+00	1.30E-09	1.60E+07	8.13E-17
4.00E+00	0.00E+00	3.80E+07	1.30E+07	2.92E+00
5.00E+00	0.00E+00	2.80E+07	9.70E+06	2.89E+00
5.00E+00	2.89E+00	1.30E-13	8.00E+06	1.63E-20
5.00E+00	-2.89E+00	1.30E-13	8.00E+06	1.63E-20

Table 21: Simple Difference Calculated among All Pairs of Points Modeled in HotSpot

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	0	-1.91667	-1.91667	-1.94118	-1.94737	1	1	-1.92308	-1.8866	1	1
Point 1	1.916667	0	0	-0.02451	-0.0307	2.916667	2.916667	-0.00641	0.030069	2.916667	2.916667
Point 2	1.916667	0	0	-0.02451	-0.0307	2.916667	2.916667	-0.00641	0.030069	2.916667	2.916667
Point 3	1.941176	0.02451	0.02451	0	-0.00619	2.941176	2.941176	0.0181	0.054579	2.941176	2.941176
Point 4	1.947368	0.030702	0.030702	0.006192	0	2.947368	2.947368	0.024291	0.06077	2.947368	2.947368
Point 5	-1	-2.91667	-2.91667	-2.94118	-2.94737	0	0	-2.92308	-2.8866	8.12E-17	8.12E-17
Point 6	-1	-2.91667	-2.91667	-2.94118	-2.94737	0	0	-2.92308	-2.8866	8.12E-17	8.12E-17
Point 7	1.923077	0.00641	0.00641	-0.0181	-0.02429	2.923077	2.923077	0	0.036479	2.923077	2.923077
Point 8	1.886598	-0.03007	-0.03007	-0.05458	-0.06077	2.886598	2.886598	-0.03648	0	2.886598	2.886598
Point 9	-1	-2.91667	-2.91667	-2.94118	-2.94737	-8.1E-17	-8.1E-17	-2.92308	-2.8866	0	0
Point 10	-1	-2.91667	-2.91667	-2.94118	-2.94737	-8.1E-17	-8.1E-17	-2.92308	-2.8866	0	0

Table 22: Comparison of Ratios across Sampling Area using a Fixed Strength of Relation

	Point 0	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8	Point 9	Point 10
Point 0	yes	no	no	no	no	no	no	no	no	no	no
Point 1	no	yes	yes	yes	yes	no	no	yes	yes	no	no
Point 2	no	yes	yes	yes	yes	no	no	yes	yes	no	no
Point 3	no	yes	yes	yes	yes	no	no	yes	yes	no	no
Point 4	no	yes	yes	yes	yes	no	no	yes	yes	no	no
Point 5	no	no	no	no	no	yes	yes	no	no	yes	yes
Point 6	no	no	no	no	no	yes	yes	no	no	yes	yes
Point 7	no	yes	yes	yes	yes	no	no	yes	yes	no	no
Point 8	no	yes	yes	yes	yes	no	no	yes	yes	no	no
Point 9	no	no	no	no	no	yes	yes	no	no	yes	yes
Point 10	no	no	no	no	no	yes	yes	no	no	yes	yes

Following the comparative representation shown in Table 22, three groups of points were created. Point 0 was given its own group, points 1, 2, 3, 4, 7, and 8 (all points along the center of the plume) were grouped together, and points 5, 6, 9, and 10 (all points off of the center of the plume) were grouped together. These groups were created simply by using the blocks of green “yes” decisions shown in Table 22. The simplicity of this model and problem and the chosen strength of relation created a simple comparison without overlapping groups. The average of the ratios of these groups was chosen as the representative value for each group. A spatial map of the sampling area was created using the nearest neighbor polygon approach. This map is shown in Figure 22.

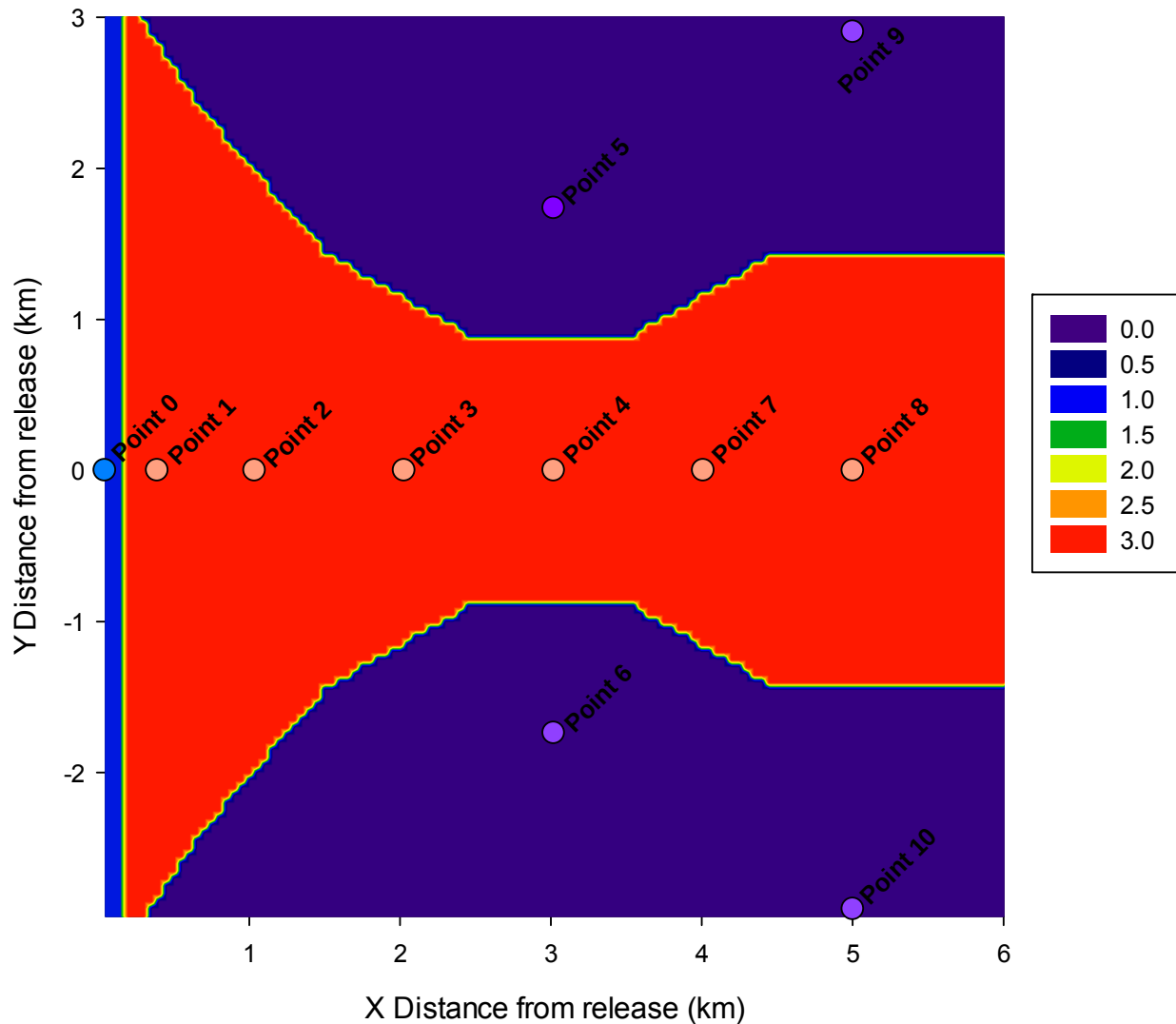


Figure 22: Spatial Representation of Grouping Created for HotSpot Data

This additional example shows that the proposed method can be applied to a variety of situations and model types with differing complexity.

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