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Quantum Chaos in Aharonov-Bohm Oscillations

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Abstract

Aharonov-Bohm oscillations in a mesoscopic ballistic
ring are considered under the influence of a resonant mag-
netic field with one and two frequencies. We investigate
the oscillations of the time-averaged electron energy at zero
temperature in the regime of an isolated quantum nonlinear
resonance and at the transition to quantum chaos, when two
quantum nonlinear resonances overlap. It is shown that the
time-averaged energy exhibits resonant behavior as a func-
tion of the magnetic flux, and has a "staircase" dependence
on the amplitude of the external field. The delocalization
of the quasi-energy eigenfunctions is analyzed.

1. Introduction

Recently persistent currents has been measured in an artificial
semiconductor ring formed in a *AlGaAs/AlAs* heterostructure [1].
The high mobility of electrons in these devices enables one to con-
sider their motion in the ring to be ballistic, and suggests the use
of the non-interacting electron gas model for the description of the
AB-oscillations. For the model of noninteracting, ballistic electrons
in a semiconducting ring, one can pose the question: Is it possible
to regulate (for example, to increase) the amplitude of the AB os-
cillations by applying an additional periodic electromagnetic field

to the system ? In [2] this problem was investigated using a non-resonant electromagnetic field with a high frequency, $\omega \gg T_0/\hbar$. In the low frequency limit the conductance in a *metallic* ring influenced by the *ac* field was studied in [3]. The first experimental results on the dynamic response of mesoscopic semiconductor rings coupled to an electromagnetic resonator, are reported in [4].

In the present article we consider a one dimensional mesoscopic ballistic ring placed into a resonant cavity. The electrons in the ring interact *resonantly* with the cavity *ac* magnetic field, and with the Aharonov-Bohm electromagnetic potential, which results in AB oscillations. The *ac* magnetic field changes significantly the electron's quasi-energy spectrum and, generally speaking, exerts a strong influence on the AB oscillations. We investigate two regimes of the resonant influence of the external *ac* magnetic field: (i) the regime of an isolated quantum nonlinear resonance (QNR), which concept was first introduced in [5] in a study of the influence of a resonant external field on quantum system with non-equidistant spectrum; and (ii) the transition to quantum chaos when two QNRs overlap. The general approach to interaction of QNRs is developed in [6,7].

We show that QNR takes place when the frequency of the external field ω resonates with the frequency of electron transitions in the vicinity of the Fermi level n_F: $\omega \approx \omega_F$ (where ω_F is the Fermi frequency). When the external field possesses two resonant frequencies, two QNRs occur in the system. Under the conditions of overlapping of two QNRs, a transition to quantum chaos takes place. In both cases, for an isolated QNR and for two overlapped QNRs, we calculate the average in time shift of the ground state energy induced by the resonant external field. We find that the energy shift exhibits characteristic resonant behavior. The structure of these resonances depends on the value of the AB magnetic flux α, on the amplitude Λ, and on the frequency ω of the *ac* magnetec field. We also investigate the behavior of the quasi-energy eigenfunctions contributing to the resonance dynamics of the system.

2. The Resonant Hamiltonian

Consider a small ballistic ring of the radius R placed in the plane (x, y) at the center of a cylindrical resonator of the radius r_0. We assume that the electrons in the ring are influenced by an external field which is described by a vector potential $\vec{A}$ consisting of two parts : $\vec{A} = \vec{A}^{AB} + \vec{A}^{RES}(t)$. Here $\vec{A}^{AB} = (0, 0, A_\varphi^{AB})$, and $A_\varphi^{AB} = \Phi/2\pi R$, describes the AB field, with Φ being the corresponding magnetic flux, whereas $\vec{A}^{RES}(t)$ describes the magnetic field created by the resonator's eigenmodes. The solution for one of the eigenmodes of the cylindrical resonator can be written in the form: $A_z^{RES} = 0$, $A_r^{RES} = -n(H/k^2 r)\sin(n\varphi)J_n(kr)\cos(\omega t - k_z z)$, $A_\varphi^{RES} = -(H/k)\cos(n\varphi)J_n'(x)|_{x=kr}\cos(\omega t - k_z z)$, where $J_n(x)$ is the Bessel function and $J'(x) \equiv dJ(x)/dx$. The frequency of this mode is $\omega^2 = c^2(k^2 + k_z^2)$. Below, we put $k_z = 0$. The wave-vector k is "quantized": $k = k_{nr}'/r_0$, where k_{nr}' is the r-th zero of the function $J_n'(x)$. For example, $k_{11}' \simeq 1.84$, $k_{12}' \simeq 5.33$, $k_{13}' \simeq 8.54$, $k_{01}' \simeq 3.83$, and $k_{02}' \simeq 7.01$. H is an arbitrary constant of order of magnitude of the magnetic field inside the resonator. In our general case, the effective Hamiltonian must take into account the resonator's two eigenmodes, with eigenfrequencies ω_1 and ω_2. Then the one-electron Hamiltonian takes the form: $\hat{H} = (\hbar^2/2mR^2)\{\partial/i\partial\varphi - \alpha - \lambda_1\sin(N\varphi)\cos\omega_1 t - \lambda_2\sin(M\varphi)\cos\omega_2 t\}^2$, where m is the electron's mass; $\alpha = \Phi/\Phi_0$, and $\lambda_{1,2} = -(2\pi R J_N'(k_{1,2}R)H_{1,2})/(\Phi_0 k_{1,2})$ are the dimensionless flux and the amplitudes of the electromagnetic waves. Because the problem we consider is defined on a ring, the wave function of an electron is periodic in the angle φ with period 2π: $\Psi(\varphi + 2\pi, t) = \Psi(\varphi, t)$. Hence, it is convenient to choose the functions $|n> = (1/\sqrt{2\pi})\exp(in\varphi)$, $(n = 0, \pm 1, \pm 2, ...)$ as a basic set. In what follows we shall consider the resonant processes in the region near the Fermi level n_F, where the values of n are rather large: $n \sim n_F \gg 1$. In addition, we assume that the amplitudes of the external field are sufficiently small: $\lambda_{1,2} \ll n_F$.

Let us now discuss some characteristic values for the parameters introduced above. Following [1], we choose $R \sim 10^{-4} cm$,

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$v_F \sim 10^7 cm/s$, $\omega_F \sim v_F/L \sim 10^{11} s^{-1}$, and $n_F \sim 10^3$. Let $r_0 \sim 10^{-2} cm$, $k'_{nr} = k'_{11} = 1.84$. Noting that $\Phi_0 \approx 4 \times 10^{-7} CGS$, we derive the relation between the dimensionless amplitude λ and the amplitude of the eigenmode of the magnetic field H in the resonator: $\lambda \approx 4H$, where the magnetic field H is measured in oersteds. Because we are dealing with the many-electron problem, we must incorporate the Pauli exclusion principle. For this purpose, we express the Hamiltonian $\hat{H}$ in terms of the operators $\hat{c}_n^\dagger$, $\hat{c}_n$ of creation and annihilation of a fermion on a given energy level n: $\{\hat{c}_n, \hat{c}_{n'}^\dagger\}_+ = \delta_{n,n'}$, $\{\hat{c}_n^\dagger, \hat{c}_{n'}^\dagger\}_+ = 0$, $\{\hat{c}_n, \hat{c}_{n'}\}_+ = 0$. Then, to lowest order in $\lambda_{1,2}/n_F$, we derive the following dimensionless Hamiltonian: $\hat{H}_e = (2mR^2/\hbar^2)\hat{H} = \sum_n (n-\alpha)^2 \hat{c}_n^\dagger \hat{c}_n + i\lambda_1(t) \sum_n n\hat{c}_{n+N}^\dagger \hat{c}_n - i\lambda_1(t) \sum_n n\hat{c}_{n-N}^\dagger \hat{c}_n + i\lambda_2(t) \sum_n n\hat{c}_{n+M}^\dagger \hat{c}_n - i\lambda_2(t) \sum_n n\hat{c}_{n-M}^\dagger \hat{c}_n$, where $\lambda_{1,2}(t) \equiv \lambda_{1,2} \cos\omega_{1,2}t$. In deriving $\hat{H}_e$, we also used the conditions $N, M \ll n_F$.

3. Isolated Quantum Nonlinear Resonance

In this section we consider the case when the resonant external field has only one eigenmode with frequency $\omega_1 \equiv \omega$, and $\lambda_1 \equiv \lambda$, $\lambda_2 = 0$. Assume that the frequency ω is resonant with the transitions between the levels with numbers n and $N + n$ of the unperturbed Hamiltonian, $\hat{H}$. Then, in the rotation wave approximation (RWA), the resonant Hamiltonian is time-independent: $\hat{\mathcal{H}}_r^{(1)} = \sum_l E_l \hat{c}_l^\dagger \hat{c}_l + i\frac{\Lambda}{2} \sum_l \hat{c}_{l+N}^\dagger \hat{c}_l - i\frac{\Lambda}{2} \sum_l \hat{c}_{l-N}^\dagger \hat{c}_l$, where $E_l = (l - \alpha)^2 + (2n_F - \omega/N\kappa)l$, $(l = n - n_F)$, and $\Lambda = \lambda n_F$. The transformation to the RWA is realized by using the unitary operator $\hat{U} = \exp\{-i(\omega/N)t \sum_n n\hat{c}_n^\dagger \hat{c}_n\}$.

4. Interaction of Two Quantum Nonlinear Resonances

Consider the case when the resonator's two eigenmodes, with frequencies ω_1 and ω_2, influence the electron's motion in the ring. In this case, two QNRs can interact strongly, and a transition to quantum chaos can occur. The equivalent resonant Hamiltonian (which describes the slow dynamics) is time-dependent and posesses 1.5 degrees of freedom. The corresponding dynamics is nonintegrable, and classically the motion is chaotic in some regions

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of phase space. In the quantum case, this kind of behavior is called "quantum chaos". Neglecting the high-frequency oscillating terms, we derive from $\hat{H}_e$ the approximate Hamiltonian describing the slow dynamics of two interacting QNRs: $\hat{\mathcal{H}}_r^{(2)} = \sum_n \left(n - [(\omega_1 - \tilde{\Delta})/2N]_{int} - \alpha \right)^2 \hat{c}_n^\dagger \hat{c}_n + i\Lambda \cos \tilde{\Delta}\tau \sum_l \hat{c}_{l+N}^\dagger \hat{c}_l - i\Lambda \cos \tilde{\Delta}\tau \sum_l \hat{c}_{l-N}^\dagger \hat{c}_l$, where $\Lambda_{1,2} \equiv n_F \lambda_{1,2}$ (for simplicity, we assumed that $\Lambda_1 = \Lambda_2 = \Lambda$), $\tilde{\Delta} = (\omega_1 - \omega_2)/2\kappa$, $\kappa = \hbar/2mR^2$. The corresponding unitary operator which realizes the transformation to the RWA has the form $\hat{U} = \exp\left\{-i\nu t \sum_n n\hat{c}_n^\dagger \hat{c}_n\right\}$, $\nu = (\omega_1 + \omega_2)/2N$.

5. Numerical Simulation

In a numerical simulation, we calculated the time-averaged energy shift: $\Delta E_0 \equiv \lim_{T\to\infty}(1/T) \int_0^T \Delta E_0(\tau)d\tau$, where $\Delta E_0(\tau) = (1/2Nn_F)[E(\tau) - E_0]$, with $E(\tau) = <\Psi(\tau)|\sum_n(n-\alpha)^2\hat{c}_n^\dagger \hat{c}_n|\Psi(\tau)>$ being the time-dependent average energy of the system, and E_0 being the energy of the ground state.

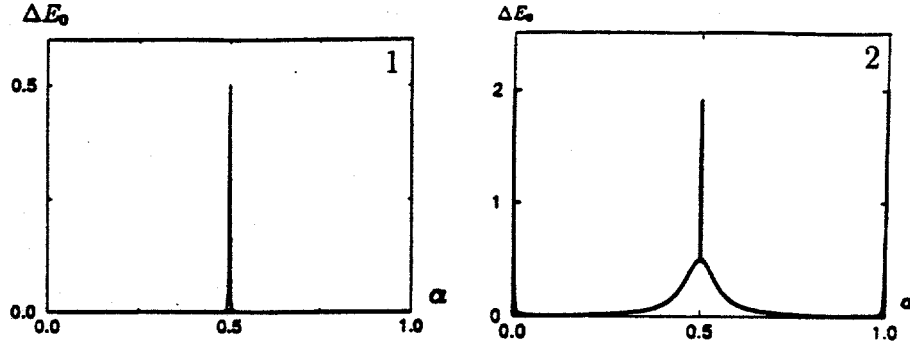Fig. 1. Dependence of the average energy shift ΔE_0 on α; $\Lambda = 0.001$; $\omega_0 = 0$; $N = 1$.

Fig. 2. Dependence of ΔE_0 on α; $\Lambda = 0.05$; $\omega_0 = 0$; $N = 1$.

Define the relation between the Fermi level n_F and the Fermi frequency ω_F: $|n_F(\alpha = 0)| = N_e = (\omega_F/2N\kappa)$, and the dimensionless detuning in frequency from ω_F: $\omega_0 = (1/2N\kappa)(\omega - \omega_F)$, where $n_e = 2N_e + 1$ is the number of electrons: in the numerical calculations presented below we assumed that the number of elec-

trons is odd. First, we discuss the numerical results on an isolated QNR considered in Sec. 3. When the resonant field with $\omega = \omega_F$ ($\omega_0 = 0$) is applied with rather small amplitude, the dynamics of electrons in the ring can be described within the two-level approximation. This is demonstrated in Fig. 1, for $\Lambda = 10^{-3}$.
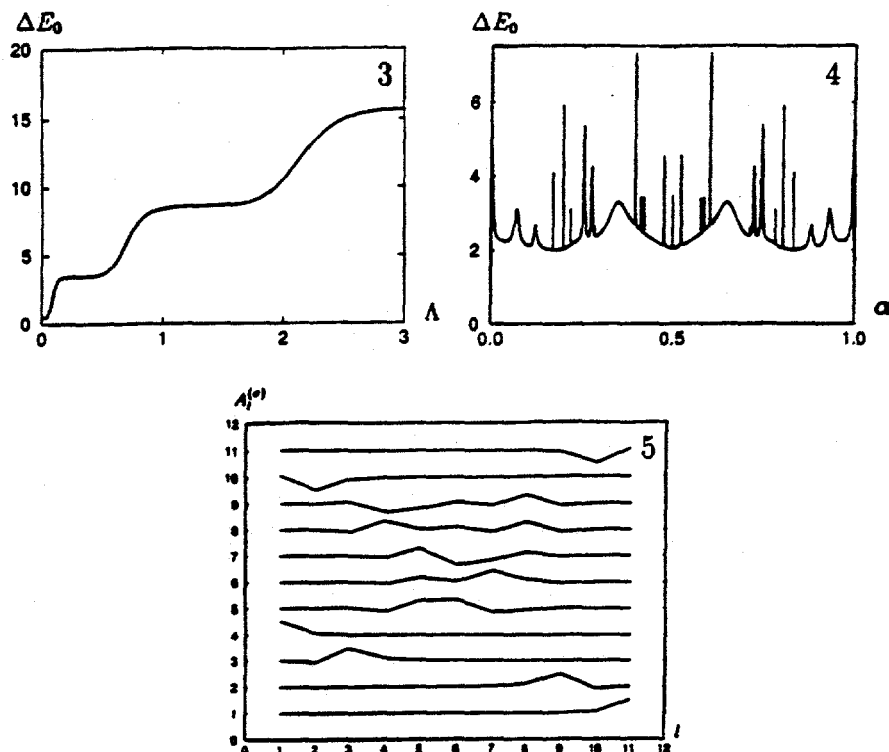

Fig. 3. Dependence of ΔE_0 on Λ; $\alpha = 0.5$; $\omega_0 = 0$; $N = 1$.

Fig. 4. Dependence of ΔE_0 on α for two interacting QNRs; $\Lambda = 1$; $\tilde{\Delta} = 1$; $\Omega = 0$; $N = 1$.

Fig. 5. Dependence of the quasi-energy functions $A_l^{(\sigma)}$ on l, for $\sigma = 1, 2, ..., 11$; $\Lambda = 1$; $\tilde{\Delta} = 1$; $\Omega = 0$; $N = 1$; $\alpha = 0.25467$ (resonant value).

In this case, when $\alpha = 0.5$, and the resonant conditions are satisfied, two levels of the unperturbed Hamiltonian contribute to the eigenfunctions, resulting in a narrow ($\delta\alpha \sim 10^{-3}$) resonance in the

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behavior of the electron energy $\Delta E_0(\alpha)$. When Λ increases, more quasi-energies are involved in the electron's dynamics, resulting in a complicated resonant structure in the dependence of ΔE_0 on α. A "double resonant" phenomenon was observed for still smaller values of Λ ($\Lambda = 0.05$), at the resonant value of α: $\alpha = 0.5$ (see Fig. 2).

Numerical calculations also show that the second resonance in Fig. 2 has complicated substructure. The "staircase" behavior of ΔE_0 on the perturbation parameter Λ is shown in Fig. 3. Notice, that the staircase behavior of the function $\Delta E_0(\Lambda)$ is well pronounced only in the vicinity of the resonance.

Let us next present the results of our numerical simulations for two interacting QNRs. Fig. 4 demonstrates the dependence of the time-averaged electron's energy $\Delta E_0(\alpha)$ on the magnetic flux α, for rather large value of the perturbation parameter Λ. This kind of resonant structure of the function ΔE_0 on α is connected with a significant modification in the structure of the quasi-energy eigenfunctions when the magnetic flux α varies. In the resonant case (Fig. 5), approximately 7 unperturbed levels give a contribution to the structure of the eigenfunction with $\sigma = 8$ (In Fig. 5, σ is the number of eigenfunction, and l is the number of the component in the unperturbed basis). This effect can be interpreted as a "delocalization" of the quasi-energy eigenfunctions in the system, when two QNRs interact strongly. Usually, the problems connected with a delocalization of the quasi-energy eigenfunctions, under the conditions of transition to quantum chaos, are investigated in the quasi-classical region of parameters: that is, for large number levels involved in the dynamics, so that $\delta n \gg 1$. In the numerical calculations presented here, we have chosen the perturbation parameter Λ not too large : $\Lambda \lesssim 3$. At these values of Λ, the characteristic number of levels is of order $\delta n \lesssim 10$. The problem of delocalization of the quasi-energy eigenfunctions under the conditions of transition to quantum chaos is not yet developed for this kind of systems.

Finally, we estimate the characteristic value of the average energy shift ΔE_0, in the presence of a resonant interaction of electrons

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in a ring with the *ac* magnetic field. For our chosen set of parameters (see Sec. 2) we find $\Delta E_0 \sim 10^{-4} - 10^{-3} eV$, which we believe indicates that this effect is within experimentally accessible limits. Thus we hope that these resonant AB oscillations can soon be investigated experimentally. The main theoretical issues for further investigations include: (a) incorporating the Coulomb interaction, which can likely be done within the framework of the Luttinger liquid model; and (b) the influence of impurities.

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