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Scaling laws in magnetized plasma turbulence

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Our work in this project concentrated on fundamental aspects of weak and strong magnetohydrodynamic turbulence, with applications to space and astrophysical plasmas. Our studies of weak MHD turbulence were devoted to analytic modeling of turbulence spectra in a strongly magnetized plasma. Weak magnetohydrodynamic turbulence is dominated by weakly interacting Alfven waves propagating in both directions along the background magnetic field. When oppositely propagating waves collide or pass each other their amplitudes deform slightly, so that it takes many independent collisions to distort a wave significantly. We discovered that such an ensemble of weakly interacting waves can spontaneously generate low frequency, nonlinear structures extended along the background magnetic field, which we call the "condensate." The spatial Fourier energy spectrum of magnetic and kinetic fluctuations in weak MHD turbulence thus contains a singular part, concentrated al low field-parallel wave numbers.

The condensate qualitatively changes the dynamics of weak MHD turbulence. In particular, it implies an excess of the magnetic energy over the kinetic energy in the inertial interval of MHD turbulence. Comparison with direct numerical simulations (also conducted by our group) reveal good agreement with the analytic prediction. In subsequent work the result was extended to the strong MHD turbulence, which is more common in natural systems. Besides its fundamental importance, our result helped to explain the observations of the solar wind turbulence, where excess of magnetic energy over the kinetic one and a slightly steeper magnetic energy spectrum are consistently observed.

Two graduate students worked on the project. They worked on analytical solution of the kinetic equations for the energies of Alfven waves in the presence of the condensate, and on comparison of the analytical predictions with the numerical data. The results were then applied to the analysis of observational solar wind data. This work lead to several publications, and numerous invited presentations at conferences:

[1] Y. Wang, S. Boldyrev, & J. C. Perez, "Residual Energy in Magnetohydrodynamic Turbulence," The Astrophysical Journal Letters, Volume 740, Issue 2, L36 (2011).

[2] S. Boldyrev, J. C. Perez, & V. Zhdankin, "Residual energy in magnetohydrodynamic turbulence and in the solar wind," in a book "Physics of the Heliosphere: a 10-year Retrospective", Jacob Heerikhuisen, Gang Li, and Gary P. Zank, Eds. (2011).

[3] S. Boldyrev, J. C. Perez, J. E. Borovsky, & J. J. Podesta, "Spectral Scaling Laws in Magnetohydrodynamic Turbulence Simulations and in the Solar Wind," The Astrophysical Journal Letters, Volume 741, Issue 1, L19 (2011).

[4] S. Boldyrev, J. C. Perez, & Y. Wang, "Residual Energy in Weak and Strong MHD Turbulence," To appear in ASP Conference Series, 6th International Conference of Numerical Modeling of Space Plasma Flows (2011), Nikolai V. Pogorelov, Ed. (2012).

Another part of the project was devoted to the strong MHD turbulence. In 2011 we analyzed the new high-resolution numerical data on strong MHD turbulence, which we obtained through the DOE 2010 IN-

CITE Award. These numerical simulations were conducted at extremely high numerical resolution (2048^3) and for very long running time (tens of large-scale dynamical times). They were the largest available direct numerical simulations of strong MHD turbulence in terms of numerical resolution and size of statistical ensemble. This study was concentrated on both balanced and imbalanced regimes of MHD turbulence (the balanced regime is the regime when the energies of Alfvén modes propagating in both directions along the guide magnetic field are the same, the imbalanced regime is when these energies are different). The goal was to test the assumptions about MHD turbulence made in currently available theories, to perform measurements of some subtle properties of MHD turbulence such as the dynamic alignment between magnetic and velocity fluctuations and the spectra of counter-propagating modes in the imbalanced case, together with the more standard measurements of the energy spectrum of MHD turbulence. This work was done in collaboration with research scientist Jean Carlos Perez, and with the University of Chicago group (Prof. Fausto Cattaneo, Dr. Joanne Mason).

Several new results were obtained. First, we confirmed that the Fourier energy spectrum of strong MHD turbulence in the presence of a strong guide field is proportional to $k^{-3/2}$. Second, we discovered that the scale-dependent alignment between the magnetic and velocity fluctuations persists not only in the inertial interval, but also throughout the whole dissipation interval, up to the smallest scales resolved in numerical simulations. We called this phenomenon "extended scaling law" in MHD turbulence. Third, we found that the spectra of co- and counterpropagating (along the background magnetic field) Alfvén modes have the same scalings, although they may have different amplitudes, if the turbulence is imbalanced. Together with the results obtained in smaller numerical simulations, this project lead to several publications:

[5] J. Mason, J. C. Perez, F. Cattaneo, & S. Boldyrev, "Extended Scaling Laws in Numerical Simulations of Magnetohydrodynamic Turbulence," The Astrophysical Journal Letters, Volume 735, Issue 2, L26 (2011).

[6] J. Mason, J.C. Perez, S. Boldyrev, & F. Cattaneo, "Numerical simulations of strong incompressible magnetohydrodynamic turbulence," Physics of Plasmas, Volume 19, 055902 (2012).

[7] J. C. Perez, J. Mason, S. Boldyrev, & F. Cattaneo, "On the energy spectrum of strong magnetohydrodynamic turbulence," Physical Review X, 2 (2012) 041005.

[8] J. C. Perez, J. Mason, S. Boldyrev, & F. Cattaneo, "Scaling Properties of Small-scale Fluctuations in Magnetohydrodynamic Turbulence," Astrophys. J. Letters, 791 (2014) L13.

[9] J. Mason, S. Boldyrev, F. Cattaneo, J.C. Perez, "The statistics of a passive scalar in field-guided magnetohydrodynamic turbulence," Geophysical & Astrophysical Fluid Dynamics, 108 (2014) 686.

Another project started in 2012 was devoted to analysis of magnetic discontinuities in MHD turbulence and in the solar wind. Recent measurements of solar wind turbulence reported the presence of intermittent, exponentially distributed angular discontinuities in the magnetic field. Observational data also pointed to the existence of two populations of such discontinuities, characterized by different typical angular shifts. The questions we studied were whether the presence of such discontinuities, and especially observations of two populations of the discontinuities, indicated that the solar wind turbulence is not universal or whether it could still be explained in the framework of universal MHD turbulence. Together with the graduate student we analyzed the data obtained in numerical simulations of MHD turbulence and compared the results with the solar wind measurements. We demonstrated that the probability density functions of the discontinuities measured in the solar wind can be explained by MHD turbulence, if the strength of the uniform background field is chosen accordingly. The required strength of the background magnetic field turns out to be in good agreement with the parameters of the solar wind. The results were presented at the annual APS Division of Plasma Physics meeting and published in Physical Review Letters:

[10] V. Zhdankin, S. Boldyrev, J. Mason & J. C. Perez, "Magnetic Discontinuities in Magnetohydrodynamic Turbulence and in the Solar Wind," Physical Review Letters, vol. 108, Issue 17, id. 175004 (2012).

This study then developed into a bigger project on the statistical analysis of the dissipation structures formed in magnetohydrodynamic turbulence. These structures have been observed in numerical simulations to resemble current sheets, which in many (although not all) instances can be associated with the sites where the topology of magnetic field lines changes abruptly (reconnection sites). These are the sites where the magnetic energy dissipates due to ohmic heating. These sites of energy dissipation and fluid heating turn out to be very intermittent. For example, 40% of the magnetic energy gets dissipated in about 2% of the volume. In high-energy astrophysical systems, a highly inhomogeneous temperature profile may result when strong prompt radiative cooling removes energy from localized dissipation sites more rapidly than it can be redistributed in the medium. Examples of such systems include quasars, accretion disks and flows, and hot X-ray gas in galaxy clusters.

A similar problem of current sheet distribution is known in the solar corona heating, the Parker "nanoflare" model. A graduate student Vladimir Zhdankin has been working on this project. He has developed a numerical algorithm allowed one to characterize the statistical properties of the intense dissipation structures: the distribution of their thicknesses, widths, and lengths. It turned out that the dominant contribution to the dissipation comes from the current sheets whose thicknesses are well inside the dissipation scale, while widths and lengths span the inertial interval (see Fig. 1). As the Reynolds number increases, the thin and extended current sheets become more numerous and tightly packed, as illustrated in Fig. (2). The thickness of the current sheets decreases, while the length and width continue to span the inertial interval. Such dissipation structures thus have the features of both "nanoflares" (as they become thinner and more numerous) and coherent structures (as they are still extended over the scales of the inertial interval). The results are published in:

[11] V. Zhdankin, S. Boldyrev, J. Mason, "Distribution of Magnetic Discontinuities in the Solar Wind and in Magnetohydrodynamic Turbulence," Astrophys. J. Lett., 760 (2012) L22.

[12] V. Zhdankin, D. A. Uzdensky, J. C. Perez, S. Boldyrev, "Statistical Analysis of Current Sheets in Three-dimensional Magnetohydrodynamic Turbulence," Astrophys. J. 771 (2013) 124.

[13] V. Zhdankin, S. Boldyrev, J. C. Perez, S. M. Tobias, "Energy Dissipation in Magnetohydrodynamic Turbulence: Coherent Structures or "Nanoflares"? Astrophys. J. 795 (2014) 127.

[14] V. Zhdankin, D. A. Uzdensky, S. Boldyrev, "Temporal Intermittency of Energy Dissipation in Magnetohydrodynamic Turbulence," Phys. Rev. Lett. 114 (2015) 065002.

Another part of the project was devoted to the study of plasma turbulence at the scales smaller than the ion gyroscale. At such scales the assumptions of one-fluid MHD breaks down and the nature of turbulence changes. At sub-proton scales the shear-Alfvén cascade transforms into the cascade of strongly anisotropic kinetic-Alfvén modes. Kinetic Alfvén turbulence attracts considerable interest due to its importance for solar wind heating, magnetic reconnection in a variety of astrophysical systems, and laboratory experiments with strongly magnetized plasmas. Such turbulence has been understood to a much lesser extent compared to MHD turbulence. Our consideration was, in part, motivated by recent measurements of small-scale magnetic and density fluctuations in the solar wind, which suggest that their Fourier energy spectrum is close to or possibly steeper than $k^{-2.8}$. The nature of such fluctuations is unclear as they are not described by existing models of either kinetic-Alfvén or electron magnetohydrodynamic turbulence, which predict the scaling $k^{-7/3}$. We have started with analyzing the so-called kinetic Alfven turbulence, and its role in the turbulent cascade at sub-proton scales. We have conducted numerical simulations using the newly developed two-fluid code. Two postdoctoral recearchers are participated in this study:

[15] S. Boldyrev & J. C. Perez, "Spectrum of Kinetic Alfven Turbulence," Astrophys. J. 758 (2012) L44.

[16] S. Boldyrev, K. Horaites, Q. Xia, J. C. Perez, "Toward a Theory of Astrophysical Plasma Turbulence at Subproton Scales," Astrophys. J. 777 (2013) 41.

[17] C. H. K. Chen, S. Boldyrev, Q. Xia, J. C. Perez, "Nature of Subproton Scale Turbulence in the Solar Wind," Phys. Re. Lett. 110 (2013) 225002.

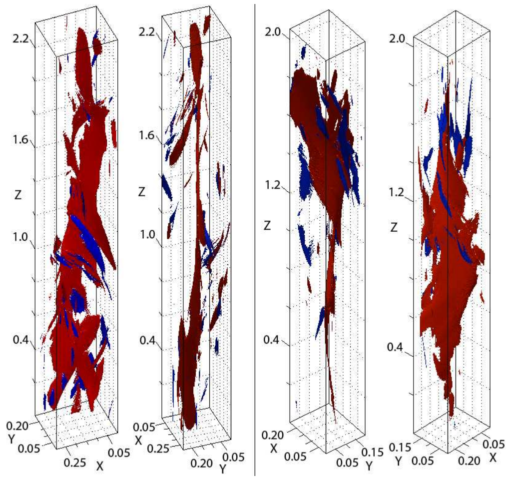

Figure 1: Samples of typical large current sheets in part of the simulation domain (in red), surrounded by several smaller structures (mostly in blue). The left panel shows two orientations of one structure, while the right panel shows two orientations of another separate structure. These samples are taken from the $Re = 1800$ case with a threshold of $j_{\mathrm{thr}}/j_{\mathrm{rms}} \approx 6.5$.

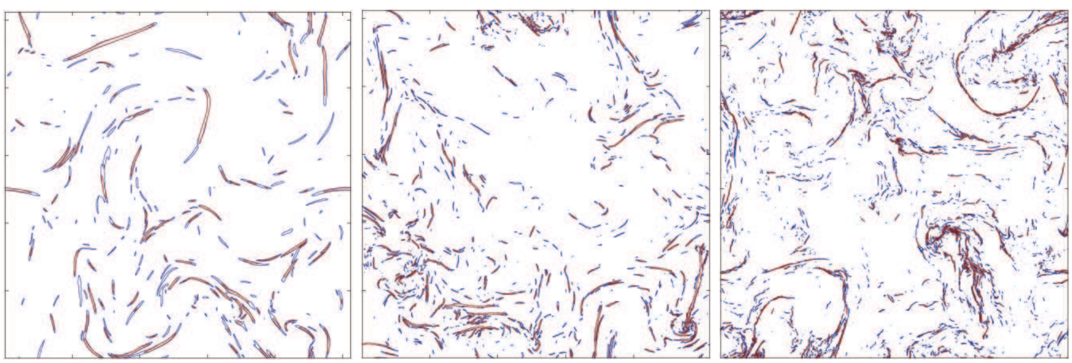

Figure 2: Contours of current density in an arbitrary plane perpendicular to the guide field. Contours are taken at $j_{\mathrm{thr}}/j_{\mathrm{rms}} = 2$ (blue) and $j_{\mathrm{thr}}/j_{\mathrm{rms}} = 3$ (red) for increasing Reynolds number (from left to right, $Re = 1000$, 3200, and 9000).

[18] C. H. K. Chen, L. Leung, S. Boldyrev, B. A. Maruca, S. D. Bale, "Ion-scale spectral break of solar wind turbulence at high and low beta," Geophysical Research Letters, 41, (2014) 8081.

Finally, a part of the roject was devoted to the study of heat conduction in the solar wind plasma. Recently, various regimes of heat conduction (collisional Sptizer-Härm, and col invariant solutions for a particular relation among the magnetic, density, and temperature scaling exponents. For the scale-invariant solution to exist, one has to require that the ratio of the electron mean free path to the temperature gradient scale is constant ($\gamma = \lambda_{mfp}/L_T$, where $\lambda_{mfp} = \mathrm{const}$). There solutions for the distribution function describe the heat flux in both Spitzer-Härm and collisionless regimes, depending on the value of γ.

We observe that the solar wind parameters are actually very close to the self-similar regime; for the Helios data at 0.3-1 AU we find $\gamma = \mathrm{const}$, and the scaling exponents for density, temperature, and heat flux are close to those dictated by the requirement of scale invariance. We find steady state solutions of the self-similar kinetic equation numerically, and show that these solutions closely reproduce the electron strahl population seen in the solar wind, as well as the transition between the Spitzer-Härm and collisionless heat flux regimes at $\gamma \approx 0.1$, see Fig. (3). The work is currently in progress, although some results have been

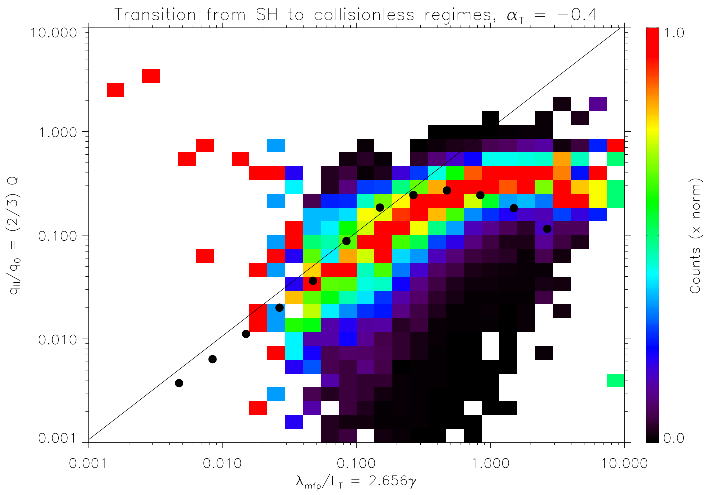

Figure 3: 2D histogram of $q/q0$ versus heliocentric distance λ_{mfp}/L_T, derived from fits to Helios electron Velocity Distribution Function data, correcting for the Parker spiral angle and assuming $T \sim r^{-2/5}$. Each column is normalized by its peak, to bring out the functional dependence. The Spitzer-Härm prediction is shown as a line. Results of our simulations are shown as dots. At about $\gamma \approx 0.3$ there observed the transition to the free-streaming regime (flattening of the curve).

already published:

[19] K. Horaites, S. Boldyrev, S. I. Krasheninnikov, C. Salem, S. D. Bale, and M. Pulupa, "Self-Similar Theory of Thermal Conduction and Application to the Solar Wind," Phys. Rev. Lett. 114 (2015) 245003.

Some other papers, with significant input from phenomenological and numerical studies of MHD turbulence, were also supported by the grant. Totally, more that 25 refereed papers were supported by this grant.