

DEVELOPMENT OF SMALL, FAST REACTOR CORE DESIGNS
USING LEAD-BASED COOLANT*

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ABSTRACT

A variety of small (100 MWe) fast reactor core designs are developed; these include compact configurations, long-lived (15-year fuel lifetime) cores, and derated, natural circulation designs. Trade studies are described which identify key core design issues for lead-based coolant systems. Performance parameters and reactivity feedback coefficients are compared for lead-bismuth eutectic (LBE) and sodium-cooled cores of consistent design. The results of these studies indicate that the superior neutron reflection capability of lead alloys reduces the enrichment and burnup swing compared to conventional sodium-cooled systems; however, the discharge fluence is significantly increased. The size requirement for long-lived systems is constrained by reactivity loss considerations, not fuel burnup or fluence limits. The derated lead-alloy cooled natural circulation cores require a core volume roughly eight times greater than conventional compact systems. In general, reactivity coefficients important for passive safety performance are less favorable for the larger, derated configurations.

I. INTRODUCTION

Several recent energy production planning studies have identified a potential market for small nuclear power plants for international deployment - to address the combination of growing energy demands in developing countries, the desire to minimize environmental impacts of greenhouse gas emissions, and the expense of transporting fossil fuels. Liquid metal cooled fast reactors (LMRs) have several advantages for this application: 1) the high power density of LMRs facilitates development of compact reactor configurations which are more easily transported, 2) the inherent reactivity feedback behavior of small LMRs facilitates a high degree of passive safety,^{1,2} and 3) modular fabrication techniques have been shown to be feasible for LMRs in the size range envisioned (~100 MWe).³

Proliferation resistance is a key reactor design concern, particularly for systems designed for international export and utilization. To prevent diversion, it is desirable to make access to the fresh and spent reactor fuel extremely difficult.

One way this can be achieved is by eliminating the need and capability for on-site fuel handling. A variety of design features which meet this goal can be envisioned. The reactor could be refueled using specialized refueling equipment which is securely stored off-site; and the refueling operations could be conducted by an outside entity under appropriate safeguards. Alternatively, the core could be designed for whole-core refueling on an infrequent basis. In this case, the design would include structural barriers to prevent the removal of individual fuel assemblies. For both these approaches, a long-lived core design is a key design goal, to reduce the frequency of refueling operations. Ideally, the core lifetime could be extended to the reactor lifetime and the system could be completely sealed, although this raises a variety of reliability and maintenance concerns. In this paper, the potential for long-lived reactor core designs is evaluated; specifically a 15-year, once-through fuel cycle (no partial reloads) is targeted.

LMRs have several advantages for applications where fuel handling is limited: 1) for typical pool designs, refueling operations are not simple, requiring significant effort to access (easily detected) and maneuver within the liquid metal pool surrounding the core, 2) in-vessel systems are typically designed for long-life and remote maintenance within the liquid metal pool, 3) with regard to extended fuel lifetime, fast reactor fuel forms and structural materials can achieve high burnup (experimental tests up to 20% atom peak burnup), 4) the use of liquid metal coolant with its excellent heat removal capabilities allows a reduced coolant volume fraction and achievable temperature increase across the core (150-200°C); the higher fuel concentration allows more compact long-lived designs, and 5) the fast neutron energy spectrum and high heavy metal loading enhance internal conversion; the fast spectrum also makes parasitic absorption in the fission products relatively unimportant. Thus, reactivity losses can be effectively compensated by the inclusion of fertile material, which tends to reduce the enrichment and excess reactivity requirements of the long-lived system.

In general, significant extension of the fuel lifetime requires derating of the core power density. Operation at

reduced power density facilitates the use of heavy liquid metals as the coolant; whereas, at high power density, the pumping power requirements become excessive. One option explored in recent design studies is the use of lead-based alloys. Recently declassified Russian submarine technology for small lead-bismuth eutectic (LBE) cooled reactors provides a technology basis; and small, transportable systems using this technology are being developed by Russian experts.⁴ Because lead alloys do not react with water, the heat transfer systems commonly employed in sodium-cooled designs may be streamlined (or possibly eliminated); concepts for system simplification and improved thermal efficiency are being investigated.⁵ Moreover, the boiling point of lead alloys is much higher than sodium and provides greater margin to coolant boiling in accident scenarios. With the prospect of future development of suitable structural materials for high temperature operation (~1000°C), the use of lead-based alloys may eventually enable reactor operation at higher temperatures with improved thermal efficiency and would pave the way for the use of advanced power conversion cycles and energy conversion technology such as hydrogen production.

II. DESCRIPTION OF FUEL CYCLE MODEL AND COMPUTATIONAL METHODS

For this study, the fuel form and fissile feed were chosen to maximize the fertile loading of the small core, which reduces the reactivity loss rate; as shown in subsequent sections, reactivity losses are the limiting design parameter for long-lived cores. Thus, metal fuel was utilized with its high heavy metal density of 14.3 g/cm³, and the fissile feed was weapons plutonium which is 94% fissile plutonium. Ternary (UPuZr) metal fuel alloy pins in HT-9 ferritic cladding have achieved 200 MWd/kg peak discharge burnup and are qualified and demonstrated for 100 MWd/kg average and 150 MWd/kg peak discharge burnup. The fluence limit for the low-swelling HT-9 steel alloy developed in the U.S. fast reactor program is 3.8-4.0x10²³n/cm².

In the fuel cycle model, a once-through fuel cycle supplied by two external feed streams, weapons plutonium and depleted uranium, was applied. The equilibrium cycle performance characteristics and material distributions were calculated using the REBUS-3 fuel cycle code.⁶ The initial plutonium enrichment was determined using the REBUS-3 search techniques.

The REBUS-3 flux calculations were performed using a three-dimensional (hexagonal-Z) nodal diffusion method⁷ and a twenty-one energy group structure. The depletion calculation utilizes burnup chains for nuclides ranging from

U-234 to Cm-246. The region dependent broad group cross sections are based on ENDF/B-V.2 data and were generated using the MC²-2⁸ and SDX⁹ codes. New group constants were generated for lead and bismuth based on a lattice calculation where the sodium coolant was replaced with LBE. Reactivity feedback coefficients were evaluated using the finite-difference diffusion theory option of the DIF3D code¹⁰ and 21 group cross sections.

III. HIGH POWER DENSITY (COMPACT) REFERENCE DESIGN

Conventional LMR systems are compact with a high power density. The core size is constrained by peak power density limits, and the fuel lifetime is constrained by both burnup and fluence limits typically to ~3-5 years lifetime. For this study, a small 100 MWe compact configuration was developed to serve as a point of comparison for the long-lived designs. A conventional tight-pitch fuel assembly was utilized with fuel, coolant, and structure volume fractions of 38, 37, and 25% respectively; and an active core height of one meter was assumed. In addition, two-region radial enrichment zoning was applied with an outer-to-inner core enrichment ratio of 1.2. A cycle length of one year at a capacity factor of 80% was employed. Several core configurations were evaluated to optimize the power peaking and burnup performance. The resulting minimum volume core configuration is shown in Fig. 1. This core is composed of only 39 driver assemblies surrounded by a row of steel reflector and a row of boron-carbide shield assemblies.

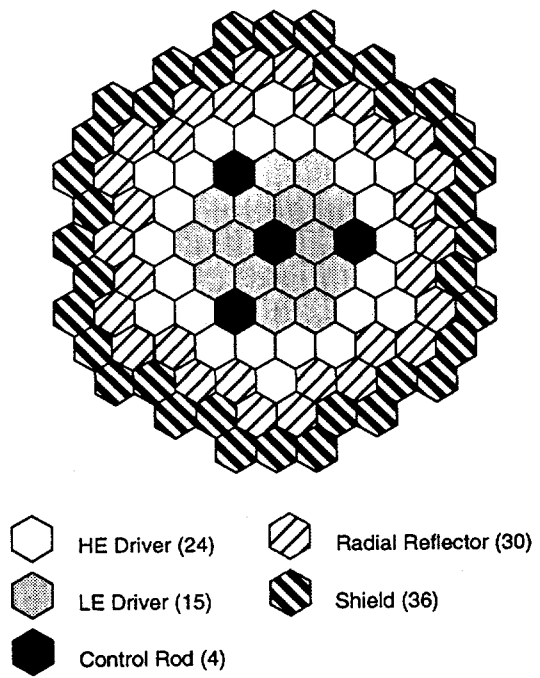


Fig. 1. 100 MWe Compact Core Configuration

Basic performance parameters for this design are summarized in Table 1. The peak linear power is 450 W/cm, near the design limit (~500 W/cm) for metal fuel pins. In addition, the peak discharge burnup and fluence are near the limits of 150 MWd/kg and 3.8×10^{23} n/cm² respectively. The burnup reactivity loss over a one year cycle is fairly high at 5.4%Δk, as expected for a small, high leakage homogeneous configuration. For comparison purposes only, the neutronic performance was also evaluated with lead-bismuth eutectic (LBE) replacing the sodium (note that direct LBE substitution is not actually feasible for compact configurations because the high coolant velocity required would lead to excessive power pumping requirements for the heavier metal). The power peaking and burnup performance is quite similar to the sodium-cooled design. Because the LBE is a superior neutron reflecting material, the enrichment decreases by 1.5%. However, this increases the flux level (to attain a similar power density) and leads to a substantial increase in neutron fluence to 4.35×10^{23} n/cm², which is beyond the design limit. Thus, use of LBE coolant in compact designs

Table 1. Performance Comparison for Compact Core Designs

Performance Parameter	Fig. 1. Vol. = 880 liters	
	Sodium	LBE
Coolant		
Enrichment, wt.% Pu/HM, Inner	19	17.5
Outer	22.5	21
Burnup Reactivity Swing, %Δk	5.43	5.41
Peak Linear Power, W/cm	449	459
Average Burnup, MWd/kg	97	97
Peak Burnup, MWd/kg	142	143
Peak Fast Fluence, 10^{23} n/cm ²	3.72	4.35

alters the burnup to fluence ratio; and fuel lifetime in compact LBE-cooled cores is limited by the allowable discharge fluence.

IV. LONG-LIVED CORE DESIGN STUDIES

Next, scoping studies to develop long-lived core designs were undertaken, and key design criteria were identified. For this study, proven fuel materials and the tight lattice design described in Section III are retained; and a long cycle length is achieved by power derating. The fuel lifetime for the compact configuration described in Section III is 4 years. This implies that the power rating would have to be reduced by roughly a factor of 4 to achieve a 15 year fuel lifetime. Using the conventional tight lattice assembly design, the power density can be increased by adding additional assemblies; results are given in Table 2 for a configuration using 150 driver assemblies. The peak linear power decreases to ~115 W/cm as expected. The increased core volume leads to a lower enrichment requirement, and yields a higher discharge fluence. If a fuel management strategy similar to the compact design is employed (four batch refueling), the burnup swing is reduced to 3.5%Δk. However, the long-core lifetime objective (15-year interval with no fuel handling) precludes all but one batch fuel management strategy. As shown in Table 2, the key performance change for a one- batch scheme is roughly a factor of 4 increase in the burnup reactivity swing, since the core is going from all fresh fuel to all fully burned fuel over its cycle. The burnup reactivity loss of 19%Δk is extremely large and would be difficult to manage with conventional reactivity compensation techniques.

The large reactivity loss of a single batch fueling scheme can be reduced to a manageable level by further increasing the core volume to add fertile material.

Table 2. Performance Comparison for Long-Lived Design Study

Performance Parameter	Compact, Fig. 1	150 Assemblies		228 Assemblies	
		Sodium	Sodium	Sodium	LBE
Coolant					
Core Volume, liters	880	3385	3385	5145	5145
Fuel Residence Time, full-power days	1168	4380	4380	4380	4380
# of Batches	4	4	1	1	1
Enrichment, wt.% Pu/HM, -					
Inner	19	13	15	12	10
Outer	22.5	16	18	15	13
Burnup Reactivity Swing, %Δk	5.43	3.64	18.7	7.17	3.30
Peak Linear Power, W/cm	449	118	108	72	75
Average Burnup, MWd/kg	97	94	94	62	61
Peak Burnup, MWd/kg	142	140	141	93	95
Peak Fast Fluence, 10^{23} n/cm ²	3.72	4.37	3.98	2.97	3.33

Additional results are shown in Table 2 for a 50% further increase in core size (to 228 assemblies). In this case, the burnup reactivity swing decreased to 7.2%Δk. Note that the fuel is no longer irradiated to its discharge burnup limit in this case since the core volume has been increased at a constant fuel residence time; the resulting average discharge burnup is ~60 MWd/kg, well below the conventional 100 MWd/kg level.

For the 228-assembly configuration, core performance was computed for direct replacement of the sodium coolant with LBE; results are given in Table 2. As observed before, the LBE reduces the enrichment requirement. The reduced enrichment implies an increased fertile loading, and this has the favorable effect of significantly decreasing the burnup reactivity swing (from 7.2%Δk to 3.3%Δk). The reduced enrichment also leads to an increased fluence, but the partial fuel burnup keeps the fluence level well below the design limit.

V. DEVELOPMENT OF AN OPTIMIZED DERATED LIQUID LEAD ALLOY COOLED DESIGN

The power derating required to achieve long-lived, single batch core designs (as demonstrated in Section IV) obviously results in an economic penalty, higher capital cost per unit power produced. However, at reduced power density it should be possible to simplify or eliminate some reactor systems, with associated economic benefits. For example, the favorable heat transfer characteristics of liquid metals may allow natural circulation cooling at derated power densities, greatly simplifying the primary cooling system. The use of lead alloy coolant offers the additional potential to remove the secondary heat transfer loop as investigated in Ref. 4. Thus, the development of natural circulation LBE cooled designs is investigated in this section.

From a core design perspective, the primary change for natural circulation systems is that the coolant volume fraction must be increased to ~60%. Based on nuclear and thermalhydraulic design considerations, a candidate assembly design was developed which has fuel, coolant, and structure volume fractions of 30, 58, and 12% respectively. This design does not employ assembly duct walls, as utilized in conventional tight lattice designs, but is an array of pins supported by a structural grid. The reduced fuel volume fraction of this assembly design requires a commensurate increase in the core volume to roughly conserve the fertile content (and limit burnup reactivity losses as shown in Section IV).

An extensive series of trade studies was performed to develop a favorable configuration for the derated natural

circulation design. The results of these trade studies provided numerous insights: 1) The neutronics impact of various lead alloys were evaluated. Bismuth is not as good of a neutron reflector as lead, and LBE has a ~1%Δk penalty in burnup swing compared to pure lead. 2) The importance of accurate fission product modeling was demonstrated. Because the core composition changes from no fission products at BOC to all fully burned assemblies at EOC, the impact of fission products on the EOC reactivity is magnified. 3) Two-region enrichment zoning reduces the burnup swing by 1.5%Δk compared to a one-zone design; the optimal enrichment split for the geometries investigated in this study was 1.40. 4) A unique aspect of the LBE cooled designs is the sensitivity to modeling of the "reflector" regions surrounding the active core. Because LBE is a superior reflector, the enrichment and burnup swing can be reduced considerably (up to 5%Δk) by utilizing LBE reflector zones; and the reflector savings extend out to thicknesses of ~one meter because the absorption cross sections are so low. However, for similar reasons LBE reflectors are not very effective in shielding the in-vessel structures. Preliminary results indicate that reflector regions of high steel content are required for structural damage considerations; thus, the superior LBE reflection could not be exploited in the outer regions.

Based on these trade studies, an optimal natural circulation core design with a core diameter of roughly 2.5 meters and an active core height of two meters was developed; the 150 assembly computational model is shown in Fig. 2. The performance parameters for this design are summarized in Table 3. The core volume is 6815 liters (7.75 times larger than the compact design in Section III). The fuel enrichment is quite low, ~10% Pu/HM. The derating leads to an average discharge burnup after 15 years at 80% capacity factor of only 58 MWd/kg. The burnup reactivity swing is quite manageable at 2.4%Δk. For comparison, the LBE coolant was replaced with sodium coolant; performance results are also presented in Table 3. With the sodium coolant, increased leakage causes the enrichment to increase by roughly 2% leading to a higher burnup reactivity loss (6.2%Δk).

Table 3. Performance Comparison for Derated Natural Circulation Designs

Performance Parameter	Fig. 2. Vol. = 6815 liters	
	LBE	Sodium
Coolant	LBE	Sodium
Enrichment, wt.% Pu/HM, Inner Outer	9.2	11
	13	15
Burnup Reactivity Swing, %Δk	2.35	6.21
Peak Linear Power, W/cm	60	60
Average Burnup, MWd/kg	58	58
Peak Burnup, MWd/kg	102	92
Peak Fast Fluence, 10 ²³ n/cm ²	3.62	2.68

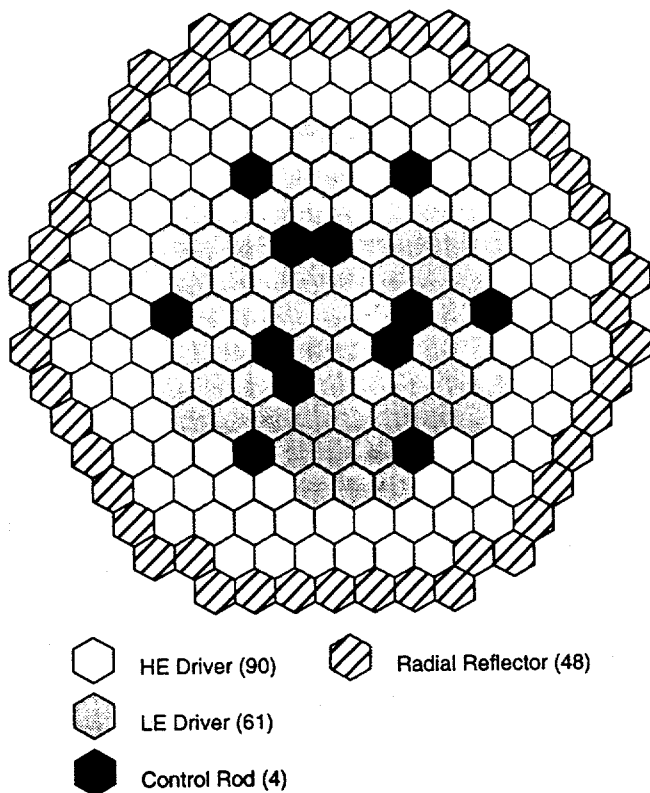


Fig. 2. Derated Natural Circulation Configuration

VI. COMPARISON OF REACTIVITY COEFFICIENTS FOR SMALL LEAD AND SODIUM-COOLED CORES

In this section, reactivity coefficients are compared for the compact (Fig. 1) and derated (Fig. 2) core design options. The compact design was developed for a tight pin lattice with sodium coolant and one year partial refueling interval, as described in Section III; results using LBE coolant were calculated for comparison. The derated design was developed for a natural circulation LBE lattice and 15-year, single batch fuel cycle, as described in Section V;

results using sodium coolant were calculated for comparison. Results are summarized in Table 4 for the kinetics parameters, beginning of cycle excess reactivity, coolant void worth, core expansion coefficients, and Doppler coefficient.

For all cases, the delayed neutron fraction is 3.2 to 3.3E-3, characteristic of a plutonium-fueled fast reactor. The excess reactivity given in Table 4 is calculated from the burnup swing results presented in previous sections; and the derated LBE design has a significantly lower burnup swing than the other cases. As observed in previous studies, the LBE coolant leads to significant reductions in the coolant void worth compared to sodium. For the compact configuration, the void worth is \$0.50 for sodium and -\$6.5 for lead. The void worths are much larger for the derated configuration where the leakage has been significantly reduced owing to the larger core volume. In the derated core, the sodium void worth is \$15, and the LBE void worth is \$3.

The Doppler coefficient is small for the high enrichment compact core (-2×10^{-3} Tdk/dT). Because the U-238 content is much higher in the derated design, the Doppler coefficient roughly doubles. Results also indicate that the Doppler is higher in sodium-cooled cores probably due to a higher flux fraction in the resonance range (greater downscattering compared to LBE).

Finally, the radial and axial expansion coefficients are dictated by core leakage effects. They are somewhat lower with LBE coolant (e.g., radial expansion coefficient is -\$4.2/cm for sodium compared to -\$3.4/cm for LBE in the compact design). In addition, the expansion coefficients are significantly lower in the derated design because of reduced leakage; both the axial and radial expansion coefficients decrease by roughly a factor of four.

Table 4. Reactivity Coefficients for Sodium and Lead Cooled Systems

Performance Parameter	Compact, Fig. 1		Derated, Fig. 2	
	Sodium	LBE	Sodium	LBE
Delayed Neutron Fraction	3.19E-3	3.19E-3	3.32E-3	3.26E-3
Prompt Neutron Lifetime, s	2.85E-7	2.86E-7	4.70E-7	4.53E-7
BOC Excess Reactivity, \$	17.0	16.9	18.7	7.21
Coolant Void Worth, \$	0.56	-6.49	14.9	3.12
Doppler, 10^{-3} Tdk/dT	-2.12	-1.63	-4.89	-2.71
Radial Expansion, \$/cm	-4.18	-3.44	-1.12	-1.10
Axial Expansion, \$/cm	-1.06	-1.04	-0.28	-0.38
CRD Expansion, \$/cm	-0.07	-0.10	-0.03	-0.04

VII. SUMMARY AND CONCLUSIONS

A variety of small (100 MWe) fast reactor core designs were studied to move systemically from conventional compact high-power density, sodium-cooled designs to derated, long-lived, single batch, lead alloy cooled natural circulation designs. Trade studies were performed to identify key modeling and design issues for long-lived and LBE-cooled systems.

The long-lived core design studies clarified several key design considerations: 1) if a one-batch fuel management strategy is employed, burnup reactivity losses (rather than fuel burnup or fluence limits) will be the limiting performance attribute, 2) the reactor size will be constrained by reactivity loss considerations, not fuel burnup or fluence limits, and 3) the superior neutron reflection capability of LBE reduces the enrichment and burnup swing compared to sodium-cooled designs (although increased discharge fluence must be accommodated).

Conventional compact core designs (high-power density with sodium coolant) for the small power ratings considered in this study (100 MWe) can be quite small - around 880 liters total active volume. However, to achieve the LBE natural circulation design with a fuel lifetime of 15 years with no refueling requires the power density to be reduced by roughly a factor of eight (a core volume of 6800 liters). Potential design simplifications to offset the economic penalties of this derating arise through the elimination of primary pumps, refueling equipment, and the intermediate heat transport loop.⁵

Reactivity coefficients were compared for sodium and LBE coolant for both compact and derated configurations. In general, reactivity coefficients important for passive safety performance are less favorable for the larger, derated configuration. The void worth is much more positive because of the reduced neutron leakage; and the geometric expansion coefficients are also reduced. Conversely, the Doppler coefficient magnitude is significantly larger in the derated design. Compared to sodium-cooled systems, the LBE cores have significantly reduced coolant void worth; however, the expansion coefficients and Doppler are less negative for the LBE-cooled system.

On the other hand, accident initiators and safety margins are likely improved for the larger, derated designs. Reliance on natural circulation eliminates the loss of pumping power accident class and the use of lead alloys vastly increases the margin to boiling. Results in this study indicate that burnup reactivity (and potential TOP initiators) can be made comparable. The overall passive safety performance of derated LBE configurations has not yet been

compared to conventional small LMR systems (where the performance is favorable) in a systematic manner and will be the focus of future work.

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