SANDIA REPORT
SAND2015-4352
Unlimited Release
Printed May 2015

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Unified Creep Plasticity Damage (UCPD) Model for Rigid Polyurethane Foams

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Abstract

Numerous experiments were performed to characterize the mechanical response of several different rigid polyurethane foams (FR3712, PMDI10, PMDI20, and TufFoam35) to large deformation. In these experiments, the effects of load path, loading rate, and temperature were investigated. Results from these experiments indicated that rigid polyurethane foams exhibit significant volumetric and deviatoric plasticity when they are compressed. Rigid polyurethane foams were also found to be very strain-rate and temperature dependent. These foams are also rather brittle and crack when loaded to small strains in tension or to larger strains in compression. Thus, a new Unified Creep Plasticity Damage (UCPD) model was developed and implemented into SIERRA with the name Foam Damage to describe the mechanical response of these foams to large deformation at a variety of temperatures and strain rates. This report includes a description of recent experiments and experimental findings. Next, development of a UCPD model for rigid, polyurethane foams is described. Selection of material parameters for a variety of rigid polyurethane foams is then discussed and finite element simulations with the new UCPD model are compared with experimental results to show behavior that can be captured with this model.

Acknowledgments

The authors are thankful for early experimental studies of polyurethane foam response to hydrostatic and tri-axial compression performed by W.A. Olsson, Sandia retired, and S.J. Bauer, 6914, which were very informative. We would also like to thank S.R. Subia, 1541, for his assistance in setting up the fully coupled thermal stress simulations and K.L. Troyer, 1526 for a thorough technical review of this report.

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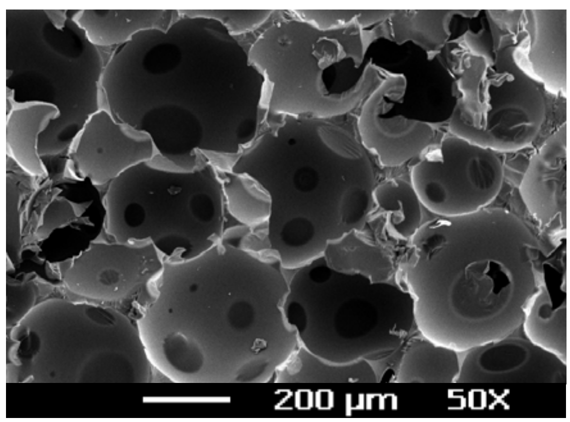
Nomenclature

DOE	Department of Energy
FR	fire retardant
GP	General Plastics
pcf	pounds per cubic foot
PMDI	Polymethylene Diisocyanate
SEM	Scanning Electron Microscopy
SNL	Sandia National Laboratories
UCPD	Unified Creep Plasticity Damage

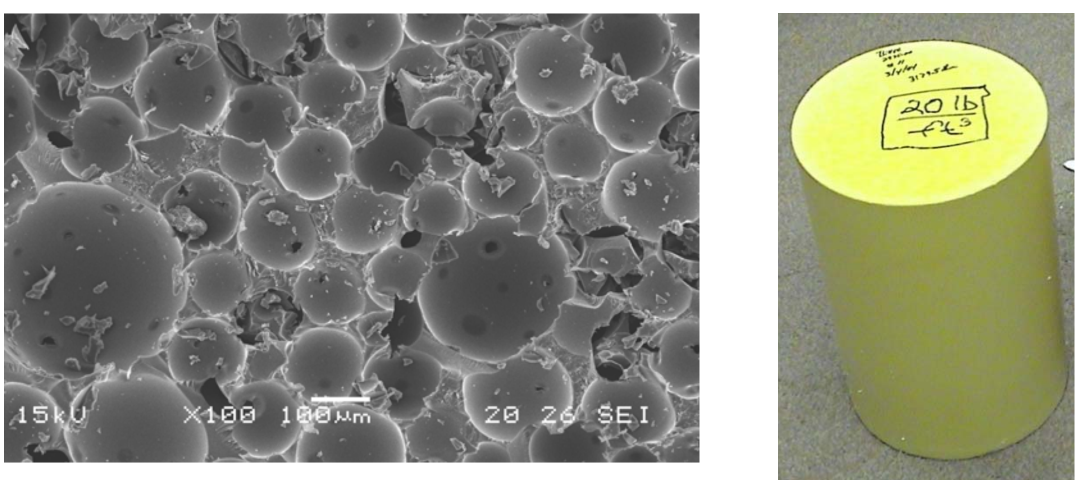
1. Introduction

Polyurethane foams are used in packaging to protect sensitive components from accidental impact events. These foams are typically designed to absorb energy during impact events by undergoing large inelastic deformation. Thus, constitutive models that describe foam response to large deformation at various rates and temperatures are needed for use in finite element analyses of impact events. Rigid, closed-cell, polyurethane foam consists of nearly spherical voids with a typical diameter of 100 to 300 microns (Figure 1). The closed cells are separated by a polymer matrix that forms cells. Voids are less spherical and walls between neighboring cells are often very thin or even ruptured in rigid polyurethane foams with densities of 192 kg/m^3 (12 pcf) or less. In higher density foams with densities of 320 kg/m^3 (20 pcf) or greater, cells are more spherical and walls between neighboring cells are typically intact.

Numerous experiments were performed to characterize the mechanical response of several different rigid polyurethane foams (FR3712, PMDI10, PMDI20, and TufFoam35) to large deformation. This report includes a description of recent experiments and experimental findings. Next, development of a Unified Creep Plasticity Damage (UCPD) model for rigid, polyurethane foams is described. Selection of material parameters is then discussed and finite element simulations with the new UCPD model are compared with experimental results to show behavior that can be captured with this model.



(a) 176 kg/m3 (11 pcf)



(b) 320 kg/m3 (20 pcf) PMDI Foam SEM and 12 inch tall billet

Figure 1. Polyurethane foam cell structure.

2. Experimental Observations

When rigid, closed-cell polyurethane foam is compressed, it exhibits an initial elastic regime followed by a plateau regime in which the load needed to compress the foam remains nearly constant (Figure 2). In the elastic regime, the foam sample is uniformly deformed. In the plateau regime, cell walls are plastically deformed and large permanent volume changes are generated. Uniform deformation is observed in foam samples that strain harden but localized deformation is often observed in foam samples that exhibit no hardening or strain softening in the plateau regime. When additional load is applied, cell walls are compressed against neighboring cell walls (Figure 3), and the stiffness and strength of the foam increases. In Figure 2, uniaxial stress and strain are plotted as positive for both compression and tension. When rigid polyurethane foam is loaded in tension, it exhibits only a very small amount of plastic deformation before it fractures. Fracture surfaces generated by uniaxial tension are oriented such that the loading axis is normal to the fracture surface (Figure 4).

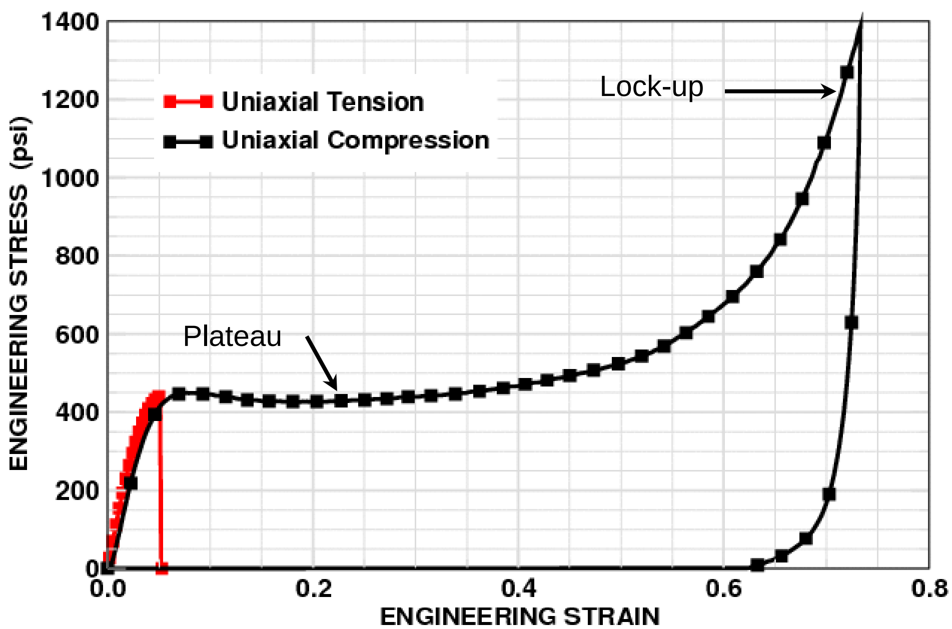
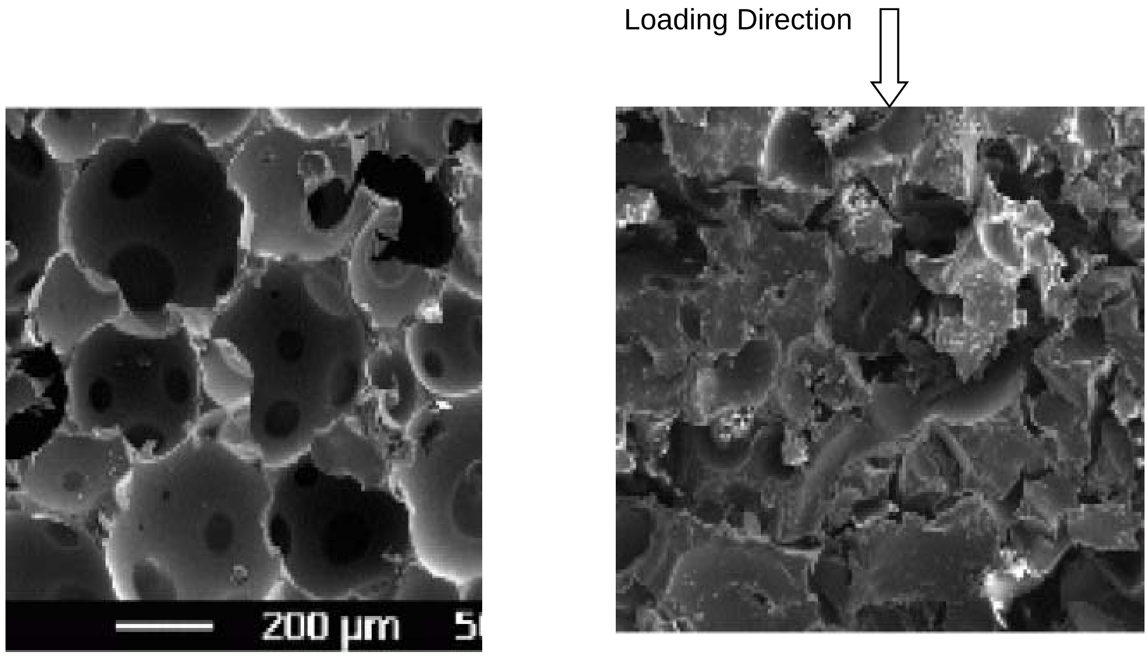


Figure 2. Typical stress-strain curves for 176 kg/m3 (11 pcf) polyurethane foam subjected to either uniaxial compression or uniaxial tension.



(a) undeformed (b) deformed shape after unloading

Figure 3. Cell walls compressed against neighboring cell walls when 176 kg/m^3 (11 pcf) foam is compressed into the lockup regime and then unloaded.

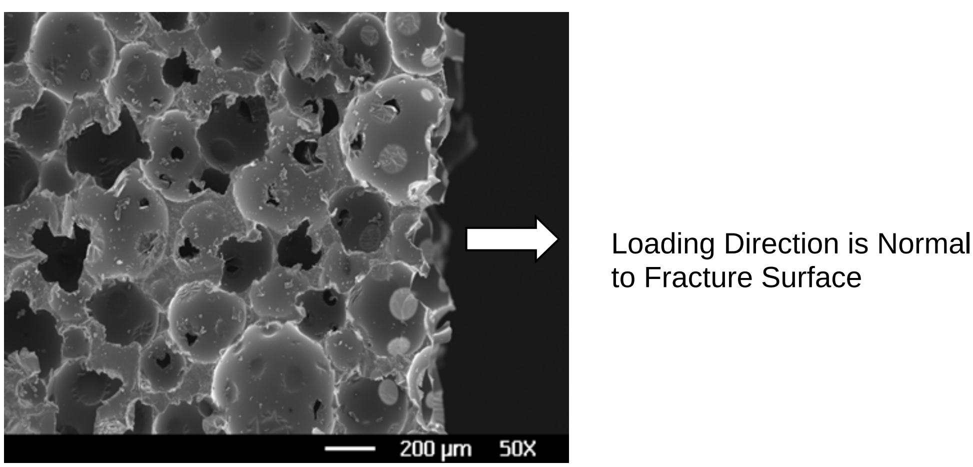


Figure 4. Fracture surface generated by uniaxial tension of 176 kg/m^3 (11 pcf) polyurethane foam in the indicated direction.

The mechanical response of polyurethane foam is also very sensitive to changes in either loading rate or temperature. For example, the crush strength of a General Plastic's FR3712 (12 pcf) rigid polyurethane foam subjected to uniaxial compression decreases significantly with increases in temperature (Figure 5). The crush strength of polyurethane foam is also observed to increase significantly with increases in loading rate (Figure 6). When FR3712 rigid polyurethane foam is subjected to hydrostatic compression, it exhibits a pressure versus volume strain curve (Figure 7) that is similar in shape to its uniaxial stress strain curve (Figure 6). There is again an initial elastic regime followed by a plateau regime and finally a lock-up regime.

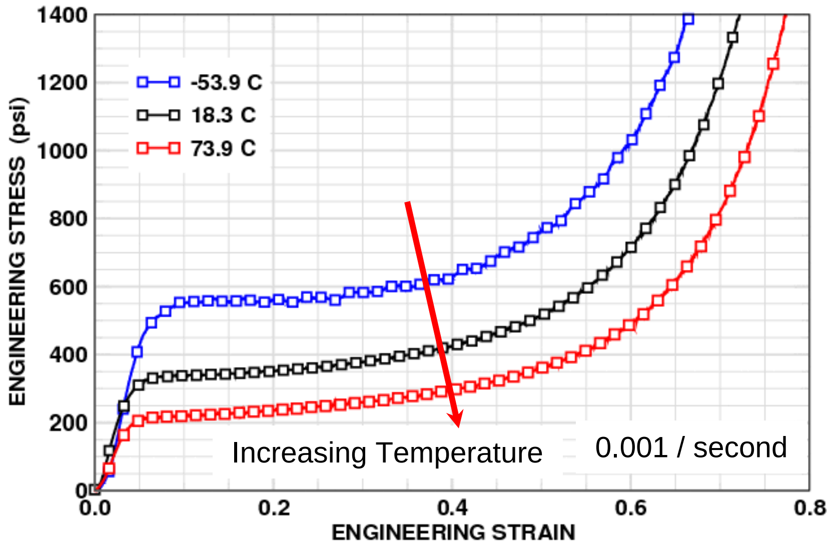


Figure 5. Stress-strain curves for 192 kg/m^3 (12 pcf) FR3712 foam subjected to uniaxial compression at engineering strain rate of 0.001 per second and at constant temperatures given by the legend.

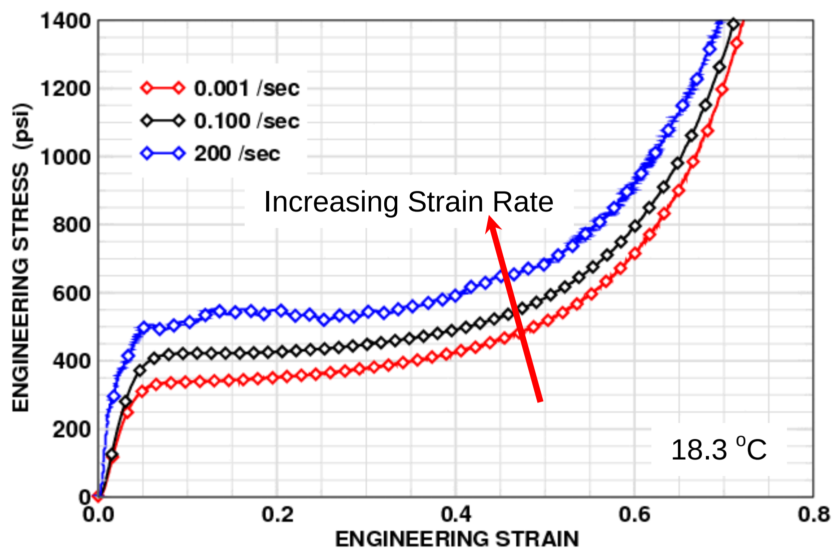


Figure 6. Stress-strain curves for 192 kg/m^3 (12 pcf) FR3712 foam subjected to uniaxial compression at an initial temperature of 18.3 ^{o}C and at constant engineering strain rates given by the legend.

In addition to hydrostatic compression, the FR3712 foam was also subjected to a variety of tri-axial compression load paths [1] in which the sample was initially subjected to hydrostatic compression and then the confining pressure was maintained while additional stress was applied in the axial direction only. Results from this series of tri-axial compression experiments were then used to generate a plot of the initial yield surface for the foam in a von Mises effective stress versus mean stress space (Figure 8). The experimental results (blue symbols in Figure 8) indicate that the initial yield surface for the FR3712 foam could be described as an ellipse in this two dimensional space (solid black line in Figure 8) or as an ellipsoid about the hydrostat in three-dimensional principal stress space.

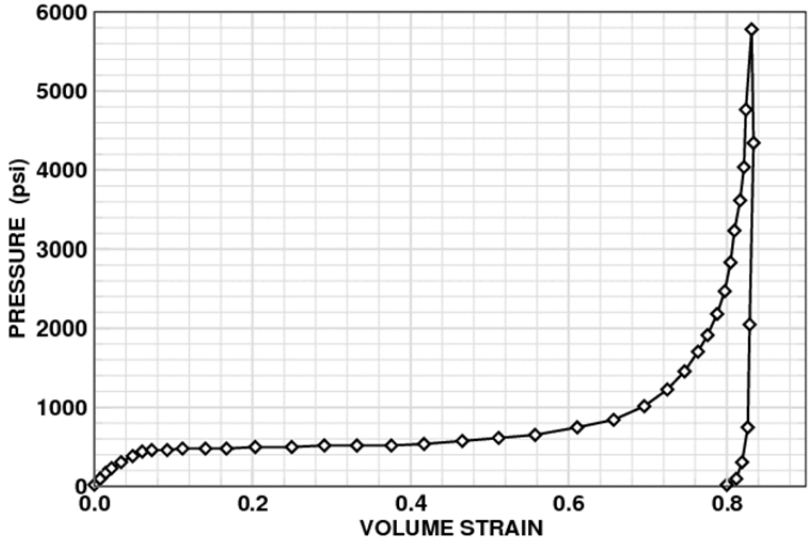


Figure 7. Pressure-volume strain curve for 192 kg/m^3 (12 pcf) FR3712 foam subjected to hydrostatic compression at a constant pressurization rate of 0.1 MPa/second.

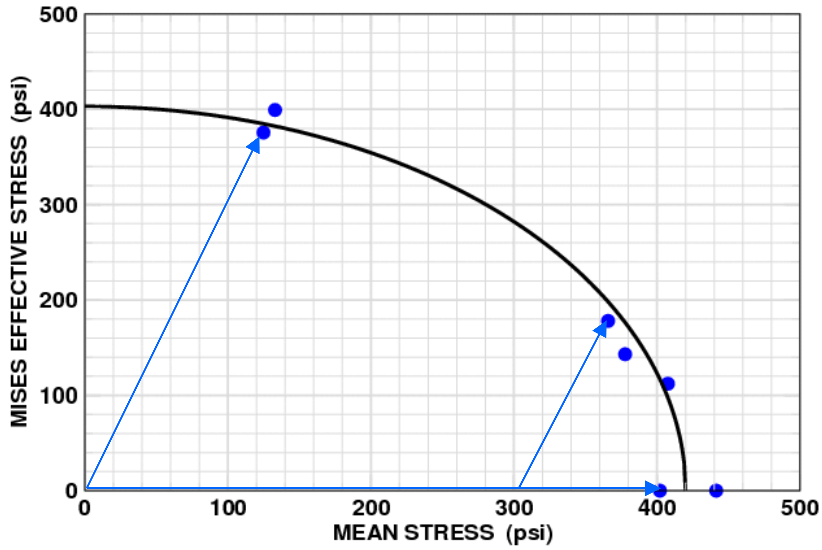


Figure 8. Yield surface obtained from a series of uniaxial compression, hydrostatic compression and tri-axial compression experiments (blue symbols) on 192 kg/m^3 (12 pcf) FR3712 foam [1].

3. Unified Creep Plasticity Damage (UCPD) Model Development

From the experimental results presented in the previous section, it is clear that a metal plasticity model which includes only deviatoric (shape-changing) plasticity would not be adequate for describing the mechanical behavior of rigid polyurethane foams. Constitutive models for foams were previously developed by a number of researchers [e.g. 1-7]. Neilsen et al. [2] developed a plasticity model for polyurethane foams with a yield surface that has a cubic shape based on the use of a principal stress yield criterion. Deshpande and Fleck [3] developed a plasticity model for metal foams with a yield surface that is an ellipsoid about the hydrostat. Deshpande and Fleck [4] subsequently developed a yield surface for polymeric foams with a yield surface that is the inner envelope of the ellipsoidal surface previously developed for metal foams and a surface based on a minimum (compressive) principal stress criterion.

The model developed here is similar to many existing foam models [e.g. 1-7]. Our current implementation in SIERRA [8], uses the unrotated Cauchy stress, $\boldsymbol{\sigma}$, and unrotated deformation rate, $\dot{\boldsymbol{\varepsilon}}$ [9, 10]. For small elastic strains, the total strain rate, $\dot{\boldsymbol{\varepsilon}}$, can be additively decomposed into elastic, $\dot{\boldsymbol{\varepsilon}}^e$, and inelastic, $\dot{\boldsymbol{\varepsilon}}^{in}$, parts as follows

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^{in} \tag{1}$$

We also assume that the elastic response is linear and isotropic such that the stress rate is given by the following equation

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}}^e = \mathbf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{in}) \tag{2}$$

where $\mathbf{E}$ is the fourth-order, isotropic elasticity tensor. Based on the experimental results shown in Figure 8, the initial yield surface is an ellipsoid about the hydrostat described by the function

$$\varphi = \frac{\bar{\sigma}^2}{a^2} + \frac{p^2}{b^2} - 1.0 = 0 \tag{3}$$

where a and b are state variables that define the current deviatoric and volumetric strengths of the foam. $\bar{\sigma}$ is the von Mises effective stress, a scalar measure of the deviatoric stress and is given by

$$\bar{\sigma} = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}} \tag{4}$$

p is the pressure or mean stress and is given by

$$p = \frac{1}{3}\boldsymbol{\sigma} : \mathbf{i} \tag{5}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress and $\mathbf{i}$ is the second-order identity tensor. $\mathbf{s}$ is the second-order deviatoric stress tensor

$$\mathbf{s} = \boldsymbol{\sigma} - p\,\mathbf{i} \tag{6}$$

Puso and Govindjee [6] and Zhang et al. [7] developed strain rate dependent models for foam that have the foam's inelastic rate given as a power-law function of stress. For the model developed here, we start with the yield function, Equation (3), rewritten as follows

$$\varphi = \sigma^* - a = 0 \tag{7}$$

where the effective stress, σ^*, is a function of the vonMises effective stress, $\bar{\sigma}$, and pressure, *p*, as follows

$$\sigma^* = \sqrt{\bar{\sigma}^2 + \frac{a^2}{b^2} p^2} \tag{8}$$

Next, using a Perzyna-type formulation, the following expression for the inelastic rate, $\dot{\boldsymbol{\varepsilon}}^{in}$, is developed

$$\dot{\boldsymbol{\varepsilon}}^{in} = \begin{cases} \dot{\lambda}\,\mathbf{g} = e^h \left\langle \dfrac{\sigma^*}{a} - 1 \right\rangle^n \mathbf{g} & when \quad \dfrac{\sigma^*}{a} - 1 > 0 \\ \mathbf{0} & when \quad \dfrac{\sigma^*}{a} - 1 \le 0 \end{cases} \tag{9}$$

where **g** is a symmetric, second-order tensor that defines the orientation of the inelastic flow. This type of model is sometimes referred to as an overstress model because the inelastic rate is a power-law function of the overstress (distance outside the yield surface). For associated flow, **g** is simply normal to the yield surface and is given by

$$\mathbf{g}_{associated} = \frac{\dfrac{\partial \varphi}{\partial \boldsymbol{\sigma}}}{\left| \dfrac{\partial \varphi}{\partial \boldsymbol{\sigma}} \right|} = \frac{\dfrac{3}{a^2}\mathbf{s} + \dfrac{2}{3b^2} p\,\mathbf{i}}{\left| \dfrac{3}{a^2}\mathbf{s} + \dfrac{2}{3b^2} p\,\mathbf{i} \right|} \tag{10}$$

When lower density foams are subjected to a simple load path like uniaxial compression, the inelastic flow direction at moderate strains appears nearly uniaxial. In other words, the flow direction is given by the normalized stress tensor as follows

$$\mathbf{g}_{radial} = \frac{\boldsymbol{\sigma}}{|\boldsymbol{\sigma}|} = \frac{\boldsymbol{\sigma}}{\sqrt{\boldsymbol{\sigma} : \boldsymbol{\sigma}}} \tag{11}$$

This type of flow is referred to as radial flow. The UCPD model has a parameter, β, which allows for the flow direction to be prescribed as a linear combination of associated and radial flow directions as follows

$$\mathbf{g} = \frac{(1-\beta)\ \mathbf{g}_{associated} + \beta\,\mathbf{g}_{radial}}{\left|(1-\beta)\ \mathbf{g}_{associated} + \beta\,\mathbf{g}_{radial}\right|} \tag{12}$$

Rigid polyurethane foams have little ductility when they are subjected to tensile stress and behave more like elastic brittle materials for this load path. Even for uniaxial compression, these foams often exhibit cracking at large strains. The damage surfaces for the UCPD model are simply 3 orthogonal planes with normals given by the positive principal stress axes in principal stress space as shown in Figure 9 and are described by the following equation

$$\varphi^i_{Damage} = \sigma^{**i} - c(1-w) = 0,\ i = 1{,}3 \tag{13}$$

where σ^{**i} is a principal stress, c is the initial tensile strength which is a material parameter, and w is a scalar measure of the damage. Damage has an initial value of 0.0 and is limited to a maximum value of 0.99 to prevent the tensile strength from going to zero or becoming negative due to numerical round-off. As damage occurs, the damage surface will collapse toward the origin and the foam will have very little tensile strength. The foam will, however, still have compressive strength. Foam that is completely damaged can be removed using element death based on the damage variable reaching a value near 0.99 but removal of fully damaged elements from a simulation is not required.

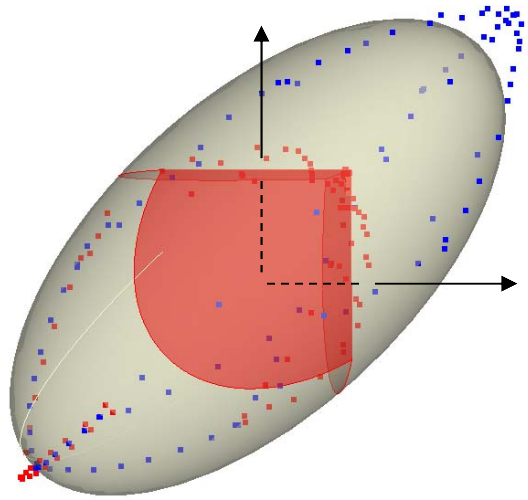


Figure 9. Yield (white) and damage (red) surfaces in principal stress space. Symbols represent results from either experiments or cell-level simulations on a representative volume of foam. Blue symbols are from cell-level simulations in which the matrix material was not allowed to crack.

Damage is given as a monotonically-increasing, user-prescribed function of damage strain, ε_{dam} , and damage strain is a function of the maximum tensile strain, $\varepsilon_{\max}$, and the plastic volume strain, ε_{vol}^{p} , as follows

$$w = w(\varepsilon_{dam}) = w\left(a_{dam}\varepsilon_{\max} + b_{dam}\,\varepsilon_{vol}^{p}\right) \tag{14}$$

where a_{dam} and b_{dam} are positive material parameters which allow the user to control the rate at which damage is generated in tension and compression. Note that in compression the plastic volume strain obtains a negative value so the maximum tensile strain needed to generate damage is larger. Damage is never allowed to decrease even if the damage strain decreases which means that once foam is damaged, healing is not allowed.

To fully capture temperature, strain-rate, and lock-up effects several material parameters are no longer simply material constants but are instead functions of temperature, θ, and/or a measure of the amount of compaction, the maximum volume fraction of solid material obtained during any prior loading, ϕ, which depends on the volume strain. Material parameters defining the foams elastic response, Young's modulus and Poisson's ratio, are functions of temperature, θ, and ϕ. To be more specific, the current Young's modulus and Poisson's ratio used in a simulation are given by

$$\begin{aligned} E &= E_r \cdot E(\theta) \cdot E(\phi) \\ v &= v_r \cdot v(\theta) \cdot v(\phi) \end{aligned} \tag{15}$$

The natural log of the reference flow rate, h, and the power law exponent, n, in Equation 9 are also functions of temperature

$$h = h_r \cdot h(\theta)$$
$$n = n_r \cdot n(\theta) \tag{16}$$

State variables that define current deviatoric and volumetric strengths, a and b in Equations 7 thru 10, are user-prescribed functions of ϕ. Also in the UCPD model, the parameter β which defines the fraction of associated and radial flow in Equation 12 is a user-prescribed function of ϕ.

Material parameters names for the Foam Damage Model are listed in Table 1. State variables that can be plotted or used in coupled-physics simulations are listed in Table 2. The plastic work rate, PWORK, is computed so fully coupled thermal stress analyses can be performed using this model in ARPEGGIO by simply defining the volumetric heating rate as some fraction of the plastic work rate.

Table 1. Parameter names and definitions.

Parameter		Definition
Youngs Modulus,	E_r	Young's modulus reference value
Poissons Ratio,	ν_r	Poisson's ratio reference value
Phi,	ϕ	Initial volume fraction of solid material
Flow Rate	h_r	Reference flow rate that is multiplied by Rate Function
Power Exponent	n_r	Sinh exponent reference value multiplied by Exponent Function
Tensile Strength	c	Initial tensile strength of material
Adam	a_{dam}	Contribution of maximum tensile strain to damage strain
Bdam	b_{dam}	Contribution of plastic volume strain to damage strain
Youngs Function	$E(\theta)$	Defines temperature dependence of Young's Modulus
Poissons Function	$\nu(\theta)$	Defines temperature dependence of Poisson's Ratio
Rate Function	$h(\theta)$	Defines temperature dependence of Flow Rate
Exponent Function	$n(\theta)$	Defines temperature dependence of Power Exponent
Shear Hardening Function	$a(\phi)$	Defines dependence of state variable a on ϕ
Hydro Hardening Function	$b(\phi)$	Defines dependence of state variable b on ϕ
Beta Function	$\beta(\phi)$	Defines dependence of β on ϕ
Youngs Phi Function	$E(\phi)$	Defines Young's Modulus dependence on ϕ
Poissons Phi Function	$\nu(\phi)$	Defines Poisson's Ratio dependence on ϕ
Damage Function	$w(\varepsilon_{dam})$	Defines damage dependence on damage strain

Table 2. State variable names and definitions.

State Variable Name	Definition	Equation
DAMAGE	Damage	14
EMAX	Maximum tensile strain used to generate damage	14
EPVOL	Plastic volume strain $\int \dot{\boldsymbol{\varepsilon}}^{in} : \mathbf{i}\, dt$	14
EQPS	Equivalent plastic strain $\int \dot{\lambda}\, dt$	9
FA	Shear strength - a	3, 7-10
FB	Hydrostatic strength - b	3, 7-10
ITER	Number of sub increments taken in subroutine	-
PHI	Maximum volume fraction of solid material obtained during prior loading history	15
PWORK	Plastic work rate $\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{in}$	-

Extensive use of user-prescribed functions in the UCPD model makes this model quite flexible for fitting data from a variety of foams; however, use of user-prescribed functions also makes the selection of material parameters more difficult because the user is no longer trying to find the best material constant but instead needs to define entire functions. When there is insufficient experimental data, the user is forced to select functions based on prior experience and good engineering judgment.

4. Foam Damage Model Parameters for FR3712

Material parameters for 192 kg/m^3 (12 pcf) FR3712 foam at temperatures between -53.9 ^{o}C and 73.9 ^{o}C, and quasi-static (0.001 per second) to dynamic (200 per second) strain rates are given in Table 3 and Figure 10. The first step in the generation of these material parameters was to determine the initial volume fraction of solid material in the foam. Since the foam has a density of approximately 192 kg/m^3 and solid rigid polyurethane has a density of 1200 kg/m^3, the foam has an initial volume fraction of solid material, ϕ_0, equal to 0.16 (0.16 = 192/1200). Unfortunately, all solid polyurethane does not have the same density. Whinnery [11] reported that solid TufFoam has a density closer to 1360 kg/m^3 (85 pcf).

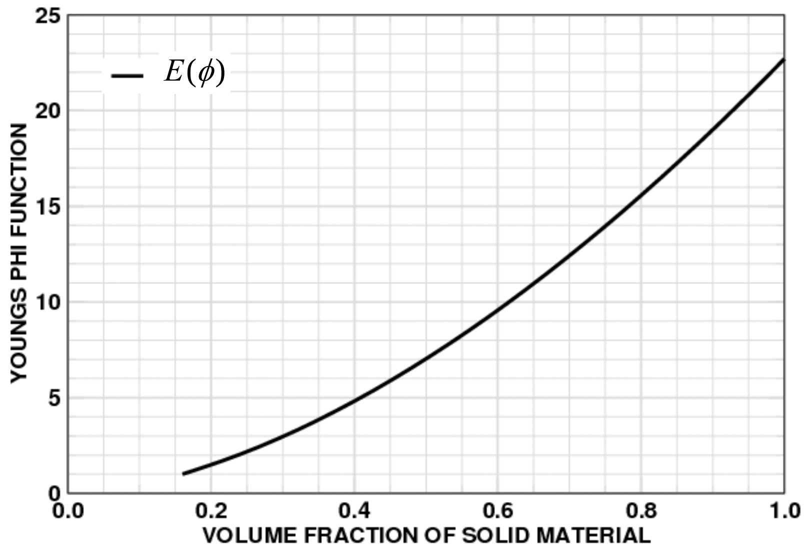
The next step in the fitting process was to plot the crush strength measured during uniaxial and hydrostatic compression experiments as a function of the current volume fraction of solid material. If we assume that the change in volume of the solid material is negligible compared with the change in volume of the foam, then the current volume fraction of solid material, ϕ, is related to the initial volume fraction, ϕ_0, of solid material as follows

$$\phi = \frac{\phi_0 V_0}{V} \tag{17}$$

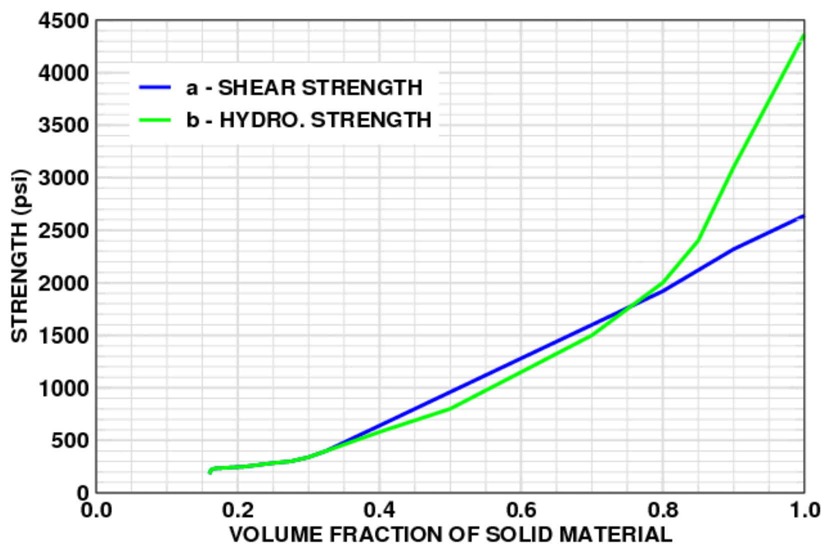
where V_o is the initial volume of foam and V is the current volume of foam. Note that in the constitutive model the ϕ that actually gets stored and used in function evaluations is the maximum volume fraction of solid material obtained during any prior loading history; however, for monotonic compressive loading the current volume fraction of solid material is the maximum volume fraction obtained during any prior loading. The consequence of using maximum volume fraction of solid material obtained during prior loading history in simulations is that once the foam is compressed it cannot be re-expanded to have the same mechanical properties it had prior to compression. Material parameter functions which were found to provide a good fit to the experimental data are shown in Figure 10. Material function β (Figure 10c) defines relative amounts of associated and radial flow, Equation 12, and is based on measurements of bulging or lateral strain during unconfined compression experiments or lateral pressure measured during elastically confined compression experiments. Unfortunately, elastically confined compression tests [12] were not performed on FR3712, so the β function for FR3712 was assumed to be identical to the β function for PMDI10 foam. A SIERRA/ADAGIO input block for this material is listed in Appendix A.

Table 3. Foam Damage Model Parameters for 192 kg/m^3 (12 pcf) foam, FR3712

Parameter		Units	Value	Value	Value
Temperature		C	-53.9	18.3	73.9
Young's Modulus	$E_r\ E(\theta)$	psi	11,550	11,550	9,240
Poisson's Ratio	ν	-	0.250		
Initial Volume Fraction Solid	ϕ_0	-	0.160		
Flow Rate	$h_r\ h(\theta)$	-	-10.0	2.60	7.50
Power Exponent	$n_r\ n(\theta)$	-	18.0	14.0	6.00
Tensile Strength	c	psi	280.0		
Adam	a_{dam}	-	1.00		
Bdam	b_{dam}	-	0.50		
Thermal Expansion Coefficient		1/C	60.0 x 10^{-6}		

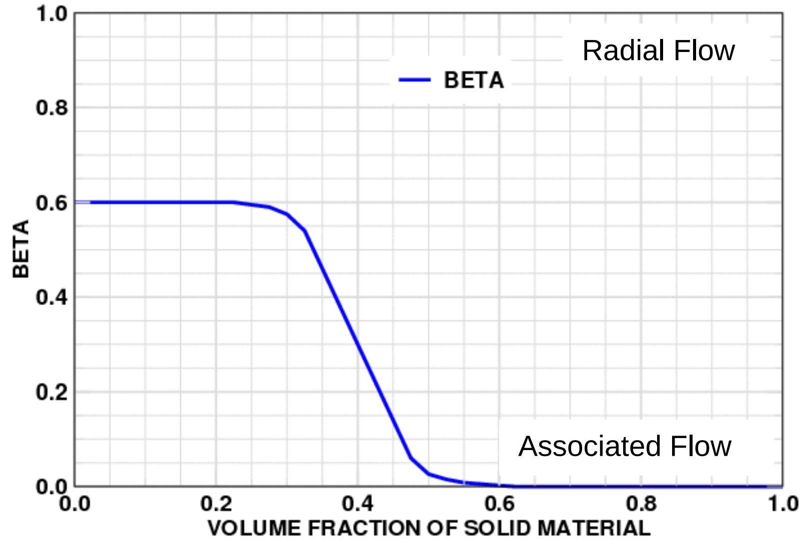


(a) $E(\phi)$ as a function of ϕ indicates foam can get up to 22.5 x stiffer when loaded into lock-up.

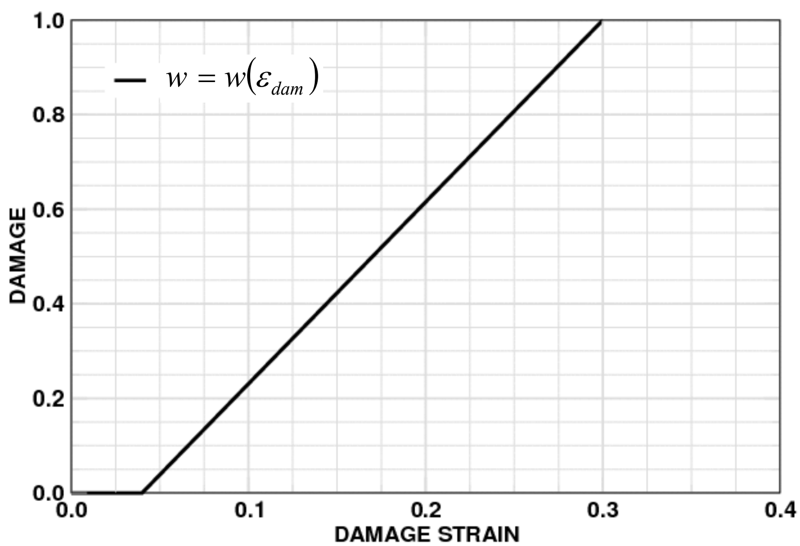


(b) shear and hydrostatic strengths increase with foam compaction

Figure 10. Material parameter functions for FR3712.



(c) flow direction becomes more associated with foam compaction



(d) damage as a function of damage strain

Figure 10 (continued). Material parameter functions for FR3712.

Next, the uniaxial and hydrostatic compression experiments were simulated using a simple 8-element model of a cube of material with a 25.4 mm (1.0 inch) edge length shown in Figure 11. In the first simulations, the unit block was subjected to uniaxial compression in the x-direction by preventing x-displacement of nodes on the left plane and displacing nodes on the right plane. The stress-strain curves generated by these simulation are compared with the experimental data in Figures 12 and 13. For all temperatures and strain rates the fit is good.

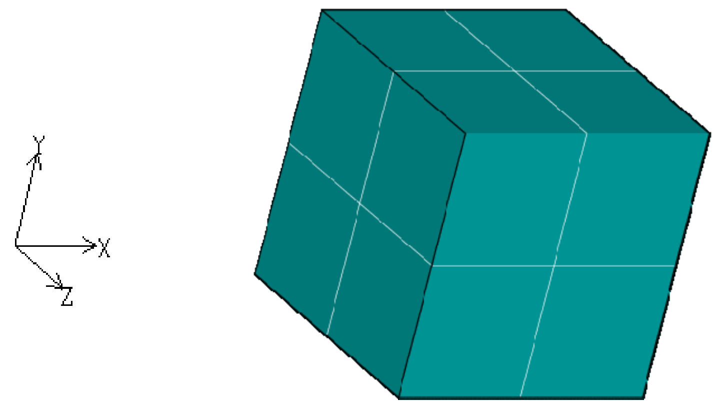


Figure 11. 8-element finite element model used for uniaxial and hydrostatic compression simulations.

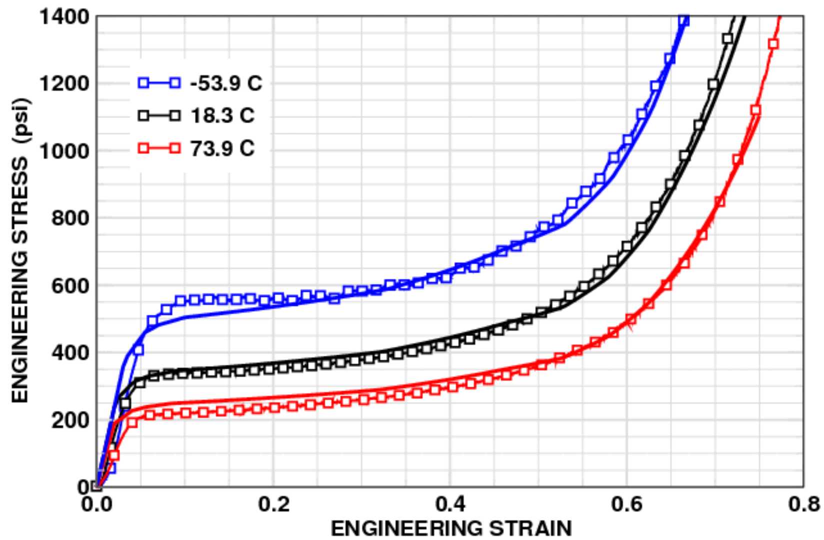


Figure 12. FR3712 uniaxial compression experiments (symbols) and simulations (solid lines) at three different temperatures and a constant engineering strain rate of 0.001 per second.

The hydrostatic compression experiment was then simulated using the finite element model shown in Figure 11. Symmetry boundary conditions were applied to the left, back, and bottom faces and pressure was applied to the right, front and top faces. The pressure was increased at a constant rate of 0.1 MPa/sec (14.5 psi/second) to match the experiment. The UCPD Model prediction matched the experiment well (Figure 14). Unfortunately, there was no uniaxial tension data available for this foam so the damage parameters selected for FR3712 were simply based on experience with other rigid polyurethane foams with similar density.

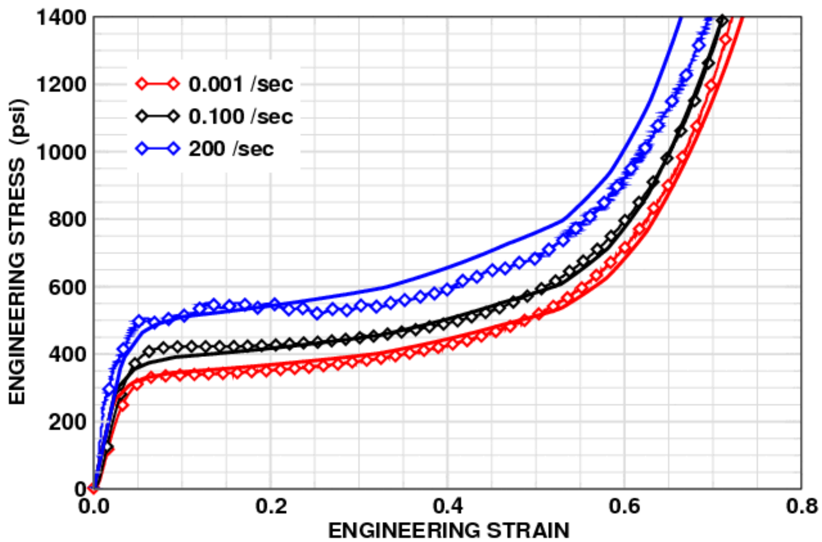


Figure 13. FR3712 uniaxial compression experiments (symbols) and simulations (solid lines) at three different engineering strain rates and a constant temperature of 18.3 ^{o}C.

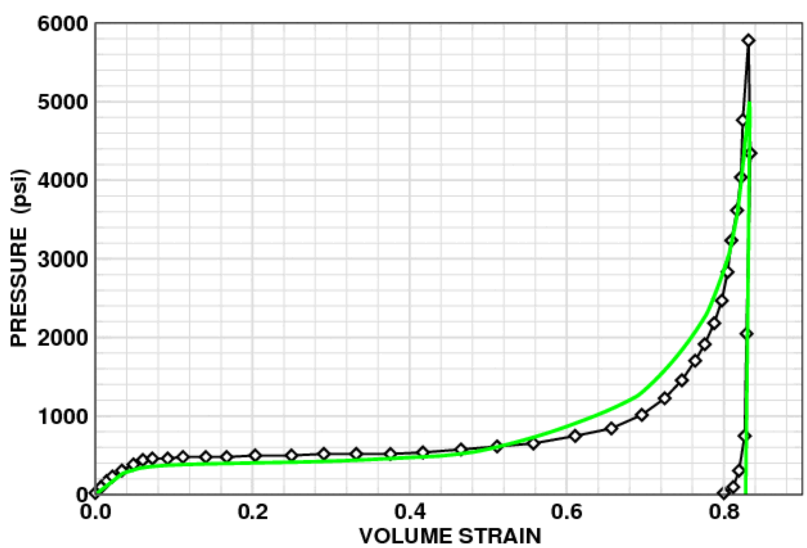


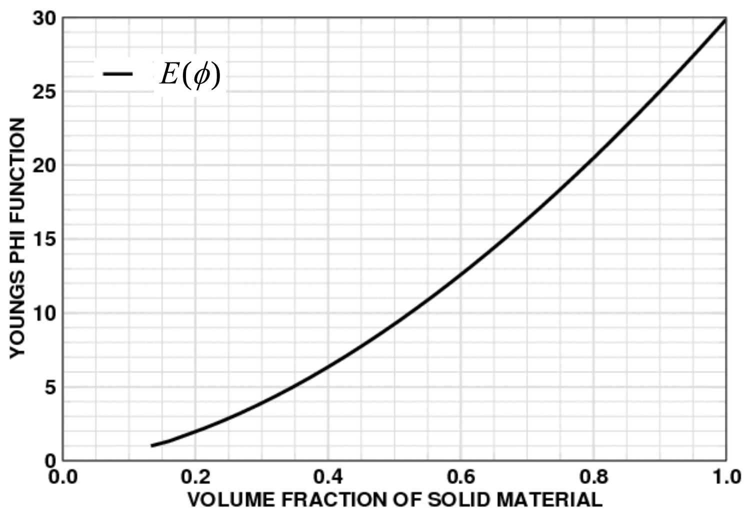
Figure 14. FR3712 hydrostatic compression experiment (symbols) and simulation (solid green line) at room temperature.

5. Foam Damage Model Parameters for PMDI10

Material characterization experiments were performed by Wei-Yang Lu on a variety of rigid polyurethane foams [12]. Experiments performed on nominally 160 kg/m^3 (10 pcf) PMDI10 foam included uniaxial compression at temperatures between -53.9 ^{o}C and 73.9 ^{o}C, uniaxial tension at 21.1 and 82.2 C, and elastically confined compression at 21.1 C. Parameters for the Foam Damage model based on these experiments are given in Table 4 and Figure 15. Since PMDI10 has a nominal density of 160 kg/m^3 and solid rigid polyurethane has a density of 1200 kg/m^3, this foam has an initial volume fraction of solid material, ϕ_0, equal to 0.133 (0.133 = 160/1200). Note that the PMDI10 foam samples had densities that actually varied between 128 and 158 kg/m^3 (8.0 to 9.86 pcf) [12] so it would be more accurate to say these are parameters for PMDI foam with an actual density of about 144 kg/m^3 (9.0 pcf). A SIERRA/ADAGIO input block for this material is listed in Appendix B.

Table 4. Foam Damage Model Parameters for 160 kg/m^3 (10 pcf) foam, PMDI10

Parameter		Units	Value	Value	Value
Temperature		C	-53.9	21.1	73.9
Young's Modulus	$E_r\ E(\theta)$	psi	10,534	8,778	7,022
Poisson's Ratio	ν	-	0.250		
Initial Volume Fraction Solid	ϕ_0	-	0.133		
Flow Rate	$h_r\ h(\theta)$	-	-12.0	2.80	5.80
Power Exponent	$n_r\ n(\theta)$	-	16.0	13.5	9.0
Tensile Strength	c	psi	280.0		
Adam	a_{dam}	-	1.00		
Bdam	b_{dam}	-	0.50		
Thermal Expansion Coefficient		1/C	60.0 x 10^{-6}		

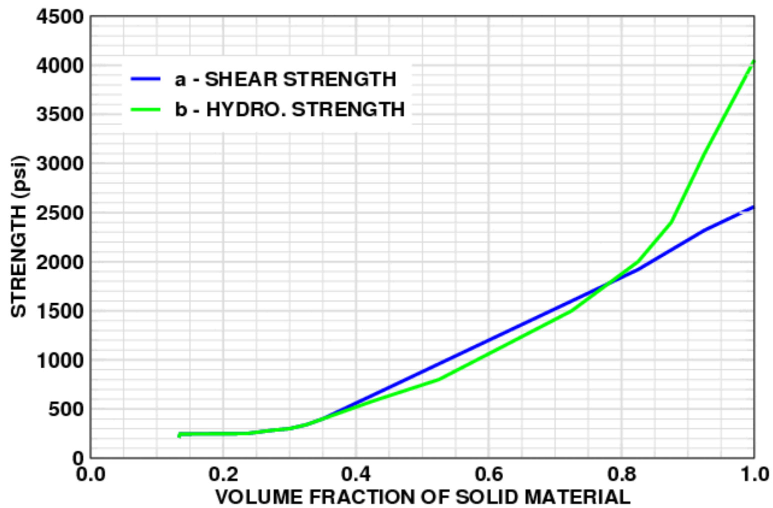


(a) $E(\phi)$ as a function of ϕ foam can get up to 30.0 x stiffer when loaded into lock-up.

Figure 15. Material parameter functions for PMDI10.

The curve used to define damage as a function of damage strain was identical to the curve used for FR3712 foam (Figure 10d). The hardening curves for PMDI10 (Figure 15b) are similar to the hardening curves for FR3712 (Figure 10b).

Next, the uniaxial compression experiments [12] were simulated using the simple unit cube model shown in Figure 11. For all temperatures and strain rates the fit is good (Figure 16). However, at -53.9 C the sample cracked at a small strain and at higher temperatures the samples cracked at much larger strains. The Foam Damage model assumes that damage is only a function of damage strain and not a function of temperature and, thus, will not be able to capture this change in strain to failure with temperature.



(b) shear and hydrostatic strengths increase with foam compaction

Figure 15 (cont). Material parameter functions for PMDI10.

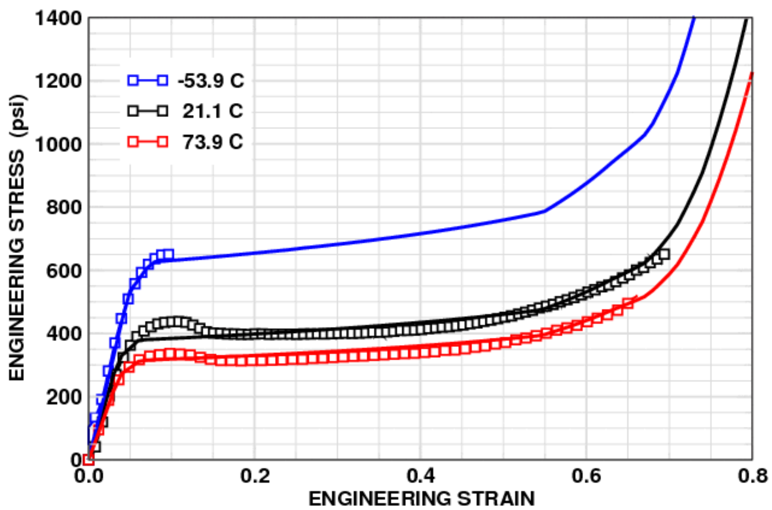


Figure 16. PMDI10 uniaxial compression experiments (symbols) and simulations (solid lines) at three different temperatures and a constant engineering strain rate of 0.01 per second.

The uniaxial tension experiments [12] were then simulated using the finite element models shown in Figure 17. These finite element models represent the middle 1.0 inch tall gage section of the sample and do not include the parts that were gripped. To ensure failure occurred in the gage section the width and thickness at the middle of the gage section was reduced from 0.250 inch to 0.249 inch. The fine model had an element edge length of ~20 mils, the second model an edge length of ~40 mils, and the coarse model an edge length of ~80 mils. All models predict the correct crack orientation and the predicted load displacement is not very sensitive to mesh refinement (Figure 18). However, the predicted strain to failure is too large for the high temperature simulation indicating that in future versions of this model, the damage evolution equation should be improved to capture effects of temperature on damage evolution.

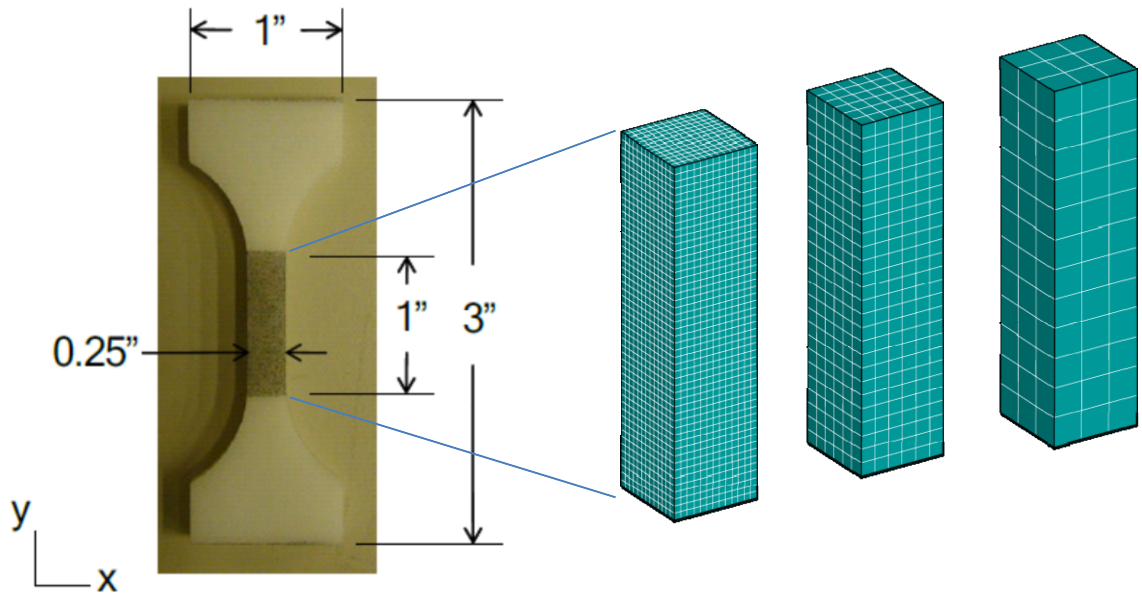


Figure 17. Finite element models for uniaxial tension simulations. Experimental image from [11].

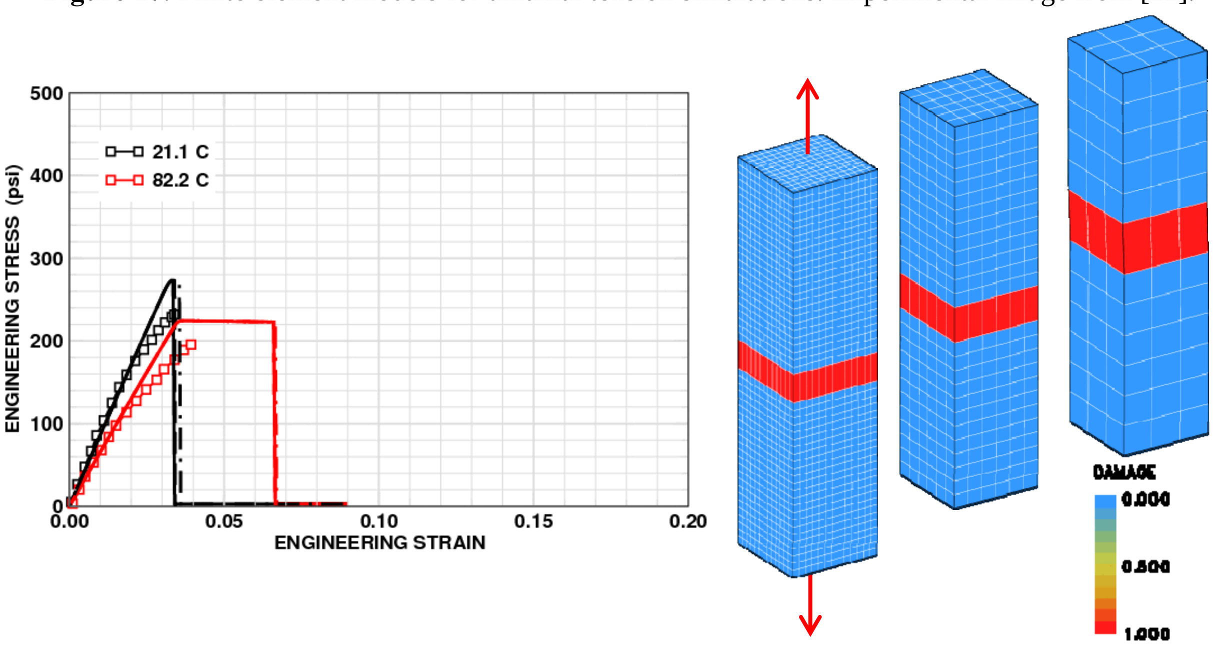


Figure 18. PMDI10 uniaxial tension experiments (symbols) and simulations (dot-dash line = coarse mesh, dashed line = intermediate mesh, solid line = fine mesh). All models predict crack perpendicular to applied loading which was observed experimentally.

Finally, the elastically confined compression test [12] was simulated using the test setup and finite element model shown in Figure 19. Since the sample, thin tube and loading are all axisymmetric; the finite element model included only a 10 degree slice with appropriate axisymmetric boundary conditions. Also, the model assumed that prior to loading the radial gap between the foam sample and the thin tube was 0.001 inch. Results from this simulation showed that experimental loads and strain in the thin tube, which is a measure of the radial pressure being generated in the foam sample, are in reasonably good agreement with the simulation (Figure 20). This was expected because the material function β (Figure 10 c) used to define flow direction was chosen to predict the correct amount of lateral expansion in this experiment. The model predicting a slightly higher initial load and radial strain and also locking up slightly sooner than the experiment suggests that the experiment may have had a slightly larger initial radial gap between sample and thin tube than the 0.001 inch used in the simulation.

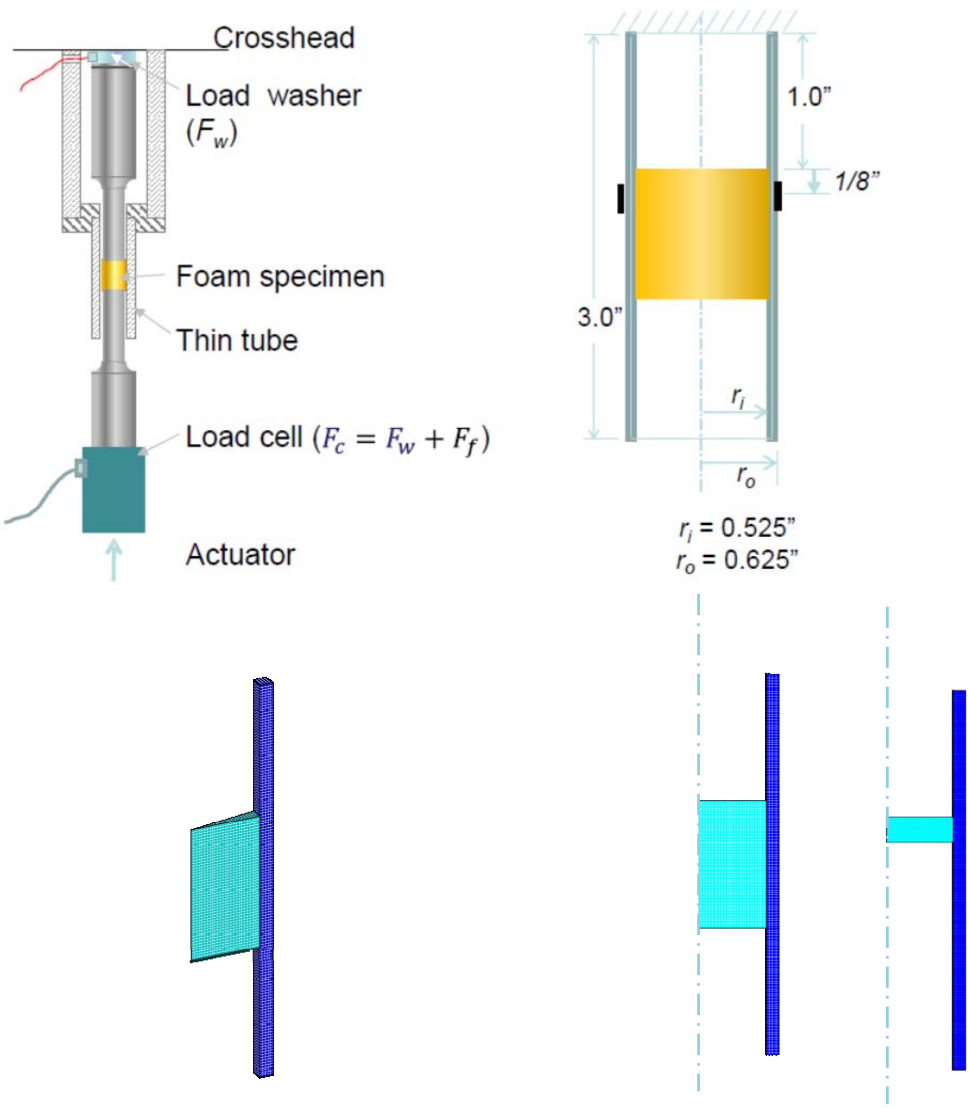
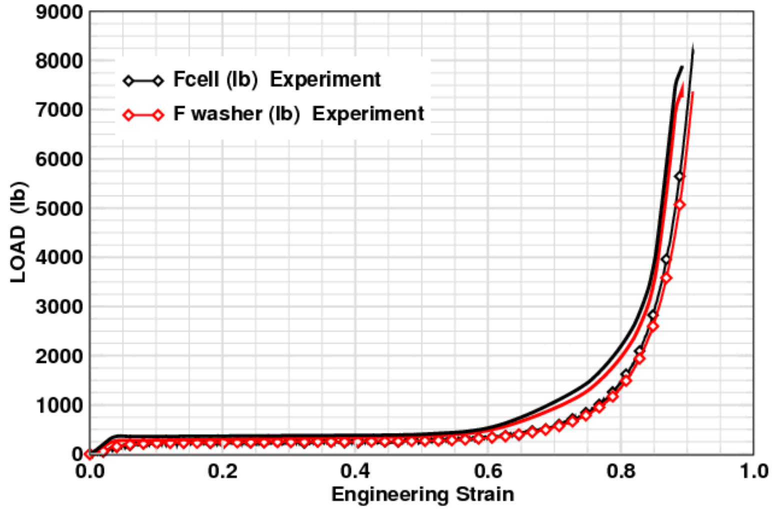
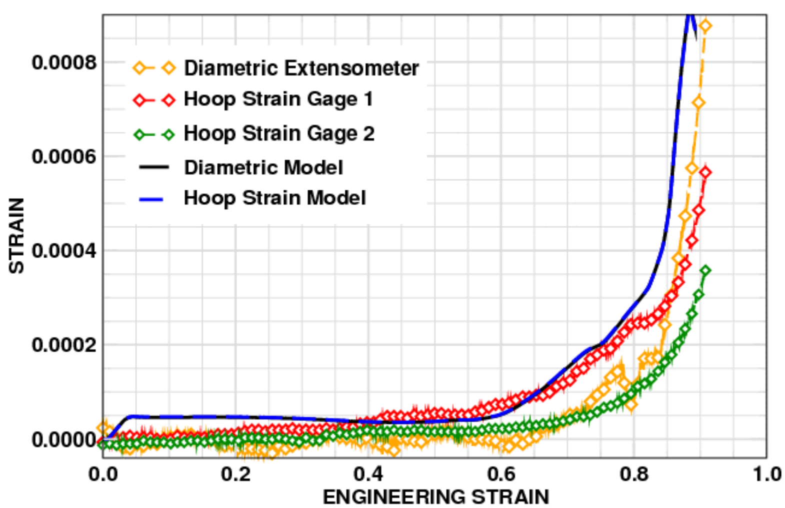


Figure 19. Elastically confined compression test setup [12], finite element model of an axisymmetric 10 degree slice, and deformed shape of model with foam compressed to engineering strain of 0.80.



(a) loads as a function of axial engineering strain in foam



(b) hoop strain in thin tube as a function of axial engineering strain in foam.

Figure 20. Results from simulation of elastically confined compression test with a 0.001 inch initial radial gap between sample and thin tube.

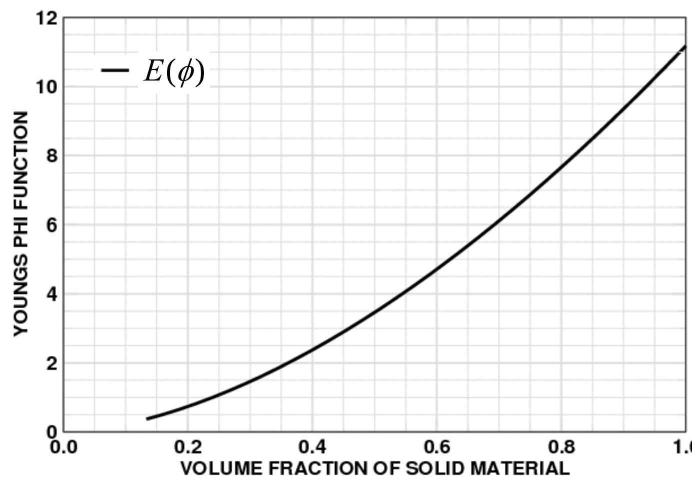
6. Foam Damage Model Parameters for PMDI20

Foam Damage model parameters for 288 kg/m^3 (18 pcf) PMDI20 foam were obtained next. Note that this foam was intended to have a nominal density of 20 pcf; however, most of the samples that were actually tested had an average density closer to 18 pcf. Since this foam has a density similar to General Plastic's FR3720 foam, we would expect these foams to exhibit similar behavior.

Material parameters for PMDI20 foam at temperatures between -53.9 ^{o}C and 73.9 ^{o}C are given in Table 5 and Figure 21. Since this foam has a density of approximately 288 kg/m^3 and solid PMDI a density of 1200 kg/m^3, this foam has an initial volume fraction of solid material, ϕ_0 ,equal to 0.240 (0.240 = 288/1200). It is interesting that the curve for β as a function of volume fraction of solid material (Figure 21c) has a shape similar to the curve for PMDI10 which indicates that there may be a master curve which would work well for all foams. Also, the curve used to define damage as a function of damage strain was identical to the curve used for FR3712, and PMDI10 foam (Figure 10d). A SIERRA/ADAGIO input block for this material is listed in Appendix C.

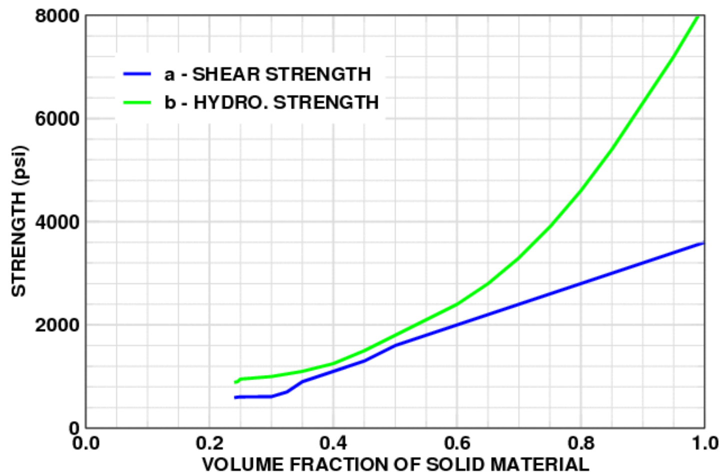
Table 5. Foam Damage Model parameters for 320 kg/m^3 (20 pcf) foams, PMDI20

Parameter		Units	Value	Value	Value
Temperature		C	-53.9	21.1	73.9
Young's Modulus	$E_r\ E(\theta)$	psi	28,905	23,500	18,800
Poisson's Ratio	ν	-	0.300		
Initial Volume Fraction Solid	ϕ	-	0.240		
Flow Rate	$h_r\ h(\theta)$	-	-8.0	2.32	6.50
Power Exponent	$n_r\ n(\theta)$	-	16.5	13.45	8.00
Tensile Strength	c	psi	950.0		
Adam	a_{dam}	-	1.00		
Bdam	b_{dam}	-	0.33		
Thermal Expansion Coefficient		1/C	60.0 x 10^{-6}		

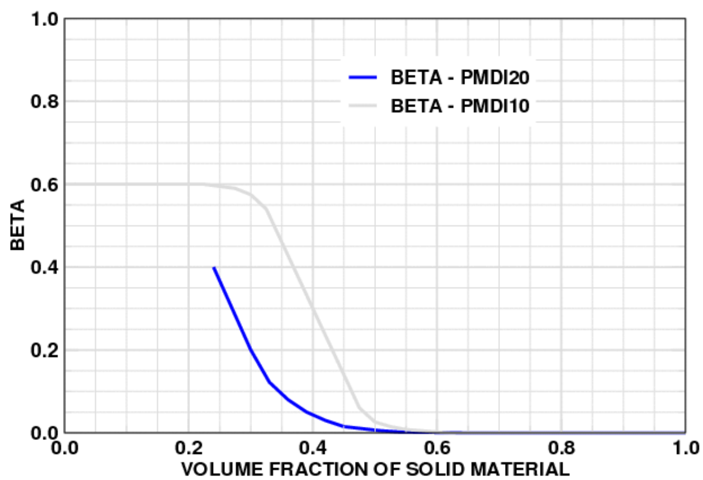


(a) $E(\phi)$ as a function of ϕ - foam can get up to 11 x stiffer when loaded into lock-up

Figure 21. Material parameter functions for PMDI20.



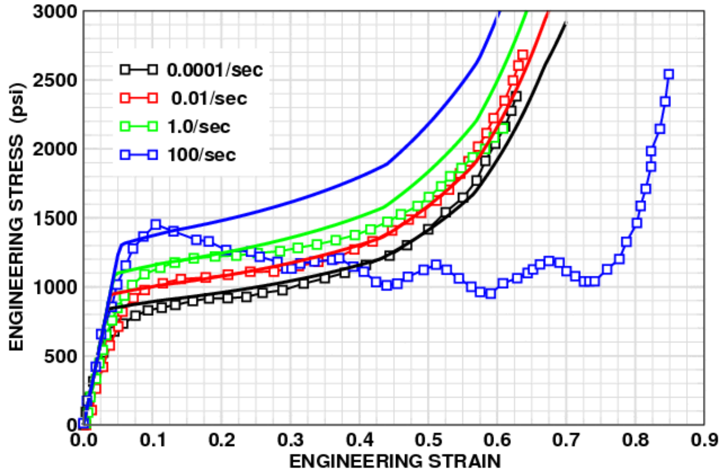
(b) shear and hydrostatic strengths increase with foam compaction



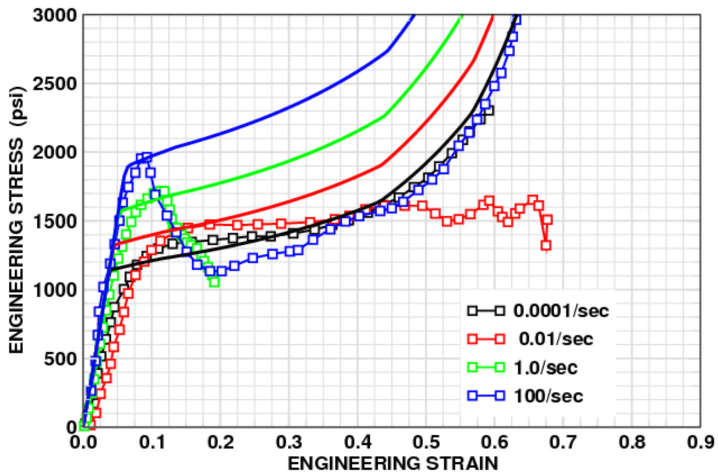
(c) flow direction becomes more associated with foam compaction

Figure 21 (cont). Material parameter functions for PMDI20.

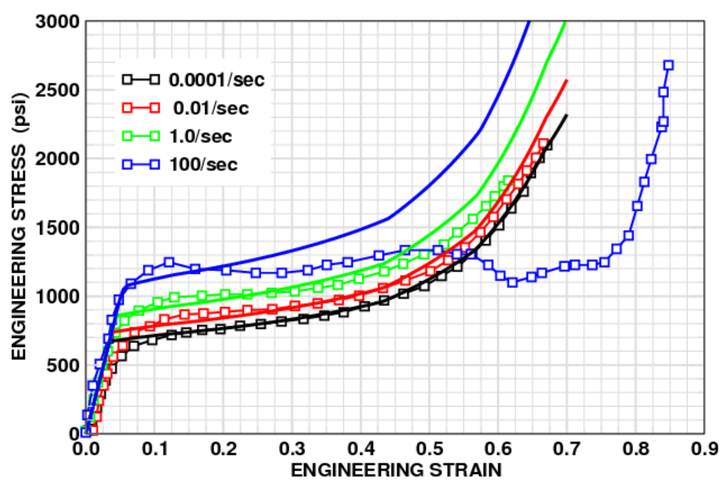
Next, the uniaxial compression experiments were simulated using the simple 8-element unit cube model shown in Figure 11. For most temperatures and strain rates the fit is reasonably good (Figure 22). However, at -53.9 ^{o}C or at high rates this foam often exhibited cracking prior to lock-up which is not captured by the current Foam Damage model in which the damage does not depend on strain rate or temperature.



(a) 21.1 C



(b) -53.9 C



(c) 73.9 C

Figure 22. PMDI20 uniaxial compression experiments (symbols) and simulations (solid lines) at three different temperatures and four different engineering strain rates.

Next the rigid confined compression experiments were simulated using the 8 element block model shown in Figure 11 with no expansion of the foam allowed in the lateral direction. Results from these simulations were in reasonably good agreement with the experiments (Figure 23). Simulation of the elastically confined compression test with the model shown in Figure 19 also generated load and metal tube strain predictions in good agreement with the experiments (Figure 24).

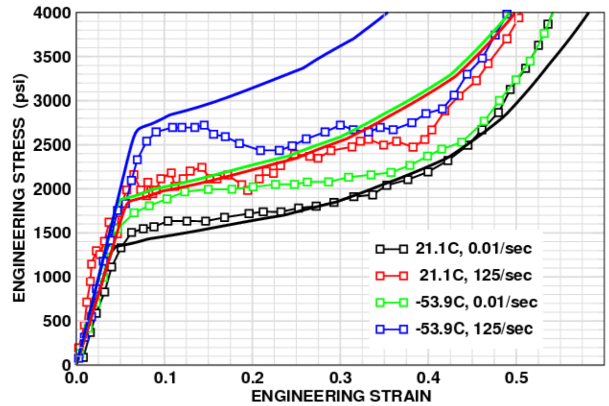
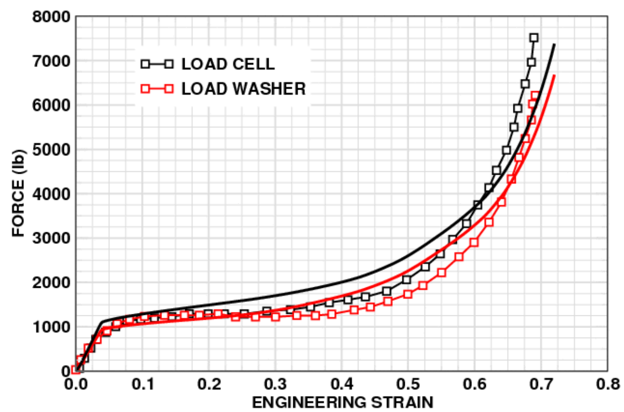
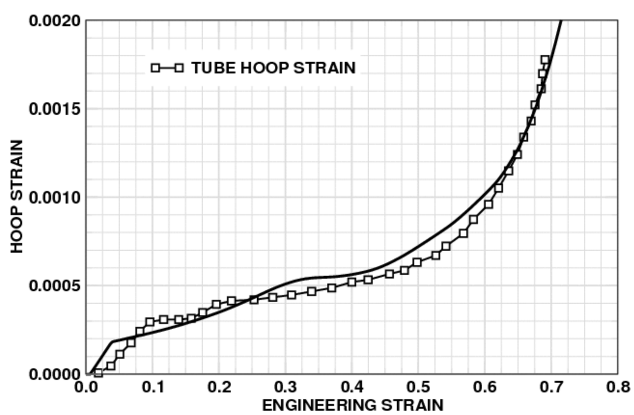


Figure 23. PMDI20 rigid confined compression experiments and simulations.



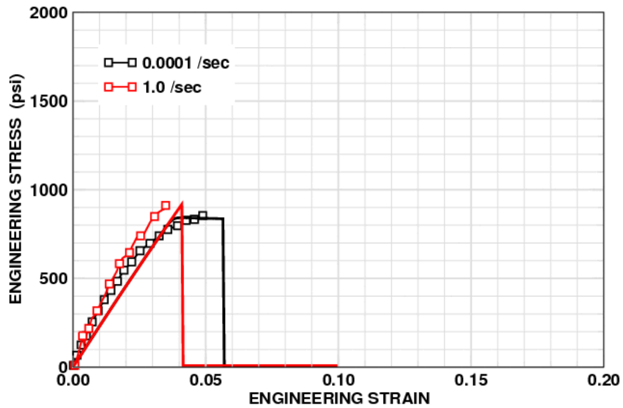
(a) loads as a function of axial engineering strain in foam



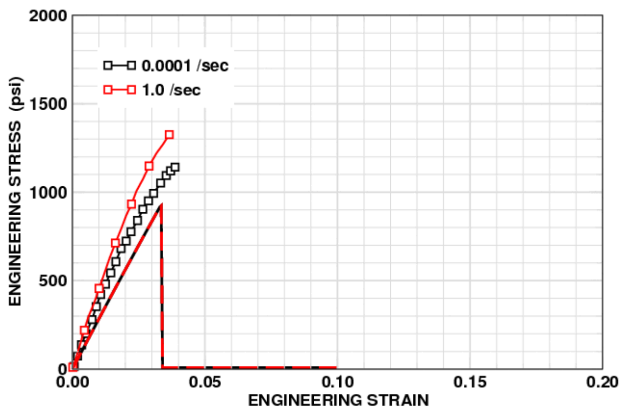
(b) hoop strain in thin tube as a function of axial engineering strain in foam.

Figure 24. PMDI20 elastically confined compression experiments and simulations.

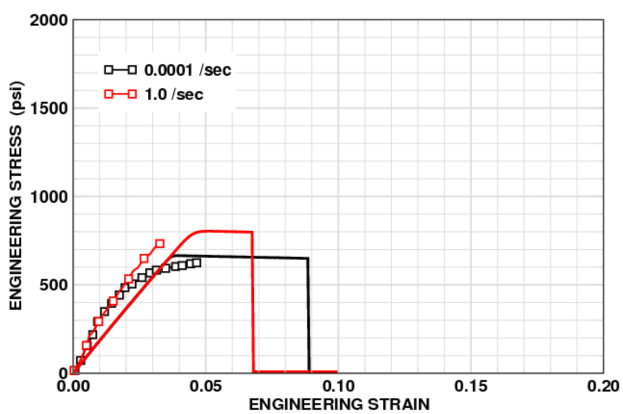
The uniaxial tension experiments on PMDI20 were then simulated using the finite element models with an element edge length of ~40 mils shown in Figure 17. All simulations predicted the correct crack orientation. However, the predicted strain to failure was too large for the high temperature simulation indicating that in future versions of this model, the damage parameters should be temperature-dependent (Figure 25).



(a) 21.1 C



(b) - 53.9 C



(c) 73.9 C

Figure 25. PMDI20 uniaxial tension experiments (symbols) and simulations (solid line).

Next, the 21.1 °C, 100 inch per second engineering strain rate, uniaxial compression experiment was simulated with a more accurate ¼ symmetry finite element model of the experimental geometry with a sample diameter of 1.0 inch and an initial sample height of 1.0 inch. For this simulation a friction coefficient of 0.12 was used for the interface between the loading platens and the foam sample. Results from this simulation indicated that the model would predict cracking of the sample which was observed in the experiment (Figure 26). The post-test experimental image shown in Figure 26 was not from this exact experiment but does show the typical cracking exhibited by 1.0 inch diameter by 1.0 inch tall PMDI20 samples subjected to uniaxial compression. At very large deformation, the contact algorithm started to have difficulty tracking all of the newly created contact surfaces due to foam cracking. Also, there were no constraints to prevent free foam particles from passing through the symmetry planes or from flying away. Improper movement of particles through the symmetry plane could be avoided by adding rigid contact surfaces at the symmetry planes but this will continue to be a challenging contact problem.

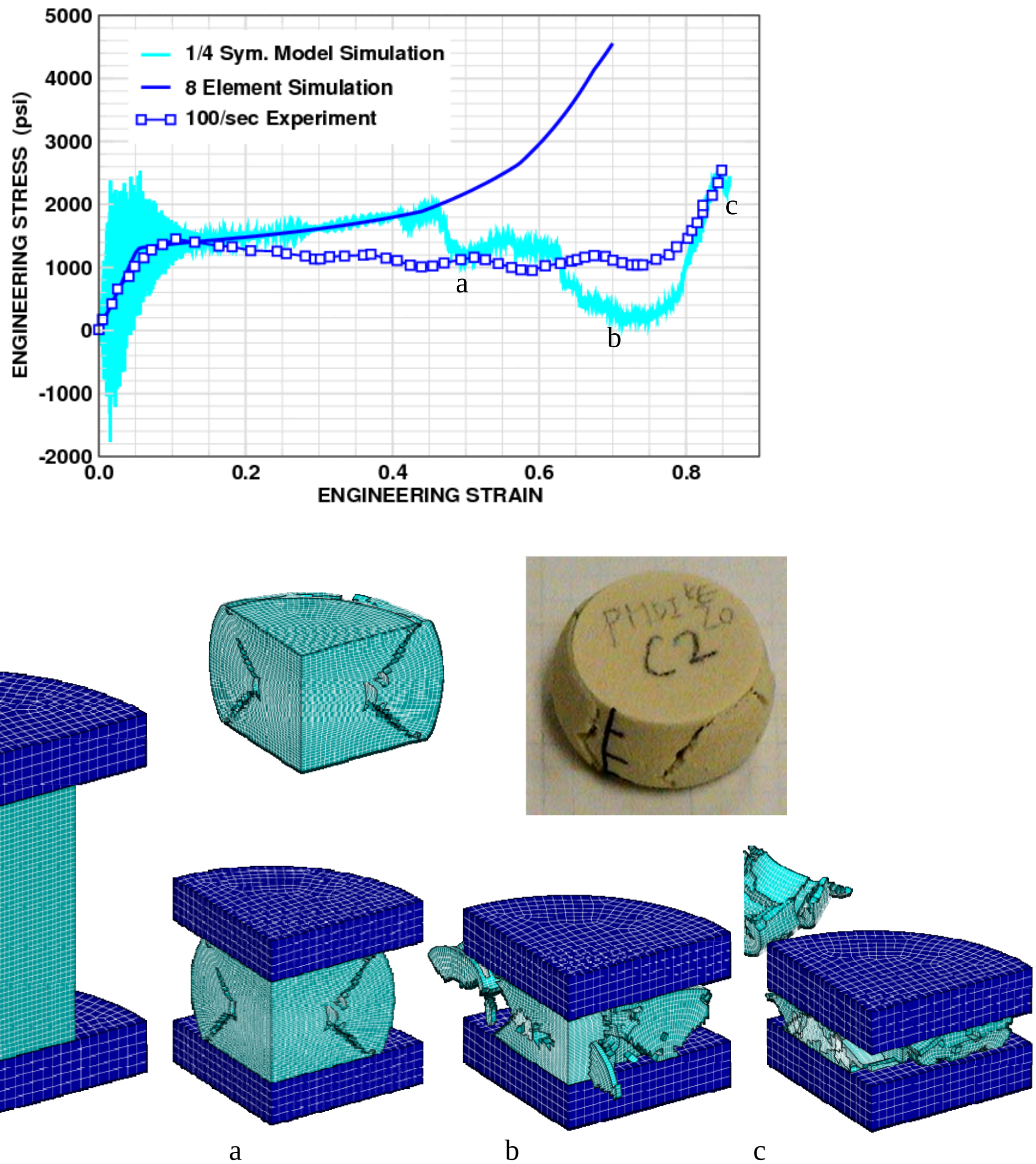


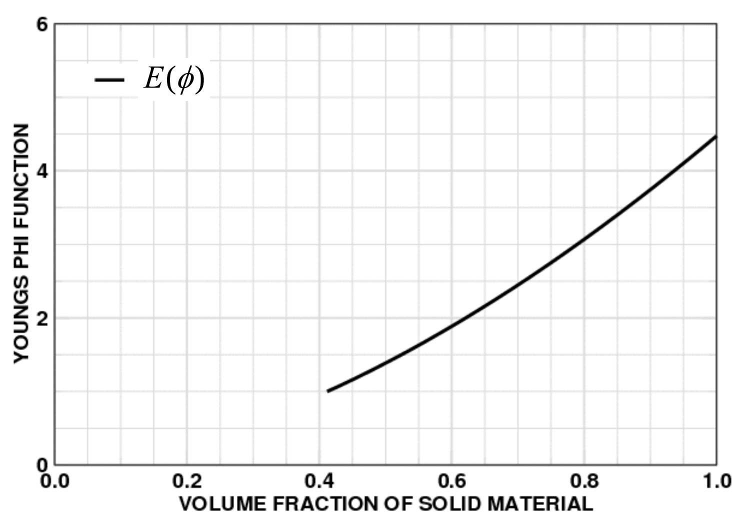
Figure 26. Large model uniaxial compression simulation on PMDI20 foam predicts cracking that was observed in experiments.

7. Foam Damage Model Parameters for TufFoam35

Foam Damage model parameters for 560 kg/m^3 (35 pcf) TufFoam35 foam were obtained next. Material parameters for TufFoam35 at temperatures between -53.9 ^{o}C and 73.9 ^{o}C are given in Table 6 and Figure 27. Since this foam has a density of 560 kg/m^3 and solid TufFoam a density of 1360 kg/m^3, this foam has an initial volume fraction of solid material, ϕ_0 ,equal to 0.4118 (0.4118 = 560/1360). For this foam, β was given a constant value of zero which means that associated flow was used (Figure 27c). Plots of β as a function of volume fraction of solid material for other foams in Figure 27c show that for all foams the flow direction becomes more associated with compaction or increased initial density. The curve used to define damage as a function of damage strain was identical to the curve used for the other foams (Figure 10d). A SIERRA/ADAGIO input block for this material is listed in Appendix D.

Table 6. Foam Damage Model parameters for 560 kg/m^3 (35 pcf) TufFoam35

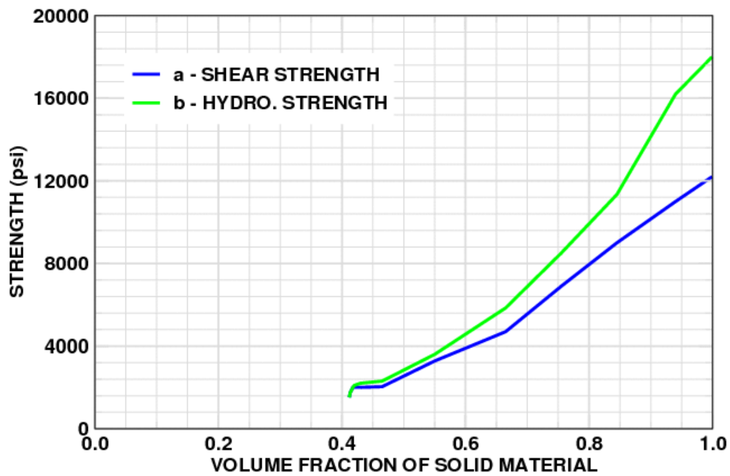
Parameter		Units	Value	Value	Value
Temperature		C	-53.9	21.1	73.9
Young's Modulus	$E_r\, E(\theta)$	psi	81,795	66,500	53,200
Poisson's Ratio	v	-	0.300		
Initial Volume Fraction Solid	ϕ	-	0.4118		
Flow Rate	$h_r\, h(\theta)$	-	-11.0	4.00	10.0
Power Exponent	$n_r\, n(\theta)$	-	15.5	13.45	6.00
Tensile Strength	c	psi	2750.0		
Adam	a_{dam}	-	1.00		
Bdam	b_{dam}	-	0.33		
Thermal Expansion Coefficient		1/C	60.0 x 10^{-6}		



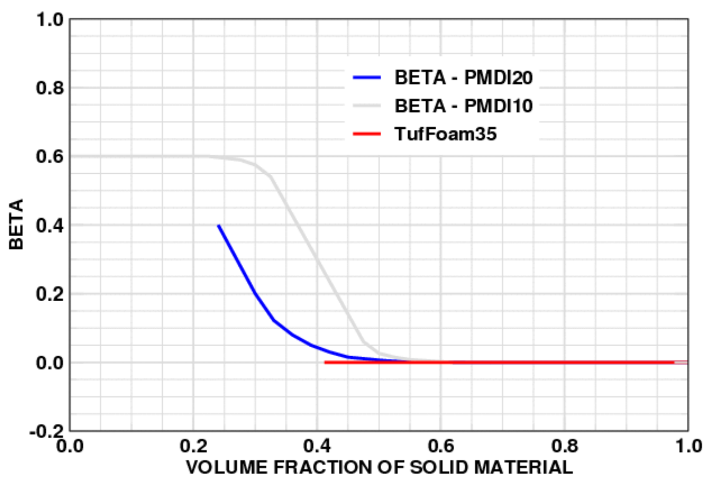
(a) $E(\phi)$ as a function of ϕ - foam can get up to 4.5 x stiffer when loaded into lock-up

Figure 27. Material parameter function for TufFoam35.

Next, uniaxial compression experiments were simulated using the simple 8-element unit cube model shown in Figure 11. For most temperatures and strain rates the fit is good (Figure 28). However, at the cold temperature of -53.9 ^{o}C the foam exhibited cracking prior to lock-up which is not captured by the current Foam Damage model in which the damage does not depend on strain rate or temperature. Next, simulation of the elastically confined compression test with the model shown in Figure 19 also generated load and tube strain predictions in good agreement with the experiments (Figure 29).



(b) shear and hydrostatic strengths increase with foam compaction



(c) flow direction for TufFoam35 is associated

Figure 27 (cont). Material parameter functions for TufFoam35.

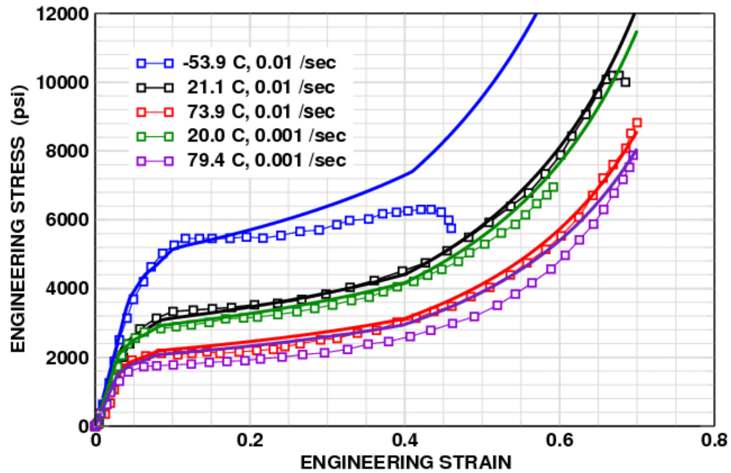
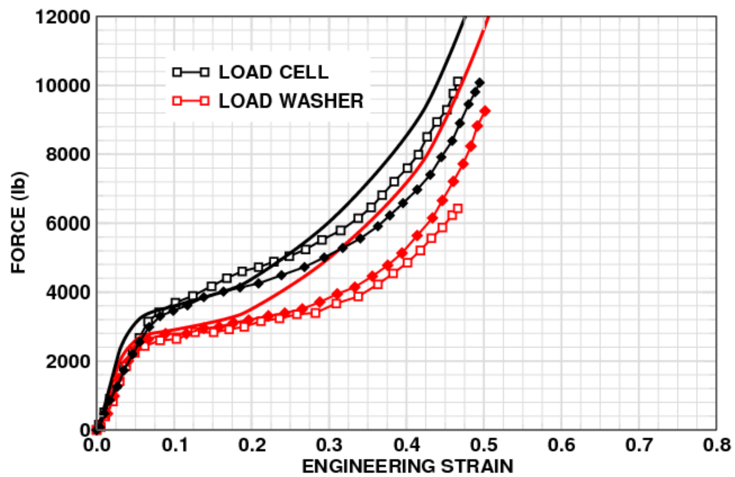
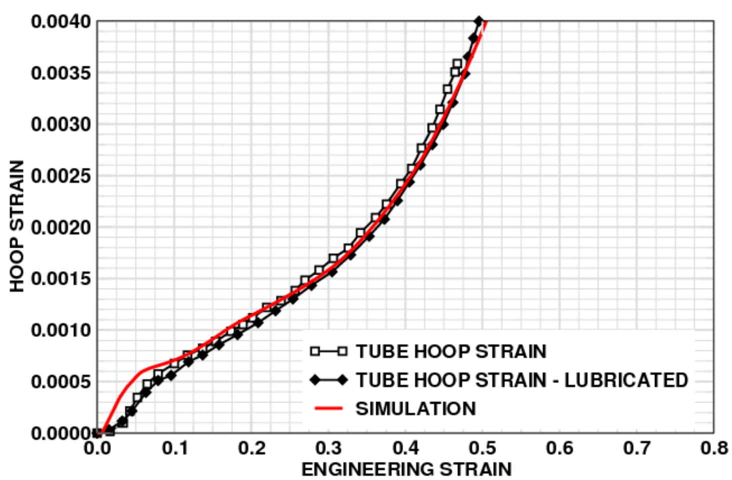


Figure 28. TufFoam35 uniaxial compression experiments (symbols) and simulations (solid lines) at three different temperatures and a constant engineering strain rate of 0.01 per second.



(a) loads as a function of axial engineering strain in foam (solid lines = model)



(b) hoop strain in thin tube as a function of axial engineering strain in foam.

Figure 29. TufFoam35 elastically confined compression experiments and simulations.

The uniaxial tension experiments on TufFoam35 were then simulated using the finite element models with an element edge length of ~40 mils shown in Figure 17. All simulations predicted the correct crack orientation. However, the predicted strain to failure was too large for the room temperature simulation (Figure 30).

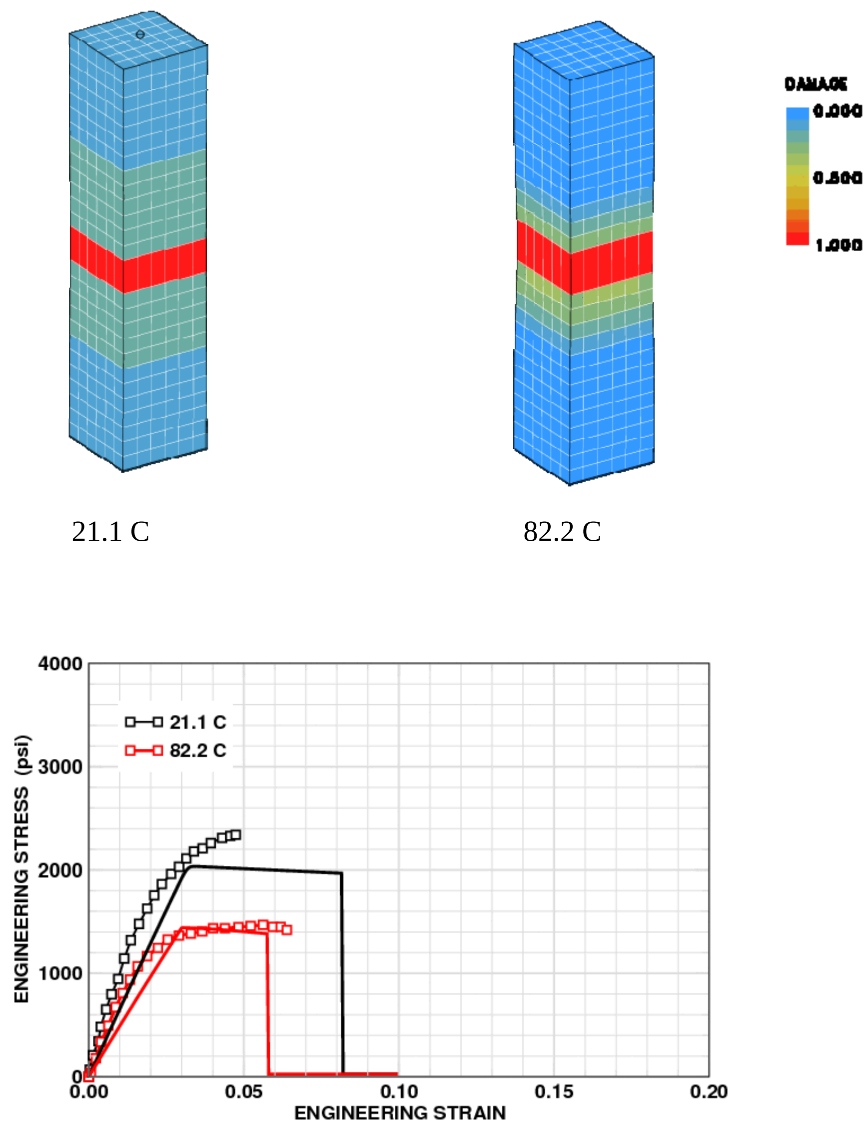


Figure 30. TufFoam35 uniaxial tension experiments (symbols) and simulations (solid line), and deformed shapes with damage contours at an applied engineering strain of 0.10.

8. Fully-Coupled Thermal Stress Analyses

Finally, fully-coupled thermal stress simulations were performed on the PMDI20, room-temperature, uniaxial compression experiments using the simple 8-element unit cube model shown in Figure 11. Plastic work generated by the inelastic deformation was used as a volumetric heat source in the thermal analysis using the following equation

$$\dot{Q} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{in} \tag{18}$$

where $\dot{Q}$ is the volumetric heating rate, $\boldsymbol{\sigma}$ is Cauchy stress and $\dot{\boldsymbol{\varepsilon}}^{in}$ is the inelastic rate. Since PWORK from the FOAM DAMAGE model has units of in-lb/(in^3-sec) it must be scaled by the conversion 10.71x10^{-5} BTU/in-lb before it is used as the volumetric heating rate in ARIA. Thermal properties used in these fully-coupled thermal stress analyses were from [13] and are summarized in Table 7.

Results from these analyses are compared with the original solid mechanics only analyses in Figure 31. As expected, heating due to plastic work causes the predicted stress to be reduced. In all simulations, the samples were compressed to an engineering strain of 0.70 and the sample boundaries were insulated so conduction into the loading platens which could be significant at the slower rates was not included. Peak temperatures of 41.4, 47.0, 51.2, and 57.4 ^{o}C were predicted for tests at applied engineering strain rates of 0.0001, 0.01, 1.0, and 100.0 per second, respectively. These results suggest there can be significant heating of the foam due to plastic work. The ARPEGGIO input file used for the first fully-coupled simulation at a constant engineering strain rate of 0.0001 per second is listed in Appendix E.

For adiabatic heating, the temperature rate, $\dot{\theta}$, is given by [14]

$$\dot{\theta} = \frac{\eta\, \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{in}}{\rho\, c} \tag{19}$$

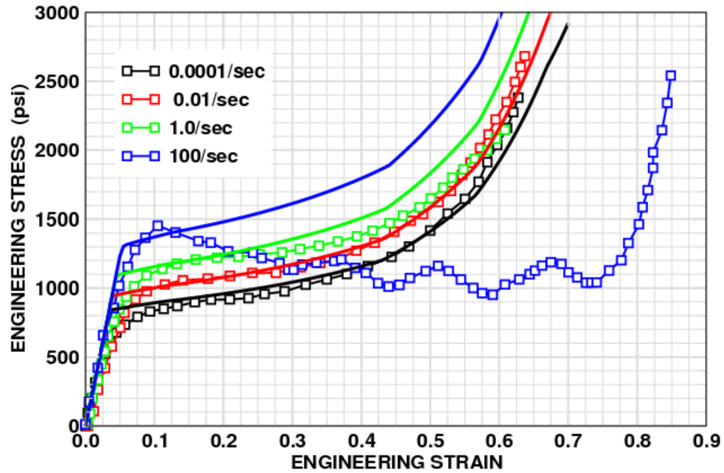
where η is the fraction of plastic work that goes into generating heat. If we assume that all plastic work generates heat and use an average value for the foam density since it increases during the test, then the temperature change from the high-rate uniaxial compression experiment compressed to an engineering strain of 0.70 or a true logarithmic strain of 1.2 (Figure 31) would be approximately

$$\Delta\theta = \frac{\eta\, \sigma_{11} \varepsilon_{11}^{p}}{\rho\, c} = \frac{1 \times 1500 \times 1.2 \dfrac{lb}{in^2}}{0.0131 \times 0.378 \dfrac{lb\ BTU}{in^3\ lb\ C}} \times 10.71 \times 10^{-5} \frac{BTU}{in - lb} = 38.9\, C \tag{19}$$

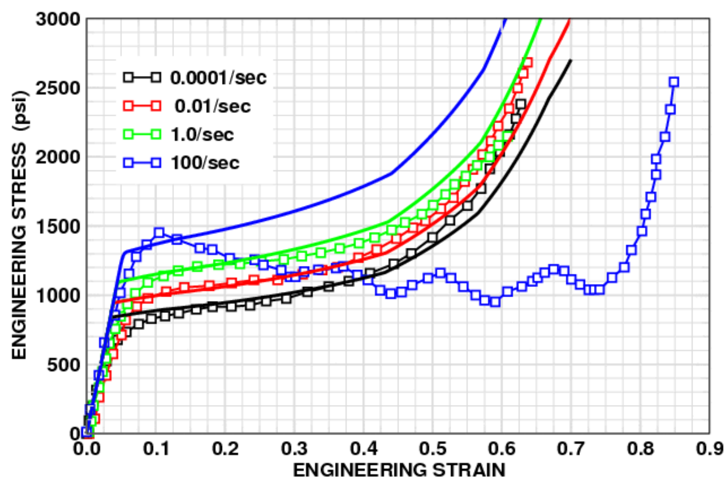
which is a little larger than the temperature change from the high rate simulation (57.4 - 20.0 = 37.4 C) indicating that, as expected, little conduction took place during the high rate test.

Table 7. Thermal properties for 320 kg/m^3 (20 pcf) foams, PMDI20 [13].

Parameter	Units	Value
Density	lb/in^3	0.01042
Thermal Conductivity	BTU/in-sec-C	1.0 x 10^{-6}
Specific Heat	BTU/lb-C	0.3780



(a) original simulations at a constant 21.1 C



(b) simulations with volumetric heating caused by plastic work

Figure 31. PMDI20 uniaxial compression experiments (symbols) and simulations (solid lines) at three different engineering strain rates and a constant temperature of 21.1 ^{o}C or from fully coupled thermal stress analyses in which temperature increases due to plastic work.

9. Summary

A new Unified Creep Plasticity Damage (UCPD) model was developed to describe the mechanical response of rigid polyurethane foams to loading experienced during accidental impact events. Experiments were performed to characterize the mechanical response of several different rigid polyurethane foams to large deformation. In these experiments the effects of load path, loading rate, and temperature were investigated. Results from these experiments indicated that, as expected, these foams exhibit significant volumetric and deviatoric plasticity when they are compressed. Mechanical response of these foams is also significantly affected by changes in either loading rate or temperature. A new UCPD model for foam called the Foam Damage model was developed to capture the effects of load path, loading rate, and temperature on foam mechanical response. The new Foam Damage model also predicts foam cracking for both compressive and tensile loading.

The new Foam Damage model captures most behavior exhibited by polyurethane foams but could be improved to more accurately capture foam response to tensile loading. Experiments revealed that rigid polyurethane foams actually exhibit a small amount of strain hardening prior to failure when they are subjected to uniaxial tension but the Foam Damage model assumes that the foam is elastic perfectly plastic prior to failure in tension. The model should be modified to have hardening depend on not only inelastic volume strain but also on perhaps the equivalent plastic strain for small strains in either tension or compression. The Foam Damage model also assumes that the damage parameters are material constants and the experiments show that foam cracking (damage) depend on the loading rate and temperature. A more complicated evolution equation for damage is needed to capture the effects of loading rate and temperature on damage.

The current foam damage model should provide accurate predictions for the deformation and energy dissipation of rigid polyurethane foams during impact events. The Foam Damage model outputs a state variable, PWORK, which can be used in fully-coupled thermal stress analyses to compute temperature increases in the foam due to plastic work which will, in turn, affect the mechanical response of the foam. Element death based on foam damage reaching a critical level can be used to simulate complex foam cracking in either tension or compression. However, the newly created contact surfaces from foam cracking could prove to be challenging for the contact algorithm. Foam density depends on the foaming process and will vary and unfortunately, foam properties are sensitive to changes in initial foam density so product performance will also be sensitive to variations in foam density. This is an area where uncertainty quantification (UQ) analysis could play a key role in understanding expected variability in product performance. It will be interesting to see how far we can push this new modeling capability.

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Appendix A: FR3712 Input for SIERRA/ADAGIO

```
##
##  Input block for FR3712 Foam Damage Model
##
 begin property specification for material fr3712
      density = 1.80e-5   # lb - s2/in4
      thermal strain function = fr3712_Thermal

      begin parameters for model elastic
           youngs modulus   = 11550.0
           poissons ratio   =   0.250
      end parameters for model elastic

      begin parameters for model foam_damage
           youngs modulus    =    11550.0   # psi
           poissons ratio    =      0.250
           phi               =      0.160
           flow rate         =      1.000
           power exponent    =      1.000
           tensile strength  =      280.0
           adam              =      1.000
           bdam              =      0.500
           youngs function   =        fr3712_Modulus
           poissons function =        fr3712_Constant
           rate function     =        fr3712_Rate
           exponent function =        fr3712_Expo
           shear hardening function = fr3712_Shear
           hydro hardening function = fr3712_Hydro
           beta function     =        fr3712_Beta
           youngs phi function =      fr3712_E
           poissons phi function =    fr3712_Constant
           damage function =          fr3712_Damage
      end parameters for model foam_damage
 end property specification for material fr3712

begin definition for function fr3712_Damage
  type is piecewise linear
   begin values
     0.00000     0.00000
     0.04000     0.00000
     0.30000     1.00000
   100.00000     1.00000
   end values
end definition for function fr3712_Damage

begin definition for function fr3712_Beta
  type is piecewise linear
   begin values
          0.000         0.600
          0.200         0.600
```

```
          0.225          0.600
          0.250          0.595
          0.275          0.590
          0.300          0.575
          0.325          0.540
          0.350          0.460
          0.375          0.380
          0.400          0.300
          0.425          0.220
          0.450          0.140
          0.475          0.060
          0.500          0.026
          0.525          0.015
          0.550          0.008
          0.600          0.002
          0.620          0.000
         10.000          0.000
   end values
end definition for function fr3712_Beta

begin definition for function fr3712_E
  type is piecewise linear
   begin values
     0.00000  1.0000000
     0.16000  1.0000000
     0.18667  1.3282045
     0.21333  1.6647727
     0.24000  2.0317727
     0.26667  2.4281364
     0.29333  2.8528864
     0.32000  3.3052045
     0.34667  3.7843864
     0.37333  4.2897500
     0.40000  4.8207273
     0.42667  5.3767500
     0.45333  5.9573409
     0.48000  6.5620455
     0.50667  7.1904318
     0.53333  7.8421136
     0.56000  8.5167273
     0.58667  9.2139318
     0.61333  9.9333864
     0.64000  10.674818
     0.66667  11.437932
     0.69333  12.222409
     0.72000  13.028068
     0.74667  13.854636
     0.77333  14.701818
     0.80000  15.569500
     0.82667  16.457386
     0.85333  17.365295
     0.88000  18.293045
```

```
     0.90667  19.240432
     0.93333  20.207295
     0.96000  21.193455
     0.98667  22.198727
     1.00000  22.708477
    10.00000  24.000000
   end values
end definition for function fr3712_E

begin definition for function fr3712_Thermal
      type is piecewise linear
      ordinate is strain
      abscissa is temperature
      begin values
      -500.0    -0.0300
         0.0     0.0000
       500.0     0.0300
      end values
end definition for function fr3712_Thermal

begin definition for function fr3712_Constant
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -500.00         1.0
         0.00         1.0
       500.00         1.0
      end values
end definition for function fr3712_Constant

begin definition for function fr3712_Modulus
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90          1.0
       18.30          1.0
       73.90          0.8
      end values
end definition for function fr3712_Modulus

begin definition for function fr3712_Rate
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90       -10.00
       18.30         2.60
       73.90         7.50
      end values
end definition for function fr3712_Rate
```

```
begin definition for function fr3712_Expo
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -53.90         18.0
      18.30         14.0
      73.90          6.0
     end values
end definition for function fr3712_Expo

begin function fr3712_Shear
     type is piecewise linear
     begin values
          0.000         160.0
          0.160         180.0
          0.162         215.0
          0.164         225.0
          0.166         229.0
          0.168         233.0
          0.210         250.0
          0.250         284.0
          0.275         300.0
          0.300         340.0
          0.325         400.0
          0.350         480.0
          0.375         560.0
          0.400         640.0
          0.500         960.0
          0.600        1280.0
          0.700        1600.0
          0.800        1920.0
          0.900        2320.0
          1.050        2800.0
         10.000       20000.0
     end values
end function

begin function fr3712_Hydro
     type is piecewise linear
     begin values
          0.000         160.0
          0.160         180.0
          0.162         215.0
          0.164         225.0
          0.166         229.0
          0.168         233.0
          0.210         250.0
          0.250         284.0
          0.275         300.0
          0.300         340.0
```

```
          0.325          400.0
          0.350          460.0
          0.375          520.0
          0.400          580.0
          0.500          800.0
          0.600         1150.0
          0.700         1500.0
          0.800         2000.0
          0.850         2400.0
          0.900         3100.0
          1.050         5000.0
         10.000        20000.0
     end values
end function

###
### end of input block for fr3712 foam
###
```

Appendix B: PMDI10 Input for SIERRA/ADAGIO

```
##
##  Input block for PMDI10 Foam Damage Model
##
 begin property specification for material pmdi10
      density = 1.50e-5   # lb - s2/in4
      thermal strain function = pmdi10_Thermal

      begin parameters for model elastic
           youngs modulus   =  8778.0
           poissons ratio   =   0.250
      end parameters for model elastic

      begin parameters for model foam_damage
           youngs modulus    =    11550.0   # psi
           poissons ratio    =      0.250
           phi               =      0.133
           flow rate         =      1.000
           power exponent    =      1.000
           tensile strength  =      280.0
           adam              =      1.000
           bdam              =      0.500
           youngs function   =        pmdi10_Modulus
           poissons function =        pmdi10_Constant
           rate function     =        pmdi10_Rate
           exponent function =        pmdi10_Expo
           shear hardening function = pmdi10_Shear
           hydro hardening function = pmdi10_Hydro
           beta function     =        pmdi10_Beta
           youngs phi function =      pmdi10_E
           poissons phi function =    pmdi10_Constant
           damage function =          pmdi10_Damage
      end parameters for model foam_damage
 end property specification for material pmdi10

begin definition for function pmdi10_Damage
  type is piecewise linear
   begin values
     0.00000     0.00000
     0.04000     0.00000
     0.30000     1.00000
   100.00000     1.00000
   end values
end definition for function pmdi10_Damage

begin definition for function pmdi10_Beta
  type is piecewise linear
   begin values
          0.000         0.600
          0.200         0.600
```

```
          0.225         0.600
          0.250         0.595
          0.275         0.590
          0.300         0.575
          0.325         0.540
          0.350         0.460
          0.375         0.380
          0.400         0.300
          0.425         0.220
          0.450         0.140
          0.475         0.060
          0.500         0.026
          0.525         0.015
          0.550         0.008
          0.600         0.002
          0.620         0.000
         10.000         0.000
   end values
end definition for function pmdi10_Beta

begin definition for function pmdi10_E
  type is piecewise linear
   begin values
     0.00000  0.7600000
     0.13300  0.7600000
     0.16000  1.0000000
     0.18667  1.3282045
     0.21333  1.6647727
     0.24000  2.0317727
     0.26667  2.4281364
     0.29333  2.8528864
     0.32000  3.3052045
     0.34667  3.7843864
     0.37333  4.2897500
     0.40000  4.8207273
     0.42667  5.3767500
     0.45333  5.9573409
     0.48000  6.5620455
     0.50667  7.1904318
     0.53333  7.8421136
     0.56000  8.5167273
     0.58667  9.2139318
     0.61333  9.9333864
     0.64000  10.674818
     0.66667  11.437932
     0.69333  12.222409
     0.72000  13.028068
     0.74667  13.854636
     0.77333  14.701818
     0.80000  15.569500
     0.82667  16.457386
     0.85333  17.365295
```

```
     0.88000  18.293045
     0.90667  19.240432
     0.93333  20.207295
     0.96000  21.193455
     0.98667  22.198727
     1.00000  22.708477
    10.00000  24.000000
   end values
end definition for function pmdi10_E

begin definition for function pmdi10_Thermal
     type is piecewise linear
     ordinate is strain
     abscissa is temperature
     begin values
     -500.0    -0.0300
        0.0     0.0000
      500.0     0.0300
     end values
end definition for function pmdi10_Thermal

begin definition for function pmdi10_Constant
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -500.00         1.0
        0.00         1.0
      500.00         1.0
     end values
end definition for function pmdi10_Constant

begin definition for function pmdi10_Modulus
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -53.90          1.2
      21.10          1.0
      73.90          0.8
      82.20          0.76
     end values
end definition for function pmdi10_Modulus

begin definition for function pmdi10_Rate
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -53.90       -12.00
      21.10         2.80
      73.90         5.80
```

```
      82.20          6.31
     end values
end definition for function pmdi10_Rate

begin definition for function pmdi10_Expo
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -53.90          16.0
      21.10          13.5
      73.90           9.0
      82.20           8.25
     end values
end definition for function pmdi10_Expo

begin function pmdi10_Shear
     type is piecewise linear
     begin values
          0.000          200.0
          0.133          210.0
          0.135          245.0
          0.235          250.0
          0.275          284.0
          0.300          300.0
          0.325          340.0
          0.350          400.0
          0.375          480.0
          0.400          560.0
          0.425          640.0
          0.525          960.0
          0.625         1280.0
          0.725         1600.0
          0.825         1920.0
          0.925         2320.0
          1.075         2800.0
         10.000        20000.0
     end values
end function

begin function pmdi10_Hydro
     type is piecewise linear
     begin values
          0.000          200.0
          0.133          210.0
          0.135          245.0
          0.235          250.0
          0.275          284.0
          0.300          300.0
          0.325          340.0
          0.350          400.0
          0.375          460.0
```

```
          0.400         520.0
          0.425         580.0
          0.525         800.0
          0.625        1150.0
          0.725        1500.0
          0.825        2000.0
          0.875        2400.0
          0.925        3100.0
          1.075        5000.0
         10.000       20000.0
     end values
end function

###
### end of input block for pmdi10 foam
###
```

Appendix C: PMDI20 Input for SIERRA/ADAGIO

```
##
##  Input block for PMDI20 Foam Damage Model
##  based on experiments mostly in the 18 pcf range.
##
 begin property specification for material pmdi20
      density = 2.70e-5   # lb - s2/in4
      thermal strain function = pmdi20_thermal

      begin parameters for model elastic
           youngs modulus   = 23500.0
           poissons ratio   =   0.300
      end parameters for model elastic

      begin parameters for model foam_damage
           youngs modulus    =    23500.0
           poissons ratio    =      0.300
           phi               =      0.240
           flow rate         =      1.000
           power exponent    =      1.000
           tensile strength  =      950.0
           adam              =      1.000
           bdam              =      0.330
           youngs function   =        pmdi20_modulus
           poissons function =        pmdi20_constant
           rate function     =        pmdi20_rate
           exponent function =        pmdi20_expo
           shear hardening function = pmdi20_shear
           hydro hardening function = pmdi20_hydro
           beta function     =        pmdi20_beta
           youngs phi function =      pmdi20_E
           poissons phi function =    pmdi20_v
           damage function =          pmdi20_damage
      end parameters for model foam_damage
 end property specification for material pmdi20

begin definition for function pmdi20_thermal
      type is piecewise linear
      ordinate is strain
      abscissa is temperature
      begin values
      -500.0    -0.0300
         0.0     0.0000
       500.0     0.0300
      end values
end definition for function pmdi20_thermal
```

```
begin definition for function pmdi20_constant
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -500.00         1.0
         0.00         1.0
       500.00         1.0
      end values
end definition for function pmdi20_constant

begin definition for function pmdi20_modulus
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90          1.23
       21.10          1.0
       73.90          0.80
       82.20          0.75
      end values
end definition for function pmdi20_modulus

begin definition for function pmdi20_rate
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90        -8.00
       21.10         2.32
       73.90         6.50
       82.20         7.00
      end values
end definition for function pmdi20_rate

begin definition for function pmdi20_expo
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90          16.5
       21.10          13.45
       73.90           8.0
       82.20           7.0
      end values
end definition for function pmdi20_expo

begin definition for function pmdi20_damage
  type is piecewise linear
   begin values
     0.00000     0.00000
     0.04000     0.00000
```

```
      0.30000     1.00000
    100.00000     1.00000
    end values
end definition for function pmdi20_damage

begin definition for function pmdi20_E
  type is piecewise linear
    begin values
      0.02667    0.0243182
      0.05333    0.0785476
      0.08000    0.1559543
      0.10667    0.2537081
      0.13333    0.3700307
      0.16000    0.5036914
      0.18667    0.6537171
      0.21333    0.8193696
      0.24000    1.0000000
      0.26667    1.1950827
      0.29333    1.4041366
      0.32000    1.6267590
      0.34667    1.8626032
      0.37333    2.1113336
      0.40000    2.3726705
      0.42667    2.6463344
      0.45333    2.9320902
      0.48000    3.2297143
      0.50667    3.5389942
      0.53333    3.8597396
      0.56000    4.1917716
      0.58667    4.5349225
      0.61333    4.8890244
      0.64000    5.2539430
      0.66667    5.6295331
      0.69333    6.0156379
      0.72000    6.4121681
      0.74667    6.8189892
      0.77333    7.2359561
      0.80000    7.6630126
      0.82667    8.1000134
      0.85333    8.5468691
      0.88000    9.0034900
      0.90667    9.4697756
      0.93333    9.9456476
      0.96000    10.431016
      0.98667    10.925793
      1.00000    11.176682
     10.00000    11.200000
    end values
end definition for function pmdi20_E
```

```
begin definition for function pmdi20_v
  type is piecewise linear
   begin values
     0.00000    1.000
     0.24000    1.000
     1.00000    1.000
    10.00000    1.000
   end values
end definition for function pmdi20_v

begin function pmdi20_shear
     type is piecewise linear
     begin values
               0             590
          0.2400             590
          0.2450             600
          0.2500             605
          0.3000             610
          0.3250             700
          0.3500             900
          0.4000            1100
          0.4500            1300
          0.5000            1600
          0.5500            1800
          0.6000            2000
          0.6500            2200
          0.7000            2400
          0.7500            2600
          0.8000            2800
          0.8500            3000
          0.9000            3200
          0.9500            3400
          0.9999            3600
             1.0          180000
              10          360000
     end values
end function

begin function pmdi20_hydro
     type is piecewise linear
     begin values
               0             885
          0.2400             885
          0.2450             900
          0.2500             950
          0.3000            1000
          0.3500            1100
          0.4000            1250
          0.4500            1500
          0.5000            1800
          0.5500            2100
          0.6000            2400
```

```
          0.6500              2800
          0.7000              3300
          0.7500              3900
          0.8000              4600
          0.8500              5400
          0.9000              6300
          0.9500              7200
          0.9999              8200
             1.0            180000
              10            360000
     end values
end function pmdi20_hydro

begin definition for function pmdi20_beta
  type is piecewise linear
   begin values
               0        0.400
          0.2400        0.400
          0.3000        0.200
          0.3300        0.122
          0.3600        0.080
          0.3900        0.050
          0.4200        0.030
          0.4500        0.015
          0.4800        0.010
          0.5100        0.005
          0.5400        0.002
          0.6400        0.000
              10        0.000
   end values
end definition for function pmdi20_beta


###
### end of input block for pmdi20 foam
###
```

Appendix D: TufFoam35 Input for SIERRA/ADAGIO

```
##
##  35 pcf Tuf foam - Foam_Damage Model
##
 begin property specification for material tuf35
      density = 5.247e-5   # lb - s2/in4
      thermal strain function = tuf35_thermal

      begin parameters for model elastic
           youngs modulus   = 66500.0   # psi
           poissons ratio   = 0.30
      end parameters for model elastic

      begin parameters for model foam_damage
           youngs modulus    =    66500.0
           poissons ratio    =      0.300
           phi               =     0.4118
           flow rate         =      1.000
           power exponent    =      1.000
           tensile strength  =     2750.0
           adam              =      1.000
           bdam              =      0.330
           youngs function   =        tuf35_modulus
           poissons function =        tuf35_constant
           rate function     =        tuf35_rate
           exponent function =        tuf35_expo
           shear hardening function = tuf35_shear
           hydro hardening function = tuf35_hydro
           beta function     =        tuf35_beta
           youngs phi function =      tuf35_E
           poissons phi function  =   tuf35_v
           damage function         =  tuf35_damage
      end parameters for model foam_damage
 end property specification for material tuf35

begin definition for function tuf35_damage
  type is piecewise linear
   begin values
     0.00000     0.00000
     0.04000     0.00000
     0.30000     1.00000
   100.00000     1.00000
   end values
end definition for function tuf35_damage

begin definition for function tuf35_E
  type is piecewise linear
   begin values
         0.40000    0.9499368
         0.41180    1.0000000
```

```
          0.42667    1.0595053
          0.45333    1.1739053
          0.48000    1.2930632
          0.50667    1.4168947
          0.53333    1.5453158
          0.56000    1.6782421
          0.58667    1.8156316
          0.61333    1.9574000
          0.64000    2.1035053
          0.66667    2.2538737
          0.69333    2.4084526
          0.72000    2.5672105
          0.74667    2.7300947
          0.77333    2.8970316
          0.80000    3.0680105
          0.82667    3.2429684
          0.85333    3.4218737
          0.88000    3.6046842
          0.90667    3.7913789
          0.93333    3.9819053
          0.96000    4.1762211
          0.98667    4.3743158
          1.00000    4.4747579
          2.00000    5.2631579
    end values
end definition for function tuf35_E

begin definition for function tuf35_v
  type is piecewise linear
    begin values
          0.40000         1.000
          0.41180         1.000
          1.00000         1.000
          2.00000         1.000
    end values
end definition for function tuf35_v

begin definition for function tuf35_thermal
     type is piecewise linear
     ordinate is strain
     abscissa is temperature
     begin values
     -500.0    -0.0300
        0.0     0.0000
      500.0     0.0300
     end values
end definition for function tuf35_thermal

begin definition for function tuf35_constant
     type is piecewise linear
     ordinate is temperature
     abscissa is time
```

```
      begin values
      -500.00         1.0
         0.00         1.0
       500.00         1.0
      end values
end definition for function tuf35_constant

begin definition for function tuf35_modulus
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90          1.23
       21.10          1.0
       73.90          0.80
       82.20          0.75
      end values
end definition for function tuf35_modulus

begin definition for function tuf35_rate
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90       -11.00
       21.10         4.00
       73.90        10.00
       82.20        12.00
      end values
end definition for function tuf35_rate

begin definition for function tuf35_expo
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90          15.50
       21.10          13.45
       73.90           6.00
       82.20           5.00
      end values
end definition for function tuf35_expo

begin function tuf35_shear
      type is piecewise linear
      begin values
           0.0000       1480.0
           0.4118       1520.0
           0.4134       1760.0
           0.4150       1840.0
           0.4175       2000.0
           0.4200       2002.0
```

```
          0.4300        2004.0
          0.4650        2036.0
          0.5500        3280.0
          0.6650        4700.0
          0.7550        6900.0
          0.8450        9000.0
          0.9400       11000.0
          0.9990       12200.0
          1.0000      180000.0
         10.0000      180000.0
     end values
end function tuf35_shear

begin function tuf35_hydro
     type is piecewise linear
     begin values
          0.0000        1480.0
          0.4118        1520.0
          0.4134        1760.0
          0.4150        1840.0
          0.4175        2000.0
          0.4200        2101.0
          0.4300        2204.0
          0.4650        2308.0
          0.5500        3600.0
          0.6650        5850.0
          0.7550        8505.0
          0.8450       11340.0
          0.9400       16200.0
          0.9990       18000.0
          1.0000      270000.0
         10.0000      270000.0
     end values
end function tuf35_hydro

begin definition for function tuf35_beta
  type is piecewise linear
   begin values
          0.0000       0.0
          0.4118       0.0
         10.0000       0.0
   end values
end definition for function tuf35_beta


###
### end of input block for tuf foam
###
```

Appendix E: Fully-Coupled Thermal Stress Analysis Input for PMDI20

```
Begin Sierra foam_test

  title  Tensile Specimen 0.0001 inch per second arpeggio

  begin definition for function constant_zero
    type is piecewise linear
    ordinate is value
    abscissa is time
    begin values
           0.0    0.0
       10000.0    0.0
    end values
  end definition for function constant_zero

  begin definition for function delta
    type is piecewise linear
    begin values
         0.0   0.00
      7000.0  -0.70
    end values
  end definition for function delta

  define direction x with vector 1.0 0.0 0.0
  define direction y with vector 0.0 1.0 0.0
  define direction z with vector 0.0 0.0 1.0

 begin aria material pmdi20
 density               = mass_preserving rho = 0.0104200 ##lb/in^3
 thermal conductivity = constant k          = 0.0000010 ##BTU/in-sec-C
 specific heat        = constant cp         = 0.3780000 ##BTU/lb-C
 heat conduction      = Fouriers_law
 end

##
##  Input block for PMDI20 Foam Damage Model
##  based on experiments mostly in the 18 pcf range.
##
 begin property specification for material pmdi20
     density = 2.70e-5   # lb - s2/in4
     thermal strain function = pmdi20_thermal

     begin parameters for model elastic
          youngs modulus   = 23500.0
          poissons ratio   =   0.300
     end parameters for model elastic

     begin parameters for model foam_damage
          youngs modulus    =    23500.0
          poissons ratio    =      0.300
```

```
          phi                =      0.240
          flow rate          =      1.000
          power exponent     =      1.000
          tensile strength   =      950.0
          adam               =      1.000
          bdam               =      0.330
          youngs function    =        pmdi20_modulus
          poissons function =         pmdi20_constant
          rate function      =        pmdi20_rate
          exponent function =         pmdi20_expo
          shear hardening function = pmdi20_shear
          hydro hardening function = pmdi20_hydro
          beta function      =        pmdi20_beta
          youngs phi function =       pmdi20_E
          poissons phi function =     pmdi20_v
          damage function =           pmdi20_damage
     end parameters for model foam_damage
 end property specification for material pmdi20

begin definition for function pmdi20_thermal
     type is piecewise linear
     ordinate is strain
     abscissa is temperature
     begin values
        0.0     0.0000
     1000.0     0.0600
     end values
end definition for function pmdi20_thermal

begin definition for function pmdi20_constant
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
        0.00         1.0
     1000.00         1.0
     end values
end definition for function pmdi20_constant

begin definition for function pmdi20_modulus
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     219.10          1.23
     294.10          1.0
     346.90          0.80
     355.20          0.75
     600.00          0.05
     end values
end definition for function pmdi20_modulus
```

```
begin definition for function pmdi20_rate
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      219.10        -9.00
      294.10         2.32
      346.90         4.50
      355.20         5.00
      600.00        15.74
      end values
end definition for function pmdi20_rate

begin definition for function pmdi20_expo
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      219.10          15.5
      294.10          13.45
      346.90           6.0
      355.20           5.0
      600.00           2.0
      end values
end definition for function pmdi20_expo

begin definition for function pmdi20_damage
  type is piecewise linear
   begin values
     0.00000     0.00000
     0.04000     0.00000
     0.30000     1.00000
   100.00000     1.00000
   end values
end definition for function pmdi20_damage

begin definition for function pmdi20_E
  type is piecewise linear
   begin values
     0.02667    0.0243182
     0.05333    0.0785476
     0.08000    0.1559543
     0.10667    0.2537081
     0.13333    0.3700307
     0.16000    0.5036914
     0.18667    0.6537171
     0.21333    0.8193696
     0.24000    1.0000000
     0.26667    1.1950827
     0.29333    1.4041366
     0.32000    1.6267590
     0.34667    1.8626032
```

```
      0.37333    2.1113336
      0.40000    2.3726705
      0.42667    2.6463344
      0.45333    2.9320902
      0.48000    3.2297143
      0.50667    3.5389942
      0.53333    3.8597396
      0.56000    4.1917716
      0.58667    4.5349225
      0.61333    4.8890244
      0.64000    5.2539430
      0.66667    5.6295331
      0.69333    6.0156379
      0.72000    6.4121681
      0.74667    6.8189892
      0.77333    7.2359561
      0.80000    7.6630126
      0.82667    8.1000134
      0.85333    8.5468691
      0.88000    9.0034900
      0.90667    9.4697756
      0.93333    9.9456476
      0.96000    10.431016
      0.98667    10.925793
      1.00000    11.176682
     10.00000    11.200000
    end values
end definition for function pmdi20_E

begin definition for function pmdi20_v
  type is piecewise linear
    begin values
      0.00000    1.000
      0.24000    1.000
      1.00000    1.000
     10.00000    1.000
    end values
end definition for function pmdi20_v

begin function pmdi20_shear
     type is piecewise linear
     begin values
              0              590
         0.2400              590
         0.2450              600
         0.2500              605
         0.3000              610
         0.3250              700
         0.3500              900
         0.4000             1100
         0.4500             1300
         0.5000             1600
```

```
          0.5500              1800
          0.6000              2000
          0.6500              2200
          0.7000              2400
          0.7500              2600
          0.8000              2800
          0.8500              3000
          0.9000              3200
          0.9500              3400
          0.9999              3600
             1.0            180000
              10            360000
     end values
end function

begin function pmdi20_hydro
     type is piecewise linear
     begin values
               0               885
          0.2400               885
          0.2450               900
          0.2500               950
          0.3000              1000
          0.3500              1100
          0.4000              1250
          0.4500              1500
          0.5000              1800
          0.5500              2100
          0.6000              2400
          0.6500              2800
          0.7000              3300
          0.7500              3900
          0.8000              4600
          0.8500              5400
          0.9000              6300
          0.9500              7200
          0.9999              8200
             1.0            180000
              10            360000
     end values
end function pmdi20_hydro

begin function pmdi20_hydro2
     type is piecewise linear
     begin values
               0               600
          0.2400               630
          0.2420               750
          0.5000              1800
          0.5500              2100
          0.6000              2400
          0.6500              2800
```

```
          0.7000            3300
          0.7500            3900
          0.8000            4600
          0.8500            5400
          0.9000            6300
          0.9500            7200
          0.9999            8200
             1.0          180000
              10          360000
      end values
end function

begin definition for function pmdi20_beta
  type is piecewise linear
   begin values
               0        0.400
          0.2400        0.400
          0.3000        0.200
          0.3300        0.122
          0.3600        0.080
          0.3900        0.050
          0.4200        0.030
          0.4500        0.015
          0.4800        0.010
          0.5100        0.005
          0.5400        0.002
          0.6400        0.000
              10        0.000
   end values
end definition for function pmdi20_beta
###
### end of input block for pmdi20 foam
###
  begin finite element model mesh_arpeggio
    Database Name = testadagio.g
    Database Type = exodusII

    begin parameters for block block_1
      material pmdi20
      solid mechanics use model foam_damage
    end

  end finite element model mesh_arpeggio

  Begin Aztec Equation Solver solve_temperature
    solution method         = gmres
    preconditioning method  = jacobi
    maximum iterations      = 9000
    residual norm scaling   = none
    residual norm tolerance = 1.0e-6
##    matrix reduction        = fei-remove-slaves
    debug output level      = 0
```

```
  end

  Begin Trilinos Equation Solver Direct_Solver
    Solution Method = amesos-superlu
  End

  Begin Trilinos Equation Solver Iterative_Solver
    Maximum Iterations      = 1000
    Solution        Method  = Gmres
    Residual Norm Tolerance = 1e-4
    Preconditioning Method  = Dd-Ilut
    Residual Norm Scaling   = Rhs
    Param-Int AZ_Kspace Value 1000
  End

  Begin Procedure The_Procedure

    Begin Solution Control Description

      Begin Initialize Init
        Advance AriaRegion
        Transfer Aria_to_Adagio_Init
        Advance AdagioRegion
      End

      Use System Main

      Begin System Main
        Use Initialize Init
        Begin Transient Time_Block_1
          Transfer Adagio_to_Aria
          Transfer Adagio_Status_to_Aria
          Advance AriaRegion
          Transfer Aria_to_Adagio
          Advance AdagioRegion
        End
      End

      # Time-stepping parameters
      Begin Parameters for Transient Time_Block_1
        start time = 0.0
        termination time = 7000.0
        Begin Parameters for Aria Region AriaRegion
          Initial Time Step Size = 10.0
          Time Step Variation    = Fixed
        End
        Begin Parameters for Adagio Region AdagioRegion
          time increment = 10.0
        End
      End
End Solution Control Description
```

```
Begin Transfer Aria_to_Adagio_Init
  Copy Volume Nodes from AriaRegion to AdagioRegion
  Send Field solution->TEMPERATURE State NEW to Temperature State NEW
  Send Field solution->TEMPERATURE State NEW to Temperature State OLD
End

Begin Transfer Aria_to_Adagio
  Copy Volume Nodes from AriaRegion to AdagioRegion
  Send Field solution->TEMPERATURE State NEW to Temperature State NEW
End

Begin Transfer Adagio_to_Aria
  Copy Volume Nodes from AdagioRegion to AriaRegion
  Send Field DISPLACEMENT State New to Solution->Mesh_Displacements
State New
End

    Begin Transfer Adagio_Status_to_Aria
      Copy Volume elements from AdagioRegion to AriaRegion
      Send Field pwork State New to pwork State None
##      Send Field life State New to sm_death_status State None
    End

    Begin Adagio Region AdagioRegion

      Use Finite Element Model mesh_arpeggio
      options = thermalstrain

## output description ###
   begin results output output_1
     Database Name = arp1.e
     Database Type = exodusII
     At Step 0, Increment = 1
     nodal Variables = displacement as displ
     nodal Variables = temperature as temp
     element Variables = stress
     element Variables = volume
     element Variables = iter
     element Variables = epvol
     element Variables = phi
     element Variables = fa
     element Variables = fb
     element Variables = damage
     element Variables = emax
     element Variables = pwork
     global Variables = timestep as timestep
     global Variables = breact
     global Variables = preact
     global Variables = bdisp
     global Variables = pdisp
   end results output output_1
```

```
   begin user output bload
     node set = nodelist_5
     compute global breact as sum of nodal reaction
     compute global bdisp as average of nodal displacement
   end

   begin user output pload
     node set = nodelist_6
     compute global preact as sum of nodal reaction
     compute global pdisp as average of nodal displacement
   end

      begin node based time step parameters nbase
        step interval = 100
        time step limit = 0.10
      end

      begin fixed displacement
        node set = nodelist_4
        components = x
      end fixed displacement

      begin fixed displacement
        node set = nodelist_1
        components = y
      end fixed displacement

      begin fixed displacement
        node set = nodelist_5
        components = z
      end fixed displacement

      begin prescribed displacement
        node set = nodelist_6
        direction = z
        function = delta
        scale factor = 1.0
      end prescribed displacement

#--------- Initial Conditions ---------
      begin initial condition
        include all blocks
        initialize variable name = temperature
        variable type = element
        magnitude = 294.1
      end initial condition

        ### --------------------- ###
        ###   Solver Definition   ###
        ### --------------------- ###

        begin solver
```

```
          begin loadstep predictor
            type = scale_factor
            scale factor = 1.0
          end loadstep predictor
          begin cg
            iteration print                  = 100
            target relative residual         = 0.0001
            acceptable relative residual     = 0.0005
            maximum iterations               =   1000
            orthogonality measure for reset = 0.5
            line search secant
            preconditioner = elastic
          end cg
        end solver
    End Adagio Region AdagioRegion

    Begin Aria Region AriaRegion

      Use Finite Element Model mesh_arpeggio
      Use Linear Solver solve_temperature
      Nonlinear Solution Strategy = Newton
      Maximum Nonlinear Iterations = 500
      Nonlinear Residual Tolerance = 1e-6

      EQ Energy for Temperature on all_blocks using Q1 with MASS DIFF
      EQ Mesh for Mesh_Displacements on all_blocks using Q1 with Xfer
##
##  plastic work goes to heating 10.71e-5 BTU / in-lb
##
      Begin volume heating uniform
        element variable = pwork
        field scaling = 10.71e-5
        add volume block_1
      End volume heating uniform

      Begin User Variable pwork
        type = element real length = 1
        initial value = 0.0
        add part block_1
      End

      Begin Initial Condition ic1
        All Volumes
        Temperature = 294.1
      End

    End Aria Region AriaRegion
  End Procedure The_Procedure
End Sierra foam_test
```

Distribution

1	MS0557	C.D. Croessmann	01520
1	MS0557	T. Simmermacher	01523
1	MS0346	D. Peebles	01526
1	MS0346	M.K. Neilsen	01526
1	MS0346	R.S. Chambers	01526
1	MS0346	D. Vangoethem	01526
1	MS0346	M. Negus	01526
1	MS0557	B. Song	01558
1	MS0840	J. Redmond	01550
1	MS0840	H.E. Fang	01554
1	MS0840	J. Bean	01554
1	MS0840	T.D. Hinnerichs	01554
1	MS0840	C.S. Lo	01554
1	MS0840	N.L. Breivik	01554
1	MS0840	L.N. Kmetyk	01554
1	MS0840	S. Grange	01554
1	MS0840	C. Hammetter	01554
1	MS0840	J. Pott	01555
1	MS1135	D. Epp	01531
1	MS9042	M.L. Chiesa	08250
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1	MS9042	J.J. Dike	08259
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