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Plasma Physics Approximations in Ares

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(U) Plasma Physics Approximations in Ares

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Abstract

Lee & More derived analytic forms for the transport properties of a plasma. Many hydro-codes use their formulae for electrical and thermal conductivity. The coefficients are complex functions of Fermi-Dirac integrals, $F_n(\mu/\theta)$, the chemical potential, μ or $\zeta = \ln(1+e^{\mu/\theta})$, and the temperature, $\theta = kT$. Since these formulae are expensive to compute, rational function approximations were fit to them. Approximations are also used to find the chemical potential, either μ or ζ. The fits use ζ as the independent variable instead of μ/θ.

New fits are provided for $A^\alpha(\zeta), A^\beta(\zeta), \zeta, f(\zeta) = (1 + e^{-\frac{\mu}{\theta}})F_{\frac{1}{2}}\left(\frac{\mu}{\theta}\right), F'_{\frac{1}{2}}/F_{\frac{1}{2}}, F_c^\alpha$, and F_c^β In each case the relative error of the fit is minimized since the functions can vary by many orders of magnitude. The new fits are designed to exactly preserve the limiting values in the non-degenerate and highly degenerate limits or as $\zeta \to 0$ or ∞.The original fits due to Lee & More[6] and George Zimmerman [9] are presented for comparison.

1 Introduction

Lee & More[6] derive transport properties of a plasma. Ares uses their formulae for electrical and thermal conductivity. The electrical conductivity is given by

$$\sigma = \frac{3\theta^{3/2}\langle Z_{\text{eff}}\rangle A^\alpha F_c^\alpha}{2^{3/2}\pi\langle Z_{\text{eff}}^2\rangle e^2 m_e^{1/2}\ln\Lambda}\left(1+e^{-\frac{\mu}{\theta}}\right)F_{\frac{1}{2}}\left(\frac{\mu}{\theta}\right) \tag{1}$$

In thermal diffusion the partial differential equation to be solved is

$$\rho\frac{\partial\mathcal{E}}{\partial T}\frac{dT}{dt} - \vec{\nabla}\cdot\left(\kappa\vec{\nabla}T\right) = 0 \tag{2}$$

The coefficient κ is given by

$$\kappa = \frac{3k\theta^{5/2}\langle Z_{\text{eff}}\rangle A^{\beta} F_c^{\beta}}{2^{3/2}\pi\langle Z_{\text{eff}}^2\rangle e^4 m_e^{1/2}\ln\Lambda}\left(1+e^{-\frac{\mu}{\theta}}\right)F_{\frac{1}{2}}\left(\frac{\mu}{\theta}\right) \tag{3}$$

where k is Boltzmann's constant, $\theta = kT$, μ is the chemical potential, m_e is the electron mass, e is the electron charge, Z_{eff} is the effective charge, $\ln\Lambda$ is the "Coulomb" log lambda, $F_{1/2}$ is a Fermi-Dirac integral, F_c^{β} is a correction term for electron–electron scattering, and A^{β} is a coefficient that depends on $\frac{\mu}{\theta}$.

In the sections that follow I will evaluate the errors present in the approximations that are used to evaluate A^{α}, A^{β}, $\zeta = \ln(1+e^{\mu/\theta})$, the common term $f = \left(1+e^{-\mu/\theta}\right)F_{1/2}$, $F'_{1/2}/F_{1/2}$, F_c^{α}, and F_c^{β}. New approximations that have smaller relative errors are generated. The Igor Pro application (Wavemetrics, Lake Oswego, OR, USA) was used to generate the fits for this paper.

2 Electrical conductivity—A^{α}

A^{α} determines the electrical conductivity and is given by Eqn. 25a in Lee & More[6]. There is an error in the printed formula; in the numerator F_3 should be F_2. This change makes the formula agree with the limits given in Eqns. 28a and 30a and with the fitting formula given in Eqn. A2 and Table VII of Lee & More[6].

$$A^{\alpha}\left(\frac{\mu}{\theta}\right) = \frac{4}{3}\frac{F_2\left(\frac{\mu}{\theta}\right)}{\left(1+e^{-\mu/\theta}\right)\left[F_{1/2}\left(\frac{\mu}{\theta}\right)\right]^2} \tag{4}$$

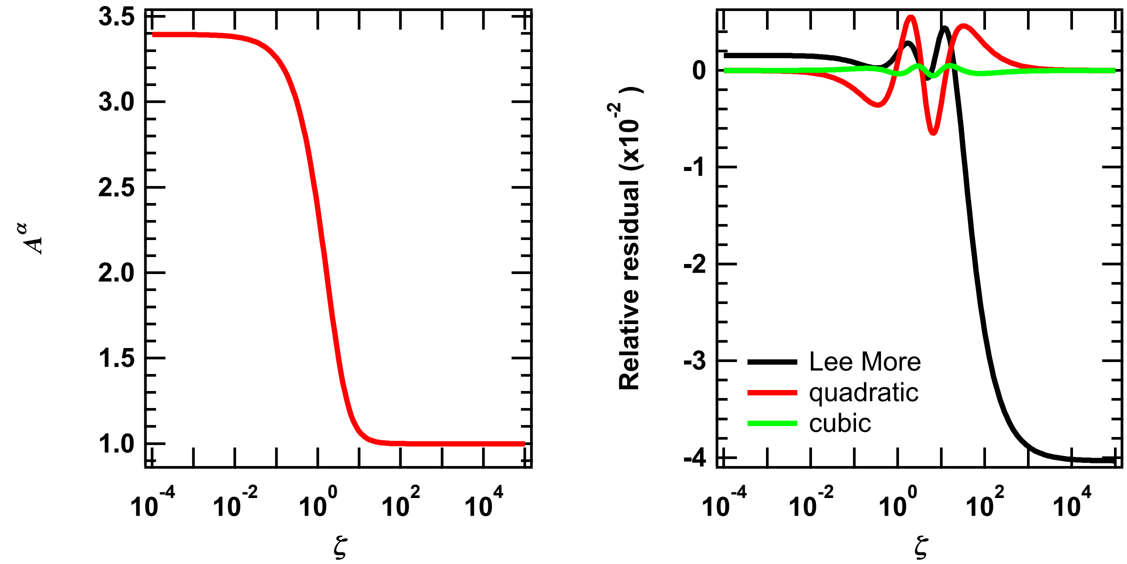
This coefficient can be approximated by rational functions. Lee & More[6] used the variable $\zeta = \ln\left(1+e^{\mu/\theta}\right)$ as the independent variable for these polynomials. They also showed that in the non-degenerate, high temperature limit, ($\frac{\mu}{\theta}\to-\infty$, $\zeta\to 0$) that $A^{\alpha} = \frac{32}{3\pi}$ and in the degenerate, high temperature limit, ($\frac{\mu}{\theta}\to\infty$, $\zeta\to\infty$) that $A^{\alpha} = 1.0$. Since these limits are constants the degree of the numerator and denominator of the rational functions must be the same. In order to get the correct limiting values for $\zeta = 0$ and ∞ the constant term in the numerator must be $\frac{32}{3\pi}$ and the coefficients for the highest degree terms must be equal. Here is a comparison of the Lee & More fit with fits of degree 2 and 3.

$$A^{\alpha}_{LM} = \frac{3.39 + 0.347\zeta + 0.129\zeta^2}{1.0 + 0.511\zeta + 0.124\zeta^2} \tag{5}$$

$$A^{\alpha}_{quad} = \frac{\frac{32}{3\pi} + 0.47429\zeta + 0.17638\zeta^2}{1.0 + 0.53326\zeta + 0.17638\zeta^2} \tag{6}$$

$$A^{\alpha}_{cubic} = \frac{\frac{32}{3\pi} + 0.80656\zeta + 0.16996\zeta^2 + 0.03226\zeta^3}{1.0 + 0.6581\zeta + 0.16813\zeta^2 + 0.03226\zeta^3} \tag{7}$$

The error in the Lee & More fit, Eqn. 5, is in the range $(-0.040, 0.00044)$, due to the highest degree coefficients not being equal. In the quadratic approximation, Eqn. 6, the error range is $(-0.0065, 0.0056)$, and in Eqn. 7 the error range is $(-0.00056, 0.00050)$. In order to generate the fits in Eqns. 6 and 7 a dataset was generated that sampled ζ from 10^{-9} to 10^{6} uniformly in log space with 20 points per decade. A^α was calculated at each ζ value and the coefficients were generated by a least squares fit that minimized the relative errors, $\chi^2 = \sum_i \left(\frac{A^\alpha - A^\alpha_{fit}}{A^\alpha}\right)^2$. See Fig. 1.



(a) The form of A^α **(b) Relative residuals**

Figure 1: The least squares fits for A^α

3 Thermal conductivity—A^β

A^β is given by Eqn. 25b in the Lee & More[6] paper:

$$A^\beta\left(\frac{\mu}{\theta}\right) = \frac{20}{9}\,\frac{F_4\left(\frac{\mu}{\theta}\right)\left[1 - \frac{16F_3^2\left(\frac{\mu}{\theta}\right)}{15F_4\left(\frac{\mu}{\theta}\right)F_2\left(\frac{\mu}{\theta}\right)}\right]}{\left(1+e^{-\mu/\theta}\right)\left[F_{1/2}\left(\frac{\mu}{\theta}\right)\right]^2} \tag{8}$$

This coefficient can also be approximated by rational functions with ζ as the independent variable. Lee & More[6] showed that in the non-degenerate, high temperature limit, ($\frac{\mu}{\theta} \to -\infty$, $\zeta \to 0$) that $A^\beta \to \frac{128}{3\pi}$ and in the degenerate, high temperature limit, $\left(\frac{\mu}{\theta} \to \infty,\ \zeta \to \infty\right)$ that $A^\beta \to \frac{\pi^2}{3}$. In order to get the correct limiting values for $\zeta = 0$ and ∞ the constant term in the numerator must be $\frac{128}{3\pi}$ and the ratio of the coefficients for the highest degree terms must be $\frac{\pi^2}{3}$. Here is a comparison

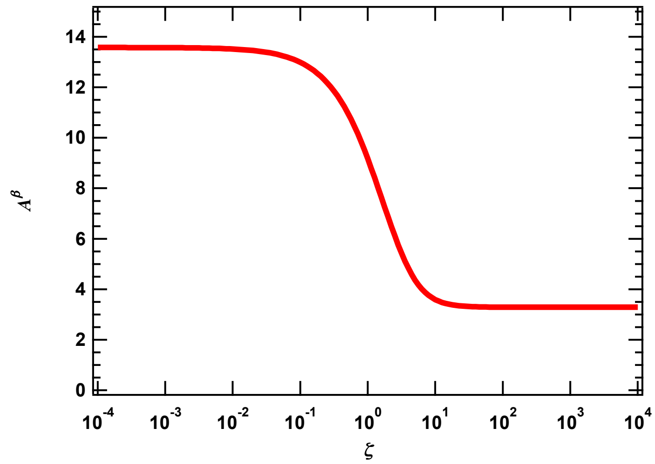


Figure 2: The form of A^{β}

of the degree 2 fits by Lee & More[6]and George Zimmerman[9] with a degree 3 fit.

$$A^{\beta}_{LM} = \frac{13.5 + 0.976\zeta + 0.437\zeta^2}{1.0 + 0.510\zeta + 0.126\zeta^2} \tag{9}$$

$$A^{\beta}_{GZ} = \frac{13.566 + 1.408\zeta + 0.565\zeta^2}{1.0 + 0.525\zeta + 0.171\zeta^2} \tag{10}$$

$$A^{\beta}_{cubic} = \frac{\frac{128}{3\pi} + 2.4905\zeta + 0.53536\zeta^2 + 0.089107\zeta^3}{1.0 + 0.63389\zeta + 0.15998\zeta^2 + \frac{3}{\pi^2}0.089107\zeta^3} \tag{11}$$

The error in the Lee & More fit, Eqn. 9, is in the range $(-0.178, 0.0812)$, due to the limited number of significant figures used. In Zimmerman's approximation, Eqn. 10, the error range is $(-0.0402, 0.0381)$, and in Eqn. 11 the error range is $(-0.00315, 0.00292)$. In order to generate the fit in Eqn. 11 a dataset was generated that sampled ζ from 10^{-4} to 10^4 uniformly in log space with 20 points per decade. A^{β} was calculated at each ζ value and the coefficients were generated by a least squares fit that minimized $\chi^2 = \sum_i (A^{\beta} - A^{\beta}_{cubic})^2$. See Fig. 3.

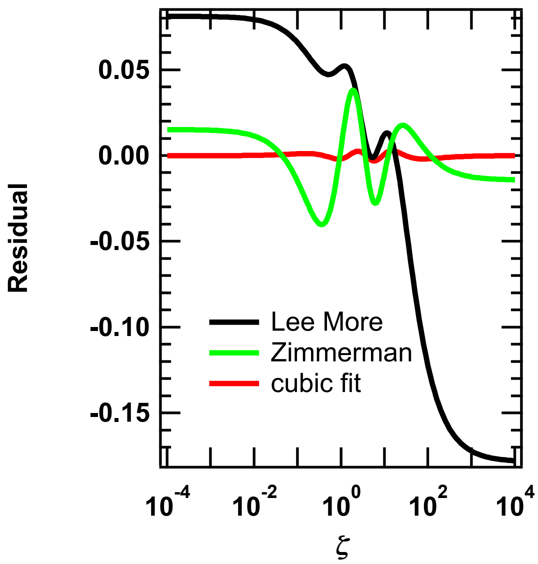
4 Chemical Potential, $\zeta(\xi)$

To calculate the electrical or thermal conductivity we first must determine the electron chemical potential, μ or ζ. To do this we must invert

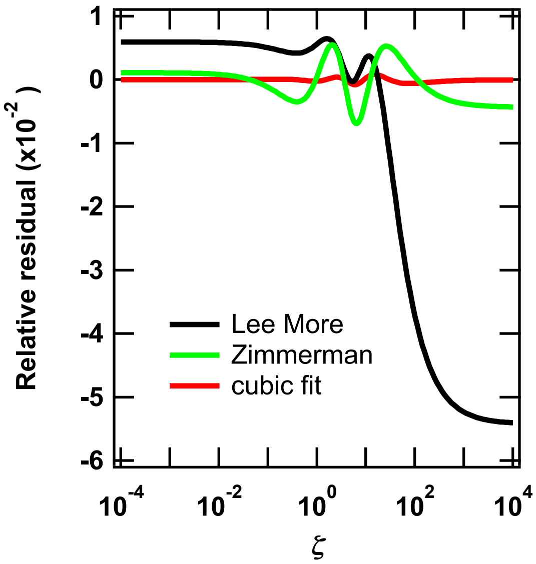
$$F_{\frac{1}{2}}(\zeta) = \frac{2}{3}\left(\frac{E_{\text{fermi}}}{\theta}\right)^{3/2} \tag{12}$$

where E_{fermi} is given by

$$E_{\text{fermi}} = \frac{\hbar^2}{2m_e}(3\pi^2 n_e)^{2/3} \tag{13}$$



(a) Residuals



(b) Relative residuals

Figure 3: The least squares fits for A^β

To find an appropriate fitting function we need to know the solution to Eqn. 12 in the degenerate and non-degenerate limits. In the non-degenerate, high temperature limit, $\zeta \to 0$, the Unger approximation[8] gives $F_{\frac{1}{2}} \to \frac{\sqrt{\pi}}{2}\ln(1+e^{\mu/\theta}) = \frac{\sqrt{\pi}}{2}\zeta$, resulting in $\zeta = \frac{4}{3\sqrt{\pi}}\left(\frac{E_{\text{fermi}}}{\theta}\right)^{3/2}$. In the degenerate, low temperature ($\zeta \to \infty$) limit the Sommerfeld approximation[7] gives $F_{\frac{1}{2}} \to \frac{2}{3}\left(\frac{\mu}{\theta}\right)^{3/2} = \frac{2}{3}\left[\ln(e^{\zeta}-1)\right]^{3/2} = \frac{2}{3}\zeta^{3/2}$, resulting in $\zeta = \frac{E_{\text{fermi}}}{\theta}$.

Following Zimmerman[9] we will use $\xi = \sqrt{E_{\text{fermi}}/\theta}$ as the independent variable of the fit function. Then we see that as $\zeta \to 0$ we have $\zeta = \frac{4}{3\sqrt{\pi}}\xi^3$ and as $\zeta \to \infty$ we have $\zeta = \xi^2$. Zimmerman's form for ζ is

$$\zeta_{GZ} = \frac{0.7531 + 0.1679\xi + 0.3108\xi^2}{1 + 0.2676\xi + 0.2280\xi^2 + 0.3099\xi^3}\xi^3 \tag{14}$$

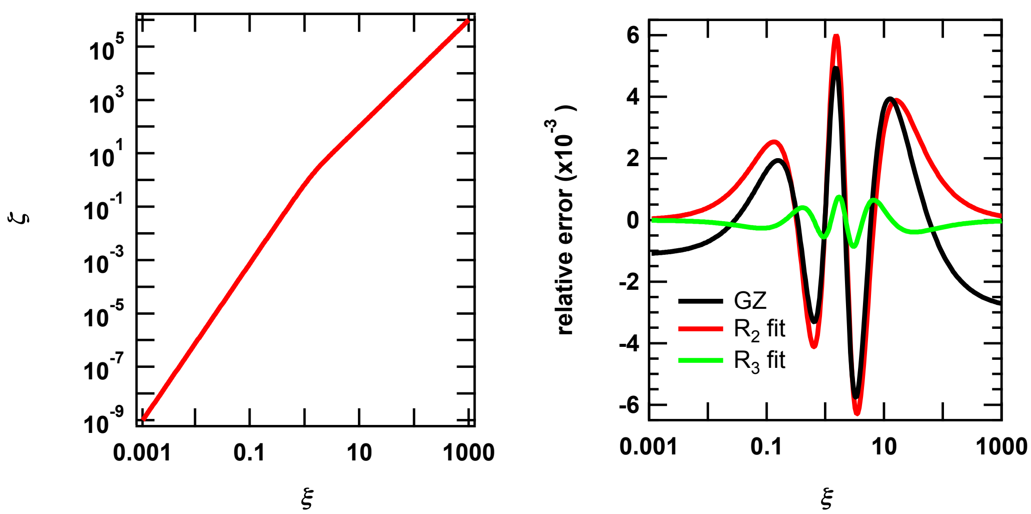
The required limits are preserved in fits based on

$$R_m(\xi) \quad = \quad \frac{\frac{4}{3\sqrt{\pi}} + \sum_{i=1}^{m} a_i\xi^i}{1 + \sum_{i=1}^{m+1} b_i\xi^i}\xi^3 \qquad b_{m+1} = a_m \tag{15}$$

Fitting with this form for $m = 2$ gives relative errors that are slightly larger than Zimmerman's form, see Fig. 4. The relative errors for $m = 3$ are about an order of magnitude smaller. The coefficients of the fits and error ranges are in Table I. The error range for Zimmerman's fit is $(-0.0058, 0.0050)$. These fits were done with a data set where ζ was sampled with 20 points per decade from 10^{-9} to 10^6 and minimized the relative error $\chi^2 = \sum_i \left(\frac{\zeta-\zeta_{fit}}{\zeta}\right)^2$.

Table I: Fit coefficients for R_2 and R_3

R_2 – error $= (-0.0063, 0.0060)$				R_3 – error $= (-0.00086, 0.00075)$			
a_1	0.19474	b_1	0.30156	a_1	0.19972	b_1	0.25829
a_2	0.33121	b_2	0.24073	a_2	0.17258	b_2	0.28756
		b_3	0.33121	a_3	0.145	b_3	0.16842
						b_4	0.145



(a) $\zeta(\xi)$ **(b) Relative residuals**

Figure 4: The least squares fits for ζ

5 A common term, f

In both the electrical and thermal conductivities, Eqns. 1 and 3, the term $(1 + e^{-\mu/\theta})F_{1/2}$ appears. Since this includes the Fermi–Dirac integral we approximate this term and call it f.

$$f\left(\frac{\mu}{\theta}\right) = \left(1 + e^{-\frac{\mu}{\theta}}\right) F_{\frac{1}{2}}\left(\frac{\mu}{\theta}\right) \tag{16}$$

In the non-degenerate, high temperature limit, ($\frac{\mu}{\theta} \to -\infty$, $\zeta \to 0$) the first term becomes $e^{-\mu/\theta}$ and using the Unger approximation[8] $F_{\frac{1}{2}} \to \frac{\sqrt{\pi}}{2}\ln(1 + e^{\mu/\theta}) \to \frac{\sqrt{\pi}}{2}e^{\mu/\theta}$ and therefore $f \to \frac{\sqrt{\pi}}{2}$. In the degenerate, low temperature limit, ($\frac{\mu}{\theta} \to \infty$, $\zeta \to \infty$) the first term goes to 1 and using the Sommerfeld approximation[7] $F_{\frac{1}{2}} \to \frac{2}{3}\left(\frac{\mu}{\theta}\right)^{3/2} = \frac{2}{3}\left[\ln(e^{\zeta} - 1)\right]^{3/2} = \frac{2}{3}\zeta^{3/2}$. Since $F_{\frac{1}{2}}$ in the large ζ limit is proportional to $\zeta^{3/2}$ Zimmerman[9] chose to approximate f with a cubic polynomial in $\zeta^{1/2}$. Another choice is to use a function that is the constant value $\frac{\sqrt{\pi}}{2}$ plus $\zeta^{1/2}$ times a rational function whose numerator has a degree one higher than the denominator. To get the correct limit as $\zeta \to \infty$ the coefficient of the highest degree in the numerator should be $\frac{2}{3}$ times the coefficient of the highest degree in the denominator. Another possibility that was investigated was to use $\left(\frac{\sqrt{\pi}}{2} + \frac{2}{3}\zeta^{3/2}\right)$ times a rational function that goes to 1 for $\zeta = 0$ and ∞. This did not result in smaller errors than the previous option and was dropped.

Here are the approximations given by Zimmerman and those I found using a least squares fit that minimizes the relative error $\chi^2 = \sum_i \left(\frac{f-f_{fit}}{f}\right)^2$ for various forms for f_{fit}. The same values of ζ are used for these fits as for A^β in §3.

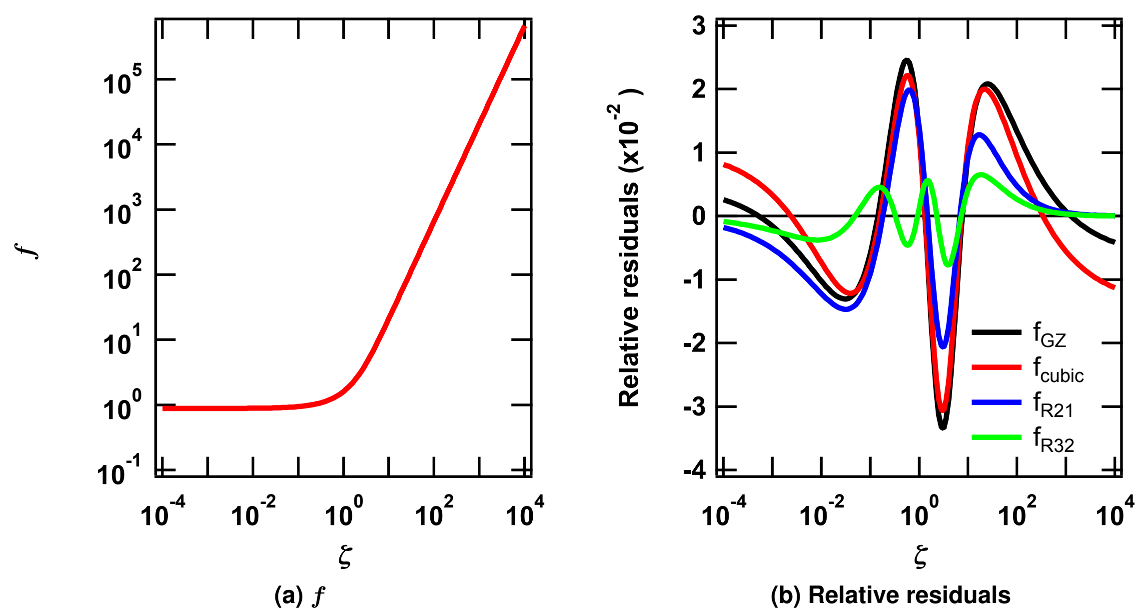
$$f_{GZ} = 0.88 - 0.16\zeta + (0.2 + 0.67\zeta)\zeta^{1/2} \tag{17}$$

$$f_{cubic} = 0.87678 + 0.22868\zeta^{1/2} - 0.18732\zeta + 0.67603\zeta^{3/2} \tag{18}$$

$$f_{R21} = \frac{\sqrt{\pi}}{2} + \zeta^{1/2}\frac{0.1611 + 0.55453\zeta + (2/3)0.26945\zeta^2}{1 + 0.26945\zeta} \tag{19}$$

$$f_{R32} = \frac{\sqrt{\pi}}{2} + \zeta^{1/2}\frac{0.080897 + 0.99341\zeta - 0.20639\zeta^2 + (2/3)1.071\zeta^3}{1 + 0.11\zeta + 1.071\zeta^2} \tag{20}$$

The error range in Zimmerman's form is $(-0.033, 0.025)$. The cubic fit has an error range of $(-0.031, 0.022)$. The two rational function fits ($m = 2, 3$) have error ranges of $(-0.031, 0.020)$ and $(-0.0077, 0.0065)$ respectively. See Fig. 5.



(a) f (b) Relative residuals

Figure 5: The least squares fits for $f = \left(1 + e^{-\frac{\mu}{\theta}}\right) F_{\frac{1}{2}}$

6 Coulomb Log term—$F'_{\frac{1}{2}}/F_{\frac{1}{2}}$

Lee & More use the Debye–Hückel radius in the calculation of the Coulomb log term. The Debye–Hückel radius includes a correction for degeneracy that involves the term $F'_{\frac{1}{2}}/F_{\frac{1}{2}}$. The Debye–Hückel radius is also used when accounting for electron screening in thermonuclear burn rate calculations.

In the Coulomb log calculation Lee & More use

$$\frac{F'_{\frac{1}{2}}}{F_{\frac{1}{2}}} = \frac{T}{\sqrt{T^2 + T_f^2}} \tag{21}$$

To see how good an approximation this is we need to know how to evaluate $F'_{\frac{1}{2}}$. It turns out there is a simple relationship

$$\frac{dF_n(\eta)}{d\eta} = nF_{n-1}(\eta) \qquad \text{thus} \tag{22}$$

$$\frac{dF_{\frac{1}{2}}(\eta)}{d\eta} = \frac{1}{2}F_{-\frac{1}{2}}(\eta) \tag{23}$$

Using Eqn. 12 and letting $E_{\text{fermi}} = \frac{3}{2}kT_f$ we find that

$$\frac{T}{T_f} = \frac{3}{2}\left(\frac{3}{2}F_{\frac{1}{2}}\right)^{-2/3} \tag{24}$$

The Lee & More form in Eqn. 21 can be written in terms of T/T_f. Managan had a rational form used when more accuracy was required. Two more rational forms are considered for comparison. Let $G = F'_{\frac{1}{2}}/F_{\frac{1}{2}}$. The fits R_2 and R_3 use the same data set based on $\zeta \in (10^{-4}, 10^4)$ with 20 points per decade and minimize the relative error $\chi^2 = \sum_i \left(\frac{G - G_{fit}}{G}\right)^2$.

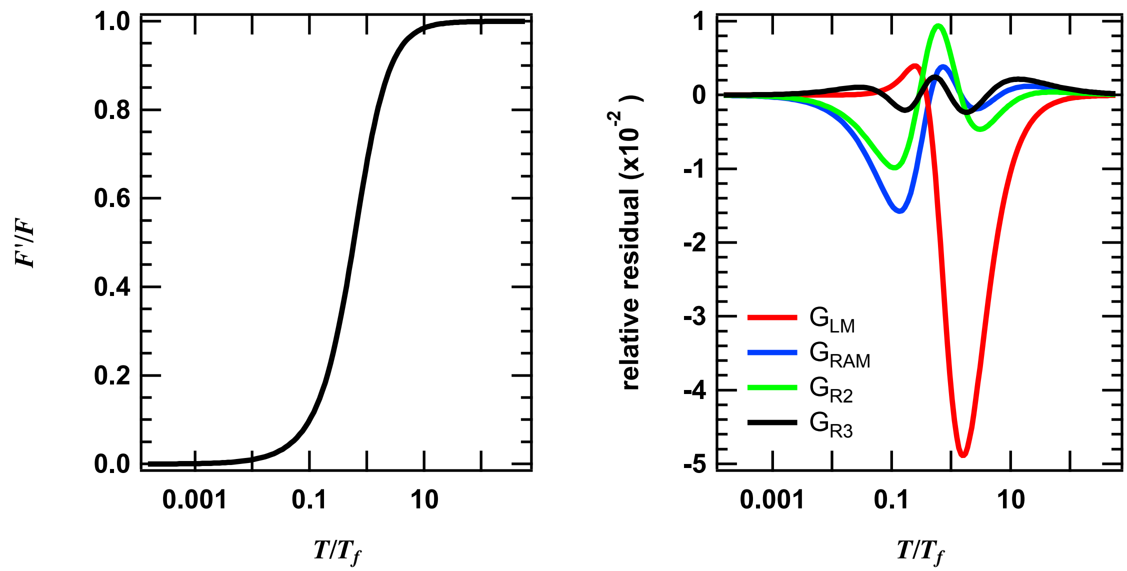
$$G_{LM} = \frac{T/T_f}{\sqrt{1 + (T/T_f)^2}} \tag{25}$$

$$G_{RAM} = \frac{\frac{T}{T_f} + 1.4126\left(\frac{T}{T_f}\right)^2}{1.0 + 1.1453\frac{T}{T_f} + 1.4126\left(\frac{T}{T_f}\right)^2} \tag{26}$$

$$G_{R_2} = \frac{\frac{T}{T_f} + 1.2952\left(\frac{T}{T_f}\right)^2}{1.0 + 1.098\frac{T}{T_f} + 1.2952\left(\frac{T}{T_f}\right)^2} \tag{27}$$

$$G_{R_3} = \frac{\frac{T}{T_f} + 6.6262\left(\frac{T}{T_f}\right)^2 + 9.0247\left(\frac{T}{T_f}\right)^3}{1.0 + 6.7128\frac{T}{T_f} + 7.7439\left(\frac{T}{T_f}\right)^2 + 9.0247\left(\frac{T}{T_f}\right)^3} \tag{28}$$

The error bounds for G_{LM} are $(-0.049, 0.004)$, for G_{RAM} they are $(-0.016, 0.0039)$, for G_{R_2} $(-0.0098, 0.0094)$, and for G_{R_3} $(-0.0023, 0.0025)$. For calculating a Coulomb logarithm G_{LM} is adequate since the magnitude of the error is reduced by taking the logarithm.



(a) F'/F (b) Relative residuals

Figure 6: The least squares fits for $F'_{\frac{1}{2}}/F_{\frac{1}{2}}$

7 Electrical Conductivity Z^* correction—F_c^α

In the electrical conductivity Zimmerman[9] also includes a correction factor for the the effect of finite Z. This correction to the Lee & More paper is added to account for electron-electron scattering which is ignored by Lee & More. However, it should be pointed out that the calculations with electron-electron scattering have all been done for plasmas whose distribution function is approximately Maxwellian. The Lee & More results do not have scattering but are done for plasmas whose distribution function is approximately Fermi–Dirac and allow for degeneracy. The lack of including scattering means the Lee & More results are good in the limit of large Z. Therefore the dependence on Z is added as a normalized correction factor to add the Z dependence from the Maxwellian results to the Fermi–Dirac results of Lee & More. In 1999, Brown and Haines[2][3] calculated the transport coefficients with electron-electron scattering for partially degenerate magnetized plasmas. They do not tabulate their results or fit them. This work could be used to generate new fits that depend on ζ, $\vec{B}$, and Z. This would remove the need for the correction factor to account for the effect of Z.

This correction comes from data in Table 2 of Braginskii[1] on page 251 which give values for $Z = 1, 2, 3, 4,$ and ∞. Zimmerman fit cubics to the data in Braginskii's table to go through the data for $Z = 1, 2, 4,$ and ∞.

$$F_c^\alpha = \frac{0.295}{1 - \frac{\alpha_0'}{\delta_0}} \tag{29}$$

$$= \frac{0.295}{1 - \frac{0.0678+0.4924x+0.976x^2+0.3008x^3}{0.0961+0.7778x+1.5956x^2+1.3008x^3}} \tag{30}$$

The results of Ji and Held[5] give a fit that is

$$\hat{\alpha}_{\parallel} = 1 - \frac{Z^{2/3}}{1.46Z^{2/3} - 0.330Z^{1/3} + 0.888} \tag{31}$$

The correction factor would be

$$F_c^{\alpha} = \frac{3\pi/32}{1 - \frac{Z^{2/3}}{1.46Z^{2/3} - 0.330Z^{1/3} + 0.888}} \tag{32}$$

$$= \frac{3\pi/32}{1 - \frac{1}{1.46 - 0.330Z^{-1/3} + 0.888Z^{-2/3}}} \tag{33}$$

$$= \frac{3\pi/32}{1 - \frac{1}{1.46 - 0.330x^{1/3} + 0.888x^{2/3}}} \tag{34}$$

This fit has a maximum at $x = 0.0065$ which corresponds to $Z = 154$. This Z will never be used; see Fig. 7. This new approximation is not being used in Ares at this time.

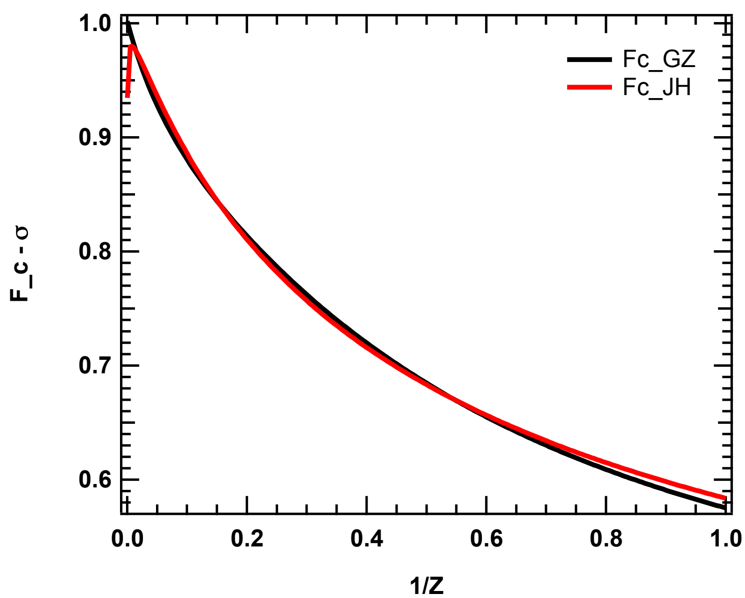


Figure 7: Comparison of fits for F_c^{α}

8 Thermal Conductivity Z^* correction—F_c^{β}

An empirical correction to account for electron-electron scattering is also added to the thermal conductivity. The same concerns as raised in §sec:FcAlpha apply here as well. The Ares version comes from George Zimmerman[9] and is just a renormalized version of what Zimmerman uses.

$$x = \frac{\langle Z_{\text{eff}} \rangle}{\langle Z_{\text{eff}}^2 \rangle (1+\zeta)} \tag{35}$$

$$F_{c,Ares} = \frac{1.0 + 4.50x + 3.67x^2 + 0.756x^3}{1.0 + 8.09x + 16.6x^2 + 13.5x^3} \tag{36}$$

$$F_{c,GZ} = \left(\frac{0.0961}{1.2}\right) \frac{1.2000 + 5.4053x + 4.4080x^2 + 0.9067x^3}{0.0961 + 0.7778x + 1.5956x^2 + 1.3008x^3} \tag{37}$$

The $F_{c,GZ}$ is a ratio of fits to the data in Table 2 of Braginskii[1] on page 251 which give values for $Z = 1$, 2, 3, 4, and ∞. The ratio is for $\gamma_0 = \frac{\gamma_0'}{\delta_0}$. The cubic coefficients are derived to pass through the data for $Z = 1$, 2, 4, and ∞. The relative error for $Z = 3$ in γ_0' is 3.5×10^{-5} and in δ_0 is -9.7×10^{-4}.

More recent results are given by Epperlein–Haines[4] and Ji–Held[5] for these quantities. Ji–Held[5] give a formula for their thermal conductivity coefficient (note that at the end of §III.B that they say that Braginskii's coefficient is their's times Z. I have included that extra factor of Z here)

$$\hat{\kappa}_{\parallel}^{e} = \frac{13.5 + 54.4Z^{-1} + 25.2Z^{-2}}{1 + 8.35Z^{-1} + 15.2Z^{-2} + 4.51Z^{-3}}. \tag{38}$$

This formula would argue that a more accurate correction factor would be

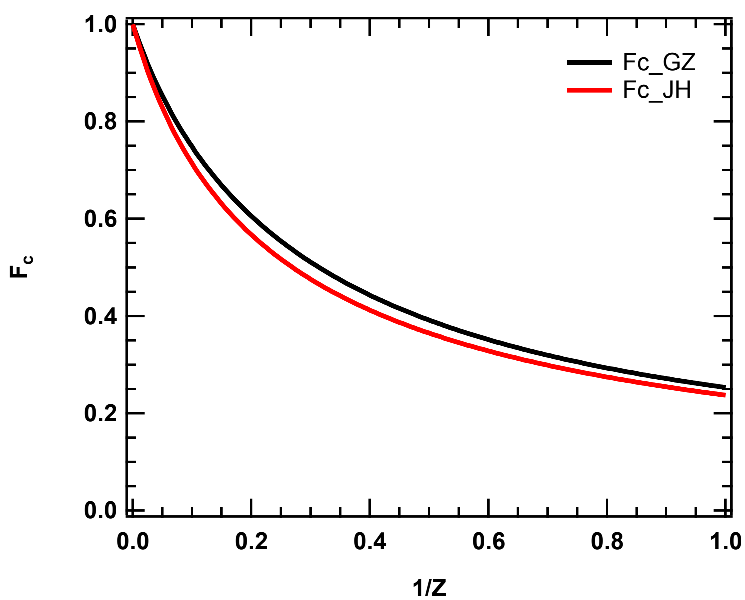


Figure 8: Comparison of fits for F_c^β

$$F_{c,JH} = \frac{1}{13.5} \frac{13.5 + 54.4x + 25.2x^2}{1 + 8.35x + 15.2x^2 + 4.51x^3} \tag{39}$$

See Figure 8 for the comparison. The two factors agree for large Z where the Lee & More calculations are accurate. This new approximation is not being used in Ares at this time.

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