Award Number: DOE Grant DE-SC0002349

Project Title:

"Parameter Estimation and Model Validation of Nonlinear Dynamical Networks"

Principal Investigator & Co-PI: Henry Abarbanel, Philip Gill

Your institute: University of California, San Diego

Period of Performance: September 15, 2009—September 14, 2013

Final Technical Report

In the performance period of this work under a DOE contract, the co-PIs, Philip Gill and Henry Abarbanel, developed new methods for statistical data assimilation for problems of DOE interest, including geophysical and biological problems.

This included numerical optimization algorithms for variational principles, new parallel processing Monte Carlo routines for performing the path integrals of statistical data assimilation.

These results have been summarized in the monograph: "Predicting the Future: Completing Models of Observed Complex Systems" by Henry Abarbanel, published by Spring-Verlag in June 2013. Additional results and details have appeared in the peer reviewed literature.

Award Number:
DOE Grant DOE-SC0002349

Project Title:
"Parameter Estimation and Model Validation in Nonlinear System"

Principal Investigator & Co-PIs:
Henry Abarbanel, Philip Gill

Your institute:
University of California, San Diego

Period of Performance:
September 15, 2009—September 14, 2012

June 1, 2011

1 Technical Progress

The overall thrust of our work is to combine the applied mathematics focus, numerical optimization, and the physics focus, state and parameter estimation, of the principals to make contributions to the use of variational methods in the evaluation of models of core interest in DOE problems.

To date these scientific problems include: (a) the upgrading of existing numerical optimization software to the use of Hessian information, (b) the extension of these programs for use on parallel and GPU computers, (c) the extension of the methods for use on partial differential equations (PDEs) of interest in geophysical fluid dynamics—with a special interest in numerical weather prediction and climate modeling, and (d) the extension of the variational methods to allow the calculation of uncertainty. This permits the estimation of states and parameters which has been the core focus of our efforts but also the evaluation of RMS variations about these mean states and parameters which now allows a quantification of the uncertainty (QU) in the outputs of these models. To the extent that the use of models in the description of complex processes, climate prediction, combustion reactions, weather forecasting, etc, impacts policy decisions, our new capabilities in QU will play an important role.

While each PI has worked on their own aspect of these problems, we meet on a regular basis both with each other and with each other's students to make sure that progress in each area is incorporated in the overall research.

1.1 Abarbanel

In this reporting period we have further developed the script used for an interface with numerical optimization packages and used that to explore design of experiments on neural circuits using a variety of stimuli. The methods for using numerical optimization front-end scripts allowing minimization of nonlinear functions $f(x)$ subject to ordinary differential equations satisfied by $x(t)$ is summarized in the paper [11] by Bryan Toth, a UCSD Physics graduate student of Abarbanel.

In a series of papers submitted to the journal *Biological Cybernetics*, written along with the members of the University of Chicago Laboratory of Professor Daniel Margoliash, "Dynamical Estimation of Neuron and Network Properties I: Variational Methods," Bryan A. Toth, Mark Kostuk, C. Daniel Meliza, and Henry D. I. Abarbanel, we have tested the optimization methods of Dynamical State and Parameters Estimation (DSPE) on a standard set of neurobiological models in an effort to design experiments that would allow us to determine all the parameters and unobserved state variables in a current model for the observations carried out in the Margoliash Laboratory in April and May, 2011. The lessons learned in that paper are now being applied to the data from that laboratory.

The extension of that work requires numerical calculations that are often best done in parallel. We have explored the use of GPU computing methods for this purpose, and as reported in [12], we report speedups of up to 300 using GPU methods on the estimation problems of interest to us. These calculations utilize the CUDA extensions of C, C^{++}, and Fortran developed by NVIDIA, the designer of the GPU elements.

Based on that set of results, we joined the group of Professor Mike Holst of the Center for Computational Mathematics (CCoM) in purchasing a 10-GPU cluster of GTX 580 devices from NVIDIA. This consists of three boxes, one the "head node" connected to the user, and two nodes of 4 GPUs each for accelerating computing. One can easily add similar nodes as appropriate as they are connected together by 10 Gb/s Infiniband communications. Also as NVIDIA upgrades their GPU devices from the GTX 580 (with 512 CUDA nodes) we can easily upgrade the machine that will soon be installed in the CCoM chilled room at UCSD.

Finally, in this reporting period we have devised a self-consistent test of our methods for dynamical state and parameter estimation. Those methods, as all such methods, require an assumption about the distribution of errors in the models. We developed and tested a method for identifying whether that assumption is self-consistent with the estimations made with the model and available data. A paper entitled "Self-Consistent Stochastic Model Errors in Data Assimilation," by PI Abarbanel was submitted to the Quarterly Journal of the Royal Meteorological Society in April, 2011.

Our plans for the next funding period include extending the successful scripts of Toth to `SNOPT` and extending the existing work to the analysis of data from the Margoliash laboratory as well as working with the shallow water equations which are the core of the PDEs within all large climate and weather models.

1.2 Gill

Co-PI Gill has continued his work on the formulation, analysis and testing of optimization methods for the estimation of model parameters using the DSPE approach. The aim of this research continues to be the formulation algorithms that can exploit highly parallel multicore and GPU architectures with sufficient speed-up that the real- or near real-time estimation of model parameters is possible.

In joint work with graduate student Elizabeth Wong, the Fortran f90 interface `SNCTRL` [13] has been developed for the large-scale optimization code `SNOPT`. The `SNCTRL` package converts the optimal control problem into a finite dimensional nonlinear program by discretizing the ordinary differential equations using the method of collocation. Two collocation methods are available in the `SNCTRL` interface, the trapezoid method and the Hermite-Simpson method. Once the problem has been discretized, the interface sets up the relevant structures and information needed by `SNOPT`. The large-scale QP software package `icQP`, which was developed in the first year of DOE funding, has been considerably revised and extended. Substantial numerical results have been obtained using the the linear system solver packages `MA57`, `MUMPS`, `PARDISO` and `UMFPACK`. These results were obtained as part of Elizabeth Wong's Ph.D. research, which she will defend in June 2011. In the next funding period, additional results will be obtained on the 10-GPU cluster of GTX 580 devices by using solvers tailored to exploit machines with GPU-based processing units.

With regard to theoretical developments, a new dual QP method has been formulated for solving convex quadratic programming problems [10]. Work is underway to use this method in conjunction with a primal method to formulate a new primal-dual method for convex QP. This method shows early promise of out-performing the criss-cross method, which is the one of the best-known and widely used primal-dual methods for QP. Dual methods are an important component of a general-purpose sequential quadratic programming (SQP) toolbox for nonlinear optimization. In mixed-integer nonlinear programming (NLP) branch and bound methods it is necessary to solve a sequence of relaxed NLPs that differ by a single constraint. If the SQP method is implemented with a dual QP solver, and is warm started with the primal-dual solution of the previous relaxation, then the dual variables are feasible, and only one branched variable is infeasible. The infeasible variable can be moved towards feasibility immediately.

All methods developed under the auspices of this DOE award are formulated so that *regularization* is both implicit and automatic. This is vital for the algorithms that employ of-the-shelf linear algebra because many solver packages are extremely fast, but are unable to control ill-conditioning or detect singularity. Unfortunately, over the course of many hundreds of iterations, performed with matrices of varying degrees of conditioning, an SQP method can place even the most robust equation solver under considerable stress. (Even a relatively small collection of difficult problems can test the reliability of a solver. Gould, Scott, and Hu: *A Numerical Evaluation of Sparse Direct Solvers for the Solution of Large Sparse Symmetric Linear Systems of Equations, ACM Trans. Math. Software, 33 (2007), pp. Art. 10, 32*, report that none of the 9 general-purpose solvers tested was able to solve all of the 61 systems in their collection.)

Two new regularized QP methods have been developed for general quadratic programming. These methods are both "single-phase" methods in the sense that no separate phase-one procedure is needed to find a feasible point for the constraints. Single-phase methods move towards feasibility and optimality simultaneously.

The advanced QP solvers discussed above are to be used in conjunction with new SQP method [16] developed jointly with Daniel Robinson at Johns Hopkins University. The proposed method employs a primal-dual generalized augmented Lagrangian *line-search merit function* [8] to obtain a sequence of improving estimates of the solution. This function is a primal-dual variant of the conventional augmented Lagrangian proposed by Hestenes and Powell in the early 1970s. Gill and Robinson show that each subproblem is equivalent to a quadratic program with regularized constraints. A crucial feature of this method is that the QP subproblem is convex, but formed from the exact Hessian of the Lagrangian. This is in contrast to `SNOPT`, which uses a less accurate quasi-Newton approximation. Additional benefits of this approach include: (i) the ability to control the quality of the dual variables during the solution of the subproblem; (ii) the availability of improved dual estimates on early termination of the subproblem; and (iii) the ability to regularize the subproblem by imposing explicit bounds on the dual variables. Preliminary numerical experiments on a subset of problems from the CUTEr test collection indicate that the proposed SQP method is significantly more efficient than our current SQP package `SNOPT`.

2 Key Accomplishments

- A new theoretical framework for the formulation of active-set methods for general quadratic programming (QP) has been developed [8,9]. This framework defines a class of methods in which a primal-dual search pair is the solution of an equality-constrained subproblem involving a "working set" of linearly independent constraints. This framework may be considered in the context of two classes of active-set method for quadratic programming: *binding-direction methods* and *nonbinding-direction methods*. Broadly speaking, the working set for a binding direction method consists of a subset of the active constraints, whereas the working set for a nonbinding direction method may involve constraints that need not be active (nor even feasible).

  This framework has allowed us to recast a binding-direction method for general QP first proposed by Fletcher in 1971, as a nonbinding-direction method. This reformulation gives the primal-dual pair as the solution of a so-called KKT-system of linear equations that is formed from the QP Hessian and the gradients of the constraints in the working set. It is shown that, under certain circumstances, the solution of this KKT-system may be updated using a simple recurrence relation, thereby giving a significant reduction in the number of KKT systems that need to be solved. The nonbinding direction method has been extended to quadratic programs with constraints in "standard form", where the inequality constraints are simple upper and lower bounds on the variables. It is shown that in the special case of a convex QP with zero quadratic term, the method is equivalent to a variant of the primal simplex method in which the π-values and reduced costs are updated at each iteration.

  A new method for convex quadratic programming has been developed that applies a nonbinding direction method to a certain regularized dual quadratic program. The resulting method does not require the assumption of strict convexity and gives a method equivalent to the dual simplex method when applied to a QP with a zero quadratic term. In addition to its proposed use within an SQP solver for optimal tracking, the method is particularly useful for use with branch and bound techniques in mixed-integer nonlinear programming [9].

- An important supplemental activity has been the development of software embodying the above algorithms and its dissemination within the manufacturing, engineering and scientific community. Many of the algorithms developed under the auspices of this award are implemented in the software package `icQP`, which is designed for the solution of general and convex quadratic programming problems. This code has been implemented and tested with the linear system solver packages `MA57`, `MUMPS`, `PARDISO` and `UMFPACK`. We are currently investigating the use of linear algebra packages that can exploit GPU-based architectures. The QP solvers will form the basis of new Fortran 2003 SQP software based on the use of interoperable linear solvers.

- We have user-friendly software that acts as a front-end to numerical optimization packages such as `SNOPT` and `IPOPT`. These permit the user to specify the desired equations of motion for the problem and the objective function to be minimized to establish parameter and states for the observed system. The front-ends allow all information to be introduced as ASCII input, and using symbolic packages such as Python, create a working program in Fortran or C^{++} to solve the problem at hand. This material will be published soon, and it is available on request from the PIs.

- We have developed an exact formulation of the state and parameter estimation problem in the form of a path integral. The saddle path integration of the path integral is precisely the variational problem we have been solving using the numerical optimization packages `SNOPT` and `IPOPT`.

- The path-integral formulation allows the evaluation of the uncertainty in the estimation of states and parameters, thus permitting any policy decisions based on the models to incorporate error bars and probabilistic forecasts, and a natural outcome of the model.

3 Comprehensive list of publications

1. Abarbanel, H. D. I., P. Bryant, P. E. Gill, M. Kostuk, J. Rofe, Z. Singer, B. Toth, and E. Wong, Dynamical parameter and state estimation in neuron models. in D. Glanzman and M. Ding (eds), An Exploration of Neuronal Variability and its Functional Significance, 139–180, Oxford University Press, Oxford and New York (2011).

2. Abarbanel, H. D. I., D. R. Creveling, R. Farsian, and M. Kostuk, "Dynamical State and Parameter Estimation," *SIAM J. Appl. Dyn. Syst.* **8**, 1341-1381 (2009) .

3. Quinn, J. C., P. H. Bryant, D. R. Creveling, S. R. Klein, and H. D. I. Abarbanel, "State and Parameter and State Estimation of Experimental Chaotic Systems Using Synchronization," *Physical Review E* **80** 016201 (2009).

4. Abarbanel, H. D. I., "Effective actions for statistical data assimilation," *Physics Letters A* **373**, 4044-4048 (2009). doi:10.1016/j.physleta.2009.08.072

5. Abarbanel, H. D. I., M. Kostuk, and W. Whartenby, "Data Assimilation with Regularized Nonlinear Instabilities," published in *Quarterly Journal of the Royal Meteorological Society*, March, 2010.

6. Quinn, J. and H. D. I. Abarbanel, "State and Parameter Estimation using Monte Carlo Evaluation of Path Integrals," under review *Quarterly Journal of the Royal Meteorological Society*, April, 2010.

7. Erway, J. B. and P. E. Gill, A Subspace Minimization Method for the Trust-Region Step, *SIAM Journal on Optimization* **20**, 1110-1131 (2009).

8. Gill, P. E., and D. P. Robinson, A primal-dual augmented Lagrangian, *Comput. Optim. Appl.* **47** 1-25 (2010).

9. Gill, P. E. and E. Wong, Sequential quadratic programming methods, in J. Lee and S. Leyffer (eds), *Mixed-Integer Nonlinear Optimization: Algorithmic Advances and Applications*, IMA Volumes in Mathematics and its Applications, 60pp, to appear, Springer Verlag, Berlin, Heidelberg and New York (2010).

10. Gill, P. E. and E. Wong, *Methods for Convex and General Quadratic Programming*. UCSD, Department of Mathematics, Technical Report NA 10-1, September 2010.

11. Toth, B. A., "Python Scripting for Dynamical Parameter Estimation in IPOPT", SIAG/OPT Views-and-News 21:1, 1-8 (2010).

12. Quinn, J. and H. D. I. Abarbanel, "Data Assimilation using a GPU Accelerated Path Integral Monte Carlo Approach," submitted to the Journal of Computational Physics.

13. Gill, P. E. and E. Wong, *A User's Guide for* `SNCTRL`*: a Control Optimization Interface for SNOPT*, Report NA 11-1, Department of Mathematics, University of California, San Diego.

14. Gill, P. E. and E. Wong, *A User's Guide for* `icQP`*: a Fortran Package for General Quadratic Programming*, Report NA 11-2, Department of Mathematics, University of California, San Diego.

15. Gill, P. E. and E. Wong, *Regularized Methods for General Quadratic Programming*, Report NA 11-3, Department of Mathematics, University of California, San Diego.

16. Gill, P. E., and D. P. Robinson, *Regularized Primal-Dual Sequential Quadratic Programming Methods*, Report NA 11-4, Department of Mathematics, University of California, San Diego.

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To date these scientific problems include: (a) the upgrading of existing numerical optimization software to the use of Hessian information, (b) the extension of these programs for use on parallel and GPU computers, (c) the extension of the methods for use on partial differential equations (PDEs) of interest in geophysical fluid dynamics—with a special interest in numerical weather prediction and climate modeling, and (d) the extension of the variational methods to allow the calculation of uncertainty. This permits the estimation of states and parameters which has been the core focus of our efforts but also the evaluation of RMS variations about these mean states and parameters which now allows a quantification of the uncertainty (QU) in the outputs of these models. To the extent that the use of models in the description of complex processes, climate prediction, combustion reactions, weather forecasting, etc, impacts policy decisions, our new capabilities in QU will play an important role.

While each PI has worked on their own aspect of these problems, we meet on a regular basis both with each other and with each other's students to make sure that progress in each area is incorporated in the overall research.

1.1 Abarbanel

Entering this reporting period, we had been focused on the use of numerical optimization methods for the solution of problems best posed as follows:

- given data $z_l(t)$; $l = 1, \ldots, L$ from an experiment or field observations taken over a time period $\{t_0, \ldots, T\}$, and

- given a physically based model

$$\frac{dy_a(t)}{dt} = F_a(\mathbf{y}(t), \mathbf{p}); \quad a = 1, 2, \ldots, D > L,$$

- determine the states of the model $\mathbf{y}(T)$ and the fixed parameters of the model by minimizing

$$C(\mathbf{y}(T), \mathbf{p}) = \frac{1}{2} \sum_{t=t_0}^{T} \sum_{l=1}^{L} (z_l(t) - y_l(t))^2,$$

  subject to the dynamical equations as equality constraints.

We have created "front ends" for the solution of this problem for both SNOPT and IPOPT, the first created by Gill and collaborators, and the second a product of work at Carnegie-Mellon and IBM. The front ends permit entry in plain ASCII format of the equations to be solved, the bounds on searches over states and parameters, and the data to be presented to the model. The programs then forms all Jacobian matrices and Hessian matrices required for the numerical optimization software; orders all the input ingredients to the numerical optimization software; and produces, either in Fortran or C^{++}, source code that uses the core of the numerical optimization programs to perform the optimization.

The application of these methods using SNOPT has been reported in the publications at the end of this report. The primary use of the methods, called DSPE, has been in small electrical circuit problems, including a detailed analysis of experiments on a chaotic circuit, and the analysis of small and sizeable (up to $D = 100$) geophysical problems. The latter is our start towards the application of the methods to large geophysical problems (climate and weather prediction) and other problems formulated in terms of PDEs.

The main new direction for the analysis of states and parameters has been to situations where **uncertainty** is an essential element: the data is noisy, the equations of motion have finite resolution or are embedded in a noisy environment, and the state of the system at the time data taking is started is uncertain. The exact solution to these problems has been expressed in terms of an integral along paths in state space. This formulation allows approximations to be made within an exact integral representation of the required answers, and more importantly allows one to evaluation the mean state and parameters as well as RMS error bounds on these quantities. The saddle path integration of the path integral is precisely the variational principles we have been exploring to date.

A very important aspect of the path integral formulation is the ability to evaluate in a quantitative manner the uncertainties in the estimations of states and parameters. This opens up an ability to quantify the uncertainty in the predictions of the models we use.

1.2 Gill

Co-PI Gill has been involved in the development and testing of new algorithms for the estimation of model parameters using the DSPE approach. The aim is formulate algorithms that can exploit highly parallel multicore and GPU architectures with sufficient speed-up that the real- or near real-time estimation of model parameters is possible.

Sequential quadratic programming methods and interior methods are two alternative approaches to handling inequality constraints in the large-scale nonlinear optimization problems associated with discretized optimal tracking problems. Sequential quadratic programming (SQP) methods find an approximate solution of a sequence of quadratic programming (QP) subproblems in which a quadratic model of the Lagrangian function is minimized subject to the linearized constraints. Interior methods approximate a continuous path that passes through a solution. Both interior methods and SQP methods solve a sequence of simpler subproblems—some of which may involve infeasible constraints. Both methods have an inner/outer iteration structure in which the work for an inner iteration is dominated by the cost of solving a large sparse system of symmetric indefinite linear equations. For an SQP method, these equations are defined in terms of an evolving "active set" of the variables and constraints; for an interior method, the equations have a fixed structure involving all the constraints and variables.

Research has focused on the formulation and analysis of methods that are not subject to the deficiencies of conventional methods on optimal tracking problems. The key to the development of efficient methods in this context is the observation that each optimization problem is just one of a hierarchy of problems associated with a hierarchy of discretization meshes. Each problem has a different size, but related sparsity structure. Our approach has been to combine the best features of interior methods and SQP methods. Interior-point methods are very efficient when solving large "one-off" problems, in large part because the fixed structure of the equations makes it possible to utilize sophisticated software developed by the linear algebra community. Moreover, interior methods can utilize the second derivatives of the problem functions and have a strong theoretical foundation that does not require the subproblem to be solved exactly when the outer iterates are far from the solution. On the other hand, SQP methods provide a relatively reliable "certificate of infeasibility" when a subproblem is infeasible (which common for a problem associated with a coarse mesh), Also, SQP methods require only one or two outer iterations when started near the solution—a crucial property as the mesh is refined.

Graduate student Elizabeth Wong has been working on new quadratic programming methods that can be used in conjunction with interoperable linear equation solvers. Such methods allow the timely implementation of innovative solvers tailored to advanced computer architectures, such as machines with GPU-based processing units. In a parallel study, graduate student Anna Shustrova has focused on using interior methods for the solution of the QP subproblem. This approach involves a sequence of unconstrained problems in which a new primal-dual modified barrier function is minimized. The new function has the crucial property that if good estimates of the Lagrange multipliers are known (e.g., from the previous SQP subproblem) then the barrier function may be minimized with one unconstrained minimization.

The advanced QP solvers discussed above are to be used in conjunction with new SQP methods based on the solution of a sequence of unconstrained subproblems. This type of method, first proposed by Fletcher in the late 1980's, has strong convergence properties but suffers from the disadvantage that the unconstrained function is not differentiable. This lack of smoothness makes the subproblem hard to solve and may cause slow convergence near the solution. A sequential unconstrained optimization algorithm has been formulated in which the objective function is a *smooth* unconstrained function first proposed by Gill and Robinson [8]. This function is a primal-dual variant of the conventional augmented Lagrangian proposed by Hestenes and Powell in the early 1970s. Gill and Robinson show that each unconstrained subproblem is equivalent to a quadratic program with regularized constraints. This implicit regularization makes the method particularly appropriate for use with the QP methods developed by Gill and Wong because the "black-box" solvers that are crucial for efficiency do not have a consistent way of treating ill-conditioned or singular equations.

2 Key Accomplishments

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3 Comprehensive list of publications

1. Abarbanel, H. D. I., P. Bryant, P. E. Gill, M. Kostuk, J. Rofe, Z. Singer, B. Toth, and E. Wong, "Dynamical Parameter and State Estimation in Neuron Models," Oxford University Press volume "Neuronal Variability and its Functional Significance," edited by Mingzhou Ding and D. Glanzman, November, 2009.

2. Abarbanel, H. D. I., D. R. Creveling, R. Farsian, and M. Kostuk, "Dynamical State and Parameter Estimation," *SIAM J. Appl. Dyn. Syst.* **8**, 1341-1381 (2009) .

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4. Abarbanel, H. D. I., "Effective actions for statistical data assimilation," *Physics Letters A* **373**, 4044-4048 (2009). doi:10.1016/j.physleta.2009.08.072

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6. Quinn, J. and H. D. I. Abarbanel, "State and Parameter Estimation using Monte Carlo Evaluation of Path Integrals," under review *Quarterly Journal of the Royal Meteorological Society*, April, 2010.

7. J. B. Erway and P. E. Gill, A Subspace Minimization Method for the Trust-Region Step, *SIAM Journal on Optimization* **20**, 1110-1131 (2009).

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9. P. E. Gill and E. Wong, Sequential quadratic programming methods, in J. Lee and S. Leyffer (eds), *Mixed-Integer Nonlinear Optimization: Algorithmic Advances and Applications*, IMA Volumes in Mathematics and its Applications, 60pp, to appear, Springer Verlag, Berlin, Heidelberg and New York (2010).

10. P. E. Gill and E. Wong, *Methods for Convex and General Quadratic Programming*. UCSD, Department of Mathematics, Technical Report NA 10-1, May 2010.

(Abarbanel) Presentations and invitations:

- Invited Speaker, Northwestern University, 5 October 2009.
- Invited Speaker, Physics Department, University of Chicago, 7 October 2009.
- Invited Speaker, Harvard University, October 20, 2009.
- Invited Speaker, American Nuclear Society Meeting, Washington, D.C., 16 November 2009.
- Invited Speaker, American Geophysical Union Annual Meeting, San Francisco, CA, 17 December 2009.
- Invited Speaker, Department of Physics, Michigan State University, Lansing, MI, 3 December 2009.
- Visitor, Meteorology Department, University of Reading, Reading, England, 21 January–30 January 2010.

- Invited Speaker, Department of Meteorology, University of Reading, Reading, England, 25 January 2010.
- Invited Speaker, Imperial College of Science and Technology, London, UK, 26 January 2010.
- Invited Speaker, European Centre for Medium Range Weather Forecasts, Reading, England, January 27, 2010.
- Invited Speaker, Los Alamos National Laboratory, Institute for Information Technology, 24 February 2010.
- Invited Speaker, CCOM Seminar, Department of Mathematics, UCSD, 9 March 2010.
- Invited Speaker, March Meeting of the American Physical Society, Portland, Oregon, 18 March 2010.
- Invited Speaker, Physics Colloquium, University of California, San Diego, 1 April 2010.
- Invited Speaker, National Center for Atmospheric Research, Boulder, Colorado, April 13, 2010.
- Invited Speaker, Physics Colloquium, Ohio State University, May 11, 2010.
- Keynote Speaker, Conference on Chaos and Complex Systems, Istanbul Kultural University, Istanbul Turkey, May 20–23, 2010.
- Invited Speaker, Experimental Chaos Conference, Lille, France, June 1–4, 2010.
- Invited Keynote Speaker, Condensed Matter and Materials Physics (CMMP10) Warwick University, UK, 14–16 December 2010.