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Understanding the Electrical Interplay Between a Capacitive Discharge Circuit and Exploding Metal

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Abstract

There is a significant body of work going back centuries that describes in detail the workings of metals that are rapidly transitioned from a solid to a vapor and beyond. These are known as exploding metals and have a variety of applications. A common way to cause metals to explode is through the use of a capacitive discharge circuit (CDC). In the past, methods have been used to simplify the complex, non-linear interaction between the CDC and the metal but in the process some important physics has been lost. This report provides insight into the complex interplay of the metal and the various elements of the CDC. In explaining the basic phenomena in greater detail than has been done before, other interesting cases such as "dwell" are understood in a new light. The net result is a detailed look at the mechanisms which shape the current pulses that scientists and engineers have observed for many decades.

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Chapter 1

Introduction

In the 1920's electrically exploding wires were used as intense light sources [1] in an attempt to replicate solar spectra. Anderson and Smith postulated the phase changes of the metal as it exploded using early capacitive discharge systems as the energy source [2]. In the 1960's, Tucker performed detailed studies of the changes in metal resistance as a function of electrical input [6]. It was found that the non-linearity of the interaction between the electrical energy source and the exploding metal was not desirable for understanding the exploding metal phenomenon. For that reason it was common to either investigate the discharge circuit or the metal but not both at the same time. One way of investigating the metal and "fixing" the discharge circuit was to use a cable discharge system which is very electrically "stiff" [7]. By using this system the effects of the changing metal on the discharge circuit are minimized. Unfortunately, these simplifications have ignored a rich physics that underlies the discharge circuit to exploding metal interaction.

Computational simulation is a means of investigating the details of the nonlinear interaction between exploding metal and the circuit driving energy into it. ALEGRA-MHD is a magneto-hydrodynamic finite element simulation tool designed to capture the energy transfer between the discharge circuit and the exploding metal. ALEGRA-MHD has been used previously to study electrical explosion of metals [4] as well as magnetic force acceleration of metals [5]. Through first-principles computational simulation each element in the capacitive discharge circuit (CDC) can be examined electrically while the exploding metal is undergoing phase changes.

This document details the results of a computational investigation to understand the electrical interplay between a capacitive discharge circuit and an exploding metal. Chapter 2.2 provides background information on expected results from a CDC into a fixed resistance load. Chapter 2.3 is a dissection of a current trace into three phases of evolution between the metal and the CDC. Chapter 2.4 also covers special cases for CDC and metal pairings. Chapter 3 contains a qualitative description of the transfer of energy during the three phases. The final result is a more complete understanding of how a capacitive discharge circuit drives metal to and through the exploding or "burst" process.

Chapter 2

Capacitive Discharge Circuit & Exploding Metal Interaction

This chapter describes in detail the interaction between a capacitive discharge circuit (CDC) and an exploding metal. In order to understand how the circuit elements interact, the CDC is described followed by analysis of the simple case of a discharge into a fixed load. From there, a new description of the current pulse into the exploding metal is presented. Some special cases are analyzed using the new understanding.

2.1 The Capacitive Discharge Circuit

The capacitive discharge circuit (CDC) consists of three real, physical elements: the capacitor, a discharge switch and the load which in this case is the exploding metal. Typically, two other elements are modeled: the parasitic inductance and resistance of the other circuit elements. A schematic of the CDC is shown in Figure 2.1. In the figure, the exploding metal is shown as a variable resistance as will be described in later sections.

The operation of the CDC is straightforward. The capacitor begins by being charged to a particular voltage. Once charged, the discharge switch can close which causes the capacitor to release its charge in the form of electrical current into the other circuit elements.

The parasitic inductance will be the inductance that is referred to in the remainder of this report. It is not typically a circuit element added by the circuit designer. Rather, it is "parasitic": a result of the construction of other circuit elements. It is beyond the scope of this report to discuss the source of parasitic inductance but suffice it to say that it is relatively easy to add inductance to a capacitive discharge circuit. Special methods are used to reduce the inductance if that is desired.

2.2 Ringdown into a Fixed Load

A ringdown is the discharge of a CDC into a fixed resistive load. It is useful to understand a ringdown in order to understand later phenomena. First, however, we will look at the special case

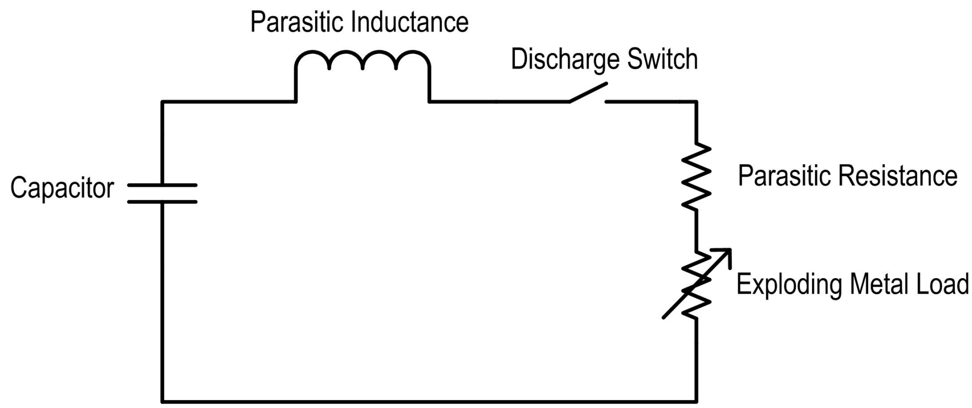

Figure 2.1. Schematic of the capacitive discharge circuit.

where the ringdown resistance is zero and the circuit consists only of a capacitor connected to an inductor. This is an unrealistic but instructive case.

The initial condition on a capacitor is to have energy stored in the form of charge. Upon "start" of this circuit the capacitor will discharge - that is, release that stored charge - in the form of current per the capacitor characteristic equation,

$$i = C\frac{dv}{dt}. \tag{2.1}$$

As soon as current flows, the inductor stores energy in the form of a magnetic field. The energy storage equation of an inductor is given by

$$E_L = \frac{1}{2}LI^2. \tag{2.2}$$

Eventually, the capacitor will release all of its initial charge and all of the energy in the system will be in the form of magnetic field stored in the inductor. At this point, the inductor will assume the role of the energy source and release its energy according to the characteristic equation for an inductor,

$$v = L\frac{di}{dt}. \tag{2.3}$$

The energy transfer between the capacitor and inductor can be observed through the voltage on the capacitor and the current through the circuit. When the capacitor is completely out of energy, equivalent to a zero voltage condition, the current transitions from increasing to decreasing. Similarly, when the current crosses zero and there is no stored magnetic energy the voltage on the capacitor transitions from increasing to decreasing. This is shown in Figure 2.2. A closed-form solution for an LC circuit is given in Appendix C.1 which illustrates mathematically the relationship between voltage and current.

The power into and out of the capacitor and inductor is shown in Figure 2.3. Power delivery from the capacitor will be discussed later when CDC interaction with the exploding metal is described. It is included here to allow contrast between the the power relationship in this ideal circuit versus the non-ideal case when the load is an exploding wire.

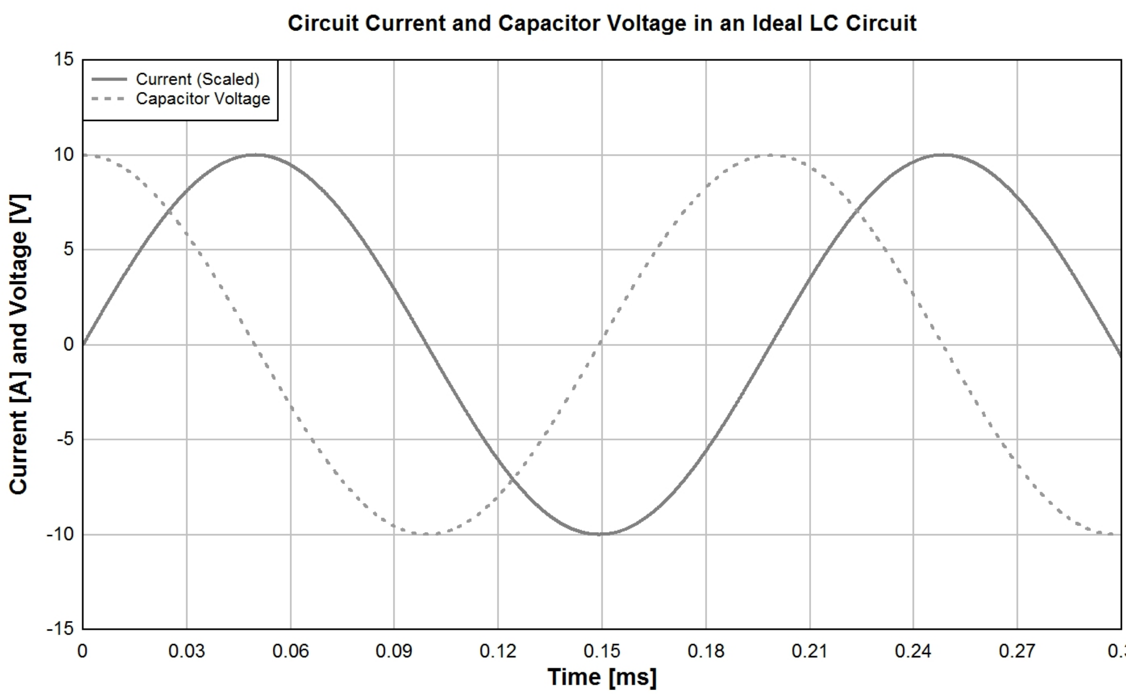

Figure 2.2. Inductor current and capacitor voltage in an LC circuit.

The oscillation of energy transfer will repeat ad infinitum in this hypothetical LC circuit because there is no energy dissipative device. If a fixed resistor is included the result is a resistor inductor capacitor (RLC) circuit where the resistor dissipates energy from the circuit as heat. The system is damped and the oscillations will die out over time as shown in Figure 2.4. A phenomenon that is difficult to notice in Figure 2.4 is that zero capacitor voltage, for example, no longer precisely correlates to a directional transition in current. Similarly, zero current does not correspond to a change in the direction in voltage amplitude (although this is somewhat difficult to see in Figure 2.4). This is an effect of the resistance and will be important later when discussing the current through an exploding metal.

The damped RLC current waveform shown in Figure 2.4 is the typical waveform observed when discharging the CDC into a fixed resistive load in order to eliminate the non-linearity of the exploding metal that will be discussed in the next section. For that reason, it is a relatively

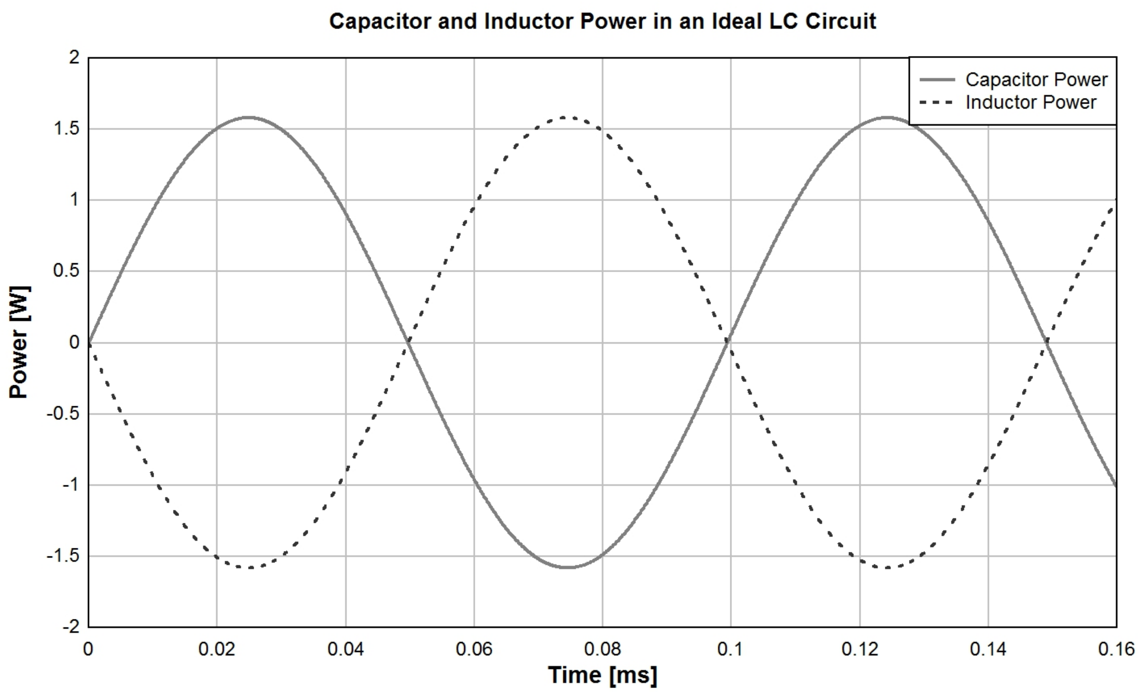

Figure 2.3. Inductor and capacitor power in an LC circuit.

important waveform but not very informative when it comes to predicting the performance of a CDC with a particular non-linear load. Derivation of a closed-form solution for this waveform is given in Appendix C.2.

2.3 Phases of the Current Trace

Having explored the basic electrical workings of a capacitive discharge circuit it is possible to apply this information to the discharge of a CDC into an exploding metal. The exploding metal phenomena, from the perspective of the metal, has been investigated to some degree or another for many decades. A diagram from Tucker and Toth shown in Figure 2.5 illustrates the widely-accepted view of the phase change of a metal as energy (or, in Tucker's case, action) is deposited into the metal [8]. It isn't intended to say whether this view of the process to explode a metal is correct but merely to show that it is typically understood from the perspective of the metal, not the metal's interaction with the CDC.

In order to examine the inner workings of a CDC discharging into an exploding metal, the ALEGRA-MHD code was used with the help of Sharon Petney (Dept. 1443, Computational Multiphysics) who developed the input deck. With ALEGRA-MHD it is possible to model the CDC and the exploding metal in a self-consistent manner. A lumped circuit representation was used including a capacitor, resistor, and inductor in series with the ALEGRA computational domain. The

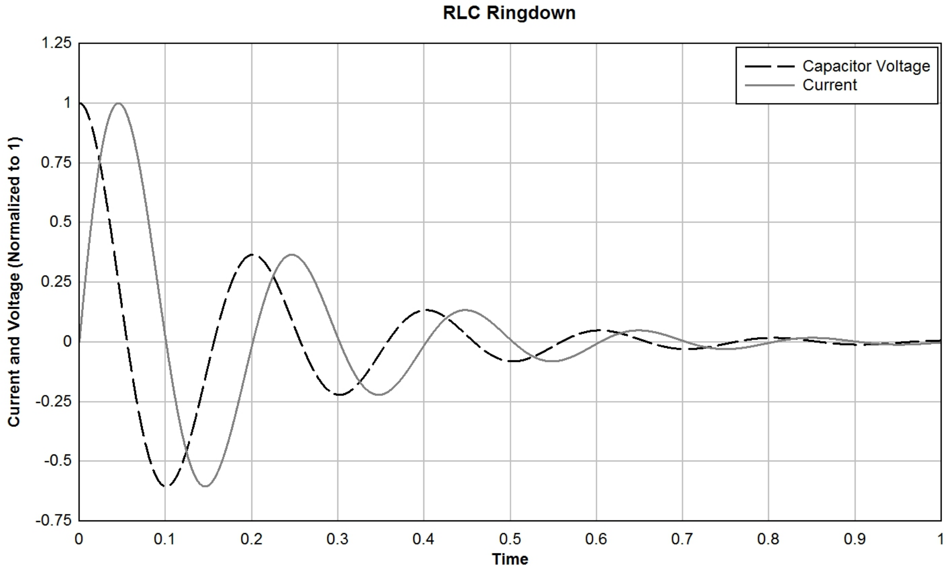

Figure 2.4. RLC circuit ringdown into a fixed resistive load.

domain included a Cartesian representation of the wire (slab symmetry) with corresponding equation of state and electrical conductivity models for gold. Using this approach each circuit element as well as the exploding metal can be examined in a way that would be impossible experimentally.

Figure 2.5 shows that a metal undergoes a rapid increase and then decrease in resistance during the process of exploding. An off-hand understanding of the CDC would lead one to believe that increasing the resistance would cause a corresponding decrease in current in accordance with Ohm's Law. While this is true to some degree, it isn't the whole story. In fact, it will be shown that the increase in resistance is critical to the ability of the system consisting of the CDC and the exploding metal to generate very high electrical powers which drive the burst process.

Past analysis of the current pulse was primarily concerned with burst - the peak of the voltage curve as shown in Figure 2.6 - as well as the amount of current recovery after burst. Little consideration was given as to what these characteristics meant or how, electrically, they were attained. The current pulse will now be described in three phases from an electrical point of view. The concepts of burst and recovery will be explained and new characteristics of the current pulse will be shown. This analysis does not consider the state of the exploding metal beyond it's resistance.

2.3.1 Phase I: From zero-time to first dI/dt = 0

Phase I of this phenomenon occurs from t=0 until $dI/dt = 0$ which in Figure 2.7(A) occurs at approximately 0.131μs and is marked by a vertical black line. At the beginning of Phase I the current (black line) closely matches the ideal current flow (dotted line) which is the current into a fixed load representing the parasitic resistance and wire resistances. It isn't until the wire melts

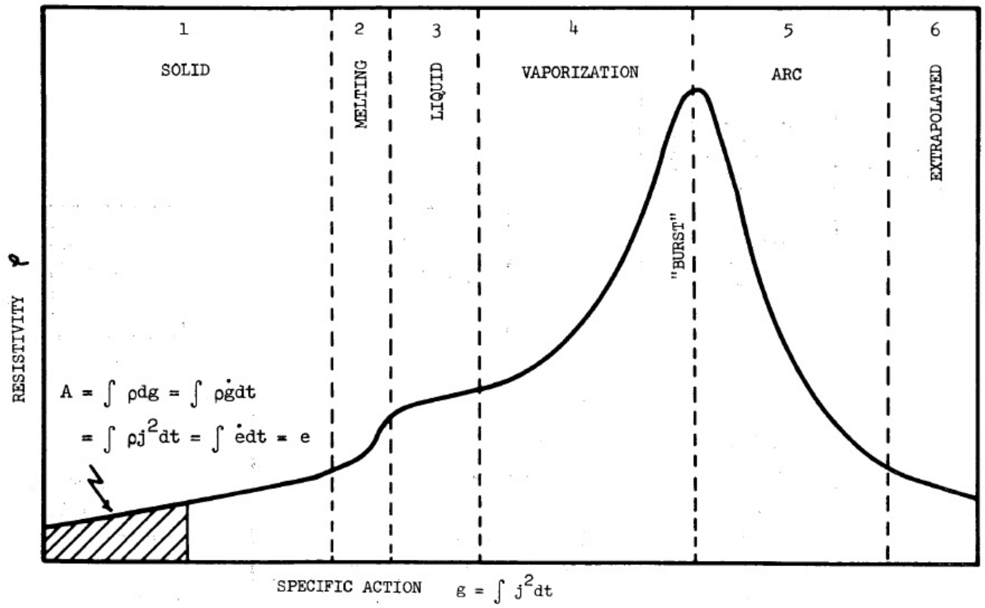

Figure 2.5. Relationship of the phases of a metal to its resistivity (reproduced from Tucker [8]).

that the ideal current and the current through the wire deviate by 1% at time t=0.093μs (marked by a grey vertical line that will be called the "1% line"). Up until that point the CDC and exploding metal system respond like the RLC system described in the previous section.

Eventually the resistance of the exploding wire increases to the point that the capacitor can no longer deliver an increasing current ($dI/dt = 0$) and as such the inductor is effectively a short circuit: the voltage across it is zero. This happens when the voltage remaining on the capacitor (V_C) is equal to the sum of the voltages across the metal (V_M), and the parasitic resistance in the entire system (V_R):

$$V_C = V_M + V_R. \tag{2.4}$$

Another way to look at it is by applying Ohm's Law, $V = IR$:

$$V_C = IR_M + IR_R = I(R_M + R_R), \tag{2.5}$$

or,

$$R_M = \frac{V_C}{I} - R_R, \tag{2.6}$$

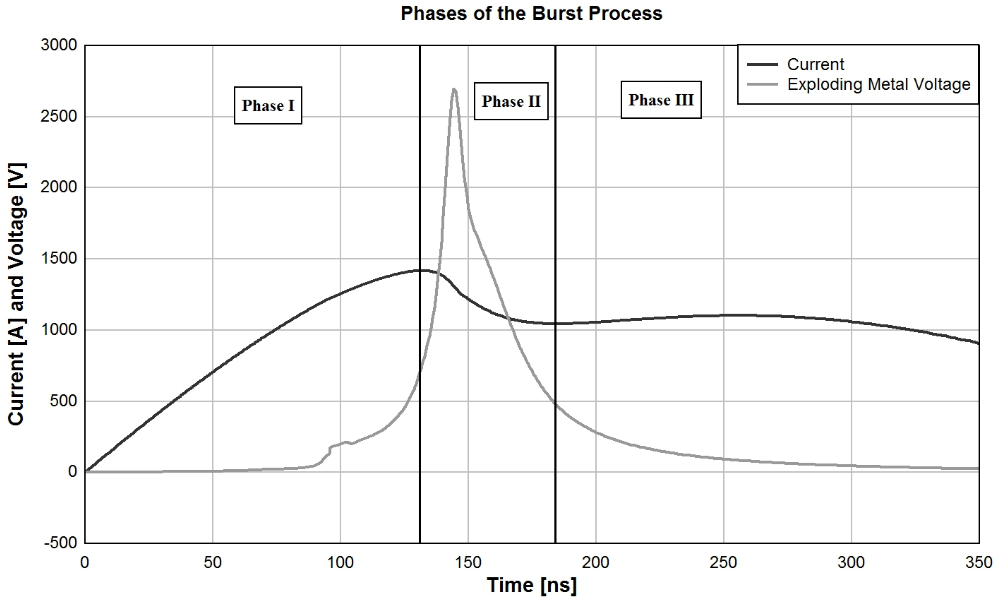

Figure 2.6. ALEGRA-MHD simulation of the phases of burst in an exploding metal driven by a CDC.

where R_M is the resistance of the exploding metal, R_R is the parasitic resistance in the circuit and I is the circuit current.

At first glance, it would appear that this is a useful result: prediction of the where $dI/dt = 0$ could be very informative and apparently only relies on voltages, currents and resistances. However, because R_M is dynamic and changing with applied energy, it is difficult to predict when it will reach a resistance where $dI/dt = 0$ without resorting to a simulation tool that incorporates an equation of state (EOS) and conductivity model.

Looking at the energy sources and sinks during this phase in Figure 2.7(B) it is observed that the inductor and wire are energy sinks (indicated by a negative energy) whereas the capacitor (minus the loss to the parasitic resistance) is an energy source. During the portion of the waveform before the 1% line almost all of the energy delivered by the capacitor is absorbed by the inductor. The wire accounts for very little energy loss. By the end of Phase I the capacitor has released 0.116J of available energy: 0.136J total minus 0.020J lost to parasitic resistance in the CDC. The inductor has absorbed 0.100J of that energy while the wire has only absorbed 0.016J. In other words, the inductor has absorbed 86% of available energy while the wire has absorbed only 14%! This is somewhat counter-intuitive and contrary to traditional thinking which would suggest that the exploding metal absorbs most of the energy released by the capacitor and as a result is close to exploding.

Finally, the power delivered by the capacitor shown in Figure 2.7(C) can be seen to closely match in amplitude but reverse sign that of the inductor. In other words, the inductor is a power sink to the capacitor's power source. For the sake of illustration, the wire is shown to have positive power which would indicate a power source but it is a power sink. After the 1% line the wire is seen to absorb more power and the inductor begins to absorb less. At the end of Phase I the inductor is neither a power source nor a sink.

2.3.2 Phase II: The Burst Process

When the current rate changes sign at t=0.131μs in Figure 2.7(C), the inductor changes from a power sink - absorbing energy - to a power source according to $V = LdI/dt$. This marks the start of Phase II. In the exploding wire simulation results, this is shown as a transition from a negative to a positive power in the inductor.

A feedback loop now occurs between the rising resistance of the wire and the inductor. The feedback loop is simply that the wire resistance serves to reduce current flow while the inductor desires to maintain it per Lenz's Law. In the end, the inductor provides a rapid power pulse to the wire by supplying a large voltage spike again because of $V = LdI/dt$. The example in Figure 2.7(C) shows that at burst - indicated here by the peak in wire power - the inductor is providing more than double the power of the capacitor. This phenomena is in contrast to that of a typical inductor-resistor (LR) circuit which does not generate large voltage and power spikes. A derivation and discussion of a typical LR circuit in Appendix C.3.

Phase II is called the *Burst Process* for this reason: there is a process between the CDC and the exploding metal that leads to the rapid deposition of energy into the metal. The rapid deposition occurs because of the inductance. In fact the low- or no-inductance case is an interesting one that will be discussed later. The capacitor is simply not able to deliver high power to the metal: Per the characteristic equation for a capacitor, $i = Cdv/dt$, the capacitor only responds to changes in it's voltage. The changing resistance of the wire would do nothing more than to stop the metal from exploding if it were a process dominated by the capacitor. *Therefore it can be said that inductance drives the burst process.*

The power delivered to the exploding metal by the inductance is much greater than what the capacitor would deliver alone and the waveshape of the power differs significantly from the ideal in Figure 2.3. Figure 2.7(C) shows for this example that the inductor delivers more than twice the power as the capacitor is delivering during burst. (The "missing" power in this figure is lost to the parasitic resistance of the system.) Without the inductance, the power output of the capacitor would go to zero or very near zero as the resistance of the metal increased.

The high power delivered by the inductor results in a significant portion of the inductor's energy being depleted. In some cases - threshold or "back-side" cases, for example - it could release all of its energy at which point the capacitor would need to have enough voltage to keep current flowing, if possible. The energy that the exploding metal absorbs in Phase II greatly increases relative to the energy it received in Phase I. The energy absorbed by the exploding metal is supplied by both the inductor and capacitor. In Figure 2.7(B), the inductor releases 50mJ, the wire absorbs 85mJ and the capacitor releases 48mJ. The parasitic resistance of the fireset accounts for the discrepancy in energy sums. The inductor and capacitor supply the wire with almost equal amounts of energy.

In high recovery discharges, the capacitor will account for a higher percentage of energy reaching the exploding metal. Conversely, nearer to no recovery the inductor supplies the majority of the energy.

Phase II in “back-side” burst cases show a precipitous drop in current as the voltage remaining in the capacitor is too low to push current through even the now-lowered resistance of the exploded metal. Burst of some sort may still occur because the inductor may have had enough energy to provide the rapid power pulse to the metal. But it would be a low peak power relative to a case where there is current recovery.

2.3.3 Phase III: From second dI/dt = 0 to complete discharge

At some point after burst the current again undergoes a direction change: $dI/dt >= 0$. Or, to be more precise, it undergoes this direction change if there is some current recovery. There may be no recovery depending on the voltage remaining in the capacitor. The conditions under which this transition to Phase III happens is when the voltage in the capacitor is equal to the sum of the voltage across the exploding metal and the parasitic resistance in the circuit (Eqn. 2.4).

This explains the concept of “recovery” that is often used as a metric for margin. Recovery is defined to be a continuation of positive-going current after burst. To attain recovery a high voltage and high capacitance is typically used. The capacitor expends very little charge and therefore voltage by the time burst has occurred. As a result the capacitor can continue to drive charge into the now low resistance load.

Conversely, in a no-recovery case there is very little or no energy left in the capacitor by the end of burst and thus no voltage to drive current through the resistance. The result is the classic ”threshold” current waveform that drops precipitously during and after burst. But there are also intermediate cases wherein recovery does happen, just at a much lower current level. The current will drop after burst until it “catches” or the voltage on the capacitor becomes greater than the sum of the voltages across the resistive elements in the circuit (again, Eqn. 2.4). Once it catches, the capacitor will provide charge to keep current flowing for a time.

It is an open question whether Phase III is important or not but this phenomenology of burst implies that it does not contribute much to the mechanics of the exploding metal. That is to say, by the time Phase II ends everything important that will happen has happened. Looking at a point long after burst in Figure 2.8, the wire has only absorbed another 21mJ (compared to the 85mJ it absorbs during the relatively short Phase II.) The inductor and capacitor return to reflecting energy to one another.

2.4 Special Cases

In this section, some special CDC discharge cases into exploding metal will be presented. Each case has some unique feature that is important to understand relative to the description of how the CDC drives the exploding metal given in the previous section.

2.4.1 Inductor-Powered Burst

One interesting special-case of the three-phase understanding of CDC to exploding metal interaction is the inductor-powered case. The waveforms for this case are shown in Figure 2.9. In Figure 2.9(A) we see that there is no current recovery but the wire does show a characteristic voltage spike. This implies that classical "burst" has occurred. A more instructive plot is Figure 2.9(C) wherein it can be seen that before burst is completed, the capacitor has nearly stopped releasing power. The inductor and wire power curves lay one atop the other as the inductor is providing all of the power from approximately the peak of the power curve to the end. Recovery does not occur in this case because the capacitor voltage is zero by the time Phase II would end. Thus there is no voltage to push current no matter how low the resistance of the wire may be.

Note that although the capacitor runs out of energy, the inductor is still able to cause a very high power pulse. Granted, it is lower than would likely be desired but it is still a burst signature. A simplified simulation given in Appendix A shows how the inductor alone can create burst in a way similar to what happened in this simulation.

2.4.2 High Current Recovery

Current recovery is an observed increase in the current through the exploding metal after burst or after Phase II ends. An example of this is shown in Figure 2.10(A). In this example, high recovery is attained by increasing the initial voltage on the capacitor. Therefore, at the end of Phase II there will be a higher voltage remaining on the capacitor and thus two effects can be observed: First, relative to the example in Figure 2.7, the drop in current during Phase II is reduced. Second, the current rises in Phase III above the peak at the end of Phase I.

2.4.3 Reduced Inductance

If the inductance of the capacitive discharge circuit is reduced but all other factors remain the same, some observations about the effect of inductance can be determined. The simulation results shown in Figure 2.11 are of the same CDC as in Figure 2.7 but with the inductance reduced from 100nH to 25nH.

With the reduced inductance, the current rise is faster and the time to Phase II is therefore reduced to slightly less than 40ns compared to 131ns for the 100nH CDC. At the start of Phase II, the lower inductance simulation exhibits a slightly higher current than the 100nH case but the inductor only has about half the energy by the start of Phase II. As a result of the lower inductance, the inductor is less able to prevent changes in current. From the start to the end of Phase II, the change in current for the low inductance case is approximately 800A while in the higher inductance case it is only approximately 320A. This shows how the inductance of the CDC attempts to prevent changes to the current.

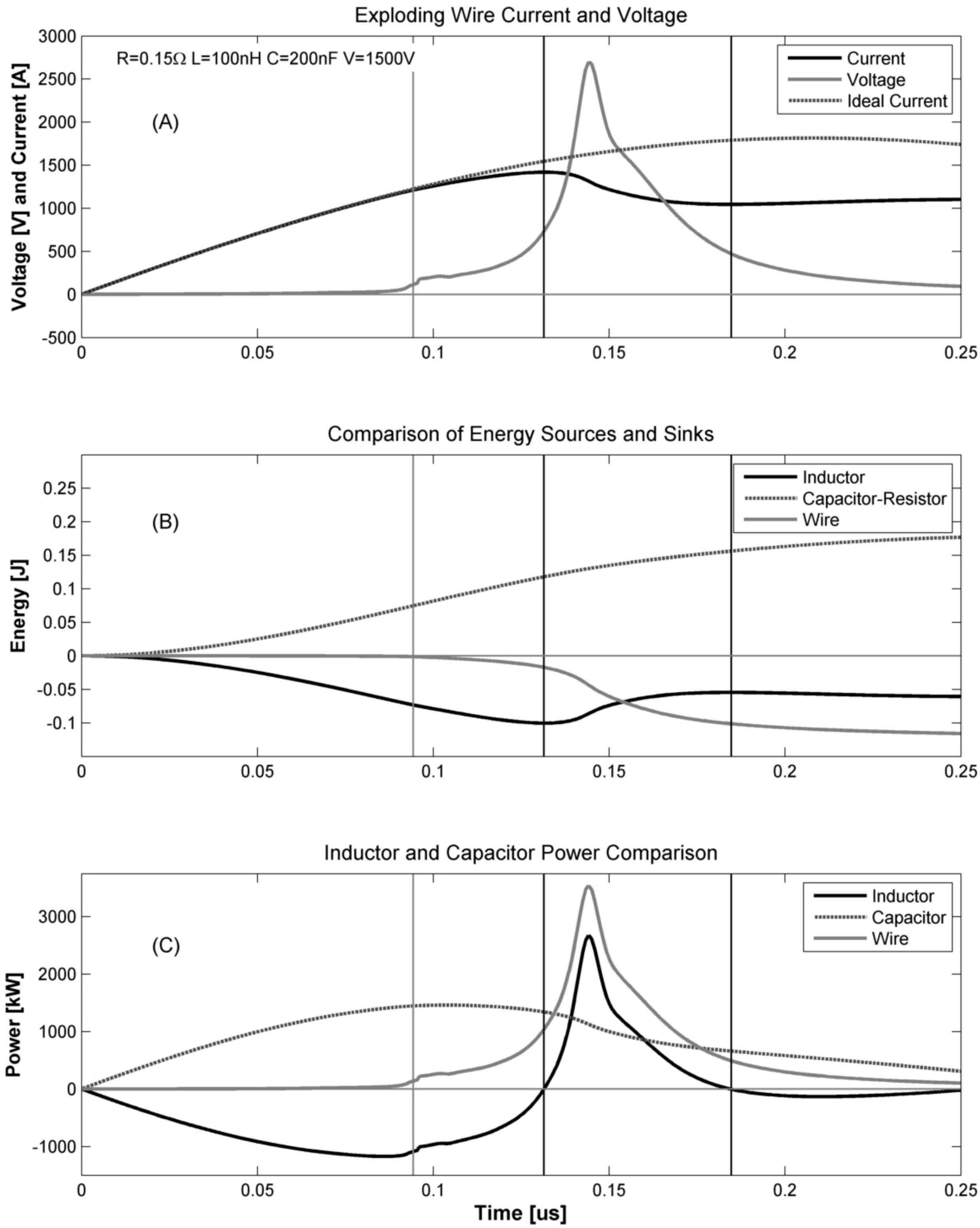

Figure 2.7. Waveforms from the simulation of a 1.2x20mil gold wire interacting with a CDC.

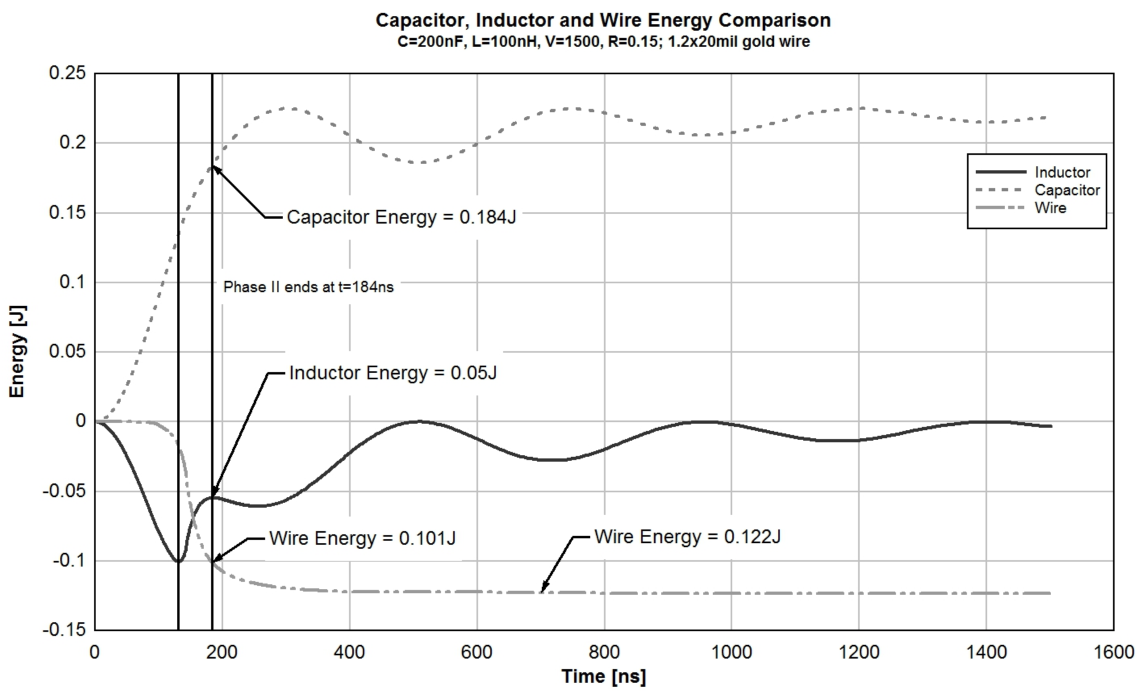

Figure 2.8. Energy absorbed in the wire long after Phase II ends.

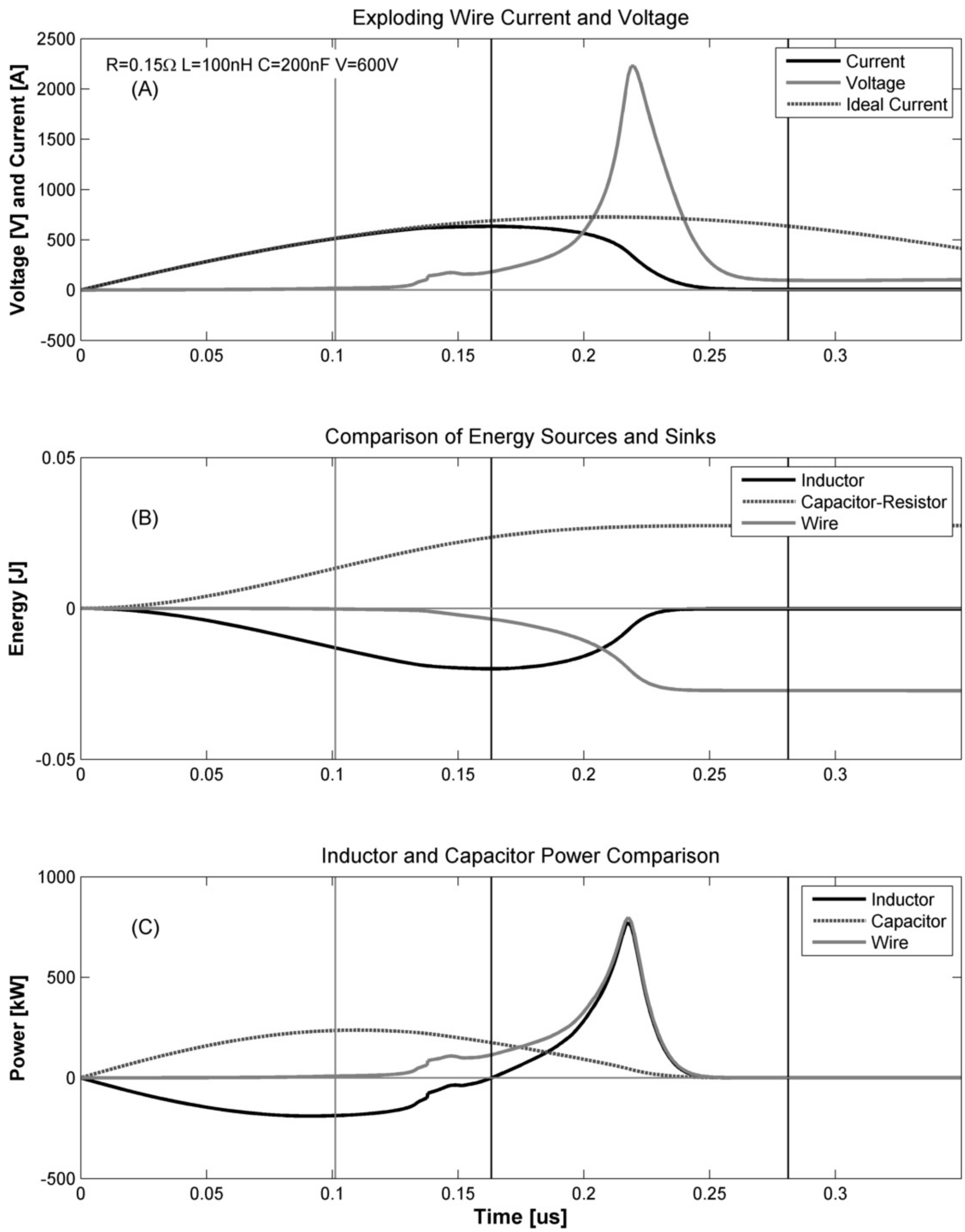

Figure 2.9. Inductor-powered burst.

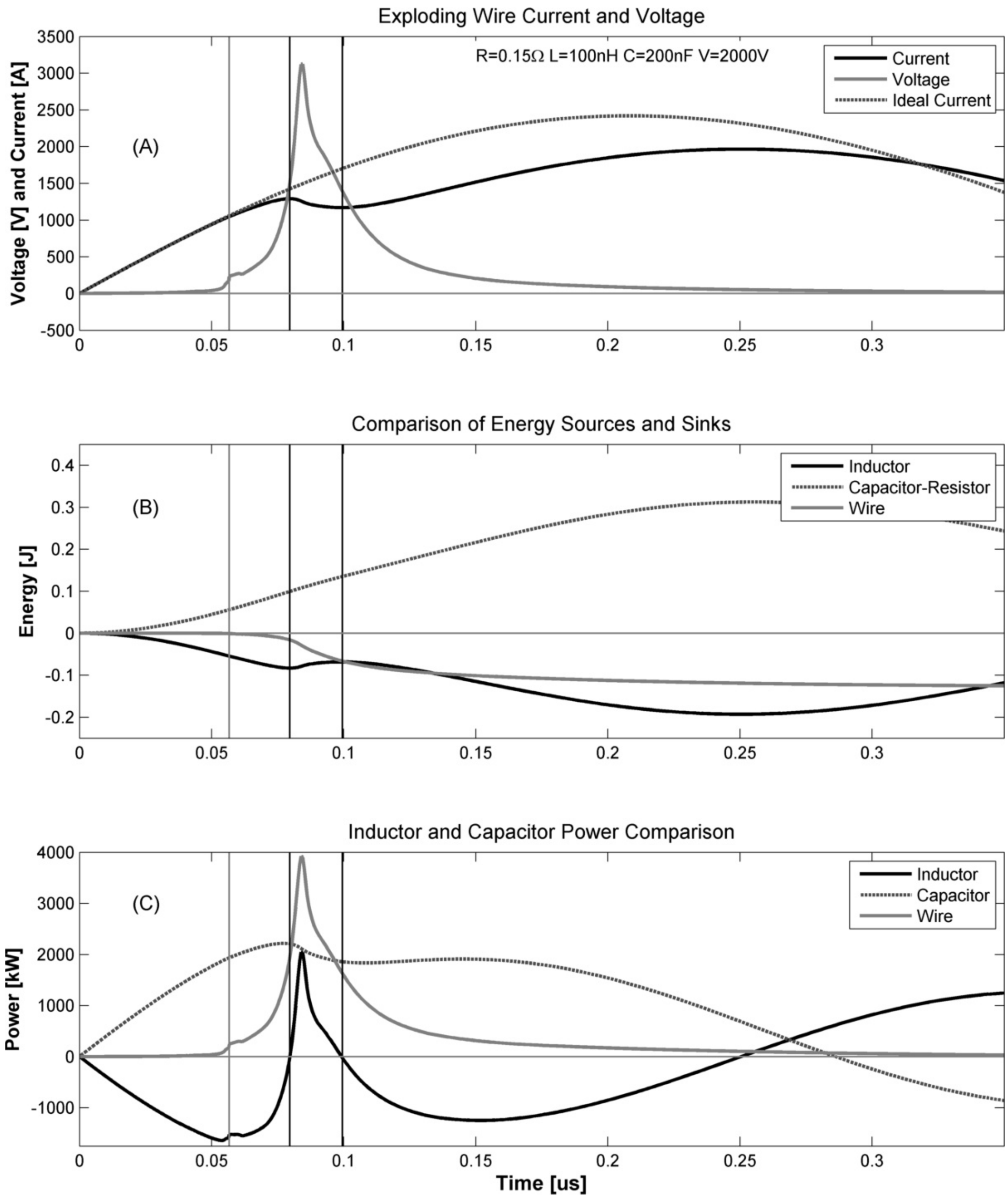

Figure 2.10. A high recovery discharge into a wire.

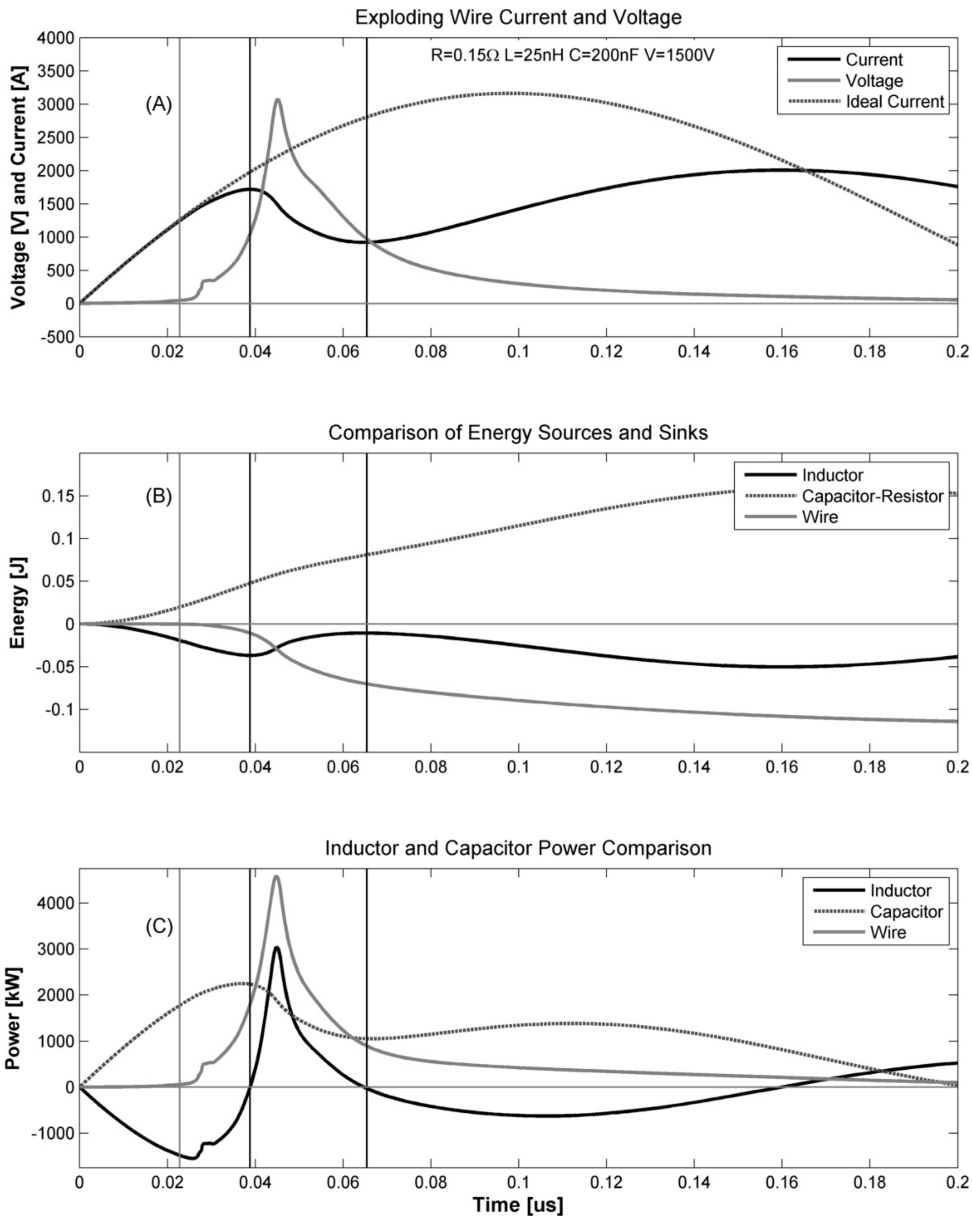

Figure 2.11. Simulation of a CDC with reduced inductance.

Chapter 3

A Qualitative Explanation of CDC Parameters as they Relate to Burst

At t=0, the capacitor has all of the energy stored statically in an electric field. At this moment, a switch closes and current begins to flow from the capacitor into the connected circuitry. Due to the inductance of the circuit, the current does not immediately jump to that predicted by Ohm's Law which would be $i = \frac{V_C}{R}$ where V_C is the initial current in the capacitor and R is the total resistance of the circuit. Rather, the inductance causes the current to be slowed as a magnetic field is created. The magnetic field slows the current flow. The exploding metal is absorbing some energy but not enough to dramatically change it's resistance. Therefore the current matches very closely with the current in an ideal ringdown.

As the current continues to rise and the metal absorbs energy, its temperature increases which in turn causes an increase in resistance. Depending on the remaining voltage on the capacitor, the resistance will only have to change a little or more dramatically to notice a deviation of the current from the ideal ringdown. The first significant and rapid change in resistance is around melt and often this is when the current flow begins to slow dramatically. For a capacitor with a lower voltage, it will happen before melt. Eventually the metal resistance is great enough to cause the change in current (dI/dt) to slow to zero.

At this point the inductor begins to source some of the energy that it has absorbed. This extra "kick" provided by the inductor allows the capacitor to keep providing power as well. While the current does drop at this point it is still flowing. The inductor is filtering out any large changes to the current flow by releasing energy stored in it's magnetic field. Energy is being absorbed by the metal and it's resistance is increasing more dramatically than before. The inductor responds by increasing it's voltage ("emf") and thus trying to keep the current flowing. As the resistance of the metal increases, the inductor's power output increases until burst occurs and the resistance again diminishes.

As the resistance diminishes, the capacitor is once again able to source current assuming that it has sufficient voltage remaining. The change in current (dI/dt) will return to being positive if it does.

During this process, some problems can occur. If the voltage in the capacitor is too low initially, if there is too little capacitance or if the inductance is too high, the energy stored in the inductor's magnetic field by the time the metal's resistance begins changing rapidly will be insufficient to drive the metal through burst. Or, to be more precise, the inductor will drive it part-way through burst and there will even be a characteristic voltage spike but the level to which it reaches will be less than is probably desired.

It appears - but is currently needing more research - that the inductor plays a role in “dwell” events. A dwell event is when current rise occurs as expected but then suddenly drops to zero while a voltage remains on the capacitor. After a while the current starts flowing again. An example is shown in Figure 3.1. What is believed to happen is that the parasitic resistance of the CDC is relatively high and this causes there to be too little energy in the inductor at the start of Phase II to push current through the high resistance of the exploding metal. The capacitor still has energy but as discussed in Section 2.3.2 it cannot use this energy to keep the exploding process going. Current stops. At some later time the resistance of the metal is reduced (possibly via expansion) and the capacitor is able to drive current through it again albeit with an overdamped waveform.

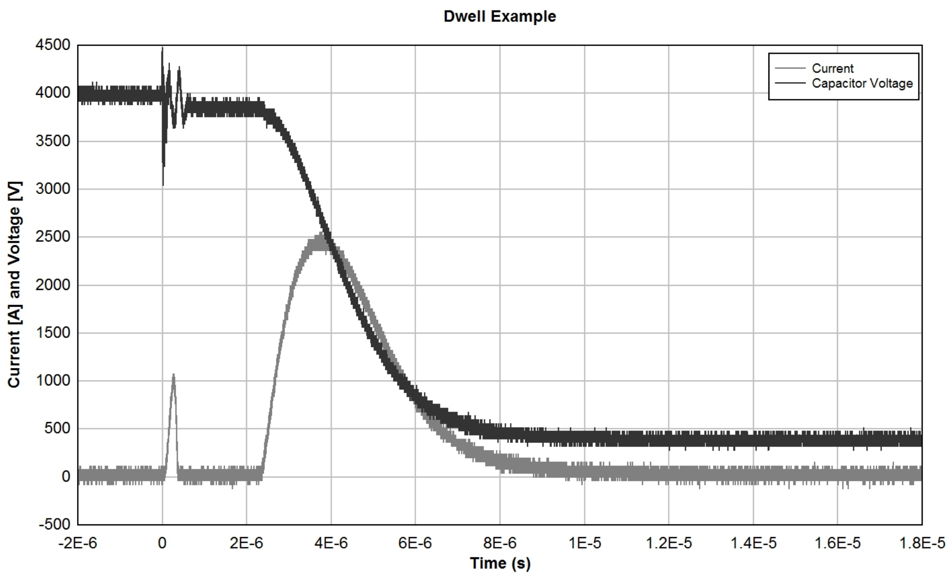

Figure 3.1. Experimental result showing a dwell event.

Chapter 4

Conclusions

The non-linear interaction between a capacitive discharge circuit and exploding metal is a complex but interesting one. It has been shown that a CDC discharging into a metal load acts electrically like a simple RLC circuit: the capacitor supplies energy primarily into the magnetic field of the inductor and to a lesser extent into the metal. At some point, the resistance of the metal begins to rapidly increase and a non-intuitive feedback loop between the energy stored in the inductor and the metal is created. This feedback loop causes the inductor to rapidly release its stored energy into the metal. During this time, the inductor may be the dominant source of power into the metal. Therefore, it can be said that the inductor, not the capacitor, in a capacitive discharge-like system is responsible for driving the metal into and through the process known as burst.

By understanding how the inductor interacts with the exploding metal, the characteristic shapes of the current and voltage waveforms can be better interpreted. It can also help understand other effects. One of those is the phenomenon of dwell that has been documented but not explained [3]. Dwell may be a case where the feedback loop between the inductor and the metal stalls - possibly as a result of the inductor running out of energy. The high resistance of the metal is unable to be countered and as a result the current drops precipitously until the resistance of the metal decreases enough to allow the capacitor to drive current again.

Ultimately, the hope is that by understanding how a capacitive discharge circuit interacts with an exploding metal improvements in devices that utilize this technique can be made.

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Appendix A

The Exploding Metal Modeled as a Positive Temperature Coefficient Resistor

In this section it will be shown that the phenomena described in this report wherein the inductance of a capacitive discharge circuit (CDC) and the exploding metal interact to generate a very high power pulse can be simulated very simply using a positive temperature coefficient (PTC) resistor. Like an exploding metal, a PTC resistor increases resistance as energy is deposited into it. Most resistors exhibit this characteristic to one degree or another as a result of Joule heating and the resultant increase in electron collisions in the lattice of the metal. Using this PTC model and connecting it in simulation to a model of first a CDC and then simply a "charged" inductor we can show that the power into the PTC is much higher than would be anticipated by standard LR circuit theory as given in Section C.3.

A PTC resistor can be modeled very simply as a piecewise defined function as shown in Figure A.1. As energy is deposited into the PTC its resistance will increase if the energy exceeds some minimum level. The model eventually switches to a negative temperature coefficient to simulate the similar effect in exploding metals although it will be shown that this phenomena may not be necessary.

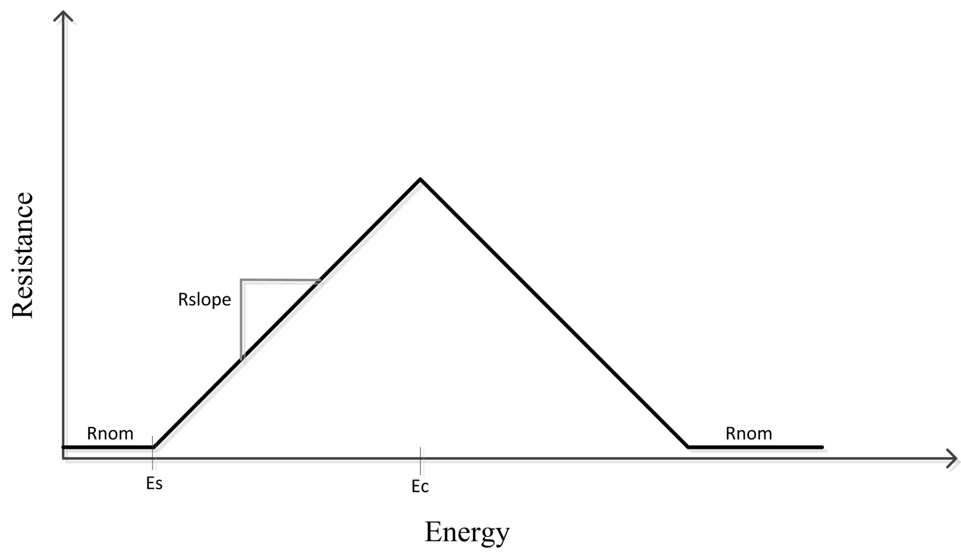

Figure A.1. Piecewise PTC resistor model.

There are a number of parameters that define this model shown in A.1.

Table A.1. Parameters of the PTC resistor model.

Parameter	Description	Units
E_S	”Starting” energy	J
E_c	Critical energy	J
R_{slope}	Slope of the resistance curve	Ohms/J
R_{nom}	Nominal resistance	Ohms

The PTC resistor model was written in VHDL-AMS (IEEE 1076.1-1999) for simulation using the Ansys Simplorer tool. The code is listed here:

```
---------- VHDLAMS MODEL ptc_resistor ----------

---------- ENTITY DECLARATION ptc_resistor ----------
LIBRARY Ieee;
use Ieee.electrical_systems.ALL;
use Ieee.fundamental_constants.ALL;
use Ieee.math_real.ALL;
ENTITY ptc_resistor IS
GENERIC (
ES : real := 0.03; --Start of PTC region
```

```
EC : real := 0.20; -- Start of negative slope region
RNOM : real := 0.01; --Initial and final resistance
RSLOPE : real := 100.0 --Slope of resistance over energy (RSLOPE=R/E)
);
PORT (
TERMINAL A :  electrical;
TERMINAL B :  electrical
);
END ENTITY ptc_resistor;

---------- ARCHITECTURE DECLARATION arch_ptc_resistor ----------
ARCHITECTURE arch_ptc_resistor OF ptc_resistor IS

QUANTITY V across I through A to B;
QUANTITY power : real := 0.0;
QUANTITY energy : real := 0.0;

BEGIN

power == V*I;
energy == power'integ;

if energy > (ES+2*EC) use
V==I*RNOM;
elsif energy > EC use
V==I*(EC*RSLOPE-((energy-EC)*RSLOPE)+RNOM);
elsif energy > ES use
V==I*((energy-ES)*RSLOPE+RNOM);
else
V==I*RNOM;
end use;

END ARCHITECTURE arch_ptc_resistor;
---------- END VHDLAMS MODEL ptc_resistor ----------
```

Simulations begin with a CDC represented by an RLC circuit discharging into the PTC model as shown in Figure A.2. At the time of burst the voltage suddenly spikes while the current shows a characteristic dip. The simulation even shows some recovery as the PTC resistance returns to a low value.

To further abstract the problem, we can eliminate the "C" in the CDC altogether and have only an inductor driving the PTC model. To give the inductor some energy an initial current condition is used. This is shown as a parameter of the inductor L1 called *I0* in Figure A.4. This is obviously an impractical case: there is no simple way of giving an inductor an initial current condition without

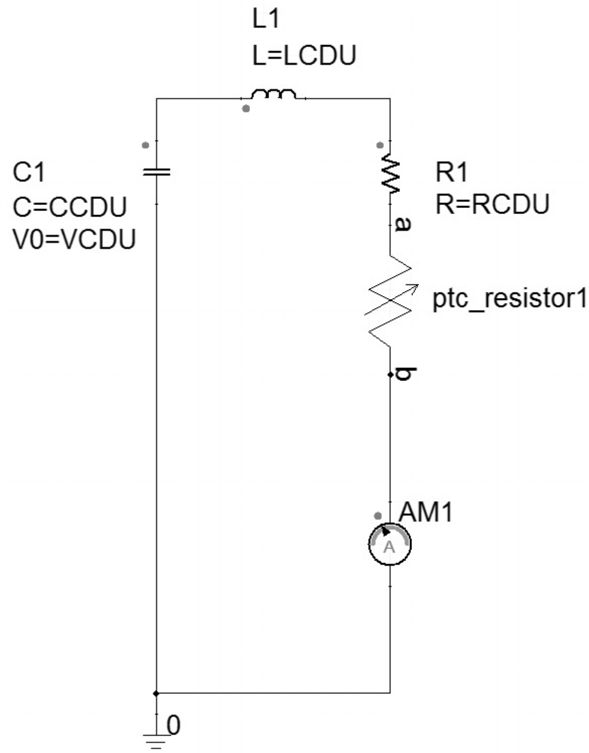

Figure A.2. Simulation model of the piecewise PTC connected to a capacitive discharge circuit.

some external source (such as a capacitor in the case of a CDC.)

This simplified model is instructive because it shows how the inductor - and the inductor alone - is responsible for the very high voltages and thus powers observed when a CDC is used with an exploding metal. Figure A.5 shows the current initially decreasing slowly as the inductor discharges into the nominal resistance of the PTC. At approximately 0.1us the PTC resistance begins to increase which leads to two effects: the voltage spike and the discharge of all of the current. The power delivered into the PTC shows the characteristic spike.

A.1 Conclusions and Observations

These simple simulations of a positive temperature coefficient resistor being driven by a CDC or a "charged" inductor show that the high power pulses observed in ALEGRA-MHD simulations and experimental data is caused by an interaction between the increasing resistance of the exploding metal and the inductor.

The simulations are instructive from a measurement planning perspective. They indicate that

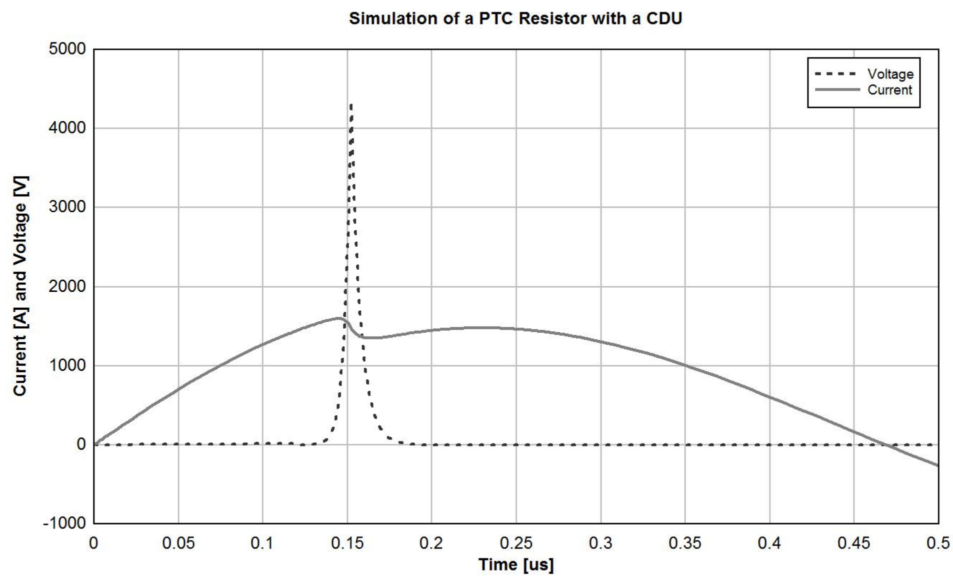

Figure A.3. Simulation of the piecewise PTC resistor model with a capacitive discharge circuit.

any attempts to measure either peak voltage or peak power will be met with frustration. Even simulations are problematic. Performing the simulation of the inductor discharging into the PTC model with a fixed step size of 10ns results in a peak voltage of 1.66kV. Changing the step size to 1ns results in a peak of 1.88kV - an increase of approximately 13%. Another way to put this is that the voltage or power pulse is of such high frequency content that any measurement system will incorrectly measure the peak.

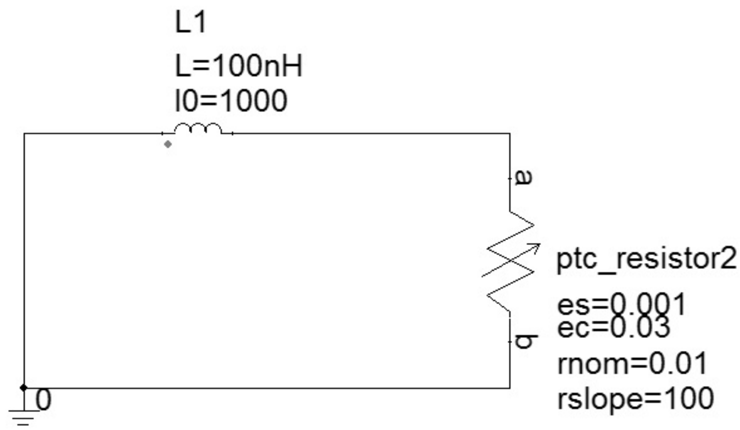

Figure A.4. Model of inductor discharge into the PTC resistor.

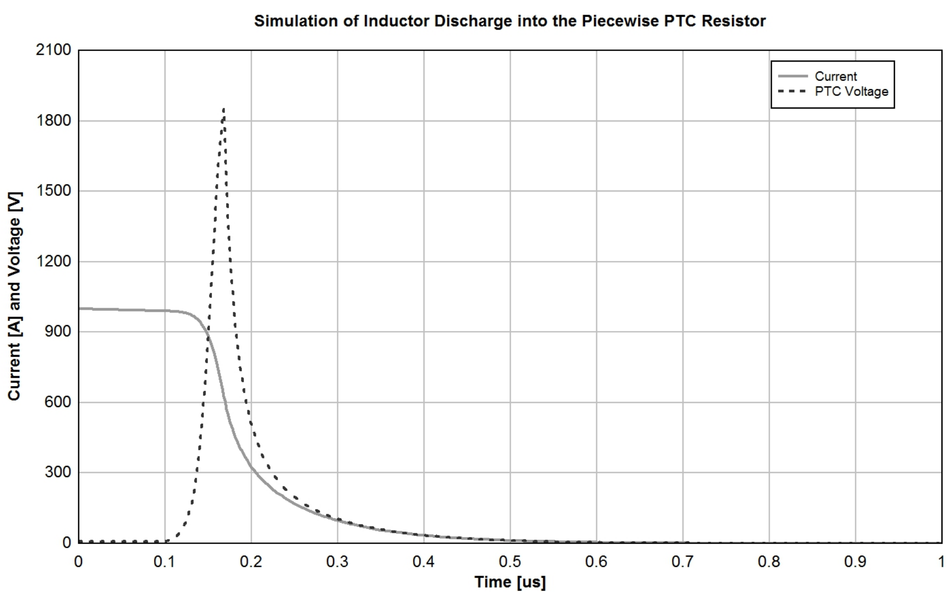

Figure A.5. Simulation of inductor discharge into the PTC resistor.

Appendix B

Understanding Electrical Circuit Elements

B.1 Resistors

Resistors dissipate energy from the system. This is in contrast to capacitors and inductors which can store energy through differing means and then release that energy back into the circuit at a later point in time. The power in a resistor is given by

$$P_R = IV \tag{B.1}$$

or, when combined with Ohm's Law,

$$P_R = \frac{V^2}{R} \tag{B.2}$$

and

$$P_R = I^2R. \tag{B.3}$$

B.2 Capacitors

Capacitors are an electrical element which can store energy statically, that is without having current flowing. A simple capacitive system consists of two plates connected through a switch, resistor, and battery. Once the switch is closed, electrons are drawn from one plate through the resistor toward the positive battery terminal resulting in the plate becoming positively charged. Electrons are also repelled by the negative terminal to the opposing plate. Electrons continue to transfer until the potential difference across the plates is equal to the battery voltage (V) which is also known as electromotive force (emf or $\mathscr{E}$). Capacitance (C) then, is a measure of the ability to store charge (Q) on the plates (storage capacity) using a voltage (V) or

$$C = Q/V. \tag{B.4}$$

The current in the circuit is determined by the amount of charge accumulated (or reduced) on the plate as a function of time, or

$$i_C = dQ/dt. \tag{B.5}$$

The current can then be related to the voltage since $Q = CV$ and

$$i_C = dQ/dt = d[CV]/dt = C\frac{dV_c}{dt}. \tag{B.6}$$

The current from the capacitor is directly related to the time rate of change of voltage across the plates. Using the relationship the transient effects of either charging or discharging the capacitor can be represented mathematically. Using Kirchoff's voltage law around a closed loop,

$$v_R + v_C = \mathscr{E}. \tag{B.7}$$

We know that $v_R = iR$ and since the current in the resistor is the same as the capacitor then $i = C\frac{dv_C}{dt}$ hence

$$RC\frac{dv_C}{dt} + v_C = \mathscr{E}. \tag{B.8}$$

The solution of this can be obtained using basic calculus resulting in

$$V_c = \mathscr{E}(1 - e^{-t/RC}). \tag{B.9}$$

The value RC can be shown to be a time constant for the simple electrical circuit by dimensional analysis:

$$[RC] = \frac{volts}{Couloumbs/sec}\frac{Coulombs}{volts} = \text{sec}.$$

Once a capacitor is charge it stores its energy (E_C) in the electric field. The stored energy can be computed by integrating the electric power (IV) over time, or

$$E_C = \int iV\,dt = \int C\frac{dv_c}{dt}V\,dt = \int CV\,dv = \frac{1}{2}CV^2. \tag{B.10}$$

B.3 Inductors

An induced emf always gives rise to a current whose magnetic field opposes the original change in the magnetic flux -Lenz's Law

Electricity and magnetism are related in that an electric current produces a magnetic field, and a magnetic field exerts a force on a moving electric charge (electric current). Futhermore, Faraday observed that a changing magnetic field can induce a current in a closed electrical system. Specifically, it is the time rate of change of the magnetic flux (ϕ) which is defined as

$$\phi_B = \int B \cdot dA, \tag{B.11}$$

or the integral of the magnetic field (B) taken over any open surface which cause a current to be induced. Faraday's experiments resulted in understanding that the induced electromotive force (emf or $\mathscr{E}$) across the inductor is equal to the time rate of change of the magnetic flux (through the inductor):

$$\mathscr{E} = -\frac{d\phi_B}{dt}. \tag{B.12}$$

For a circuit with N closely wrapped loops, then

$$\mathscr{E} = -N\frac{d\phi_B}{dt}, \tag{B.13}$$

since the induced emf of each loop adds together. The inductance of a coil is a measure of the instantaneous change in flux due to an instantaneous change in current through the coil, or

$$L = N\frac{d\phi_B}{di}. \tag{B.14}$$

The induced emf can be related to the inductance via the chain rule:

$$\mathscr{E} = -N\frac{d\phi_B}{dt} = (N\frac{d\phi_B}{di})(\frac{di}{dt}) = L\frac{di}{dt}. \tag{B.15}$$

Therefore, the greater the time rate of change of the current, the greater the emf/voltage across the inductor will be.

The energy within the inductor is stored in the magnetic field and can be also be computed by integrating the power over time:

$$E_L = \int iV\,dt = \int i(L\frac{di}{dt})dt = \int Lidi = \frac{1}{2}Li^2. \tag{B.16}$$

In contrast to the capacitor which stores the energy in the electric field statically, the inductor stores energy in the magnetic field and requires a current to do so. Additionally, since an inductor can store energy, it can discharge energy in a peculiar way when coupled with a resistor as described in Section C.3.

Appendix C

Derivation of Useful Circuit Equations

C.1 LC Circuit

For an idealized circuit with no resistance, an inductor, and capacitor plates charged to $\pm Q_0$ Coloumbs, the potential difference across each element must be equal to the other:

$$\frac{Q}{C} = -L\frac{di}{dt}. \tag{C.1}$$

As current is due solely to the capacitor, $I = dQ/dt$ which can be substituted into the above equation resulting in the second-order differential equation:

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0, \tag{C.2}$$

which has the form and solution for harmonic oscillation,

$$Q = Q_0 \cos(\omega t + \alpha), \tag{C.3}$$

where $\omega = \sqrt{1/LC}$. The current in the inductor is

$$i = \frac{dQ}{dt} = -\omega Q_0 \sin(\omega t + \alpha). \tag{C.4}$$

With no resistance in this circuit, the energy is able to pass from the discharging capacitor to the inductor at which point in time the current change is zero and the capacitor energy is zero (all the energy resides in the inductor). The inductor then discharges its energy back to the capacitor on the timescale $\sqrt{1/LC}$.

C.2 LC Circuit With Resistance (RLC)

$$\frac{Q}{C} + L\frac{di}{dt} + IR = 0. \tag{C.5}$$

As current is due solely to the capacitor, $I = dQ/dt$ which can be substituded into the above equation resulting in the second-order differential equation:

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0, \tag{C.6}$$

which has the form and solution for a damped harmonic oscillator,

$$Q = Q_0 e^{-\frac{R}{2L}t} cos(\omega' t + \alpha), \tag{C.7}$$

where $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$. The circuit current is equal to

$$i = \frac{dQ}{dt} = Q_0 e^{-\frac{R}{2L}t} \left(-\frac{R}{2L} cos(\omega' t) - \frac{1}{\sqrt{LC}} sin(\omega' t) \right). \tag{C.8}$$

C.3 LR Circuit

Let's examine the current evolution of a circuit which consists of a fixed inductance (L), and total resistance (R) which is includes an external resistor and the finite resistance from the inductor, and a dc voltage (V). When the circuit is closed, current begins to flow through the circuit, with an induced emf from the inductor which resists the change in current. The time-evolution of current can be determined analytically using Kirchoff's loop rule.

The circuit contains voltage from the battery (V), the inductor ($-L\frac{dI}{dt}$), and the resistor (IR) and are related by

$$V - L\frac{di}{dt} = IR. \tag{C.9}$$

Re-writing the equation and integrating we obtain

$$\int_{i=0}^{i} \frac{di}{V - iR} = \int_0^t \frac{dt}{L}, \tag{C.10}$$

hence

$$-\frac{1}{R} ln\left(\frac{V - iR}{V}\right) = \frac{t}{L}, \tag{C.11}$$

which reduces to

$$i = \frac{V}{R}(1 - e^{-t/\tau}), \tag{C.12}$$

where $\tau = L/R$. For this case, the current rises gradually until it reaches a steady-state after at which point in time there is no potential drop aross the inductor and all the potential drop is across the resistor. The same arguments can be made for the discharge of the inductor if a switch were applied removing it from the battery and allowing the inductor to discharge through the resistor. the inductor will exponentially decay the current on the same characteristic timescale ($\tau = L/R$).

Now let's take a look at the case where the resistance is not constant. If the resistance is varying proportionately with the current ($R = Ai$) then Kirchoff loop analysis modifies slightly to

$$\int_{i=0}^{i} \frac{di}{V - i(Ai)} = \int_{0}^{t} \frac{dt}{L}, \tag{C.13}$$

hence

$$\frac{1}{\sqrt{AV}} \arctan\left(\sqrt{\frac{A}{V}} i\right) = t/L, \tag{C.14}$$

and

$$i = \sqrt{\frac{V}{A}} \tan\left(\frac{\sqrt{AV}}{L} t\right). \tag{C.15}$$

The resulting current for this configuration has a runaway effect on the timescale ($t < \frac{\pi}{2}\frac{L}{\sqrt{AV}}$) which is much faster than L/R (the fixed resistance case). The case where $R = Ai^2$ is not considered here because of the difficulty in extracting the current term (i) from the combination of arctan and ln terms. Observation of the effect that current runaway can occur when the inductor and resistor couple nonlinearly is the most important point to conclude here.

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