Final Report

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Project title: Basic Studies of Non-Diffusive Transport in Plasmas
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Distribution: Unlimited

Executive summary:

The project expanded and developed mathematical descriptions, and corresponding numerical modeling, of non-diffusive transport to incorporate new perspectives derived from basic transport experiments performed in the LAPD device at UCLA, and at fusion devices throughout the world. By non-diffusive it is meant that the transport of fundamental macroscopic parameters of a system, such as temperature and density, does not follow the standard diffusive behavior predicted by a classical Fokker-Planck equation. The appearance of non-diffusive behavior is often related to underlying microscopic processes that cause the value of a system parameter, at one spatial position, to be linked to distant events, i.e., non-locality. In the LAPD experiments the underlying process was traced to large amplitude, coherent drift-waves that give rise to chaotic trajectories. Significant advances were made in this project. The results have lead to a new perspective about the fundamentals of edge transport in magnetically confined plasmas; the insight has important consequences for worldwide studies in fusion devices. Progress was also made in advancing the mathematical techniques used to describe fractional diffusion. New insight was obtained on the origin of the spectral signatures of chaotic dynamics. The new developments permit a more realistic description of transport phenomena in magnetically confined plasmas. The project provided a unique connection between basic experiments, recent advances in the mathematical description of nonlocal transport, modern signal-analysis methods to elucidate the transition between classical transport governed by Fick's law, and nonlocal behavior. The broader impacts included the generation of basic insight useful to magnetic fusion and space science researchers, establishing broader contacts with the nonlinear dynamics and fluid turbulence communities, completion of a Ph.D. dissertation in broad areas of national interest, and promotion of interest in science through community outreach events and classroom instruction.

Comparison of accomplishments with goals:

The project completed all the initial goals and generated far-reaching results on topics that originally were not perceived to be connected to the research plans. The mathematical methodology of fractional diffusion was successfully advanced to include finite jumps, to describe higher dimensions, and to allow for boundary conditions of experimental relevance. The new mathematical machinery was quantitatively compared with data from the major fusion devices in the world to obtain a direct test of the presence of fractional transport in fusion experiments. These were the original goals of the proposed project; their successful completion lead to three publications in major refereed journals, several presentations at scientific conferences, and to a Ph.D. dissertation. The unexpected consequences were related to the comparison with basic heat transport experiments performed in the LAPD device at UCLA. It was discovered that the subtle nonlocal transport observed was a manifestation of the underlying chaotic dynamics. This lead to new insights that have resulted in major advances in the basic understanding of the experimental signature of chaotic processes and to the demonstration, via an extensive survey, that the dynamics responsible for the anomalous transport at the edge of magnetic confinement devices is chaotic and not stochastic. These findings were reported in four additional publications and in invited talks at international conferences. An overview of these accomplishments is presented in the next entry.

Summary of project activities:
The project activities resulted in 8 refereed publications, 4 invited talks at international conferences (Latin American Workshop on Plasma Physics (LAWPP), Mar del Plata, Argentina, November 21-25, 2011; joint EPS/ICPP meeting in Stockholm, Sweden, July 2-6, 2012; EFTSOMP 2012, "Workshop on Electric Fields, Turbulence and Self-Organization in Magnetized Plasmas" in Stockholm, Sweden, July 9-10, 2012; Annual meeting of the Mexican Physical Society, San Luis Potosi, Mexico, Oct. 28-Nov.1, 2013), 5 contributed presentations at meetings of the American Physical Society (DPPP-APS), one Ph. D. dissertation (A. Kullberg), and the training of one undergraduate student (S. Taimourzadeh) who is now a graduate student at UC Irvine. The following provides a brief summary of the project activities.

i)"Transport in the spatially tempered, fractional Fokker–Planck equation". This is a basic theoretical study that explores the methodology of how to incorporate the realistic effect of finite-length flights into the description of fractional diffusion, i.e., "tempering". This is an important issue in making contact with experiments since realistic flights are always finite. To isolate the important features, a study has been performed of truncated Lévy flights in super-diffusive transport in the presence of an external potential. The study is based on the spatially tempered, fractional Fokker–Planck (TFFP) equation in which the fractional diffusion operator is replaced by a tempered fractional diffusion (TFD) operator. Harmonic (quadratic) potentials and periodic potentials with broken spatial symmetry (example in Fig.1) are considered. The main objective is to determine the dependence of the steady-state probability distribution function (PDF), and of the electric current (in the case of periodic potentials), on the level of tempering, λ, (the inverse scaled length) and on the order of the fractional derivative in space, α. An expansion of the TFD operator for large λ has been implemented to calculate the coarse-grained PDF. The steady-state PDF solution of the TFFP equation for a harmonic potential is computed numerically. In the limit $\lambda \rightarrow \infty$, the PDF approaches the expected Boltzmann distribution. However, nontrivial departures from this distribution are observed for finite ($\lambda > 0$) truncations, and $\alpha \neq 2$. In the limit $\lambda \rightarrow \infty$, the PDF converges to the Boltzmann distribution and the current vanishes. However, for $\alpha \neq 2$, the PDF deviates from the Boltzmann distribution and a finite non-equilibrium current appears for any $\lambda > 0$. The current is observed to converge exponentially in time to the steady-state value. A detailed numerical study has been done of the dependence of the current on λ, and the physical parameters of the system.

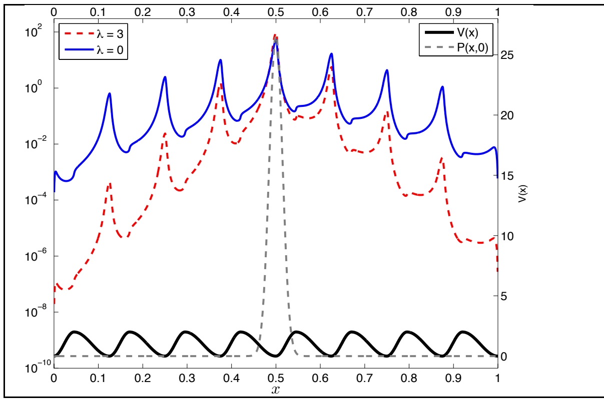

Fig.1. Spatial snapshot of time-dependent solution in a periodic potential (black line), showing the effect of "tempering" in the fractional diffusion operator on the spreading of an initial localized perturbation (dashed gray line). Blue curve is PDF for infinite-length flights, and red is for a scaled inverse tempering length $\lambda = 3$.

ii) "Extension of fractional diffusion description to two-dimensional systems with boundaries".
A two-dimensional (2D) fractional Laplacian operator has been derived and used to model nonlocal transport in magnetically confined plasmas. The new 2D fractional Laplacian is an integral-differential operator that arises in the long-wavelength, fluid description of quantities undergoing non-Brownian random walks without a characteristic length scale. To describe the behavior of plasmas bounded radially, an inhomogeneous form of the fractional Laplacian with spatially variable diffusivity is used, and an effective sheath is included at the interface with walls. Green's function solutions to the 2D fractional diffusion equation have been obtained for the ideal unbounded case. These Green's functions exhibit algebraic scaling with position and have a strong nonlocal behavior. The polar fractional Green's function (i.e., azimuthally averaged) can be related to the Cartesian fractional Green's function (in 1D) when an initial pulse is located far from the origin, i.e., in the slab approximation. Fractional diffusion in a finite circular disk with azimuthal symmetry has been studied and a time-implicit numerical integration scheme for the fractional diffusion equation in this geometry has been developed. Numerical studies of the steady-state fractional diffusion equation reveal temperature profiles that have a heat flux and gradient in the same direction ("uphill" transport) over a significant portion of the radial profile (example in Fig. 2). When an off-axis heating source is included (e.g., ECRH), the predicted steady-state temperature profiles exhibit regions of upward concavity near the origin. The fractional 2D model also shows an anomalous scaling of the confinement time τ with the system size L: $\tau_c \propto L^\alpha$ where α is the order of the fractional Laplacian. The propagation of heat pulses is found to exhibit uphill transport.

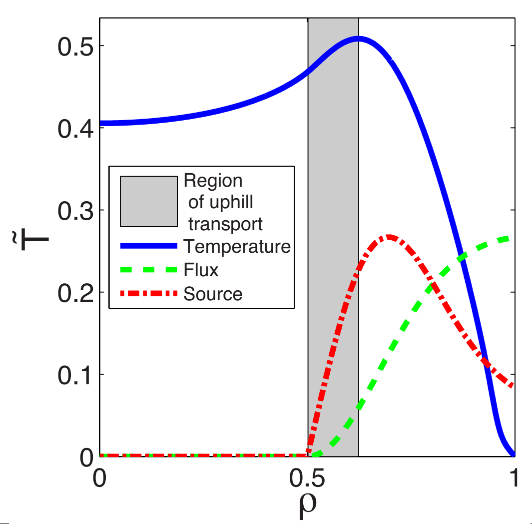

Fig. 2. Uphill fractional transport calculated from new 2D fractional diffusion model [65] for an off-axis heating source. Shown is dependence of scaled temperature (blue curve) on scaled radius. The region of uphill transport makes up about 10% of the plasma radius.

iii)" Generality of exponential spectra, Lorentzian pulses and deterministic chaos in magnetically confined plasmas". The results of comparative investigations indicate that fluctuations measured at the plasma edge, in a broad range of magnetic confinement devices (as is illustrated in Fig. 3), exhibit a power spectrum whose frequency dependence is exponential, i.e.,

$$P(\omega) \sim \exp(-2\omega\tau),$$

where τ is a time constant associated with the underlying processes. Since it has been recognized, by researchers in several disciplines, that a fluctuation power spectrum with an exponential frequency dependence is an intrinsic signature of systems whose dynamics exhibit deterministic chaos, the inference has been drawn that fluctuations at the edge of magnetized plasmas are a consequence of this type of dynamics.

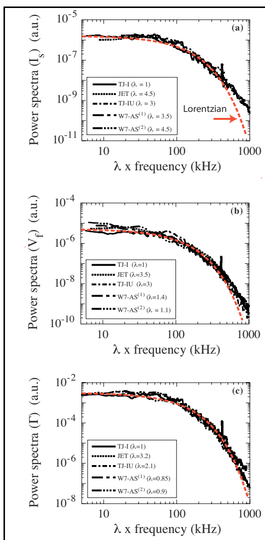

Fig. 3. Power spectra of fluctuations in ion saturation current (a), floating potential (b) and turbulent flux (c) from the survey of Pedrosa, et al. [59] , including tokamak and stellarators (black), is compared to the power spectrum of a single Lorentzian pulse with width, τ = 1 ms (red). The power spectra are clearly exponential for frequencies below 400 kHz, and the turbulent flux spectra is exponential over nearly the entire frequency range displayed. Plot is in log-log format.

The temporal signals associated with these spectra are intermittent or 'spiky', consisting of a series of apparently randomly occurring 'spikes' or pulses. Exponential spectra have been identified in widely different systems including the fluctuation in sunspot number, CO_2 chaotic forcing of ice ages , the unipolar injection hydrodynamic instability, turbulence in neurons related to Parkinson's disease , weakly turbulent Couette-Taylor flows,

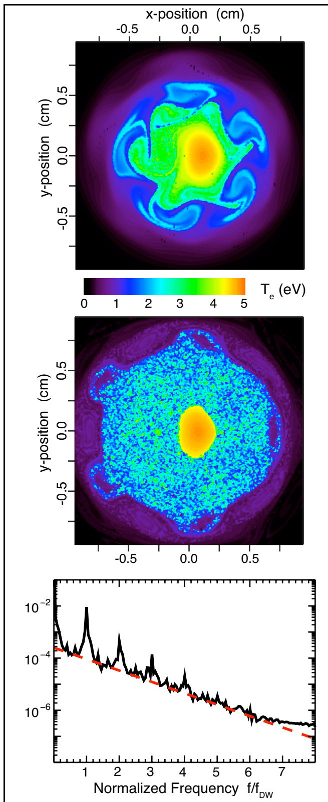

Fig. 4. Results of a deterministic chaos model of the temperature filament experiment [34]. Top: temperature contours in a filament just before the onset of chaotic behavior. Middle: contours after the onset of chaos. Bottom: power spectrum of temperature fluctuations corresponding to the middle panel shows characteristic exponential shape.

and Rayleigh-Bénard convection, among others. To illustrate the connection between deterministic chaos and exponential spectra in a magnetized plasma, Fig. 4 presents the results of a simple, two-mode (azimuthal mode numbers m = 1 and m = 6) model of the relaxation of a magnetized temperature filament of the type investigated in the LAPD device at UCLA. The amplitude of the m = 1 mode is increased adiabatically before ramping up the m = 6 mode amplitude. The interaction of the two modes leads to chaotic Lagrangian orbits once an amplitude threshold is exceeded. The top panel shows the complex, but spatially connected structures formed when the m = 1 mode is at full amplitude and the m = 6 mode amplitude is just below the threshold for chaotic behavior. The region of elevated temperature near the center (orange color) corresponds to orbits in the 'island of stability' associated with the m = 1 mode. The middle panel shows the fine-scale spatial structures that develop after the onset of chaos. The bottom panel is the frequency spectrum of the temperature fluctuations at a time corresponding to the middle panel, showing a clear exponential dependence in a log-linear display, as highlighted by the red dashes. The protruding peaks correspond to the fundamental and first few harmonics of the coherent modes driving the chaos.

iv)"Origin of Lorentzian pulses in deterministic chaos". This theoretical study provides a solution to a long-standing question within the nonlinear dynamics and fluid turbulence communities, namely what is the origin of the ubiquitous exponential power spectrum of deterministic chaos? The first part to the answer was obtained by identifying that detailed measurements in a basic linear plasma machine (LAPD at UCLA), and in a toroidal stellarator confinement device (TJ-K at Stuttgart), established a link between an exponential frequency dependence of the fluctuation power spectrum, and Lorentzian temporal pulses having average pulse width τ. The connection established by the plasma experiments ruled out that the exponential

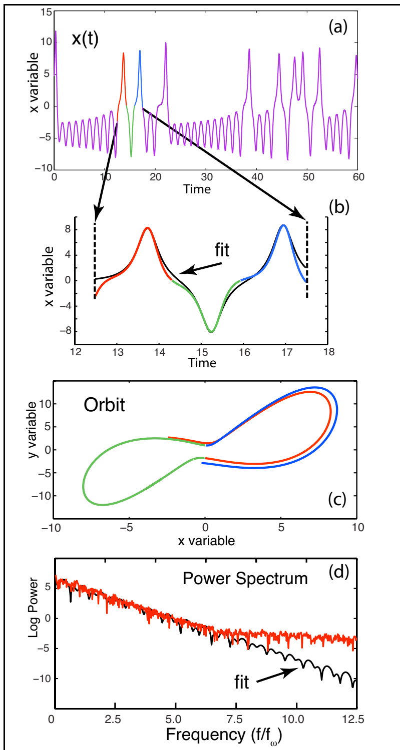

Fig 5. (a) Time series for Lorenz model variable x shows intermittent pulses. (b) Color-coded time series is fit by a sum of three Lorentzian functions (black curve). (c) Phase-space trajectory of variables (y, x) is near the limit cycle of the attractor. (d) Frequency power spectrum of time series (red) compared to spectrum (black) of the sum of three Lorentzians.

feature is a statistical property (e.g., a canonical distribution), but, rather, that it is the imprint of individual intermittent events with a unique shape. It is natural to question why pulses emerging from a chaotic system should have a Lorentzian shape. In fact, some researchers expect that such pulses should more closely follow a Gaussian form or other distorted shapes determined by random events. The study answers this question by illustrating explicitly the origin of Lorentzian pulses as chaotic dynamics near the separatrix boundaries of elliptic regions in flow fields, or, more generally, near the limit cycles of attractors in nonlinear dynamics models. Two explicit examples are considered, a bifurcation given by a potential field appropriate for drift waves in a plasma, and a case from the classic example of deterministic chaos, the Lorenz model (shown in Fig. 5).

v)" Permutation entropy analysis of temperature fluctuations". The concept of amplitude permutation probability introduced by Bandt and Pompe is used to compute the entropy and statistical complexity for time signals obtained from experimental data. The associated CH plane, which displays Jensen-Shannon complexity vs. normalized Shannon entropy as introduced by Rosso et al., is used to determine the statistical nature of the dynamics associated with temperature fluctuations measured in the temperature filament experiment at LAPD. The advantage of this modern signal analysis technique is that is permits the identification of the underlying dynamics that generates the time series without any *a priori* assumptions about the behavior of the signal. An example of a CH display is given in Fig. 6. For reference, note that, in a CH display, the lower right corner corresponds to white noise, while the lower left corner represents constant signals. The central top region is where pure deterministic chaos appears, e.g., signals generated by the logistic map. The path delineated by the green line in Fig. 6 represents the stochastic process of fractional Brownian motion (fBm) for a range of Hurst exponents between .025 and .975. The quantity “d” is the embedding dimension used in the analysis. The results shown in Fig. 6 identify that the underlying dynamics responsible for nonlocal transport in the temperature filament is deterministic chaos. This conclusion is arrived at independently of any statement about the shape of the frequency spectrum.

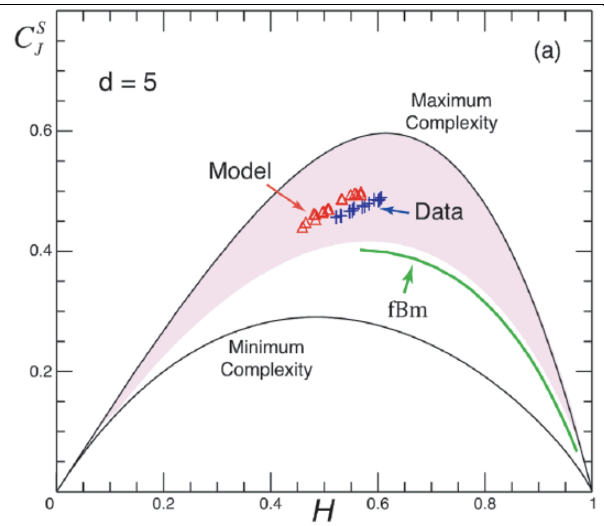

Fig. 6. Locations in the CH plane of temperature filament data [34] (blue crosses), and numerical output of chaotic advection model shown in Fig. 4 (red triangles). The shaded region delineates the region of chaotic behavior. The stochastic process of fractional Brownian motion (fBm) is also shown for comparison.

Products developed:

i) Publications

1. A. Kullberg and D. del-Castillo-Negrete, “Transport in the spatially tempered, fractional Fokker- Planck equation”, J. Phys. A : Math. Theor. 45, 255101 (2012).
2. A. Kullberg, D. del-Castillo-Negrete, G. J. Morales, and J. E. Maggs, “Isotropic model of fractional transport in two-dimensional bounded domains”, Phys. Rev. E 87, 052115 (2013).
3. A. Kullberg, G. J. Morales, and J. E. Maggs, “Comparison of a radial fractional transport model with tokamak experiments”, Phys. Plasmas 21, 032310 (2014).
4. G. Hornung, B. Nold, J. E. Maggs, G. J. Morales, M. Ramisch, and U. Stroth, “Observation of exponential spectra and Lorentzian pulses in the TJ-K stellarator“, Phys. Plasmas 18, 082303 (2011).
5. J. E. Maggs and G. J. Morales, “Generality of deterministic chaos, exponential spectra and Lorentzian pulses in magnetically confined plasmas, Phys. Rev. Lett. 107, 185003 (2011).
6. J. E. Maggs and G. J. Morales, “Origin of Lorentzian pulses in deterministic Chaos“, Phys. Rev. E 86, 015401 (R) (2012).
7. J. E. Maggs and G. J. Morales, “Exponential power spectra, deterministic chaos, and Lorentzian pulses in plasma edge dynamics“, Plasma Phys. Control. Fusion 54, 124041 (2012).
8. J. E. Maggs and G. J. Morales, “Permutation entropy analysis of temperature fluctuations from a basic electron heat transport experiment“, Plasma Phys. Control. Fusion 55, 085015 (2013).

ii) Conference presentations

1. Kullberg, A., del-Castillo Negrete, D., Morales, G.J., and Maggs, J.E., “”Non-local Transport Across Shear Flows”, 52nd Meeting of the Division of Plasma Physics of the American Physical Society, Chicago, Ill., November 8-12, 2010, APS Bull. 55, 321 (2010).

2. Kullberg, A., del-Castillo-Negrete, D.. Morales, G.J., and Maggs, J.E., “Two-dimensional non-local transport across zonal shear flows”, 53rd Meeting of the Division of Plasma Physics of the American Physical Society, Salt Lake City, Utah, Nov. 14-18, 2011, CP9.69.
3. Kullberg, A., del-Castillo Negrete, D., Morales, G. J., and Maggs, J. E., “Non-local models of nondiffusive transport in magnetically confined plasmas”, 54th Meeting of the Division of Plasma Physics of the American Physical Society, Providence, Rhode Island, Oct. 29-Nov. 2, 2012, BP8.00145
4. Kullberg, A., Morales, G. J., Maggs, J. E., and del-Castillo Negrete, D., “Comparison of a 2D fractional transport model with tokamak experiments”, 55th Meeting of the Division of Plasma Physics of the American Physical Society, Denver, Co., Nov. 11-15, 2013, GP8.10.
5. Maggs, J. E., and Morales, G. J., “Identification of chaotic and stochastic processes by permutation entropy analysis”, 55th Meeting of the Division of Plasma Physics of the American Physical Society, Denver, Co., Nov. 11-15, 2013. GO5.1.