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*Title:* Plasticity Microstructure Characteristics using 3D Voronoi Structures

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## Plasticity Microstructure Characteristics using 3D Voronoi Structures

BL Hansen, CA Bronkhorst

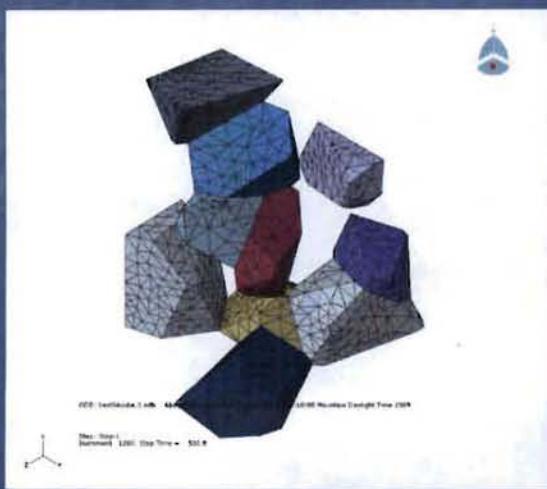
Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM

First a comparison is made between direct single crystal simulations run on 3D Voronoi single crystal structures analyzing the need to run simulations with 3D microstructures. The simulations are preformed to large deformations at high strain rates using traditional plasticity models with the finite element method. Voronoi tessellations are used due to the ability to apply finite element meshes to such structures. The concerns of meshing these structures and successful techniques will be discussed. Laminate models of dislocation structures are considered to include size effects of the crystals for future simulations.

# Plasticity Microstructure Characteristics using 3D Voronoi Structures

Benjamin L. Hansen, Curt A. Bronkhorst

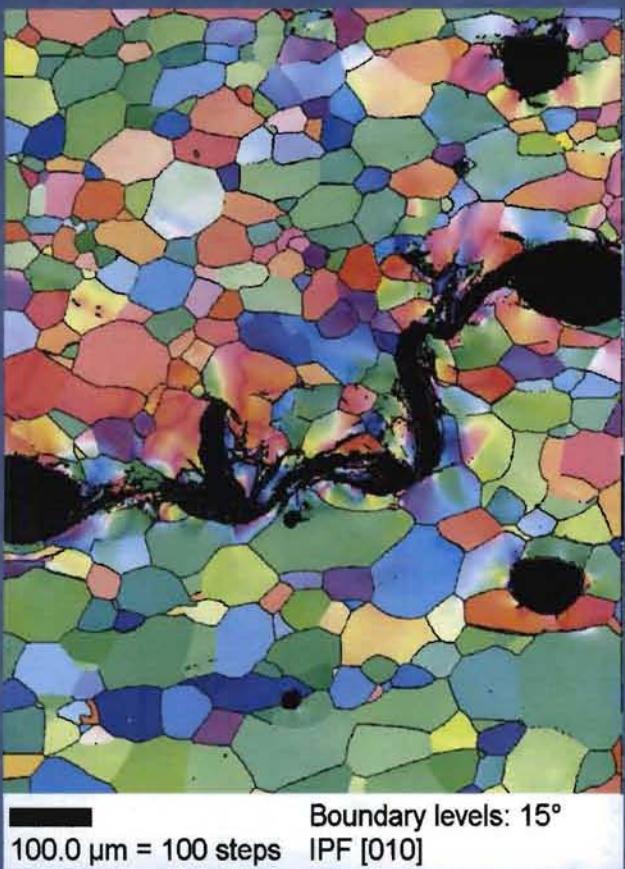
Theoretical Division  
Los Alamos National Laboratory



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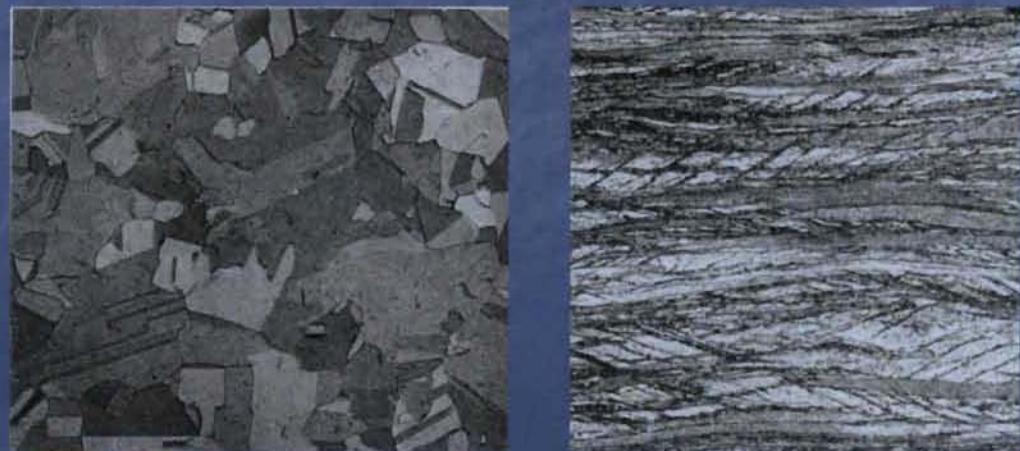
# Motivation



Tantalum Plate Impact Compression  
OIM: B. Henrie, MST-8, LANL

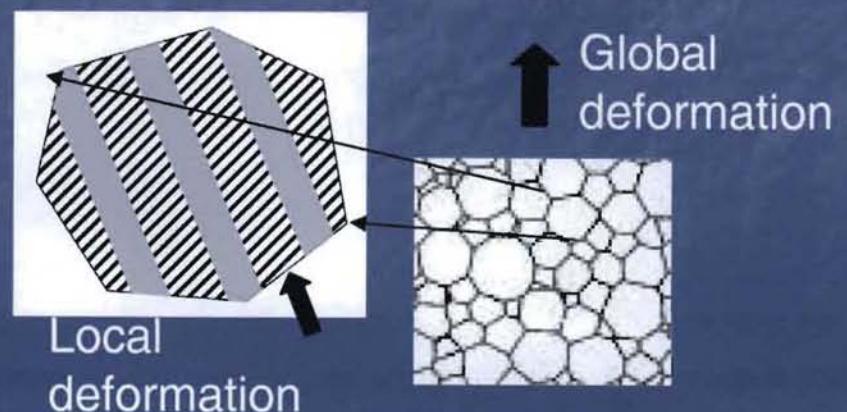
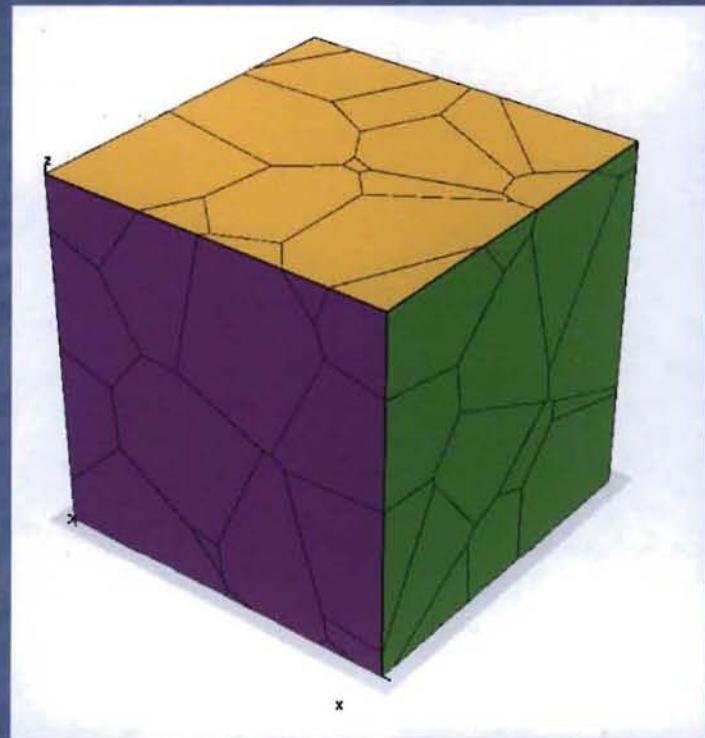
- Microstructure properties effect continuum material response.
- Material damage (e.g. intergranular corrosion cracking, void nucleation) and high deformation mechanisms (eg. shear banding) are local.
- Localizations occur because of inhomogeneities in the material.
- The innate inhomogeneities of metallic materials are caused by the underlying polycrystalline microstructure.
- Goal is to simulate statistically equivalent polycrystalline microstructures. Validate to experimental measurements.

Cu Plane Strain Compression  
Undeformed  $\varepsilon = -1.5$



# Voronoi

- Voronoi cells used as approximation of grain structure.
- Voronoi cells are based off an idealized grain growth model - provide a good approximation.
- Convex cells.
- Fewer complications in meshing.
- Currently using PyXL software from Cornell Fracture Group
- Length scale appropriate simulation involves
  - Individual distinguished grains
  - Material model accounts for deformation (including hardening and texture characteristics of grain).



# Single crystal model

Asaro & Rice (1977), Acharya & Beaudoin (2000),  
 Kothari & Anand (1998), Busso et al. (2000), Kocks (1976)  
 Kalidindi et al. (1992), Bronkhorst et al. (1992), Anand (1998)

## Stress

$$\mathbf{T}^* = \underline{\mathbf{L}} \left[ \mathbf{E}^* - \mathbf{A}(\theta - \theta_0) \right]$$

$$\mathbf{E}^* \equiv \frac{1}{2} \left( \mathbf{F}^{*T} \mathbf{F}^* - \mathbf{1} \right)$$

$$\mathbf{T}^* \equiv \mathbf{F}^{*-1} (\det \mathbf{F}^*) \mathbf{T} \mathbf{F}^{*-T}$$

$$\mathbf{F}^* = \mathbf{F} \mathbf{F}^{p^{-1}}$$

## Hardening

$$\dot{s}^\alpha = \sum_\beta h^{\alpha\beta} |\dot{\gamma}^\beta| \quad h^{\alpha\beta} = [r + (1-r) \delta^{\alpha\beta}] h^\beta$$

$$h^\beta = h_o \left( \frac{s_s^\beta - s_0^\beta}{s_s^\beta - s_0^\beta} \right)$$

$$s_s^\beta = \hat{s}_s^\beta(\dot{\gamma}, \theta) = s_{s_0} \left( \frac{\dot{\gamma}^\beta}{\dot{\gamma}_0} \right)^{\frac{k\theta}{A}}$$

## Flow Rule

$$\mathbf{L}^p = \dot{\mathbf{F}}^p \mathbf{F}^{p^{-1}} = \sum_\alpha \dot{\gamma}^\alpha \mathbf{S}_0^\alpha \quad \mathbf{S}_0^\alpha \equiv \mathbf{m}_0^\alpha \otimes \mathbf{n}_0^\alpha$$

$$\dot{\gamma}^\alpha = \dot{\gamma}_0 \exp \left[ -\frac{F_0}{k\theta} \left\langle 1 - \left\langle \frac{\left| \tau^\alpha \right| - s^\alpha \frac{\mu}{\mu_0}}{s_l^\alpha \frac{\mu}{\mu_0}} \right\rangle^p \right\rangle^q \right] \operatorname{sgn}(\tau^\alpha)$$

$$\tau^\alpha \equiv (\mathbf{C}^* \mathbf{T}^*) \cdot \mathbf{S}_0^\alpha$$

$$\mathbf{C}^* = \mathbf{F}^{*T} \mathbf{F}^*$$

$$\frac{\mu}{\mu_0} \equiv \frac{C_{12}}{C_{12_0}} = 1 + \frac{m_{12}}{C_{12_0}} \theta$$

Simmons & Wang, 1971

## Texture

$$\mathbf{m}^\alpha = \mathbf{F}^* \mathbf{m}_0^\alpha$$

$$\mathbf{n}^\alpha = \mathbf{F}^{*-T} \mathbf{n}_0^\alpha$$

## Adiabatic Heating

$$\dot{\theta} = \frac{\eta}{\rho c_p} \sum_\alpha \tau^\alpha \dot{\gamma}^\alpha$$

# Material Parameter Evaluation

## Parameters:

$$\rho = 16640 \text{ kg/m}^3$$

$$c_p = 150 \text{ J/kg-K}$$

$$\alpha = 6.5 \text{ } \mu\text{m/m-K}$$

$$\eta = 0.0, 0.95$$

$$m_{11} = -24.5 \text{ MPa/K}$$

$$C_{11_0} = 268.5 \text{ GPa}$$

$$m_{12} = -11.8 \text{ MPa/K}$$

$$C_{12_0} = 159.9 \text{ GPa}$$

$$m_{44} = -14.9 \text{ MPa/K}$$

$$C_{44_0} = 87.1 \text{ GPa}$$

$$r = 1.4$$

$$\dot{\gamma}_0 = 10^7 \text{ sec}^{-1}$$

$$s_0 = 50 \text{ MPa}$$

$$s_l = 550 \text{ MPa}$$

$$F_0 = 2.1 \times 10^{-19} \text{ J}$$

$$p = 0.34$$

$$q = 1.66$$

$$s_{s_0} = 125 \text{ MPa}$$

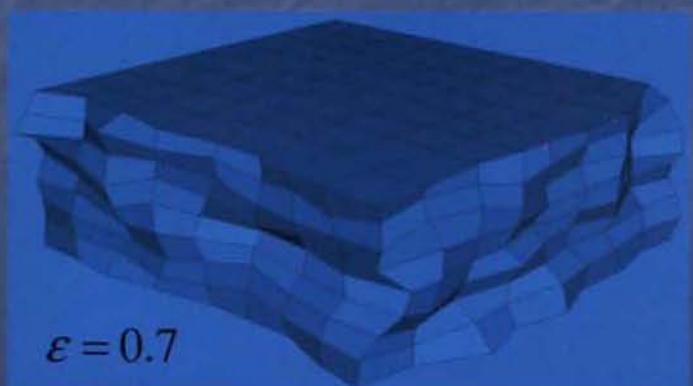
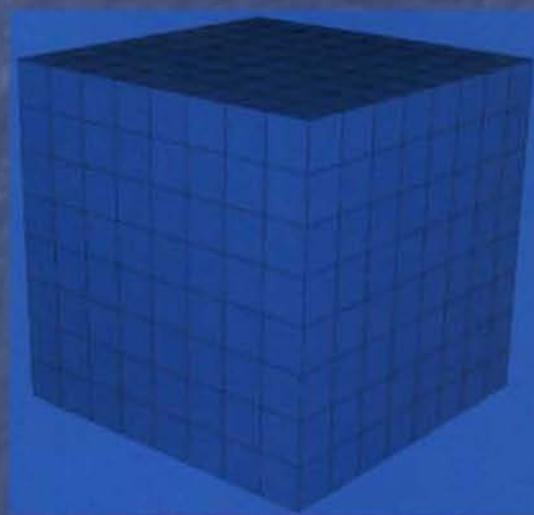
$$h_{s_0} = 300 \text{ MPa}$$

$$A = 10^{-18} \text{ J}$$

Ta - 24 BCC systems  
 $\{110\}\langle111\rangle, \{112\}\langle111\rangle$

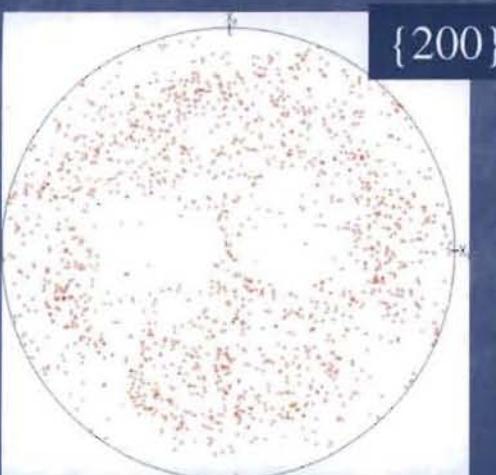
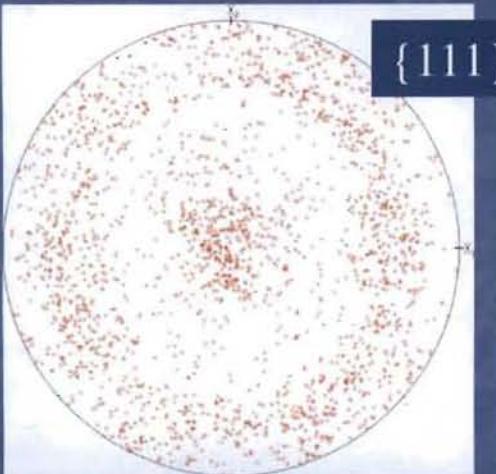
Cu - 12 FCC systems  
 $\{111\}\langle110\rangle$

Material Parameter Evaluation  
512 Elements/Grains

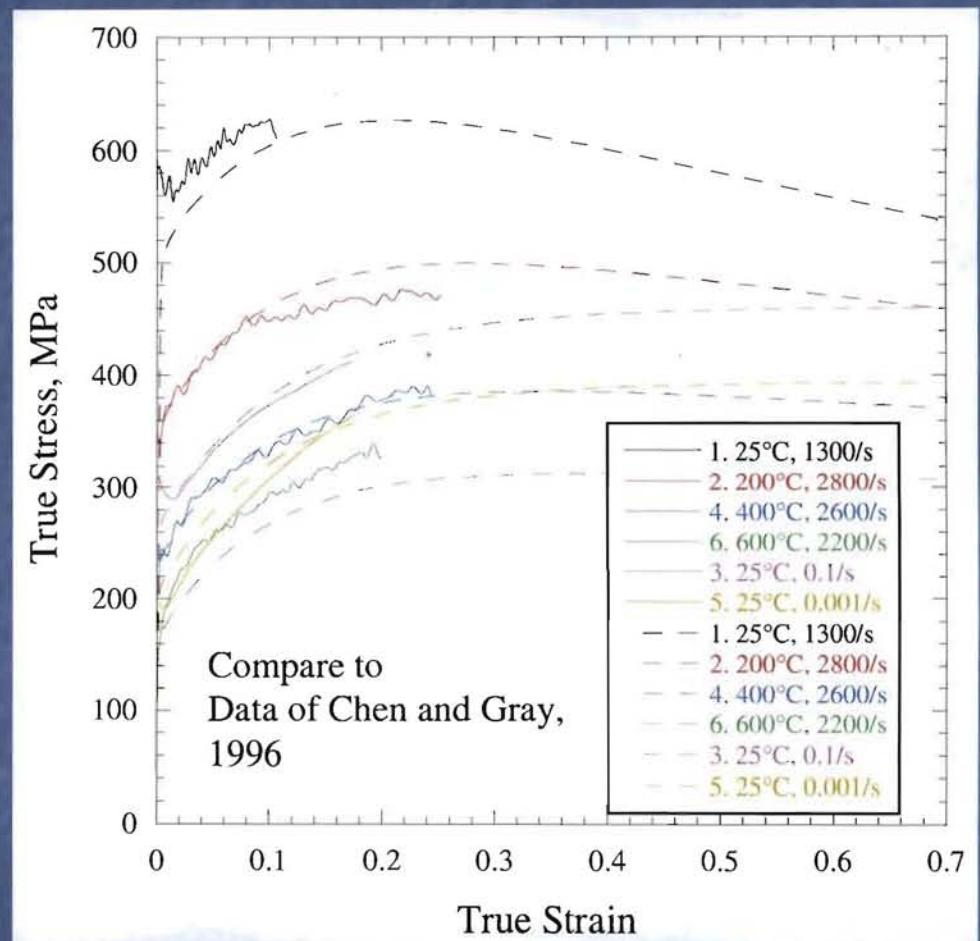


# Single crystal model - Ta

Initial Texture



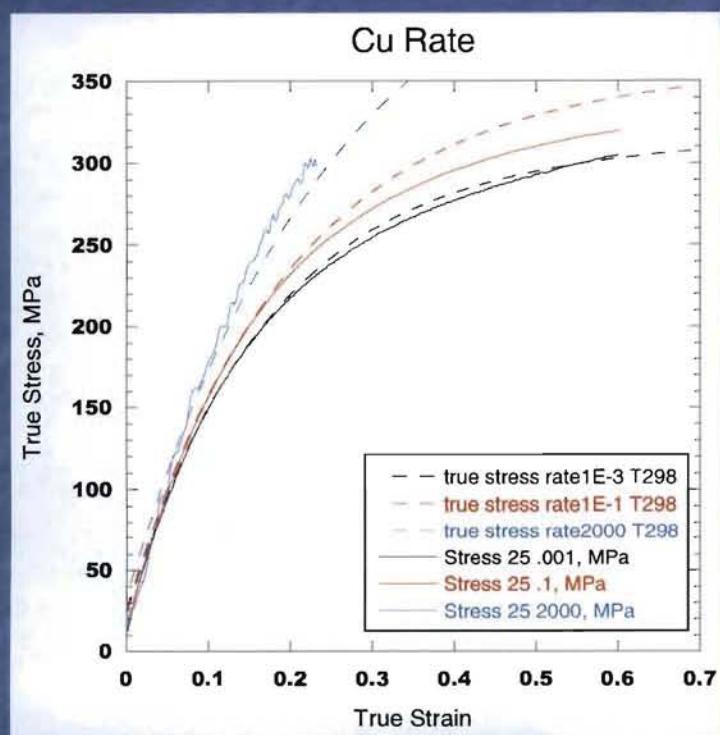
$$\begin{aligned}
 \rho &= 16640 \text{ kg/m}^3 \\
 c_p &= 150 \text{ J/kg-K} \\
 \alpha &= 6.5 \mu\text{m/m-K} \\
 \eta &= 0.0, 0.95 \\
 m_{11} &= -24.5 \text{ MPa/K} \\
 C_{11_0} &= 268.5 \text{ GPa} \\
 m_{12} &= -11.8 \text{ MPa/K} \\
 C_{12_0} &= 159.9 \text{ GPa} \\
 m_{44} &= -14.9 \text{ MPa/K} \\
 C_{44_0} &= 87.1 \text{ GPa} \\
 r &= 1.4 \\
 \dot{\gamma}_0 &= 10^7 \text{ sec}^{-1} \\
 s_0 &= 50 \text{ MPa} \\
 s_t &= 550 \text{ MPa} \\
 F_0 &= 2.1 \times 10^{-19} \text{ J} \\
 p &= 0.34 \\
 q &= 1.66 \\
 s_{s_0} &= 125 \text{ MPa} \\
 h_{s_0} &= 300 \text{ MPa} \\
 A &= 10^{-18} \text{ J}
 \end{aligned}$$



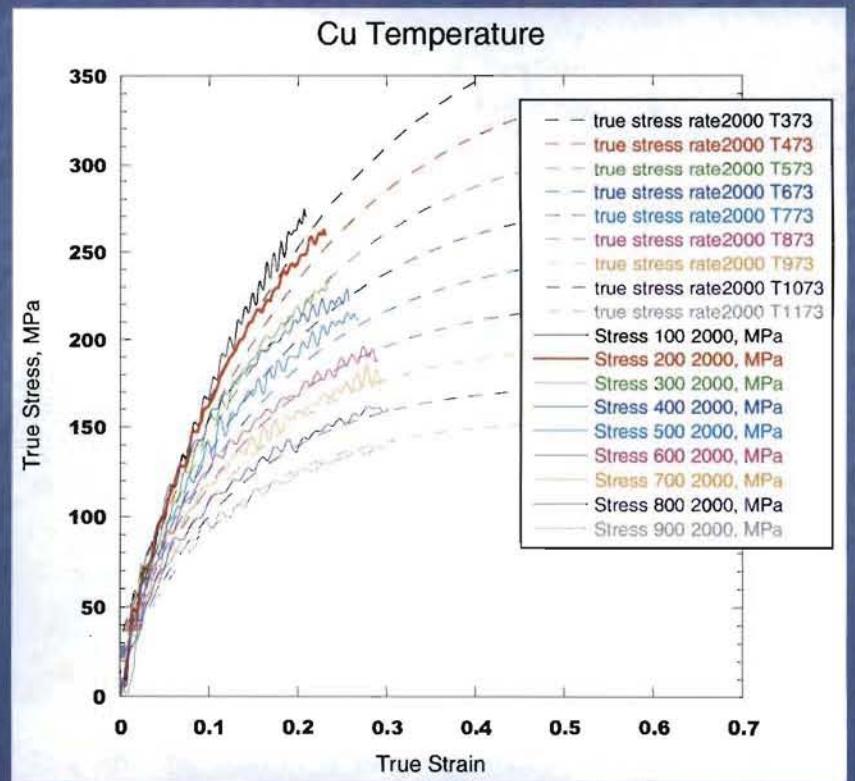
# Single crystal model - Cu

$\rho = 8960 \text{ kg/m}^3$   
 $c_p = 380 \text{ J/kg-K}$   
 $\alpha = 17.7 \mu\text{m/m-K}$   
 $\eta = 0.0, 0.95$   
 $m_{11} = -36.3 \text{ MPa/K}$   
 $C_{11_0} = 179.5 \text{ GPa}$   
 $m_{12} = -16.4 \text{ MPa/K}$   
 $C_{12_0} = 126.4 \text{ GPa}$   
 $m_{44} = -25.7 \text{ MPa/K}$   
 $C_{44_0} = 82.5 \text{ GPa}$   
 $r = 1.4$

$\dot{\gamma}_0 = 10^7 \text{ sec}^{-1}$   
 $s_0 = 1 \text{ MPa}$   
 $s_i = 20 \text{ MPa}$   
 $F_0 = 1.0 \times 10^{-18} \text{ J}$   
 $p = 0.33$   
 $q = 1.66$   
 $s_{\gamma_0} = 205 \text{ MPa}$   
 $h_{\gamma_0} = 200 \text{ MPa}$   
 $A = 1.5 \times 10^{-19} \text{ J}$

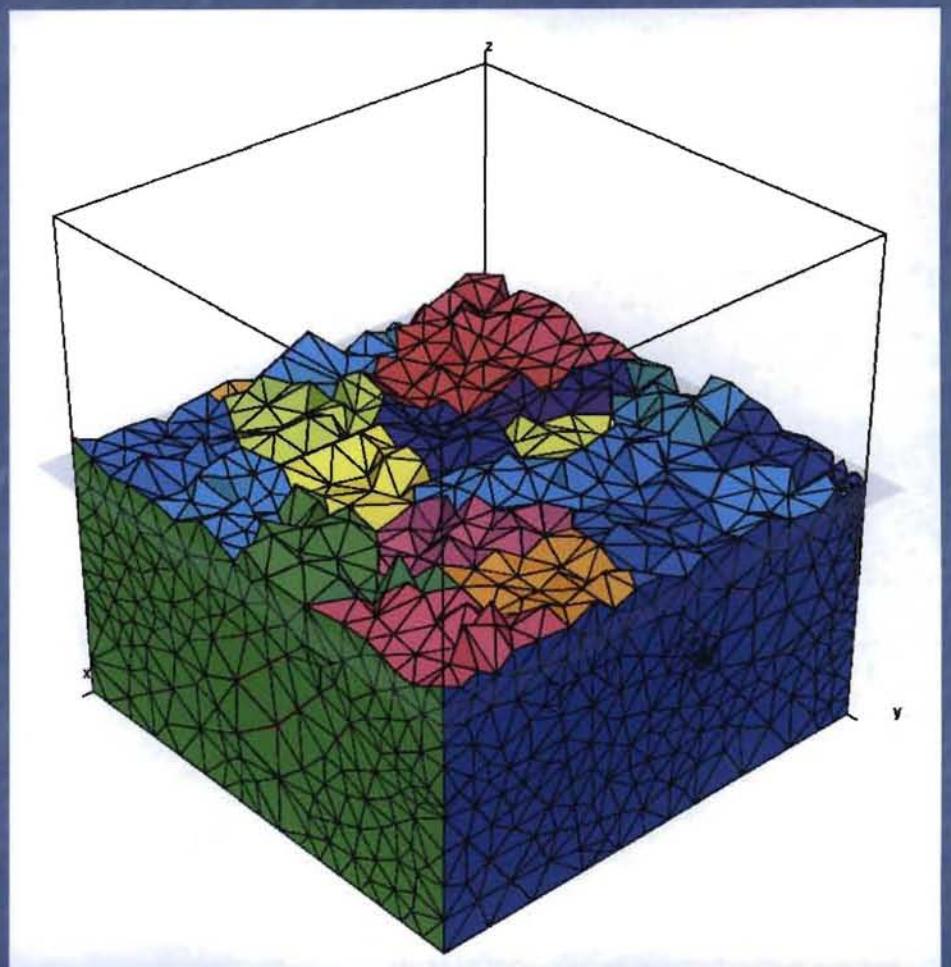


Initial Texture: Random

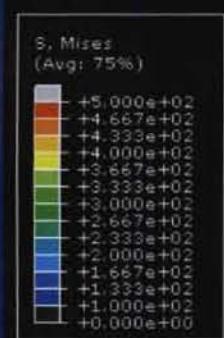


# Tetgen Meshes

- Looking for Delaunay mesh generation for convex shapes.
- Generates tetrahedral meshes in 3D with internal faces respected.
- Not all slivers always removed.
- Quality of meshes is sufficient to run simulations.
- Meshing is a difficult prospect due to the constraint placed on meshing by the intersection of grain faces.



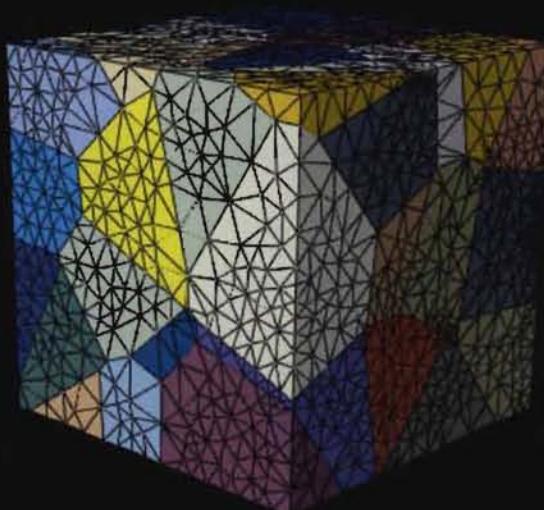
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Step: Step-1 Frame: 0

Viewport: 2 ODB: C:/Documents and Settings...ube mesh/best64cube.3.odb

Step: Step-1 Frame: 0



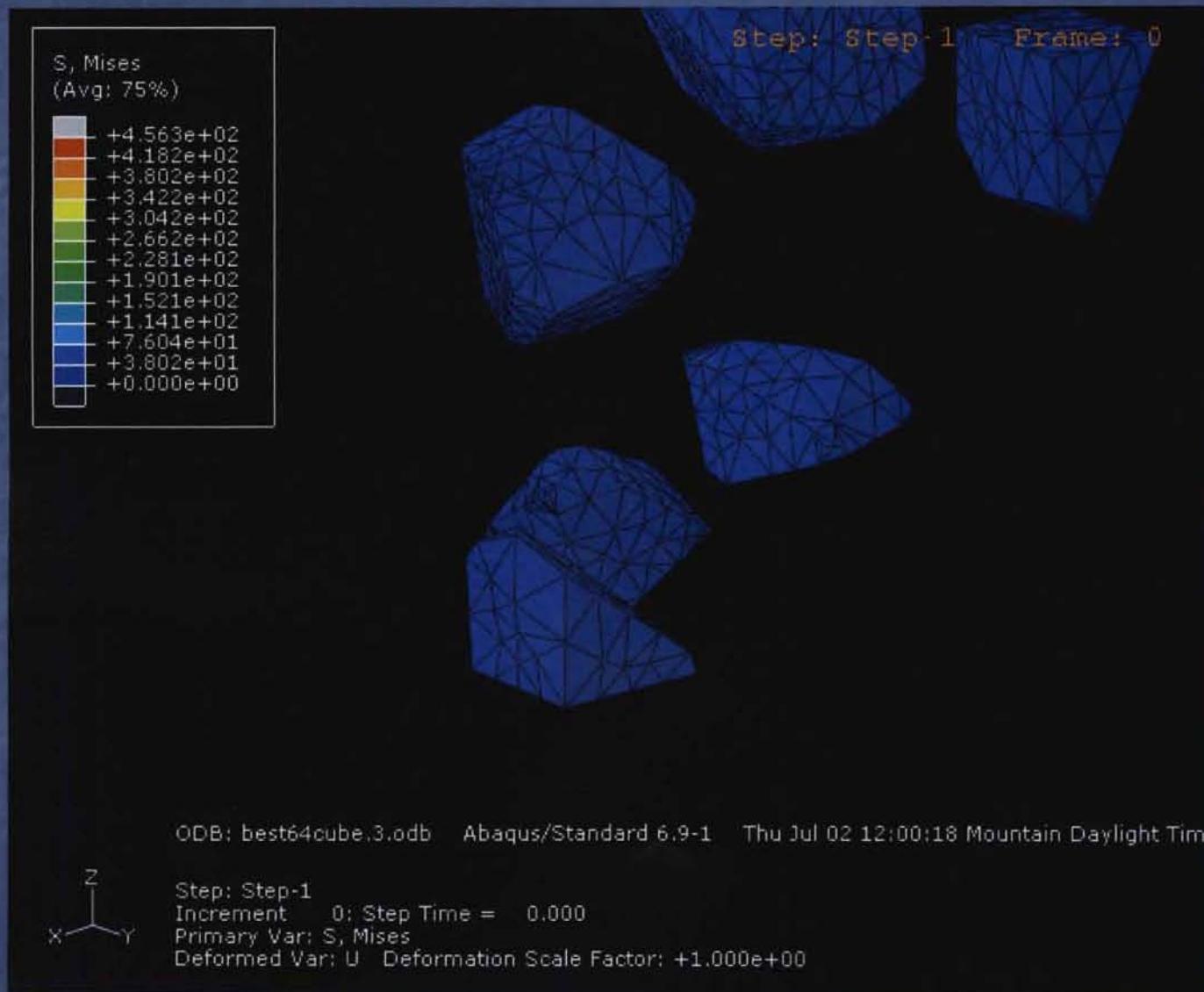
ODB: best64cube.3.odb Abaqus/Standard 6.9-1 Thu Jul 02 12:0

Step: Step-1  
Increment: 0; Step Time = 0.000  
Primary Var: S, Mises  
Deformed Var: U Deformation Scale Factor: +1.000e+00

ODB: best64cube.3.odb Abaqus/Standard 6.9-1 Thu Jul 02 12:0

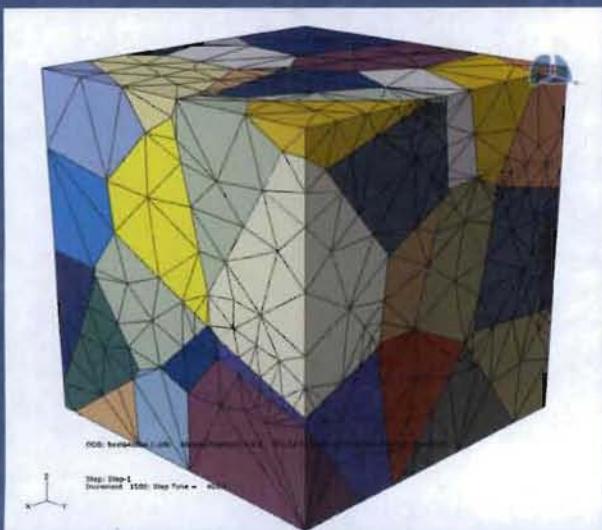
Step: Step-1  
Increment: 0; Step Time = 0.000  
Deformed Var: U Deformation Scale Factor: +1.000e+00

# Individual Grains

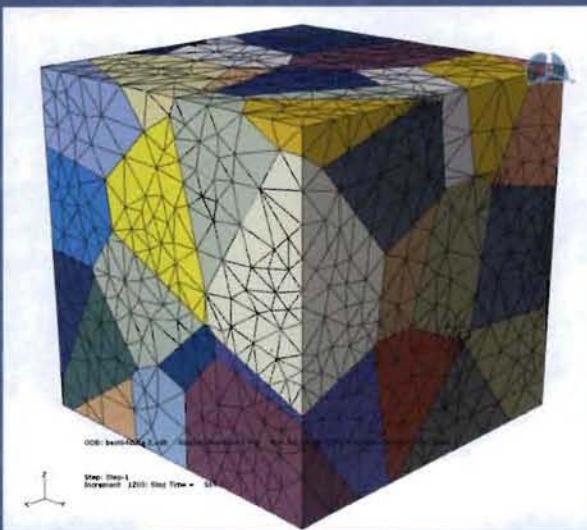


# Mesh Sensitivity

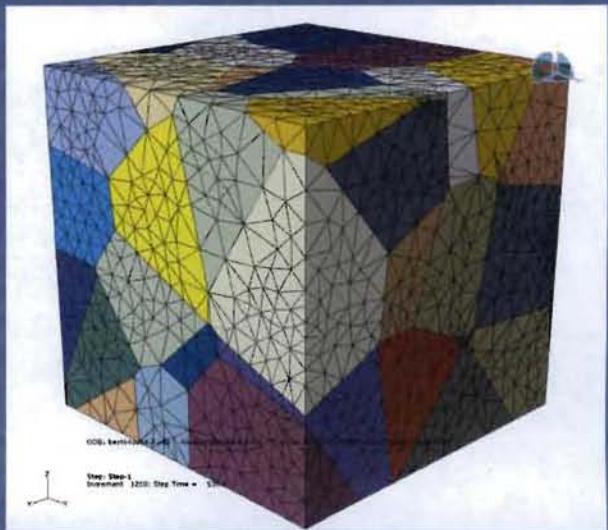
64 grains



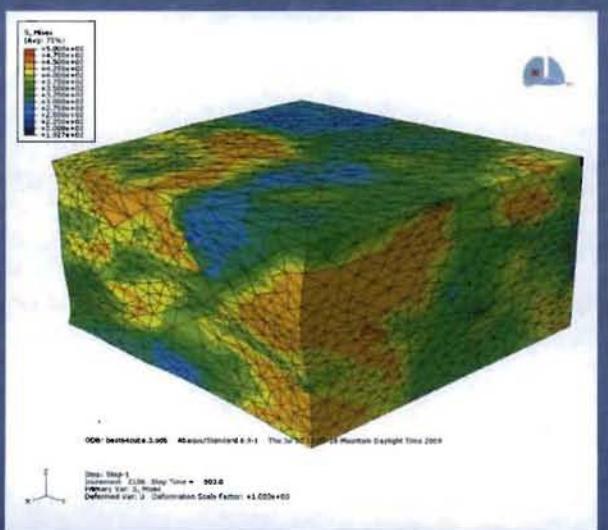
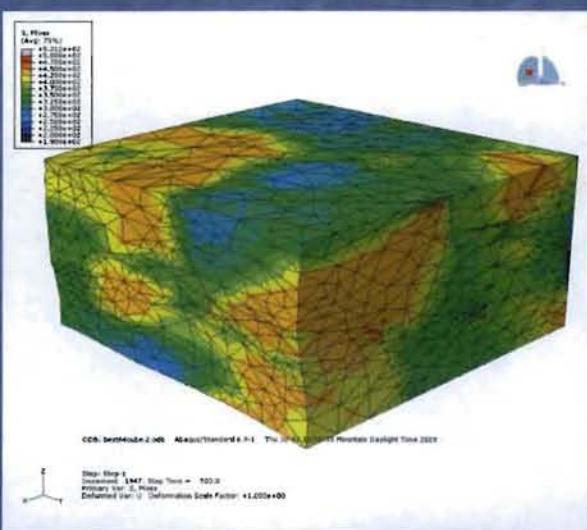
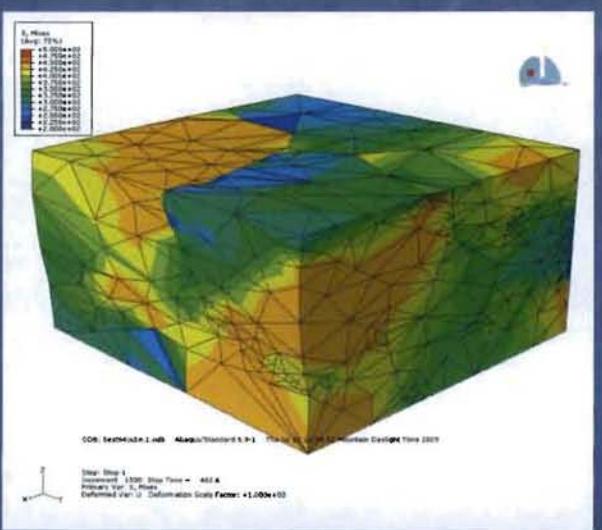
11175 Tet. Elements



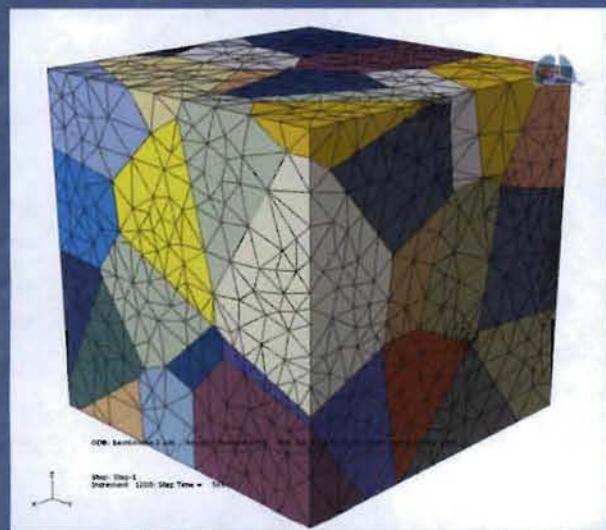
23544 Tet. Elements



43141 Tet. Elements



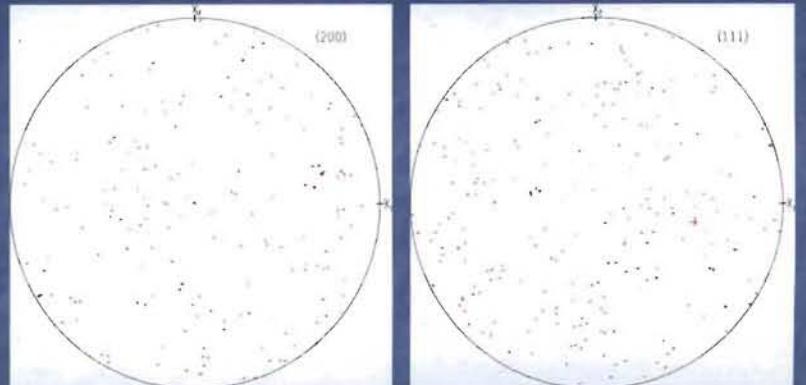
# Crystallographic Texture



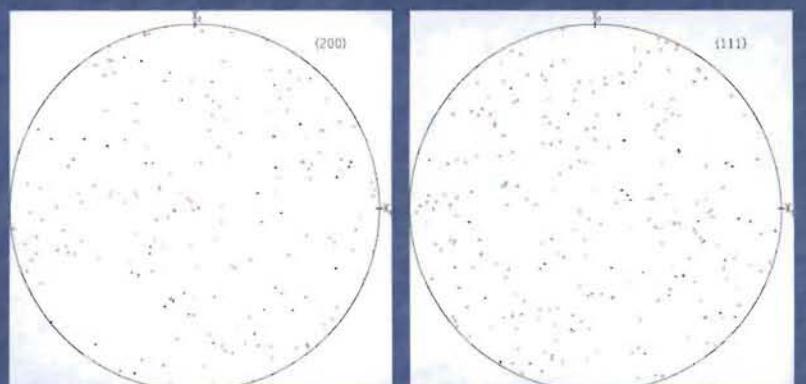
23544 Tet. Elements  
64 grains

3 sets of 64 random  
crystal orientations

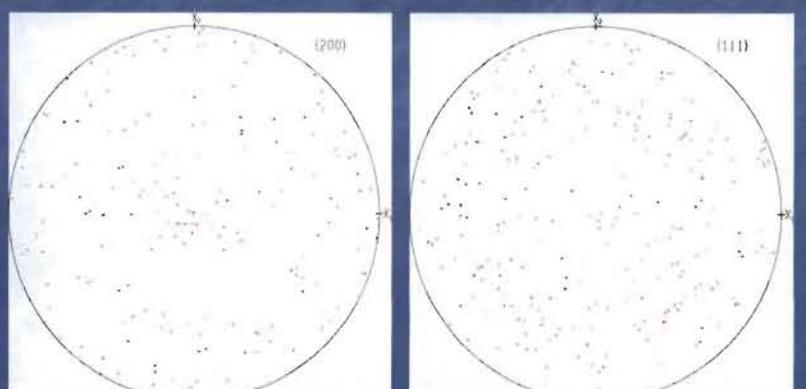
Texture 1:



Texture 2:

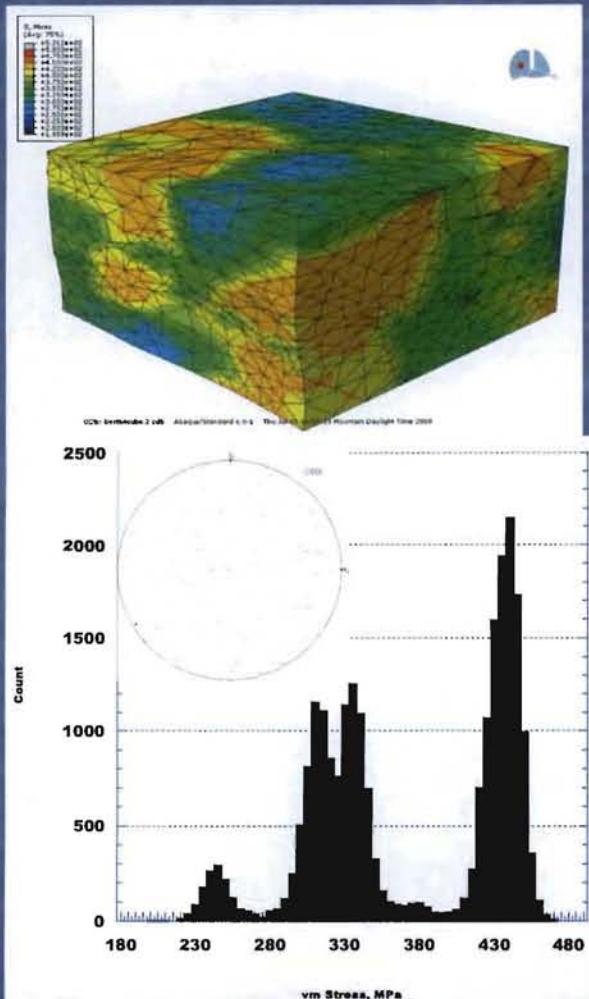


Texture 3:

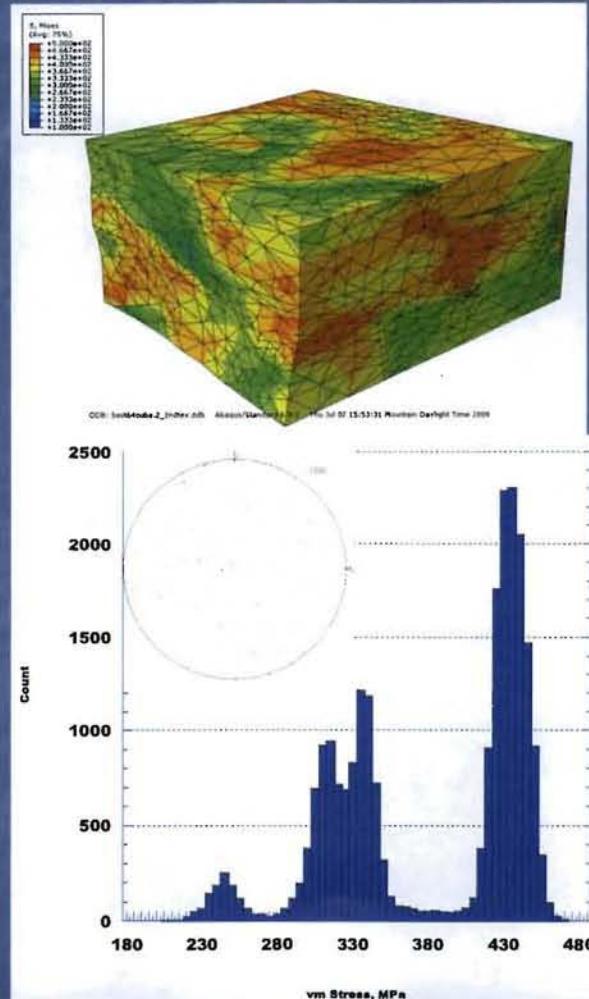


# Crystallographic Response

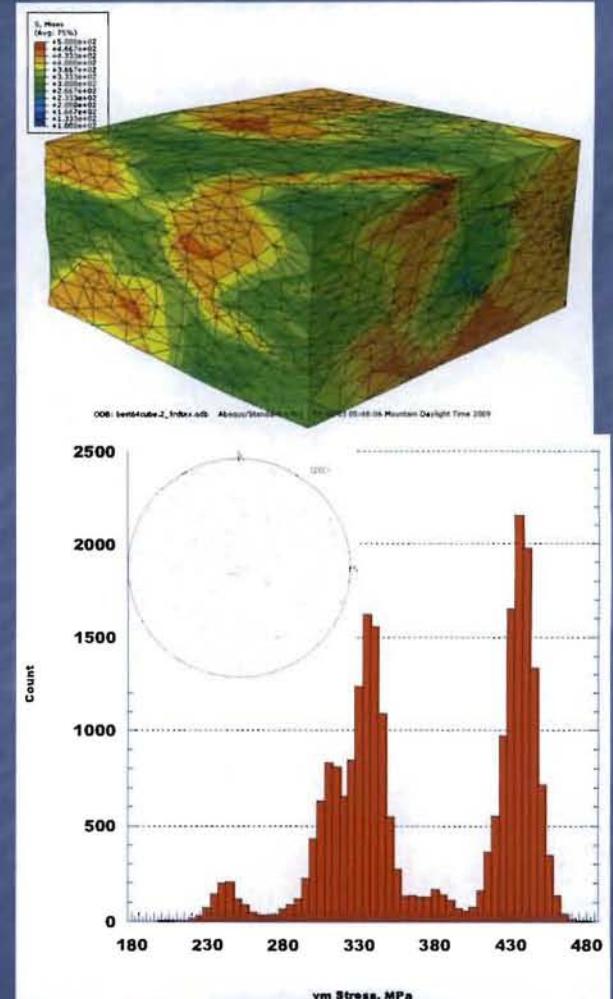
Texture 1



Texture 2

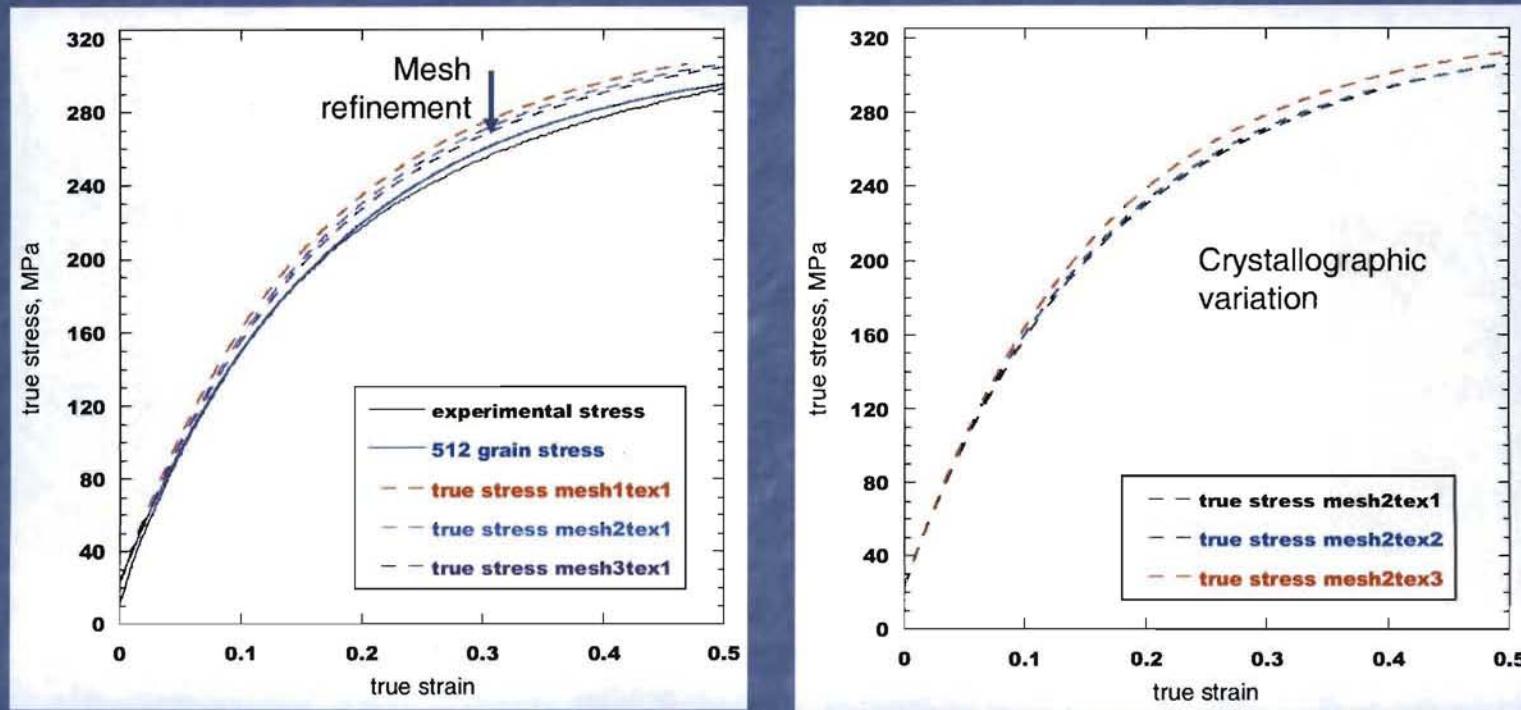


Texture 3



Von Mises Stress of elements.

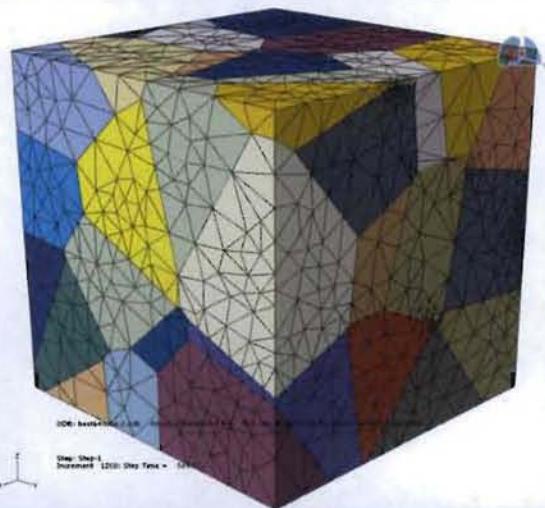
# Macroscopic Response



Macroscopic stress response decreases with a finer mesh size.

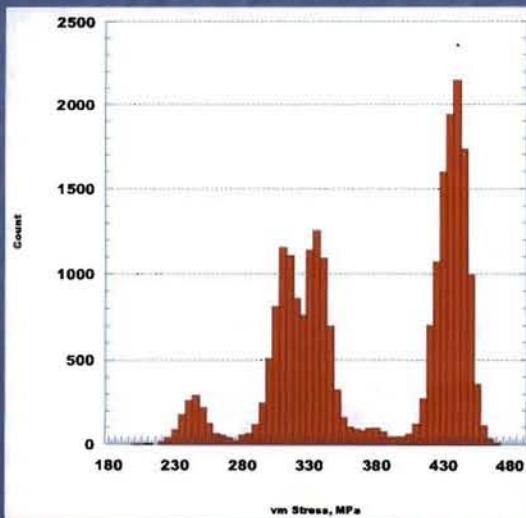
Crystallographic texture has a minor effect on the macroscopic stress response.

# Morphologic Response

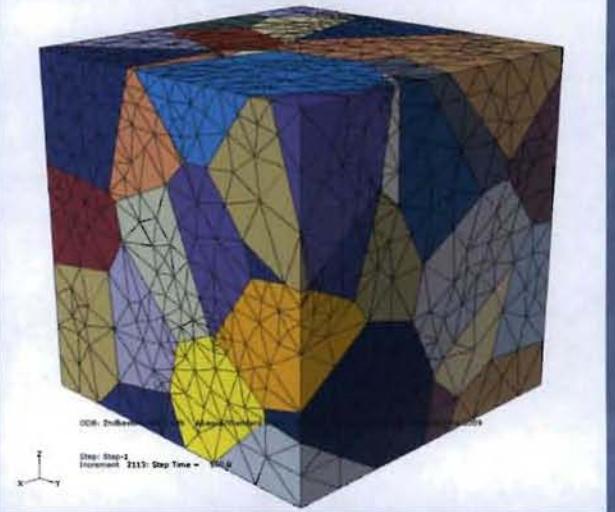


Voronoi 1

23544 Tet. Elements

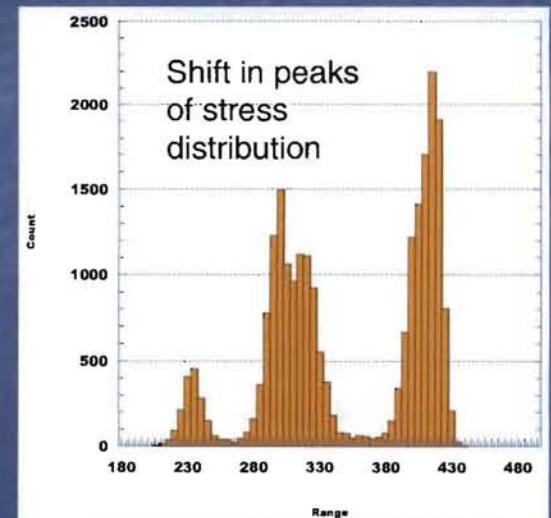
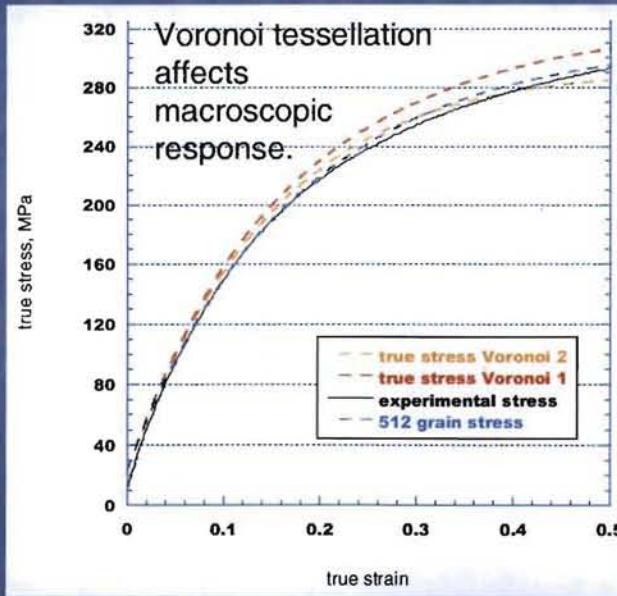


Identical 64  
crystal  
orientations



Voronoi 2

23351 Tet. Elements



# Subgrain Dislocation Structures

- Plastic deformation in metals typically involves motion of dislocations within the atomic lattice.
- Dislocation walls form by dissimilar dislocations interacting.
  - Seen at higher strain states through TEM analysis.
  - Material separates into distinct areas.
  - Act as boundaries to further dislocation motion (plastic deformation).
  - These are “geometrically necessary” boundaries, as opposed to statistical or incidental dislocation boundaries.

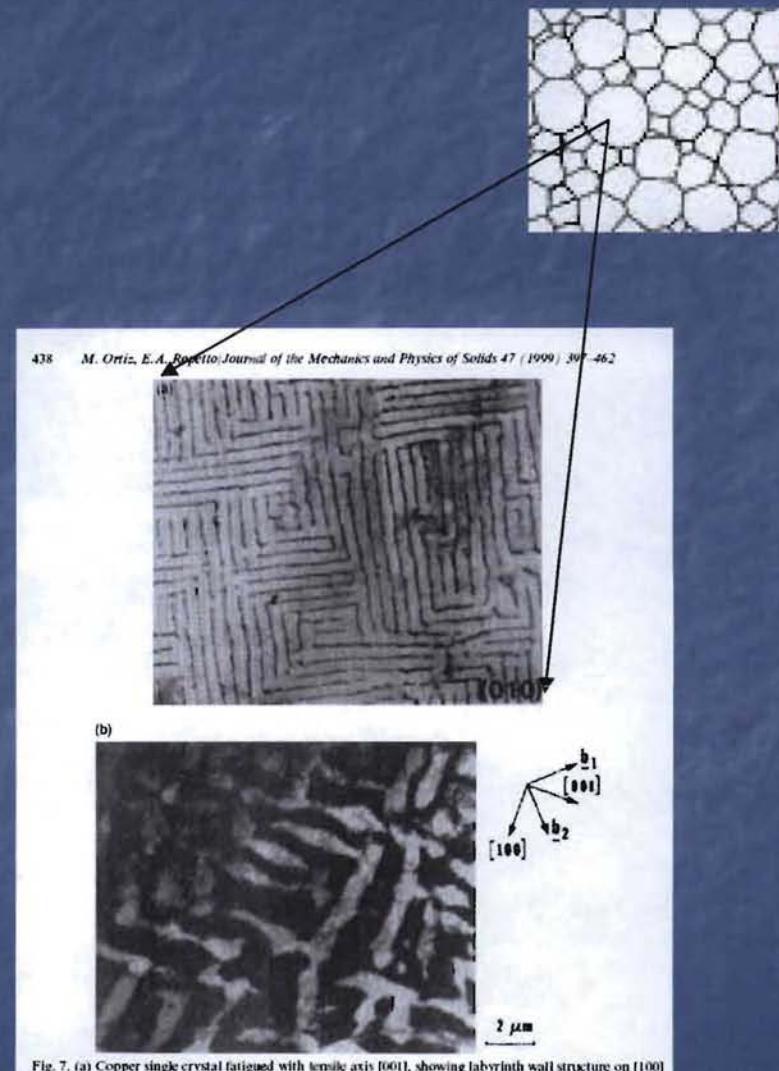
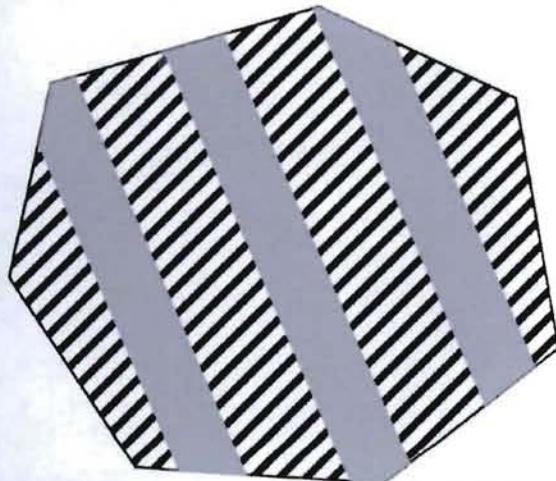


Fig. 7. (a) Copper single crystal fatigued with tensile axis [001], showing labyrinth wall structure on [100] and [001]-planes Jin and Winter (1984a). (b) (010)-cross section of a copper single crystal specimen showing labyrinth dislocation structure Ackermann et al. (1984). Reprinted by permission of Publisher.

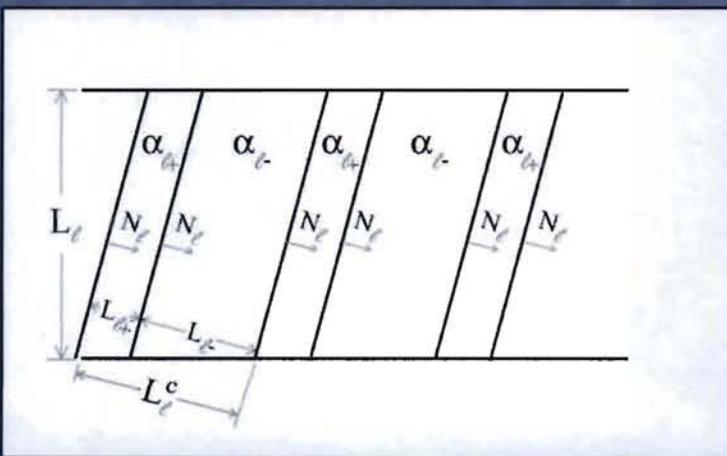
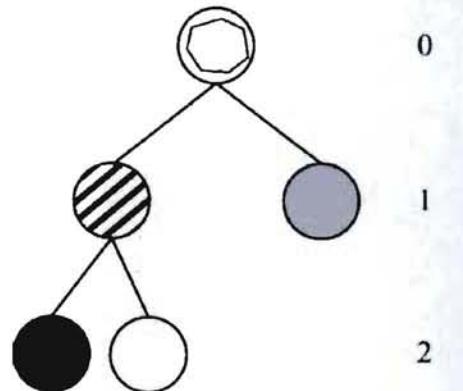
# Subgrain Laminate Microstructures

Tractable mathematically as energy minimization.

Grain



RANK



Characterized by:

$a$  = polarization vector

$\alpha$  = slip system

$N$  = wall normal

- coplanar

$\lambda$  = volume fraction

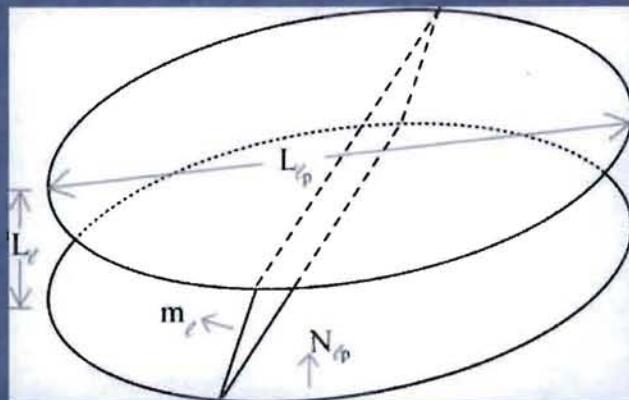
$L^c$  = length scale

Rank 1 connected deformations:

$${}_n\mathbf{F}_{l+} - {}_n\mathbf{F}_{l-} = {}_n\mathbf{a}_l \otimes \mathbf{N}_l$$

# Mean Free Path Shortening

- When dislocation walls form the mean free path of dislocations is shortened.



- The actual length depends on the angle between the slip plane and the dislocation wall:

$$A = \frac{L_l}{\sqrt{1 - (\mathbf{m}_l \cdot \mathbf{N}_{l_p})^2}}$$
$$B = L_{l_p}$$

- Coplanar slip is a special case and must be accommodated within a single laminate.
- By Orowan's relation, shortening the mean free path,  $h$ , increases the critically resolved shear stress:

$$n\tau_l^c = \tau_0^c + \frac{T}{b_n h_l}$$

# Laminate Microstructures: Non-local Energy

- The non-local energy is the combination of the energy gained from the dislocation walls and the mismatch at the boundary.

$${}^nW_l^{NL} = {}^nW_l^{DW} + 2\Upsilon \frac{{}^nL_l^c}{{}^{n-1}L_l} {}^nW_l^{BL}$$

- The laminate thickness is determined by the minimization of the non-local energy.

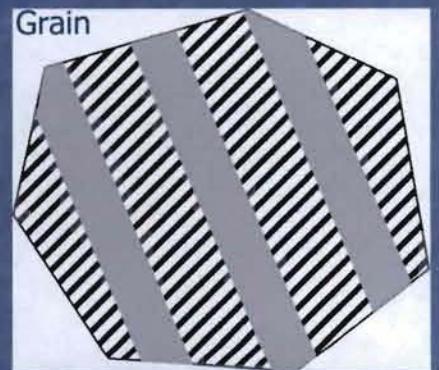
$${}^nL_l^c = \sqrt{\frac{T {}^{n-1}L_l}{2\Upsilon b {}^nW_l^{BL}} \left( \frac{{}^n\gamma_{l-}}{\lambda_{l-}} \sqrt{1 - (\mathbf{m}_{l-} \cdot \mathbf{n}_l)^2} + \frac{{}^n\gamma_{l+}}{\lambda_{l+}} \sqrt{1 - (\mathbf{m}_{l+} \cdot \mathbf{n}_l)^2} \right)}$$

$${}^nL_l^c = \begin{cases} \text{equation 3.24} & , \text{if (equation 3.24)} < {}^{n-1}L_l \\ {}^{n-1}L_l & , \text{otherwise} \end{cases}$$

# Laminate Length Evolution

- To obtain correct time evolution of hardening it is essential to update the widths within the microstructure:

$$L_l^c, \min_{l=1, \dots, Z} \sum_{l'=1}^Z W_{l'}^{NL}$$

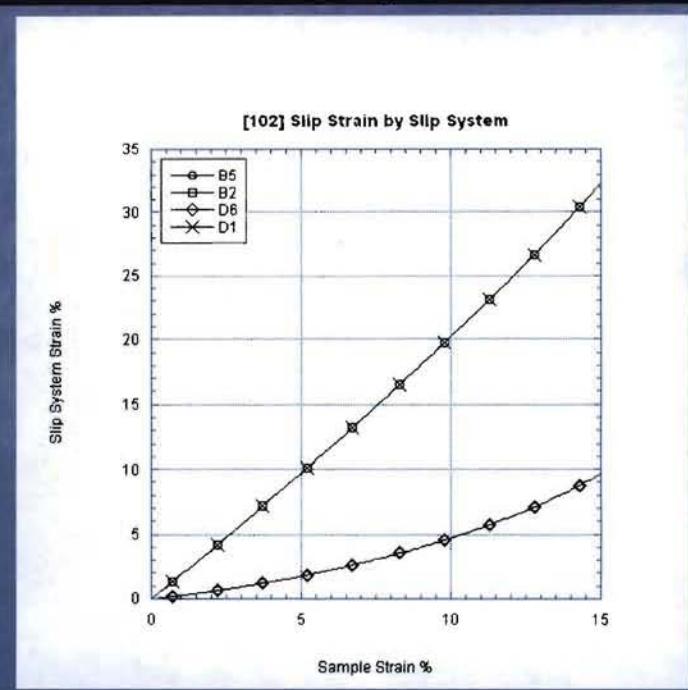
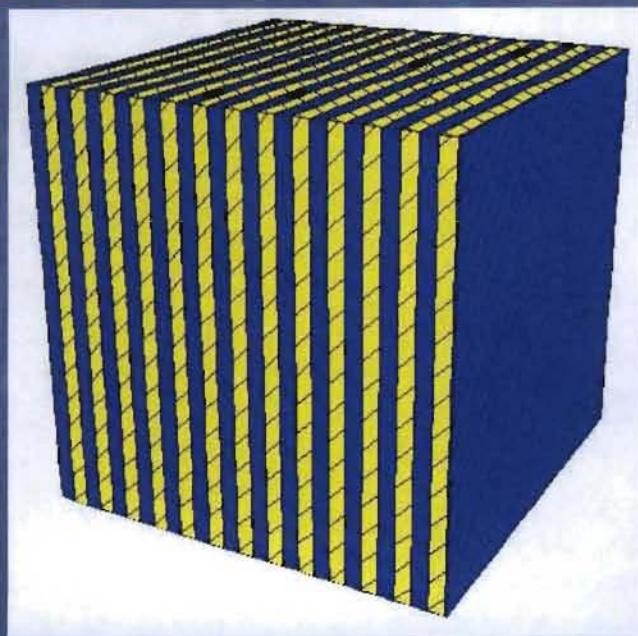
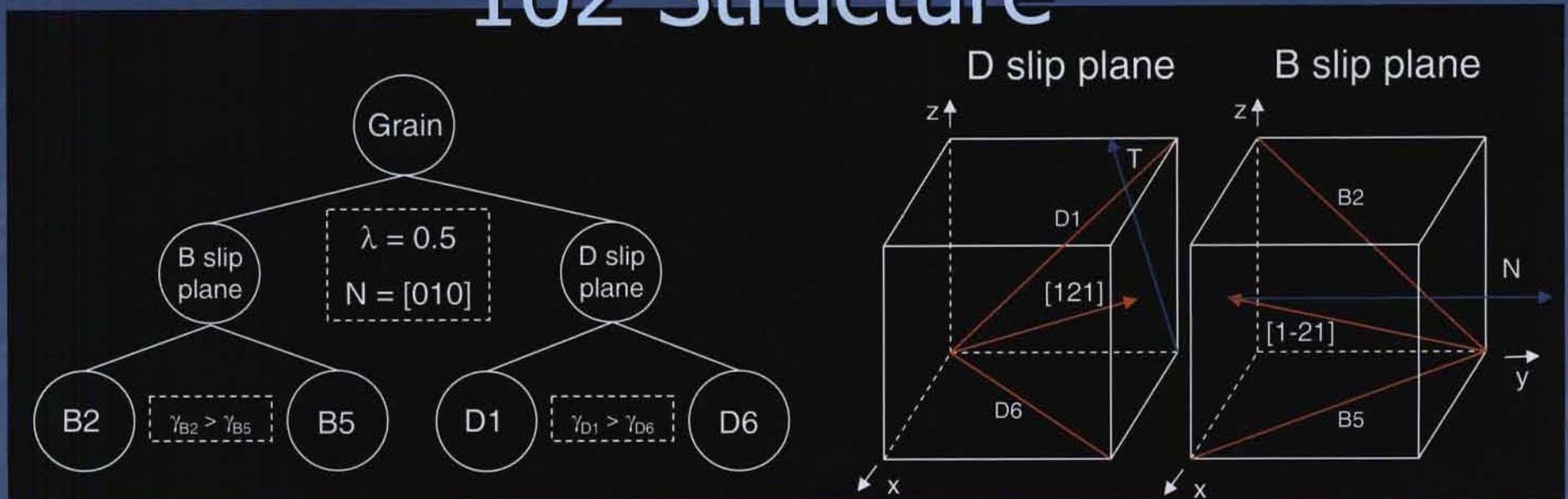


$$L_l^c = \sqrt{\left( \frac{L_{l+}^c W_{l+}^{BL}}{\lambda_{l+}} + \frac{L_{l-}^c W_{l-}^{BL}}{\lambda_{l-}} + \frac{\delta_l}{2\Upsilon} \right) \frac{\lambda_l}{W_l^{BL}} L_{l_p}^c}$$

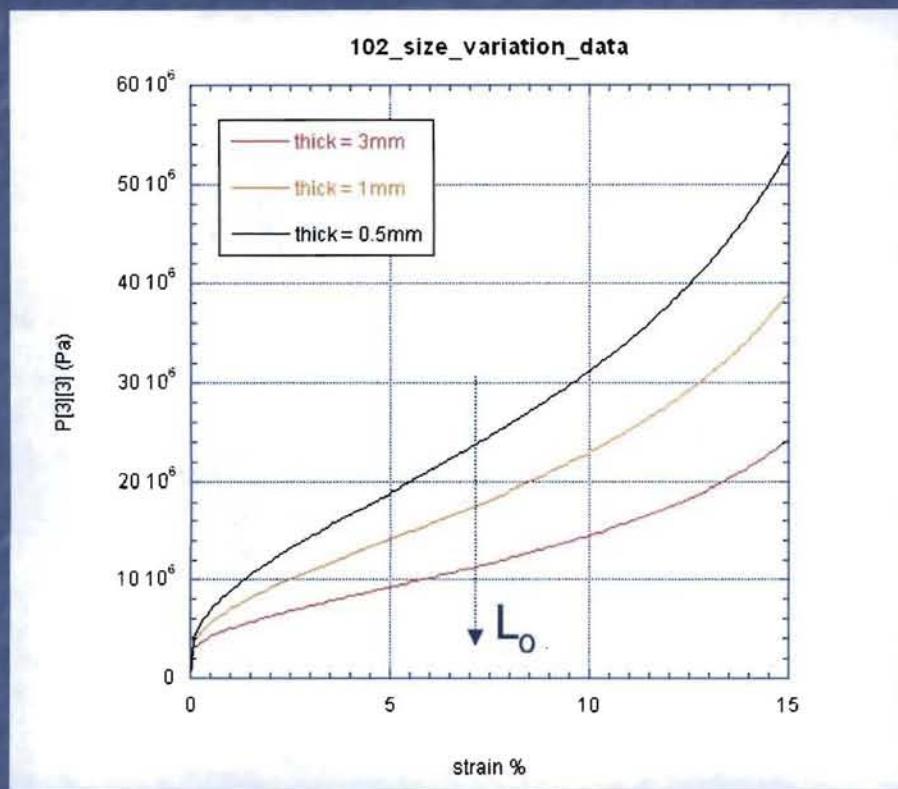
$$\delta_l \equiv \frac{T}{b\zeta} \left( \frac{\gamma_{l+}}{\lambda_{l+}} \sqrt{1 - (m_{l-} \cdot N_l)^2} + \frac{\gamma_{l-}}{\lambda_{l-}} \sqrt{1 - (m_{l-} \cdot N_l)^2} \right)$$

\*Note both traditional mechanisms of hardening are reproduced by the laminate size evolution: self hardening and multislip hardening.

# 102 Structure

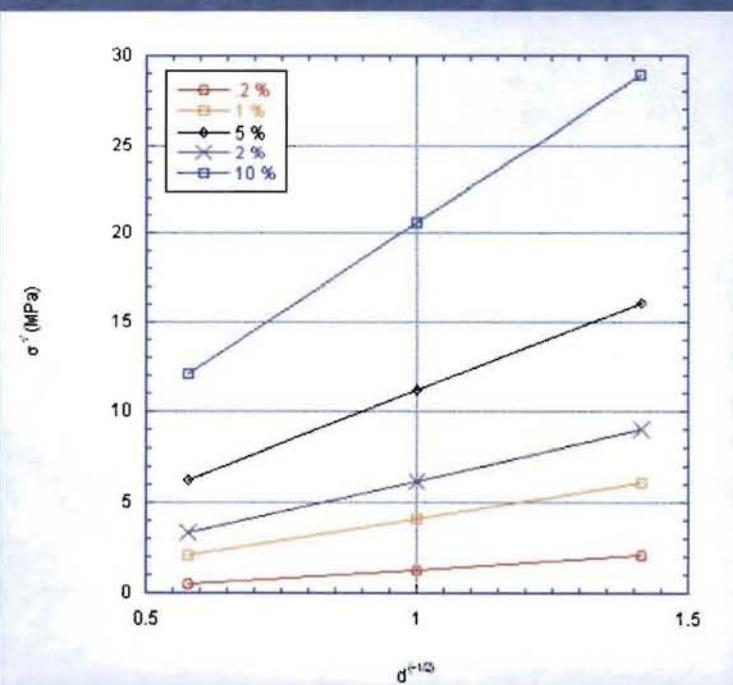


# Size Dependency



Variation in L for the 102 orientation simulations.

Relationship is Hall-Petch as predicted.



# Summary

- Conclusions
  - Mesh generation is successful
  - Required mesh resolution could result in lengthy calculations
  - Crystallographic texture appears to have minor influence on macroscopic response.
  - Morphological texture has a significant impact on stress response.
- Future work
  - Include size effects in materials model
  - Further refine meshing technique
  - Analyze traction at grain boundaries
    - Against orientation
    - Against misorientation angle
    - Against grain size
  - Refine grain model – improved Voronoi or other models
  - Further comparison to experiments
  - Tri-modal distribution

# Individual Grains

