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Robust Design of Reliability Test Plans Using Degradation Measures

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Abstract

With short production development times, there is an increased need to demonstrate product reliability relatively quickly with minimal testing. In such cases there may be few if any observed failures. Thus, it may be difficult to assess reliability using the traditional reliability test plans that measure only time (or cycles) to failure. For many components, degradation measures will contain important information about performance and reliability. These measures can be used to design a minimal test plan, in terms of number of units placed on test and duration of the test, necessary to demonstrate a reliability goal. Generally, the assumption is made that the error associated with a degradation measure follows a known distribution, usually normal, although in practice cases may arise where that assumption is not valid. In this paper, we examine such degradation measures, both simulated and real, and present non-parametric methods to demonstrate reliability and to develop reliability test plans for the future production of components with this form of degradation.

ACKNOWLEDGMENTS

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FIGURES

NOMENCLATURE

CDF	Cumulative Distribution Function
KDE	Kernel Density Estimate

1. INTRODUCTION

Often it is difficult to assess component reliability using traditional methods due to a lack of observed failures. For many such components, degradation measures, observed over time, will contain important information about performance and reliability. These measures can be used to predict remaining time to failure and to estimate the overall reliability distribution for that component. Degradation measures are inherent in situations where failure occurs due to a process of accumulation of damage. In mechanical systems, the failure (breakage of some part) may be caused by the impact of a peak load. The amount of accumulated wear or fatigue damage plays the role of the degradation measure. There are also failures in electronic parts (such as a short circuit, dielectric breakdown, breaking of a circuit, etc.) which are speeded up by some gradual process of deterioration, e.g. corrosion, mechanical deformation, chemical reactions and so on. Metrics on these processes all serve as degradation measures. Reference [1], [2] and [3] all discuss important aspects of degradation data and reliability estimation using degradation data.

Although in many cases it is reasonable to assume that the data follow a known statistical distribution, in practice situations arise where such an assumption is inappropriate. To combat this, we turn to non-parametric methods, such as simple bootstrapping or smooth bootstrapping, To make the model fitting technique robust to outliers, we apply median regression.

The purpose of this paper is to present a robust reliability test plan methodology along with case studies that rely on a degradation measure rather than time to failure data. We will adapt the methods developed in [4] to robustly estimate a reliability distribution using degradation measures. The methods will be applied to estimating the reliability of an electrical component using historical degradation data. Bootstrapping simulation techniques will then be used to investigate minimum sample size requirements and test duration necessary to demonstrate a reliability goal. It will be shown that test plans designed using this approach may require far less testing than traditional reliability test plans. The test plans developed can be used to demonstrate the reliability of future production of this component.

1.1. Degradation Measures

1.1.1. Assumptions

Several assumptions are usually made regarding a degradation measure:

1. The state (health) of the component can be characterized by a randomly changing time-dependent variable that we will denote y_t.
2. A failure of the component is defined as a certain catastrophic event ("hard" failure) whose probability of occurrence depends on the value of the variable y_t, or is defined as the variable y_t entering some critical region ("soft" failure).
3. The variable y_t is accessible for either continuous or discrete observations, i.e. a degradation "path" is obtainable.
4. The probabilistic laws governing the changes in the degradation measure y_t are known (physical model) or can be approximated (empirical model).

Under these assumptions y_t is called a degradation measure. Component tests in which degradation data are collected provide, for each unit tested, data consisting of an observed path of degradation measurements (either discrete or continuous) over time. The observed degradation path is a unit's actual degradation path, a monotonic function of time, plus some measurement error. Note also that "time" could be in units of real time or other units such as number of cycles or amount of usage. The path may be censored if the unit is taken off test before failure occurs or it may include the entire degradation path to the point of failure.

An accurate degradation model can be used to show that failure probability is small when the degradation variable is far from the critical region. With censored data, reliability can then be measured in terms of the distance of the degradation measure from the critical region. This is especially important with high reliability products, because high reliability goals can then be demonstrated with less testing than would be required using just time to failure (including censoring times) data. This advantage of degradation data will be exploited in designing a minimal reliability test plan in which no failures may be observed.

1.1.2. Estimating Reliability with a Degradation Measure

The procedure that we will use to estimate reliability using a degradation measure follows that given in [1] and [2]. The steps are:

1. Fit a general path model to each of the n sample degradation paths. Least squares estimation can be used to estimate the parameters for each path.
2. Determine the statistical distribution (using the estimates from the n sample paths) of each of the random parameters from the general path model.
3. Use the resulting distributions to solve for the time to failure distribution $F_T(t)$ if a close form expression exists.
4. If no closed form expression for $F_T(t)$ exists, use the parameter distributions from (2) to simulate a large number N of random degradation paths.
5. To estimate $F_T(t)$, compute the proportion of random paths generated in (3) that cross a pre-determined critical level (which defines failure) before time t. That proportion is the estimate of $F_T(t)$.

To quantify the uncertainty associated with the estimate of $F_T(t)$ we used a slightly different method than in [1]. The proposed procedure that we investigated is:

1. Choose a bootstrap sample (sampling with replacement) of n degradation paths from the original n sample paths.
2. Repeat steps 1-5 from the estimation procedure above to obtain an estimate of $F_T(t)$.
3. Repeat steps 1 and 2 here many times (say 1000) to obtain a distribution of values for $F_T(t)$. The central (100-α)% of this distribution defines a (100-α)% confidence interval for $F_T(t)$.
4. This interval becomes the uncertainty interval for $F_T(t)$.

These confidence intervals are constructed using the bootstrap percentile method discussed in [5]. Sensitivity to sample size was determined by varying the size n of the number of degradation paths in (1). Sensitivity to the length of the test (number of cycles of each degradation path observed) was determined by truncating the original sample paths at various points and performing the estimation and uncertainty analyses above using the truncated paths.

1.2. Statistical Methods

Our goal is to design reliability test plans that are robust with respect to both outliers and to underlying distribution. We will focus on median regression and kernel density estimation, two non-parametric statistical methods.

1.2.1. Median Regression

Median regression is the estimation of the median of a response variable conditioned on some input variable or variables. That is, rather than the more common least-squares regression, the conditional median is estimated rather than the conditional mean. To find the estimated coefficients of the median regression line, $\hat{\beta}_{.5}$, the following equation is solved:

$$\hat{\beta}_{.5} = \arg \min_{\beta \in R^k} \sum_{i=1}^{n} |Y_i - X_i\beta|$$

This may be more recognizable to some as quantile regression with $\tau = 0.5$.[6] Although the concept of median regression has existed for centuries, it has only seen widespread usage since the advent of computers, as solving this minimization requires much more complicated numerical techniques than least squares. However, median regression does have a useful advantage to least squares regression in that it is considered to be robust to outliers. That is, the estimates found in median regression are less affected by the presence of outliers than those found in least squares regression. By using median regression to model degradation paths, we then increase robustness to any outliers found in the individual degradation paths. Median regression is also a non-parametric statistical method, meaning that no assumption is being made about the distribution of the residuals from the fitted equation. This allows for improved estimation of the conditional median in the presence of, for example, skewed residuals.

In this paper, all degradation paths are assumed to be well-approximated by a linear model. That is, we use the model

$$y_{ij} = \beta_{0i} + \beta_{1i}x_{ij} + \epsilon_{ij},$$

where β_{0i} and β_{1i} represent the intercept and slope of unit i, y_{ij} is the jth observation of the output of unit i at time x_{ij}, and ϵ_{ij} is the random error at the jth observation of the output of unit i. However, as with least squares, median regression extends to other path models.

1.2.2. Kernel Density Estimation

Kernel density estimation, or KDE, is a useful method to estimate the probability density function of a collection of data that comes from an unknown distribution. If we are to estimate the probability density function, $f(x)$, of independent, identically distributed data, $x_1, x_2, \ldots, x_n$, comes from, the KDE can be expressed as:

$$\hat{f}_h(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - x_i),$$

where $K_h(z)$ is the chosen kernel and h is the selected bandwidth. Kernel density estimation results in a smooth density function from which we can make inferences about the population. Although there are several options for kernels, in this paper the kernel is considered to be Normal (Gaussian). There are also several selection criteria for the bandwidth. After performing a simulation study, we chose the selection method proposed by Sheather and Jones [7] for this paper.

2. METHODOLOGY

In order to improve robustness to outliers and to distributional assumptions, we modify the algorithm in Section 1.1.2 to incorporate the non-parametric statistical methods found in Section 1.2. To do so, we first use median regression (instead of least squares regression) in Step 1. This allows our fitted degradation paths to be more robust to random errors and to estimate the degradation path with non-Normal residuals. From the parameter estimates found in Step 1, we no longer try to determine the distribution of the random coefficients. Instead, assuming the slope and intercept coefficients are independent, we fit a univariate KDE to each set of estimated coefficients in Step 2. We then simulate from these KDEs the coefficients needed to perform Step 4 and use the simulated degradation paths to estimate the failure distribution $F_T(t)$ in Step 5. Simulating from the KDEs in this fashion is sometimes known as "smooth bootstrapping." Thus, our new steps are as follows:

1. Fit a general path model to each of the n sample degradation paths. Median regression is used to estimate the parameters for each path.
2. Fit a KDE to each of the empirical parameter distributions from the general path model.
3. Use the KDEs from (2) to simulate a large number N of random degradation paths.
4. To estimate $F_T(t)$, compute the proportion of random paths generated in (3) that cross a pre-determined critical level (which defines failure) before time t. That proportion is the non-parametric estimate of $F_T(t)$.

The uncertainty quantification algorithm remains the same as before. It is also possible to simply use bootstrapping from the random parameter estimates to simulate a large number of random degradation paths, effectively skipping Step 2. This obtains similar results to the smooth bootstrapping, particularly in large samples. Some comparisons of these methods can be found in Sections 3 and 4.

If the sample slope and intercept estimates appear to be correlated, then we can instead fit a bivariate KDE to the joint set of estimated coefficients. The coefficients for the simulated random degradation paths would then be drawn from this bivariate KDE. However, for the examples in this paper, the coefficients are either assumed or designed to be independent.

3. SIMULATED DATA ANALYSIS

To provide examples of situations where our added robustness to distributional assumption adds value to the previous methodology, we look at two simulated examples. In each case, the slopes of the degradation paths are randomly selected from a Normal distribution, as are the random errors along each path. However, the intercepts are randomly selected from non-normal distributions.

For the purpose of reliability test planning, we also are interested in the effects that test duration and of observed units on test have on failure probability estimates and their associated uncertainties. By taking into account these effects, we can to design minimal robust reliability test plans.

3.1. Skewed Intercept Distribution

First, we look at degradation paths whose intercepts are randomly drawn from a skewed distribution. In Figure 1, the 50 simulated degradation paths used in the analysis are shown on the left. A lower failure limit of 20 is also given. On the right, a KDE plot of 100,000 points of data simulated from the skewed distribution is shown, along with a Normal distributional fit. This fit places too much probability in the right-most tail, meaning that when simulating from this distribution, we would often see larger intercept values than expected.

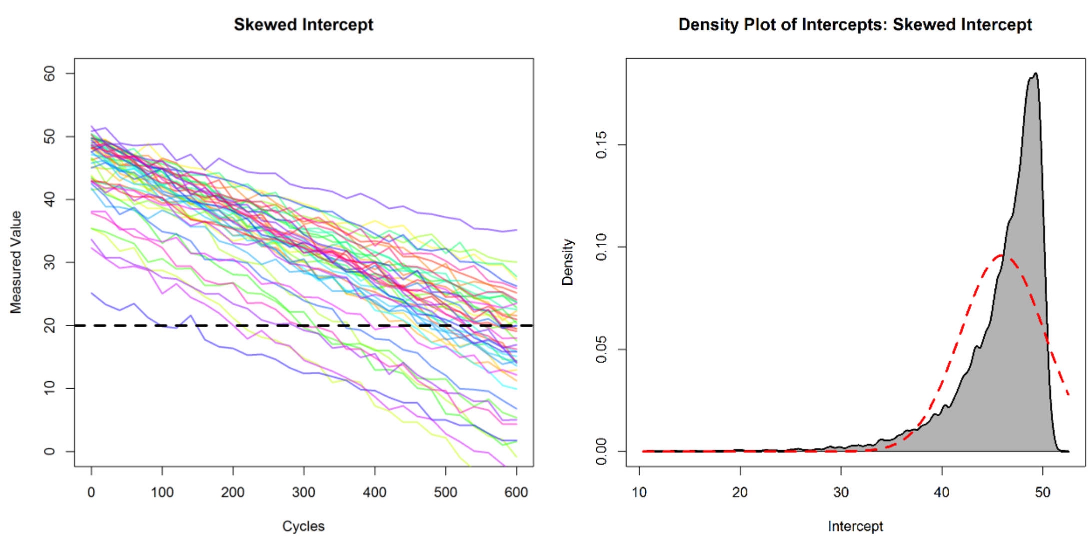

Figure 1. On the left, a plot of 50 simulated degradation paths with intercepts drawn from a skewed distribution. On right, a KDE plot of 100,000 points of data simulated from the same distribution that was used for the intercept of the random paths.

Because there are more large intercept values than expected, the failure probability tends to be underestimated. In Figure 2, on the left we see the estimated cumulative distribution functions (CDFs) for cycles to failure for the Normal distribution method, the simple bootstrap method, and the KDE bootstrap method. Also, 90% confidence intervals are also shown for each of the

CDFs. If, for example, we are looking for a reliability goal of 0.95 with 95% confidence, we would look to where the upper bound of the 90% confidence interval crosses the 0.05 value. As we can see on the left plot of Figure 1, the reliability estimates with 95% confidence are about 20 cycles apart between the KDE bootstrap method and the Normal distribution method, with the Normal method underestimating the failure probability, as expected. In fact, when we compare the expected values for these degradation paths via simulation, we see that the expected values for both the KDE bootstrap and the simple bootstrap methods are close to the true theoretic values while the Normal distribution method differs. This comparison can be found on the right in Figure 2.

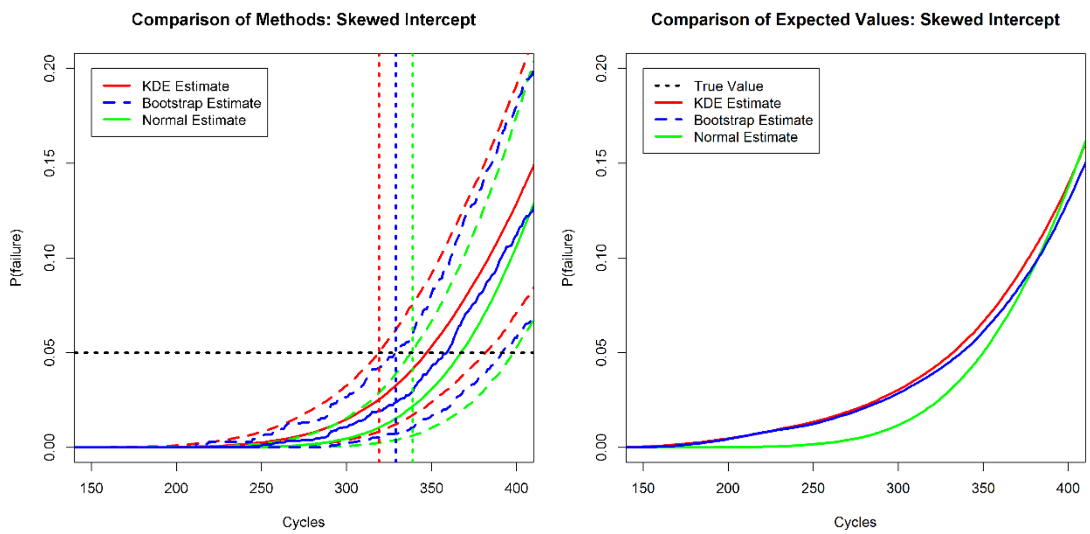

Figure 2. On the left, CDF estimates for the 50 simulated degradation paths found in Figure 1 with 90% confidence intervals for the smooth bootstrap (KDE) method, the standard bootstrap method, and the Normal distribution method. The point where the confidence interval first touches the 0.05 reliability point is marked with a vertical line. On the right, the expected failure distribution estimate for each of the three methods.

3.2. Bimodal Intercept Distribution

We also look at degradation paths whose intercepts are from a bimodal intercept. In Figure 3, the 50 simulated degradation paths are shown on the left and the bimodal distribution they come from is on the right. On the right-most plot, a Normal distribution fit is also shown. We see that in this scenario, a large amount of probability is placed in the region between the two modes, meaning that if we were to simulate from the normal distribution, we would obtain a large number of simulated data points from low probability regions. This will affect both our failure probability estimates as well as the uncertainty about those estimates.

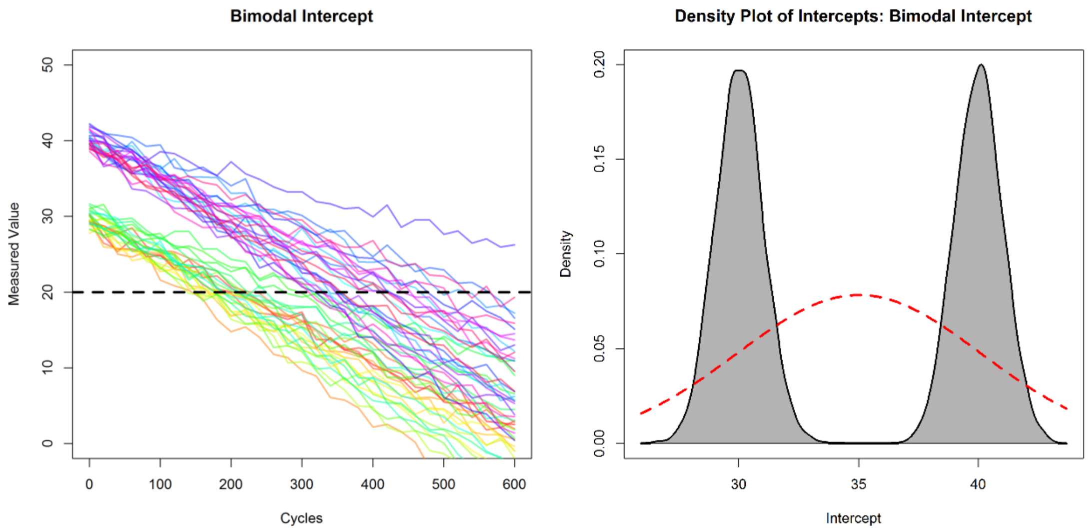

Figure 3. On left, plot of 50 simulated degradation paths with intercepts drawn from a skewed distribution. On right, KDE plot of 100000 points of data simulated from the same distribution that was used for the intercept of the random paths.

As before, we can look at what effect this type of distribution has on our failure probability estimates. On the left in Figure 4, we see that the Normal distribution estimate is noticeably different than either the simple bootstrap or KDE bootstrap estimates and the uncertainty around its estimate is larger than the uncertainty around the other two estimates. On the right in Figure 4, we see the expected values for the estimates from each method. Again, we see that the KDE bootstrap and simple bootstrap estimates are very close to the theoretic values while the normal approximation gives a poor estimate.

3.3. Effects of Test Duration and Sample Size

There are several parameters that can affect the CDF estimate of cycles to failure, including the duration of the test, sample size, time between tests, performance limit, and confidence level. In this paper, we look specifically at the effects of test duration and sample size on the CDF and its associated uncertainties. These effects will be used to determine minimal reliability test plans.

3.3.1. Test Duration

We simulated 50 degradation paths with slope, intercept, and random errors simulated from Normal distributions. Each of the paths was simulated for 600 use cycles with observations occurring every 20 cycles. To determine the effect of test duration on the estimated CDF of cycles to failure, we truncated the degradation paths at various cycle numbers, re-estimated the CDF and its uncertainties, and compared the results. In Figure 5, we see three such CDF estimates, with no cutoff, a 300 cycle cutoff, and a 100 cycle cutoff. The estimates change little between 300 cycles and all 600 cycles. However, at a certain point, the estimates begin to vary, as evidenced by the estimate at 100 cycles.

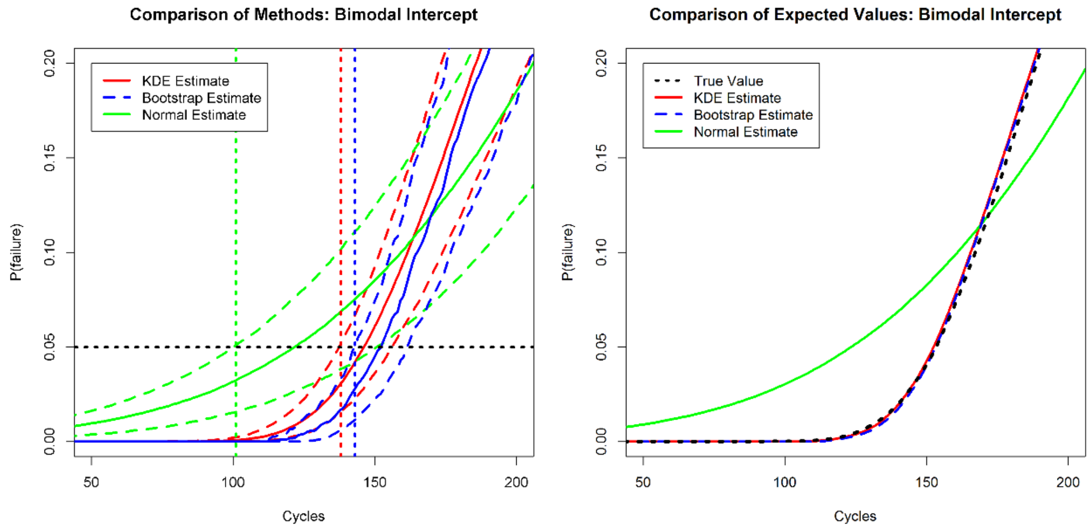

Figure 4. On the left, CDF estimates for the 50 simulated degradation paths found in Figure 3 with 90% confidence intervals for the smooth bootstrap (KDE) method, the standard bootstrap method, and the Normal distribution method. The point where the confidence interval first touches the 0.05 reliability point is marked with a vertical line. On the right, the expected failure distribution estimate for each of the three methods.

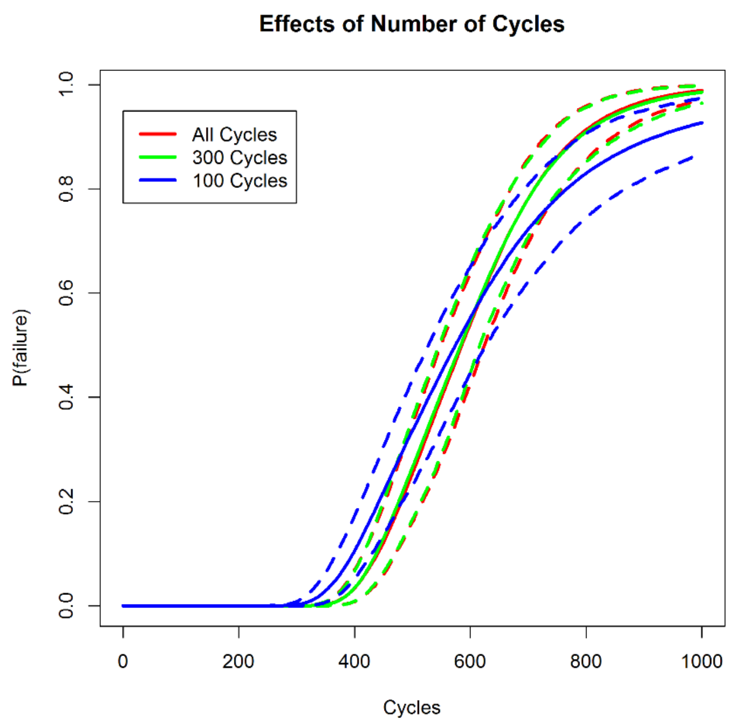

Figure 5. Comparison of CDF estimates using simulated data with a Normally distributed intercept and slope and 90% confidence intervals. Cycle counts include 100 cycles, 300 cycles, and all 600 cycles.

This is to be expected, as we know that the degradation paths follow a linear model, so we know that our model choice is correct. At a certain point, we have enough information from the paths for the estimates to stabilize. The time it takes to stabilize seems particularly dependent on the random error variability around the degradation paths. If the estimates do not stabilize, then either the units were not observed for enough cycles or the chosen path model is incorrect.

3.3.2. Sample Size

As we might expect, the number of degradation paths observed affects the uncertainty interval about the estimated CDF. As the sample size increases, the uncertainty decreases. This can be seen in Figure 6, in which we use the degradation paths from Section 3.2.1 to find our estimates. To find our estimates for the various sample sizes, instead of taking n samples from the original n degradation paths in our uncertainty quantification algorithm, we take m samples, where m is the sample size to be evaluated and $m \leq n$. This allows us to estimate the sample size needed to obtain the desired uncertainty at a given cycle time.

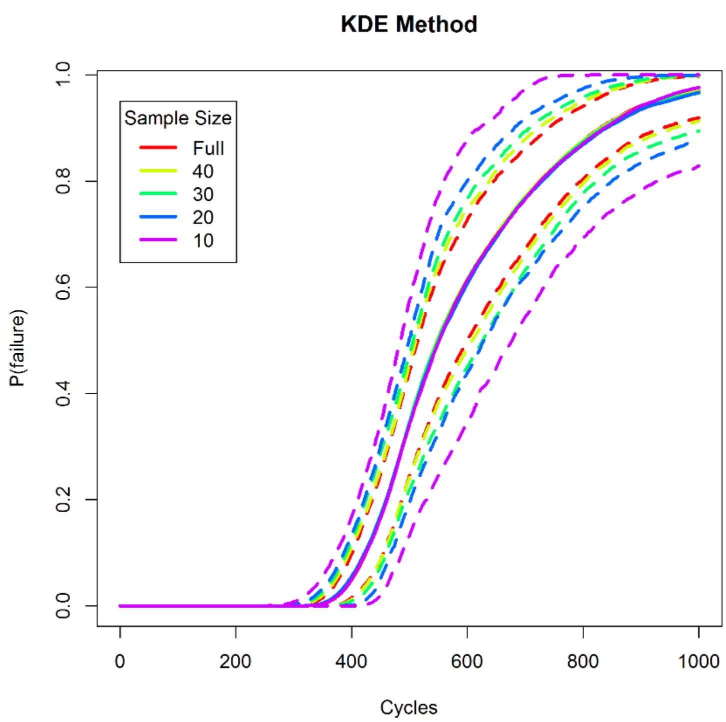

Figure 6. Comparison of CDF estimates using simulated data with a Normally distributed intercept and slope and their 90% confidence intervals for sample sizes of 10, 20, 30, 40, and 50 units.

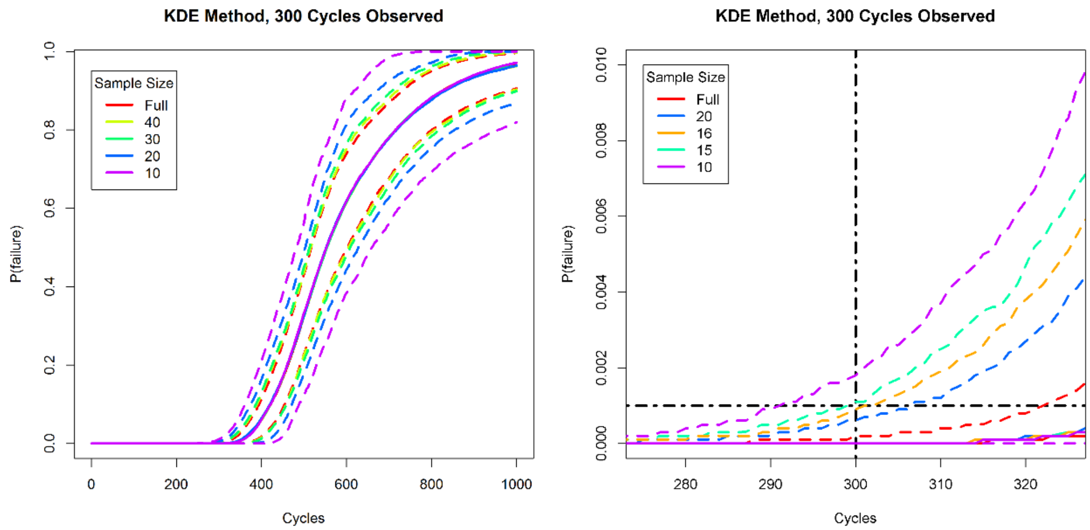

Figure 7. On the left, comparison of CDF estimates using simulated data with a Normally distributed intercept and slope coefficients and 300 cycles, with their 90% confidence intervals. Sample sizes of 10, 20, 30, 40, and 50 units are shown. On the right, we see CDF estimates and their 90% confidence intervals from the same degradation paths with more resolution at 300 cycles. Sample sizes of 10, 15, 16, 20, and 50 are shown.

3.3.3. Designing Test Plans Based on Test Duration and Sample Size

For a given maximum failure probability, confidence level, and performance requirement, we can adjust the test plan parameters to obtain an optimal test plan for future component testing. Using the degradation paths from Section 3.2.1, we first see that we can reasonably reduce the number of cycles that we observe the degradation paths from 600 to 300 with little change to our CDF estimate and the uncertainty around it. From this, we can then vary the sample size to find the minimum number of needed to demonstrate the reliability requirement with the desired level of confidence. Figure 7 shows that with a sample size of 16 degradation paths, we can demonstrate a reliability requirement of 0.999 (0.001 failure probability) with 95% confidence. Thus, our test plan would be to place 16 of these units on test for 300 cycles.

If there are other parameters, such as duration between observations, those parameters can also be varied to determine values that provide an estimate that meets the reliability requirement with the desired level of confidence. Because this may not lead to a unique solution, it is recommended to select the test plan with the lowest cost.

4. COMPONENT DATA ANALYSIS

4.1. Electrical Component Data

To demonstrate the design of a robust reliability test plan in practice, we use our methodology on a real-world, electrical component.

4.1.1. Data and Model Description

The data used in this case study come from the degradation of an electrical component used to close a high-energy circuit. A low energy pulse causes the component to discharge, forming a plasma at the probe, closing the high-energy circuit. The resulting plasma causes a carbon residue to be deposited around the probe. The degradation mechanism is the carbon buildup, and the degradation measure is probe resistance. As the carbon residue builds up due to repeated discharges, the probe resistance gradually decreases until the component fails to discharge. The probe resistance data for the components are given in Figure 8, with the probe response shown on a natural log scale.

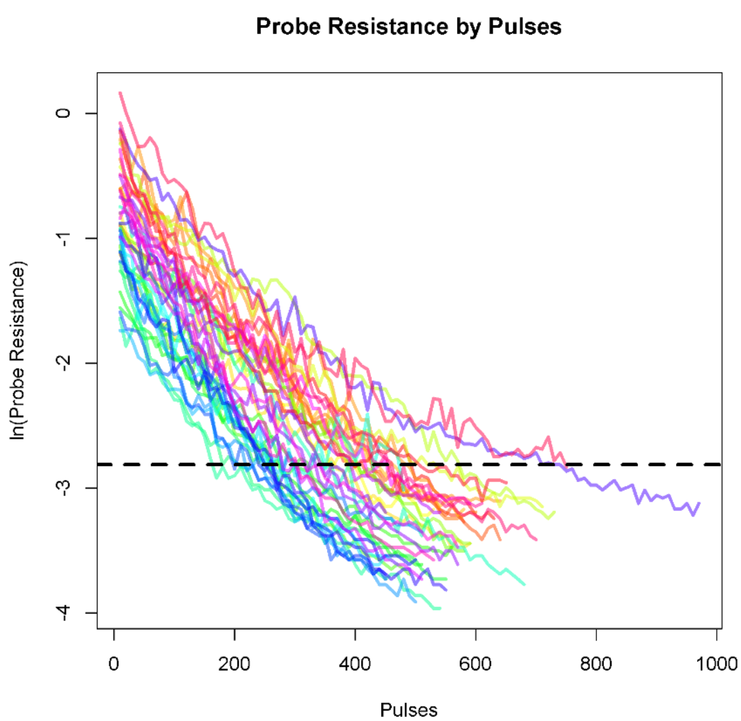

Figure 8. Degradation paths for the 42 tested electrical components. The output is shown on a natural log scale.

When we fit median regression lines to each degradation path and examine the coefficient estimates, we obtain the histograms and KDEs in in Figure 9. As we can see, shape of the histograms and densities suggest that the coefficient distributions may not be Normally distributed. It is worth noting that the coefficient tests do pass an Anderson-Darling test of Normality, however the small sample size makes it difficult to reject the assumption of Normality. Thus, we choose to not make an assumption about either coefficient distribution. When we test the slope and intercept coefficient estimates for independence, we see that the two sets of estimates are uncorrelated.

The initial objective of the testing was to demonstrate that this component design can be pulsed 100 times without failure. A reliability goal of 0.995 (0.005 unreliability) at 100 pulses was established for this component, meaning that the desired probability of surviving at least 100 pulses is 0.995 or greater.

The major cost associated with this test is the cost of the components, which are destroyed by testing to failure. Individual components may be expensive to manufacture, so any reduction of sample size results in significant savings. Additional costs are associated with the manpower and equipment required to pulse the components, so reducing the duration (number of pulses) of the test would also result in savings. To demonstrate the reliability goal of the component in future production lots, it is thus of interest to reduce both the number of components tested and the duration of the test. Reducing the number of pulses in this test corresponds to reducing “time on test” in the traditional component reliability study, an important consideration in product development

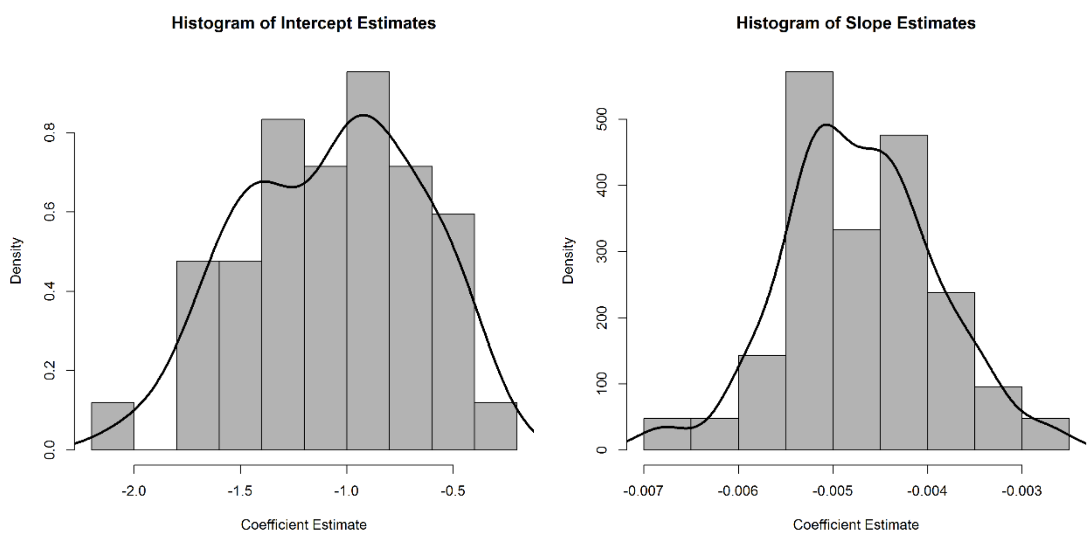

Figure 9. Histograms and KDEs for the coefficient estimates from the median regression lines fitted to the degradation paths found in Figure 8.

4.1.2. Test Plan and Analysis

To determine our reliability test plan, we first check to see if we can demonstrate that our estimates meet the reliability goal. In Figure 10, we see that the upper bound of our 90% confidence interval on the CDF estimate lies below 0.005 at 100 pulses, meaning that we have at least 95% confidence that we have a cumulative probability of failure less than 0.005 at 100 pulses. We then begin to vary the number of pulses observed to see when the estimates stabilize. As we can see in Figure 10, we start to see some slight changes early, at 600 and 700 pulses, and then a larger change at 500 pulses. We also see that at 700 pulses, we can still meet our reliability goal, with our upper confidence bound estimated to be 0.0047, but we are unable to meet our reliability goal at 600 pulses.

Because the cost of the components is the major testing cost, the decision was made to not reduce the pulse observation length. Simply setting the cutoff at 700 pulses did not shorten the observation length significantly and removed the ability to reduce the sample size, as the upper confidence bound was already very close to the desired cumulative probability of failure. Therefore, we turned our attention to reducing the sample size. In Figure 11 the estimated CDFs for 30 units, 35 units, and all 42 units are displayed. From this, we see that we are able to reduce the sample size to 35 units and still demonstrate that we meet our reliability goal. Any further reduction would not demonstrate that we meet our reliability goal. From all of this, our recommended test plan would not put a limit on observed cycle length, but reduce the sample size from 42 units to 35 units.

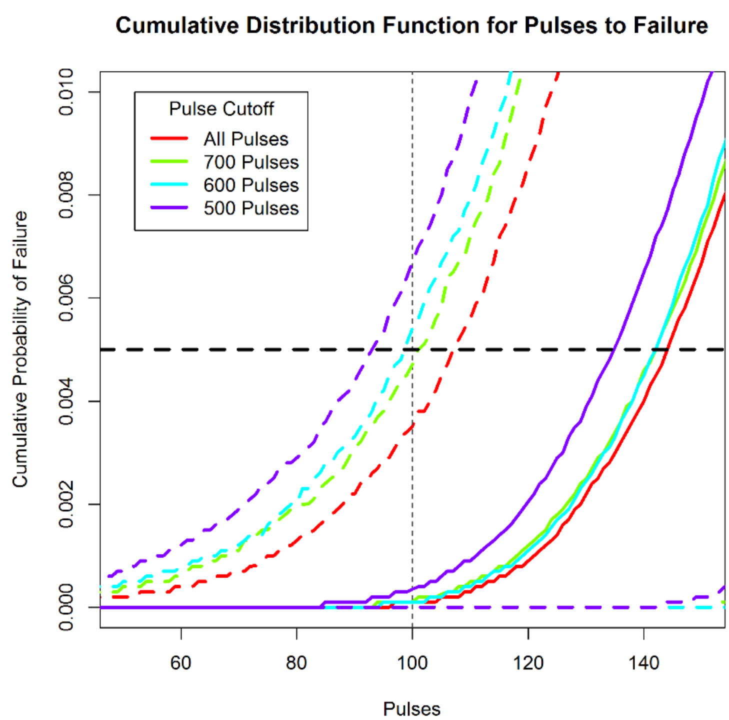

Figure 10. Comparison of CDF estimates for the electrical component using all pulses, 700 pulses, 600 pulses, and 500 pulses. 90% confidence intervals are also shown.

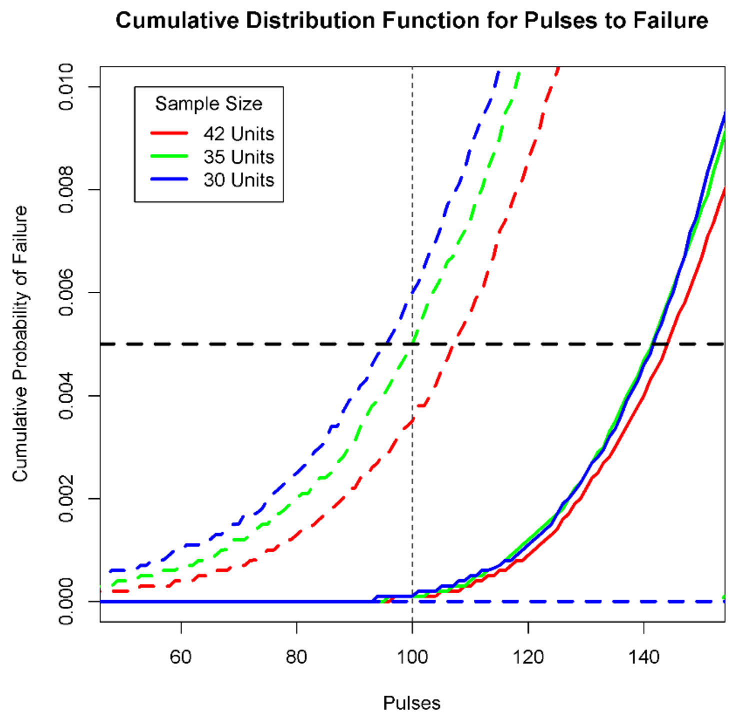

Figure 11. Comparison of CDF estimates for the electrical component using sample sizes of 42, 35, and 30 units. 95% upper confidence bounds are also shown.

5. SUMMARY AND CONCLUSIONS

In this paper, we have demonstrated, though both simulated and real examples, the use of applying non-parametric statistical methodologies to both estimate reliability and to develop minimal test plans for degradation data that does not adhere to the usual assumptions of a known distribution. The added robustness can give a more accurate estimate in the presence of skewness or a mixture of distributions. As is the case with other non-parametric methods, the increase in robustness comes with an increase in uncertainty. This means that in cases where a known distribution is a reasonable assumption for the degradation paths, using that information will allow for further reductions in the designed test plan. However, in practice, it is not uncommon to see data that does not fit any of the standard statistical distributions, making this non-parametric approach a reasonable choice to perform reliability analysis and to develop minimal test plans.

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