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Approximate Bounds for Kinetic Energy and Resistance from Z-experimental data

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OUTLINE

- What quantities are known experimentally?
- Current paths, beyond the thin single path assumption
- Review of Inductance Reconstruction Procedure (IRP)
- Lower bound for pinch kinetic energy few ns before the main x-ray pulse from energy conservation
- Upper bound for pinch resistance at stagnation from energy conservation
- Application of technique to Z shots 1233,1469 and 1735
- Needed experimental accuracy to make technique useful
- Fitting the data to a two-thin current path scenario
- Conclusions

Experimentally known quantities

- **On the Z machine the quantities measured directly are:**

V_{stack}	Stack voltages	4 levels
I_g	MITL current	Sum of 4 levels
I	Load current	6 cm from axis
P_X	Radiated power	

- **From those quantities measured directly we can construct (and considered them known experimentally)**

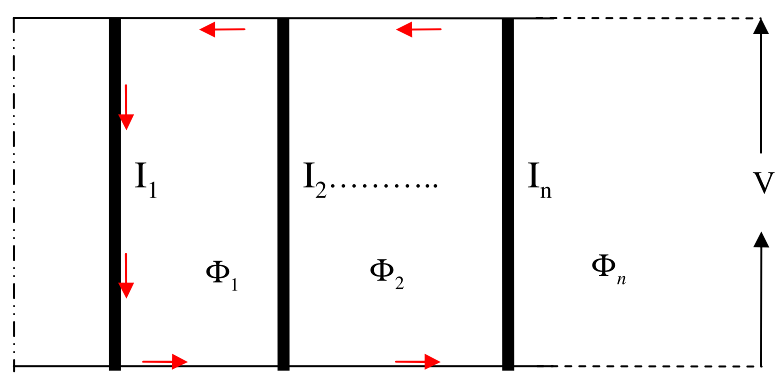
V	Convolute voltage	Inductive translation (Ref. 1)
E_X	Radiated energy $\int_0^t P_x dt$	
P_P	Poynting power VI	
E_P	Poynting energy $\int_0^t VIdt$	

- **Nominal inductances are used (cold metal, no mechanical motion; from specifications) for MITL and load currents**

Beyond the thin single path assumption

- For a discrete set of "thin straight currents" the circuit equation can be written as:

$$V = d(LI)/dt + R_1 I_1 \qquad (1)$$



Current paths, generalizing the thin single path assumption (cont.)

- L **is defined by:**

$$LI \equiv \int drdzB/c = \sum_{i=1}^{n} L_i I_i = \sum_{i=1}^{n} L_{i,i+1} C_i, \text{ where } C_i = \sum_{j=i}^{i} I_j \quad (2)$$

- L_j corresponds to the flux linked by I_j, to the convolute, and is given

by $L_j = (2l/c^2)\ln(r_0/r_j(t)) + L_c$, and $L_{j,j+1} = L_{j+1} - L_j$, where L_c is the inductance between the vacuum convolute and the initial wire array position

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Review of Inductance Reconstruction Procedure (IRP)

- **Assume that at "sufficiently high load current";**

$$V = d(LI)/dt, t \geq t_0$$

$$L(t) = (L_0 I_0 + \int_{t_0}^{t} V dt)/I \qquad (3)$$

- ***Method 1* (Ref. 1): Look for plateau, start of rapid rise determines end of ablation phase t_a . Sufficiently high load current, $I(t_0) \approx 2-5MA$ for Z**
  - **Best results for this method for 0.93 factor correction of the voltage and MITL inductance of 7.2 nH in lieu of the calculated 6.9 nH**

- ***Method 2 (used in this presentation):* Look for plateau of $(V - L_0 \dot{I})/I$ For small changes in the load inductance during the ablation phase, this quantity is the resistance of the innermost current path. Use uncorrected convolute voltage and MITL inductance. Apply Eq(3) as in *Method 1.***

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Useful inequalities for distributed current

- **For the axisymmetric case, distributed currents (positive in the discrete representation, or monotonic magnetic field in the radial direction) we have:**

  $$W = \int B^2 r dr dz / 4 \leq LI^2 / 2, \text{ where } L \text{ is the "flux inductance"}$$

  defined in Equation (2)

- **Also from Equation (1)**

  $$\tilde{L}(t) \equiv L(t) = (L_0 I_0 + \int_{t_0}^{t} V dt) / I \geq L(t), \ \forall (\mathrm{t}, \mathrm{t}_0)$$

- **Thus,** $$W \leq \tilde{L} I^2 / 2 \qquad (4)$$

Energy Conservation related to experimental data on Z

- **Energy Conservation:**

$$Q(t) \equiv E_P - E_X = K + W + E$$

- **Where** K, W, E **are the kinetic, magnetic and internal energies, respectively (** W **from the vacuum convolute to the axis)**

- **In the expression for energy conservation the left hand side is experimentally known**

- **Can we obtain estimates for** $K, W \text{ and } E$ **at relevant times of the implosion ?**

Lower bound for pinch kinetic energy few ns before stagnation

- **Using the inequality (4) and energy conservation we obtain**

$$K = Q(t) - W - E \geq Q(t) - \tilde{L} I^2 / 2 - E$$

- **If we assume that the internal energy is small,** $K \gg E$**, at** t_1 **,a few ns before stagnation, we obtain the following (approximate) inequality**

$$K(t_1) \geq Q(t_1) - \tilde{L}(t_1) I^2(t_1) / 2$$

- **Example:** $E \approx 20kJ$ for tungsten at $100eV, \rho = 0.04 g / cm^3, m = 6mg$
- **Hence, the internal energy is at least one order of magnitude smaller than the kinetic energy at** t_1

Upper bound for pinch resistance at stagnation

- At time t_2 ,when the power is about halfway down of its peak value we have; $K(t_2) \ll W(t_2)$,and thus;

$$W(t_2) \approx Q(t_2) - E(t_2)$$

- Using x-ray diagnostics, we can estimate $E(t_2)$ following Ref. 2 , to about 10% accuracy
- And use this approximate value for W at t_2 to obtain;

$$L(t_2)I(t_2) \leq 2W(t_2) / I(t_2)$$

- From the circuit Equation (1);

$$\int_{t_1}^{t_2} R_1 I_1 dt = \int_{t_1}^{t_2} V dt - L(t_2)I(t_2) + L(t_1)I(t_1)$$

Upper bound for pinch resistance at stagnation (cont.)

- Thus, recalling the inequalities for W and $\tilde{L}$, and that $I \geq I_1$ the resistively dissipated flux satisfies:

$$\int_{t_1}^{t_2} R_1 I_1 dt \leq \left(\int_{t_1}^{t_2} V dt - 2W(t_2) I(t_2) + \tilde{L}(t_1) I(t_1) \right)$$

- This bound for the dissipated flux is approximate because at $t = t_2$ we have:
  - Assumed kinetic energy to be zero
  - Uncertainty in the internal energy estimate
- Define a estimate for pinch resistance from dissipated flux;

$$R_{est} \equiv \left(\int_{t_1}^{t_2} R_1 I_1 dt \right) / \int_{t_1}^{t_2} I dt$$

Two-path assumption (trailing mass and current)

- **Assume mass and current split at end of ablation phase; (current flows in two thin current paths) located at** $r_{out}(t), r_{inn}(t), \quad t \geq t_a$

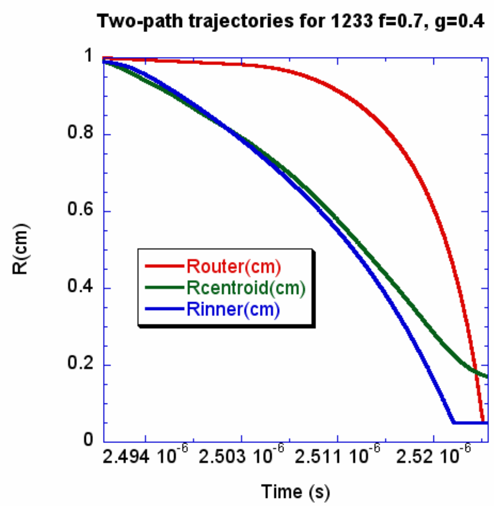
$$\mu_{out} = fg\mu_0, \text{ constant}$$
$$\mu_{inn}(t_a) = f(1-g)\mu_0$$
$$\mu_{out} du_{out}/dt = -(1-\alpha^2) I^2 / 100 r_{out}$$
$$d(\mu_{inn} u_{inn})/dt = -\alpha^2 I^2 / 100 r_{inn}$$
$$d\mu_{inn}/dt \text{ given by snowplow using rocket model}$$
$$\alpha = \ln(r_{out}/r_{bar}) / \ln(r_{out}/r_{inn})$$

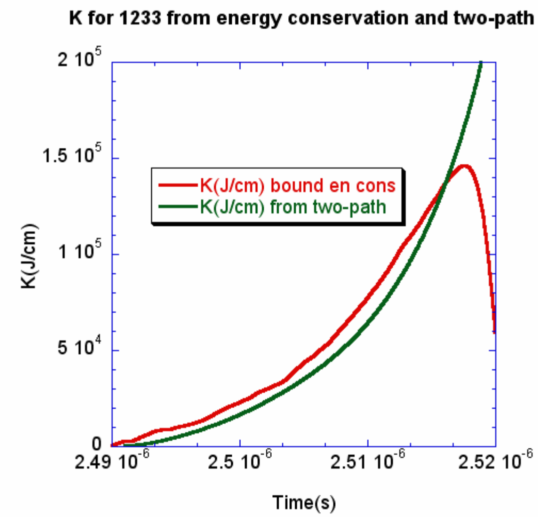
- **Output of calculation: trajectories and current fraction** $r_{out}(t), r_{inn}(t), \alpha$
- **Two adjustable parameters** f, g**, while** $I, r_{bar} \equiv \exp(-(L(t)-L(t_a))/l),\ L[nH]$ **are from experimental data and the IRP. The rocket model ablation velocity is determined by;**

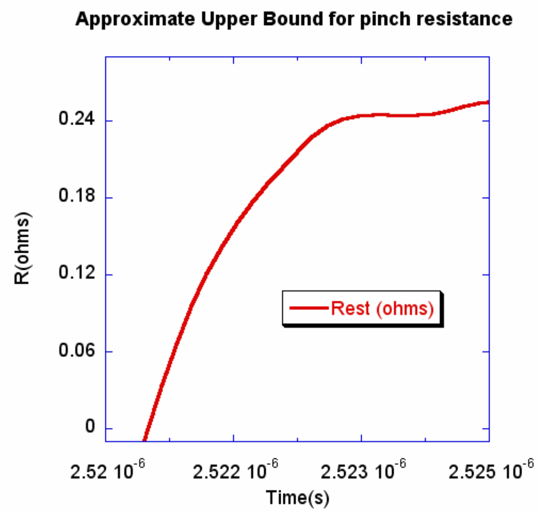
$$u_a = \left(\int_0^{t_a} I^2 / 100 r_0\right) / (f\mu_0)$$

Results for shot 1233

Ta=2489ns, Tpeak=2521ns







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Conclusions

- Employing current, voltage and radiated power data, we have obtained useful bounds for the kinetic energy a few ns before peak x-ray power, and the pinch resistance estimated at about halfway down from the peak, using the circuit and energy conservation equations
- To obtain the kinetic energy lower bound to about 10% we need about 2-3% accuracy for the electrical data. For the estimate of resistance we also need about 5-10% for the radiated power. Transit time effects should be taken into account
- Actual initial inductances should be about 5% in accuracy
- The data can be fitted by a two-path trailing current and mass procedure, using the experimental data self-consistently
- This type of analysis will be employed for ZR