LA-UR-14-26415
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Title: An hp-adaptive Predictor-Corrector Split Projection Method for Turbulent Compressible Flow

Author(s): Carrington, David Bradley

Intended for: The 15th International Heat Transfer Conference, 2014-08-10/2014-08-15 (Kyoto, Japan)

Issued: 2014-08-12

An *hp*-adaptive Predictor-Corrector Split Projection Method for Turbulent Compressible Flow

Xiuling Wang

David B. Carrington

Darrell W. Pepper

Proceeding of the 15th International Heat Transfer Conference, IHTC-15

Aug.10-15, 2014, Kyoto, Japan

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Los Alamos NATIONAL LABORATORY EST.1943

UNLV University of Nevada, Las Vegas

Nevada Center for Advanced Computational Methods NCACM

Overview

- Introduction
- Methodologies
  - FE Predictor-Corrector Split Projection Method
  - Adaptation Technology
- Simulation Results
  - Subsonic flow, transonic flow and supersonic flow over 2-D NACA0012 airfoil
  - Supersonic flow over 3-D NACA0012 airfoil
- Conclusions
- Acknowledgement

Introduction

- The Petrov-Galerkin (P-G) weighting is adopted for advection stabilization in the PCS algorithm. In addition, pressure stabilization is produced by the creation of projection system

- The PCS method employs a partially implicit system of equations, if desired, using the θ-method to switch from fully explicit to semi-implicit.

- The method is well suited for parallel computing on clusters using multi-core processors – this work is currently underway.

Introduction – cont.

- ***hp*-adaptation is a powerful numerical technique and has been proven to produce optimal numerical solutions with minimal computational cost.**

- **An *a-posteriori* error estimator based on the L_2 norm is employed to guide the adaptation procedure.**

- **Benchmark results are presented by employing the *hp PCS FEM turbulent model* in solving subsonic, transonic and supersonic flow problems.**

Predictor-Corrector Split FEM

- **Velocity predictor**

$$\{\Delta \mathbf{U}_i^*\} = -\Delta t\left[\mathbf{M}_v^{-1}\right]\left[\, [\mathbf{A}_u]\{\mathbf{U}_i\} + [\mathbf{K}_{\tau u}]\{\mathbf{U}_i\} - \{\mathbf{F}_{v_i}\} - \frac{\Delta t}{2}\left([\mathbf{K}_{char}]\{\mathbf{U}_i\} - \{\mathbf{F}_{char_i}\}\right)\right]^n$$

$\{U_i^*\}$ is an intermediate

where $\{\Delta U_i^*\} = \{U_i^*\} - \{U_i^n\}$

- **Velocity corrector**

$$U^{n+1} - U^* = \Delta t \frac{\partial P'}{\partial x_i}$$

- **A corrector - preserving mass**
  - From continuity

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho u_i}{\partial x_i} = -\frac{\partial U_i}{\partial x_i} \qquad \frac{\rho^{n+1} - \rho^n}{\Delta t} = -\frac{\partial U'_i}{\partial x_i}$$

PCS FEM - cont.

- A corrector - preserving mass

Define $U^{'} = \theta_1 U^{n+1} + (1-\theta_1) U^n$

Desire $U^{n+1} - U^{*} = \Delta t \dfrac{\partial P^{'}}{\partial x_i}$ Let $U_i^{'} = \theta_1 \left(-\Delta t \dfrac{\partial P^{'}}{\partial x_i} + U_i^{*} \right) + (1-\theta_1) U_i^n$

Then

$$\frac{1}{c^2}\Delta P = \Delta \rho = -\Delta t \frac{\partial U_i^{'}}{\partial x_i} = -\Delta t \frac{\partial}{\partial x_i}\left[\left(\theta_1 (-\Delta t) \frac{\partial P^{'}}{\partial x_i} + \theta_1 U_i^{*} \right) + (1-\theta_1) U_i^n \right]$$

$$\frac{1}{c^2}\Delta P = \Delta \rho = -\Delta t \frac{\partial U_i^{'}}{\partial x_i} = \left[\left(\Delta t^2 \theta_1 \frac{\partial^2 P^{'}}{\partial x_i^2} - \Delta t \theta_1 \frac{\partial U_i^{*}}{\partial x_i} \right) - \Delta t (1-\theta_1) \frac{\partial U_i^n}{\partial x_i} \right]$$

Let

$$P^{'} = \theta_2 P^{n+1} + (1-\theta_2) P^n$$

PCS FEM - cont.

$$\Delta U^* = U^* - U^n$$

$$\frac{1}{c^2}\Delta P = \Delta\rho = -\Delta t \frac{\partial U_i^{'}}{\partial x_i} = \Delta t^2 \theta_1 \left(\theta_2 \frac{\partial^2 P^{n+1}}{\partial x_i^2} + \left(1-\theta_2\right) \frac{\partial^2 P^n}{\partial x_i^2} \right) - \Delta t \left(\theta_1 \frac{\partial \Delta U_i^*}{\partial x_i} + \frac{\partial U_i^n}{\partial x_i} \right)$$

$$\Delta P = P^{n+1} - P^n$$

$$\Delta\rho - \theta_2 \frac{\partial^2 \Delta P}{\partial x_i^2} = \frac{1}{c^2}\Delta P - \theta_1\theta_2 \frac{\partial^2 \Delta P}{\partial x_i^2} = \Delta t^2 \theta_1 \frac{\partial^2 P^n}{\partial x_i^2} - \Delta t \left(\theta_1 \frac{\partial \Delta U_i^*}{\partial x_i} + \frac{\partial U_i^n}{\partial x_i} \right)$$

Let

$$P^{'} = \theta_2 P^{n+1} + (1-\theta_2) P^n$$

PCS FEM - cont.

FEM matrix form

$$\left(\left[\mathbf{M}_p\right]+\Delta t^2 c^2 \theta_1 \theta_2 \mathbf{H}\right)\left\{\Delta \rho_i\right\}=\left(\left[\frac{\mathbf{M}_p}{c^2}\right]+\Delta t^2 \theta_1 \theta_2 \mathbf{H}\right)\left\{\Delta P_i\right\}=$$

$$\Delta t^2 \theta_1 \mathbf{H}\left\{P_i^n\right\}-\Delta t\left(\theta_1 \mathbf{G}\left\{\Delta \mathbf{U}_i^*\right\}+\mathbf{G}\left\{\mathbf{U}_i^n\right\}\right)-\Delta t\left\{\mathbf{F}_{P_i}\right\}$$

Velocity Corrector

Now $$P^{n+1} = \Delta P + P^n$$

recall $$P^{'} = \theta_2 P^{n+1} + (1-\theta_2) P^n = \theta_2 \Delta P + P^n$$

Then $$\Delta U_i = U^{n+1} - U^n = \Delta U^* - \Delta t \frac{\partial P^{'}}{\partial x_i} = \Delta U^* - \Delta t \left(\theta_2 \frac{\partial \Delta P}{\partial x_i} + \frac{\partial P^n}{\partial x_i} \right)$$

FEM Matrix form

$$\left\{ \Delta \mathbf{U}_i \right\} = \left\{ \Delta \mathbf{U}^* \right\} - \Delta t \left[\mathbf{M}_u^{-1} \right] \left(\theta_2 [\mathbf{G}] \{ \Delta p_i \} + [\mathbf{G}] \left\{ p_i^{\,n} \right\} \right)$$

where $$\left\{ \mathbf{U}_i^{\,n+1} \right\} = \left\{ \Delta \mathbf{U}_i \right\} + \left\{ \mathbf{U}_i^{\,n} \right\}$$

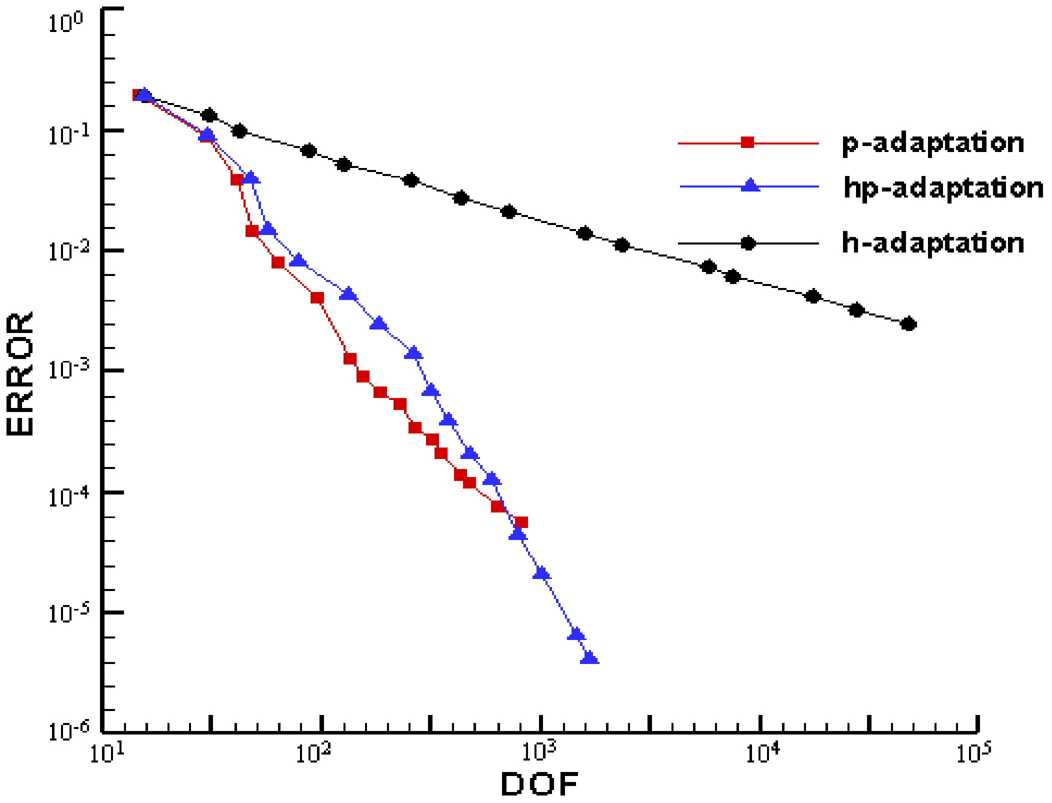
Final mass conserving velocity

$$u^{n+1} = U^{n+1} / \rho^{n+1}$$

Adaptation Methodology

	h-	*p-*	*r-*	*hp-*
element size	various	constant	various	various
DOF	various	various	constant	various
shape function	constant	various	Constant	various
advantages	elements will not become overly distorted	relative coarse mesh may be sufficient	no new nodes need to be added	exponential convergence rate
disadvantages	difficulty in dealing with constraint nodes	coding complexity	elements may become overly distorted	difficulty in dealing with constraint nodes and coding complexity

hp-adaptation



Error Estimator

- Various error estimators exist in the literature:
  - The element residual method, interpolation methods, subdomain-residual methods, and projection method.
  - The stress error measure based on L_2 norm

$$\|e_\sigma\| = \left(\int_\Omega e_\sigma^T e_\sigma \, d\Omega \right)^{1/2}$$

where the total element error and error index

$$\|e_\sigma\|^2 = \sum_{i=1}^{m} \|e_\sigma\|_i^2 \qquad \eta_\sigma = \left(\frac{\|e_\sigma\|^2}{\|\sigma^*\|^2 + \|e_\sigma\|^2} \right)^{1/2} \times 100\%$$

η_σ is the error index, in error percentage form

σ^* is the continuous solution obtained by a projection or averaging process

Error Estimator – cont.

- Both global and local error conditions have to be met
  - Global percentage error should not be greater than an maximum specified percentage error $\eta \le \bar{\eta}_{\max}$
  - Local relative percentage error of any single element should not be greater than the averaged error among all the elements in the domain

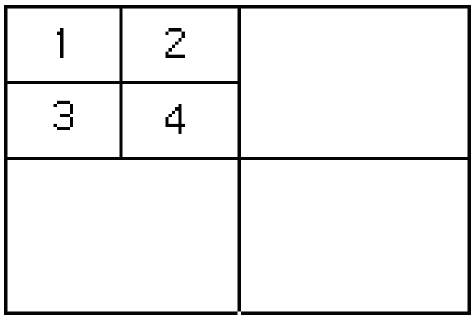
$$\left\| e_\sigma \right\|_i \le \bar{e}_{avg} \qquad \bar{e}_{avg} = \bar{\eta}_{\max} \left[\frac{\left(\left\| \sigma^* \right\|^2 + \left\| e_\sigma \right\|^2 \right)}{m} \right]^{1/2}$$

Local element refinement indicator is defined to decide if a local refinement for an element is needed.

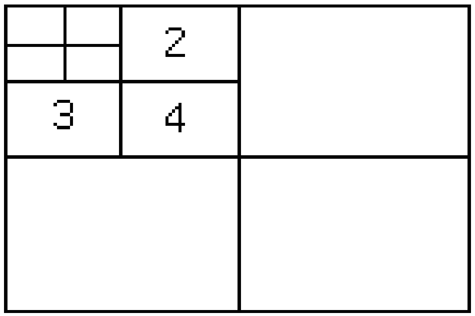
$$\xi_i = \left\| e \right\|_i / \bar{e}_{avg}$$

Adaptation Rule

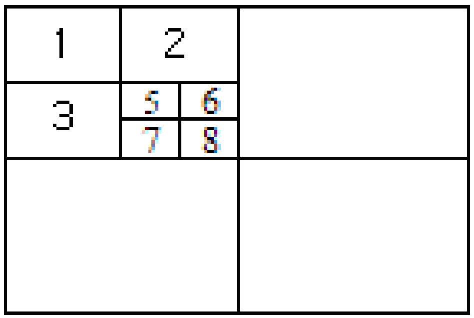
- **1-irregular mesh adaptation rule in *h*- adaptation**
  - **An element can be refined only if its neighbors are at the same or higher adaptation level**



(a) Initial *h*-adaptive mesh



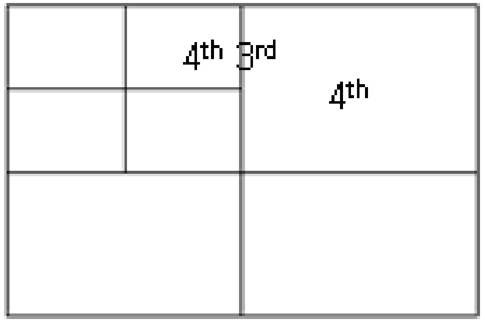
(b) Correct *h*-adaptive mesh



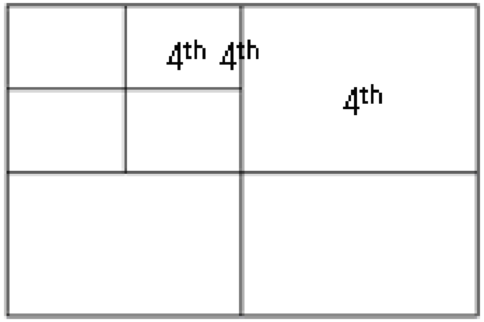
(c) Incorrect *h*-adaptive mesh

Adaptation Rule

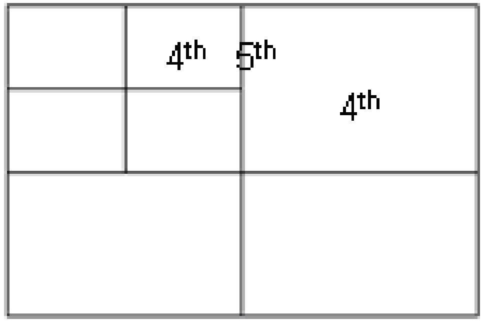
- **Minimum rule in *p*-adaptation**



(a) Initial mesh



(b) Correct *p*-adaptive mesh



(c) Incorrect *p*-adaptive mesh

Adaptation Strategies

- **If a local element refinement indicator $\xi_i > 1$, mesh need to be refined or enriched.**
- **In an *h*-adaptive process, the new element size is calculated using**

$$h_{new} = \frac{h_{old}}{\xi_i^{1/p}}$$

- **In a p-adaptive process, the new shape function order is calculated using:**

$$p_{new} = p_{old}\,\xi_i^{1/p}$$

Adaptation Strategies

- **Three steps are followed in *hp*-adaptation:**
  - **Step 1: Construct initial coarse mesh, preset target value for error**
  - **Step 2: Construct the intermediate *h*-adaptive mesh**

$$h_{new} = \frac{h_{old}}{\xi_i^{1/p}}$$

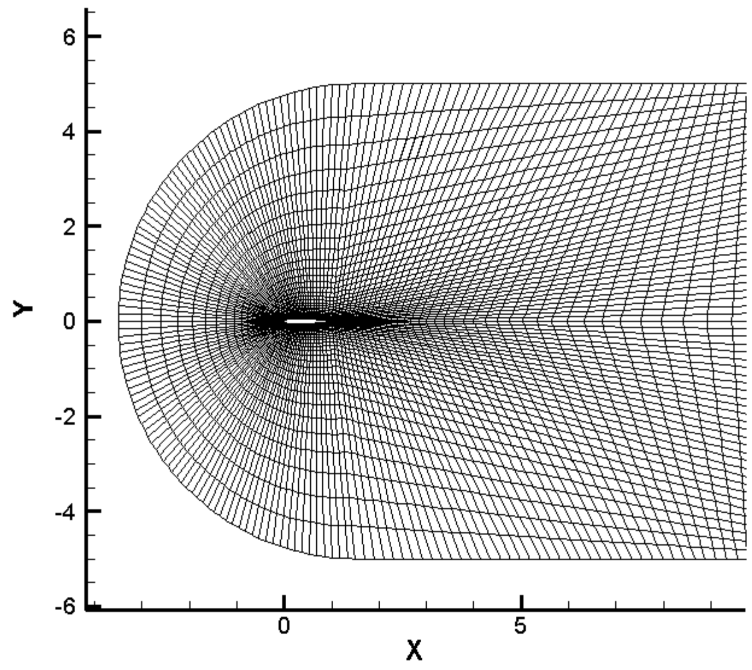
  - **Step 3: Apply *p*-adaptive enrichments on the intermediate mesh to obtain the final *hp*- adaptive mesh.**

$$p_{new} = p_{old}\,\xi_i^{1/p}$$

PCS hp-FEM Benchmark Results

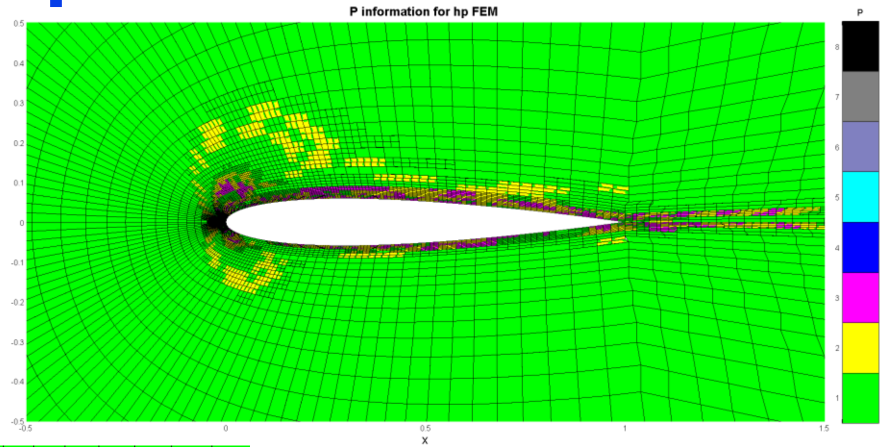
- **Subsonic flow over 2-D NACA0012 airfoil**

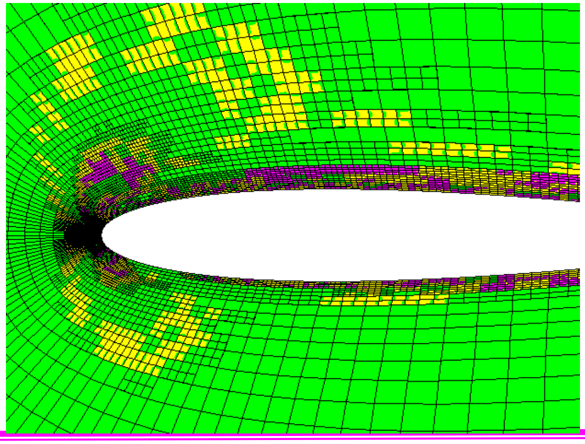
Mach = 0.502, attack angle α = 2.06º

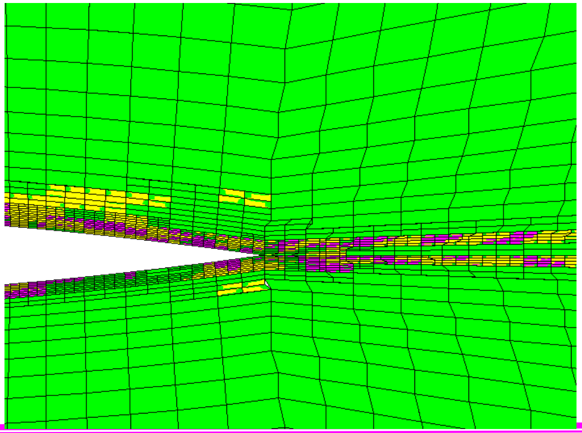


Subsonic Flow Simulation

- hp-adaptive mesh

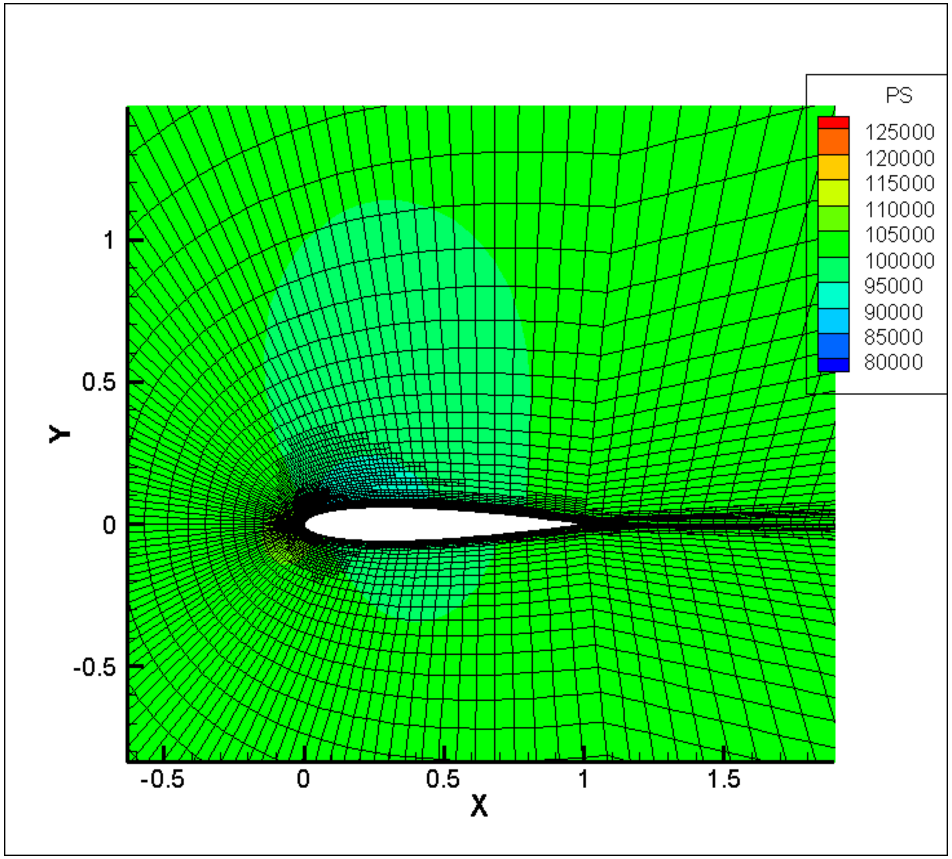


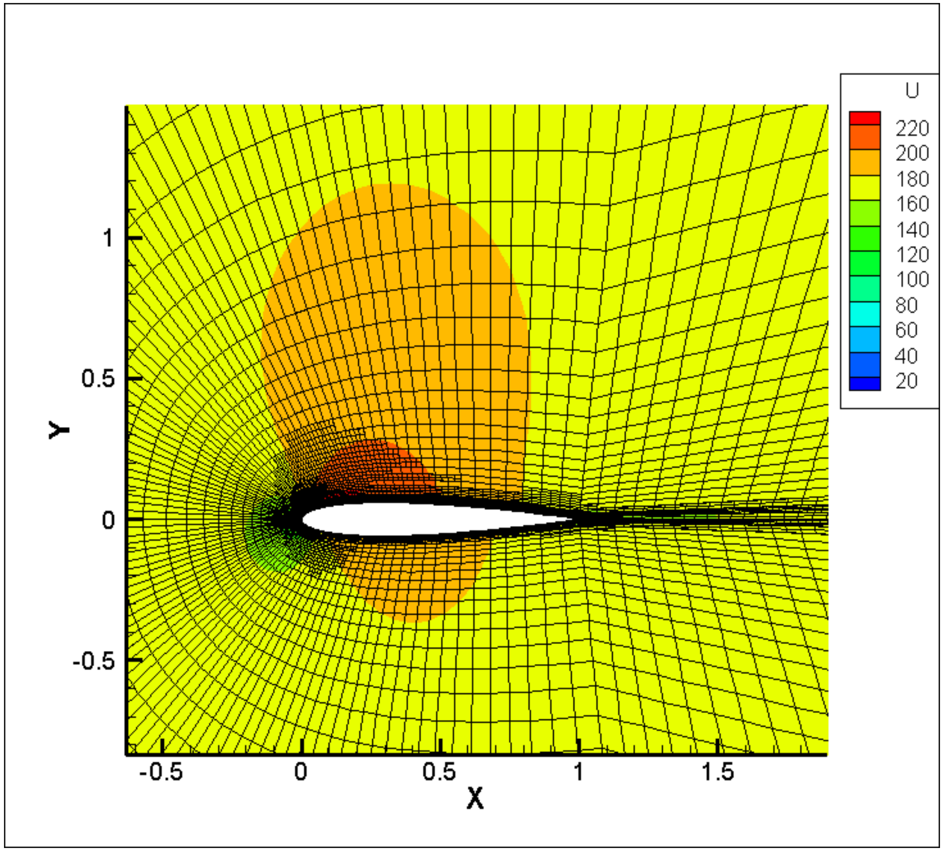




Subsonic Flow Simulation

- Pressure and Velocity Contours





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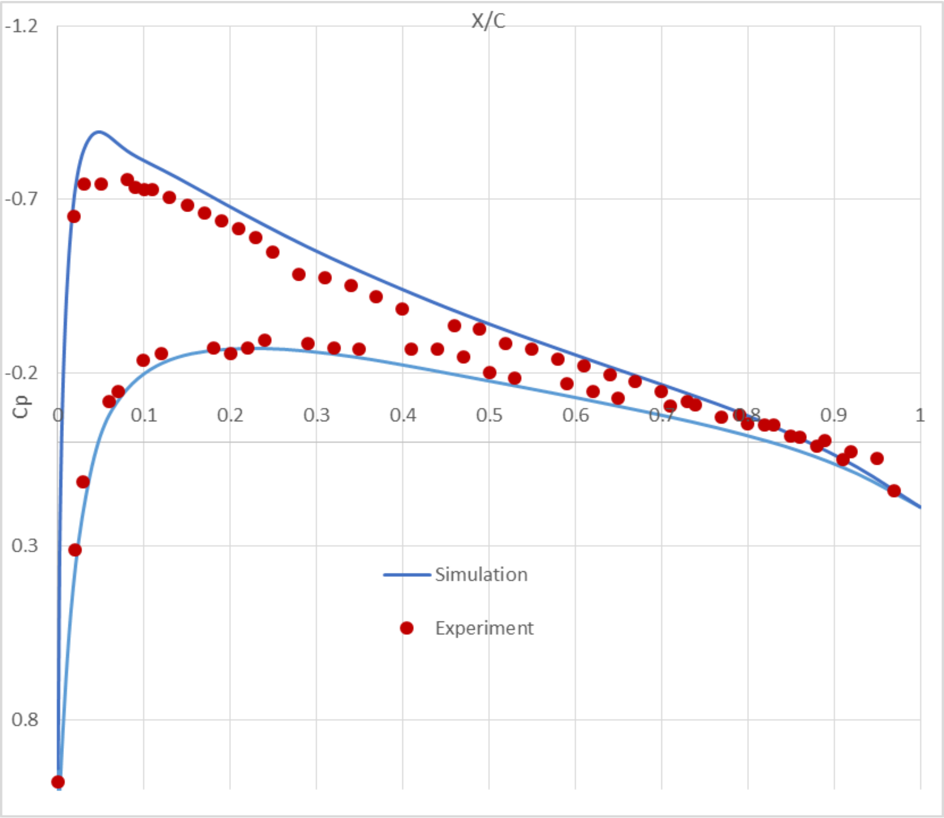
Subsonic Flow Simulation

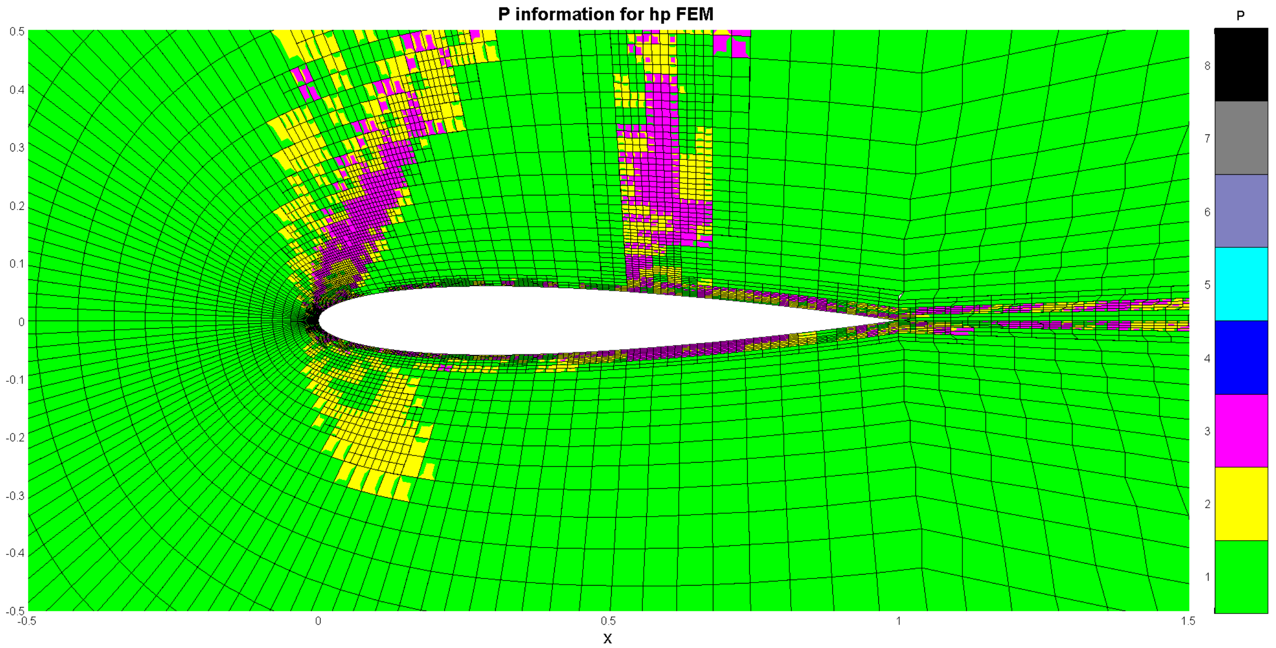
- Comparison with experimental data



KIVA-hp Benchmark Results

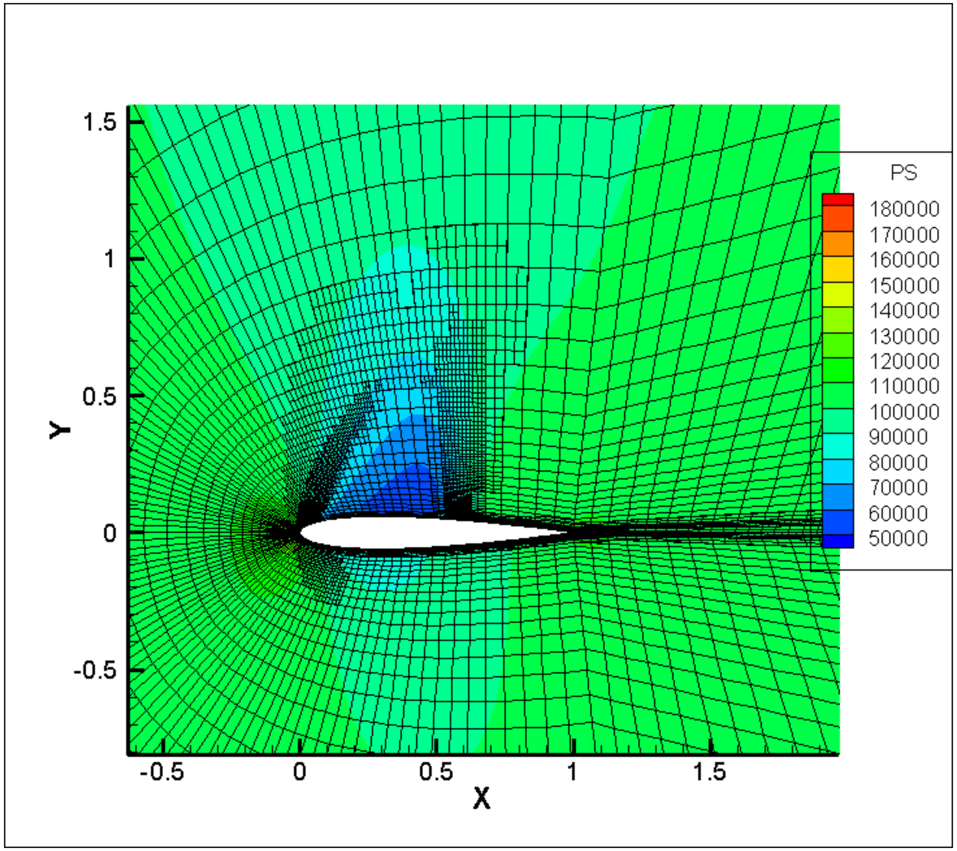
- Transonic flow over 2-D NACA0012 airfoil

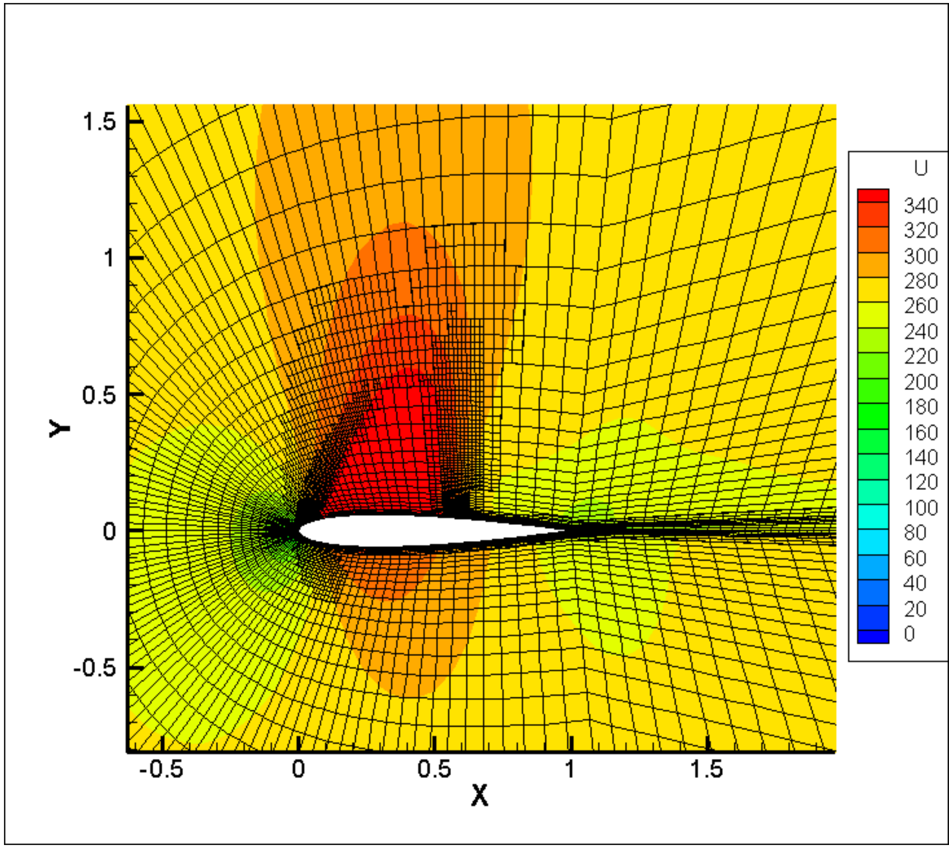
Mach = 0.775, attack angle α = 2.05°



Transonic Flow Simulation

- Pressure and Velocity Contours





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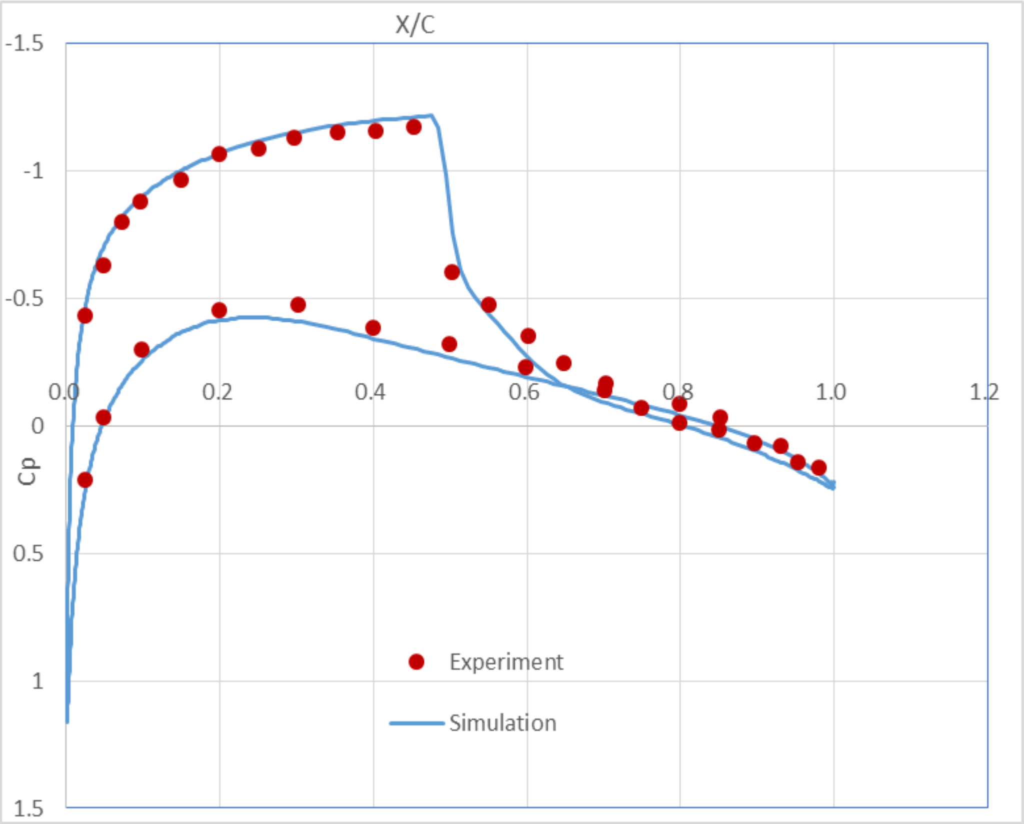
Transonic Flow Simulation

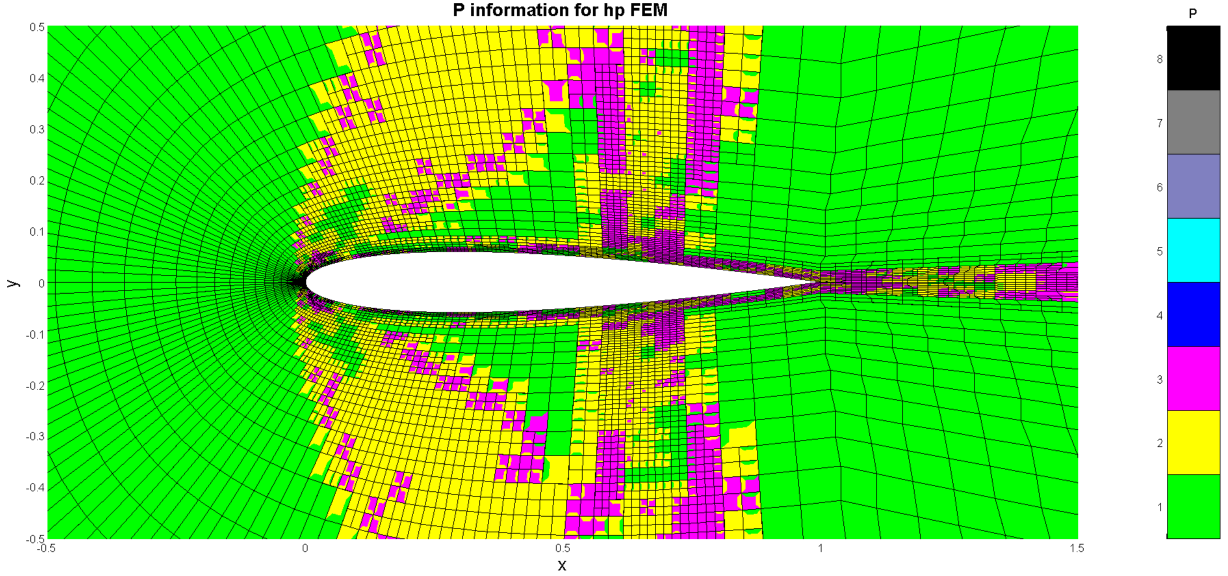
- Comparison with experimental data



KIVA-hp Benchmark Results

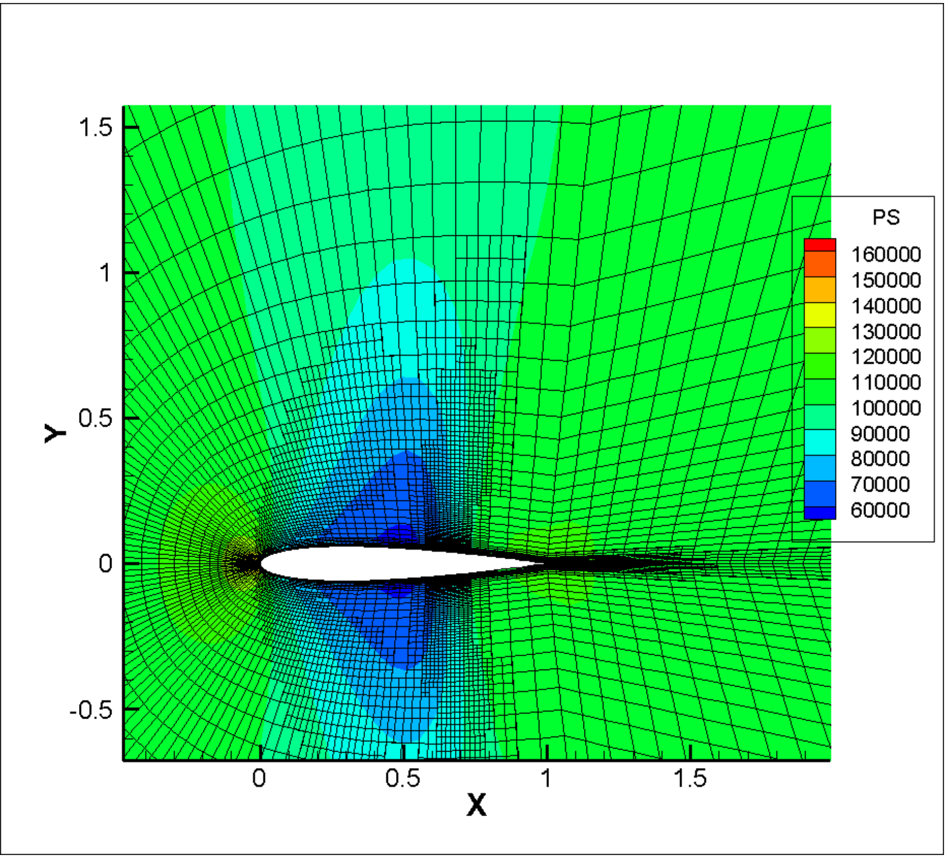
- Supersonic flow over 2-D NACA0012 airfoil

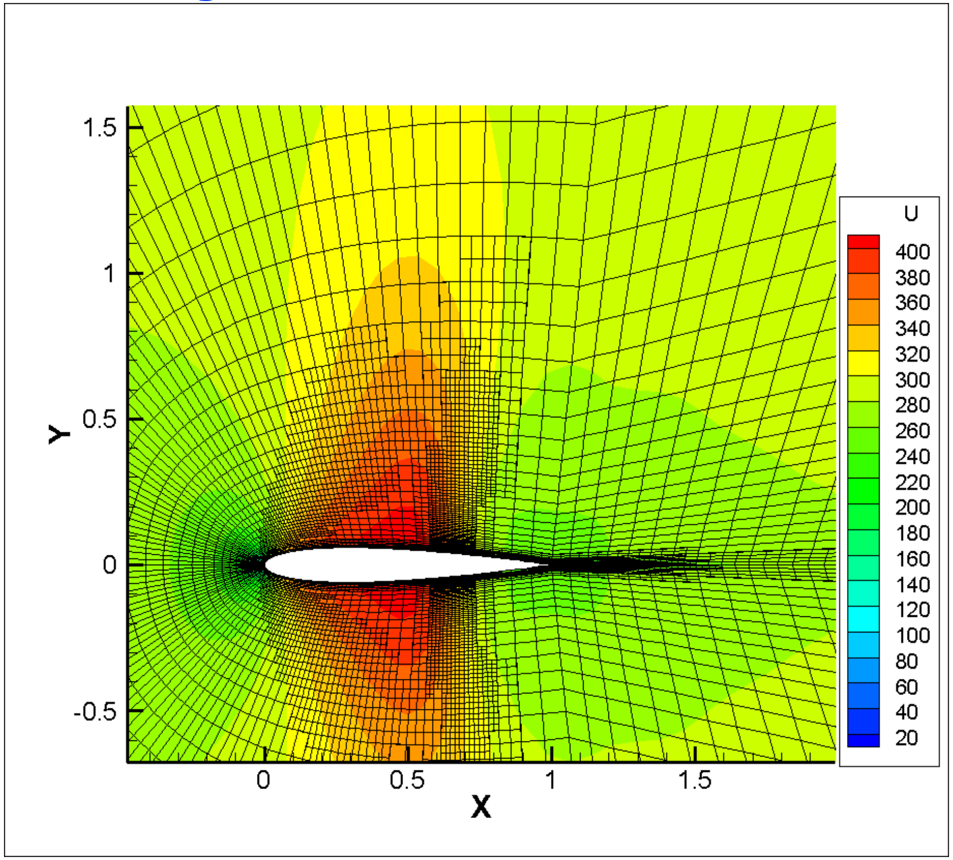
Mach = 0.829, attack angle α = 0.05º



Supersonic Flow Simulation

- Pressure and Velocity Contours





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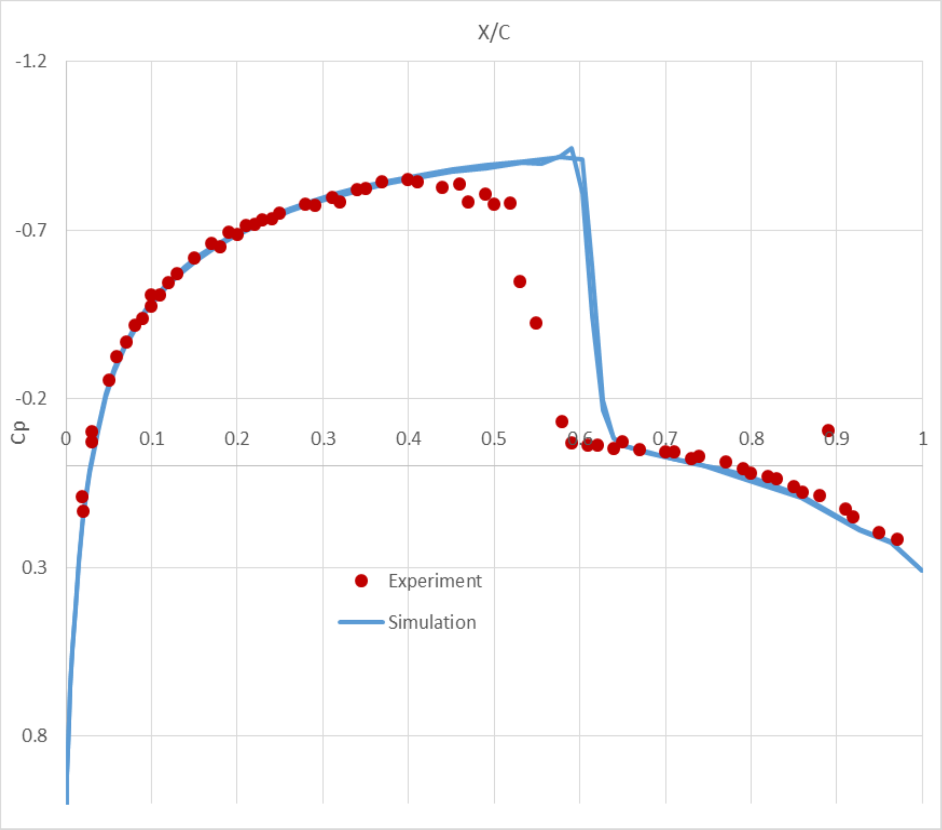
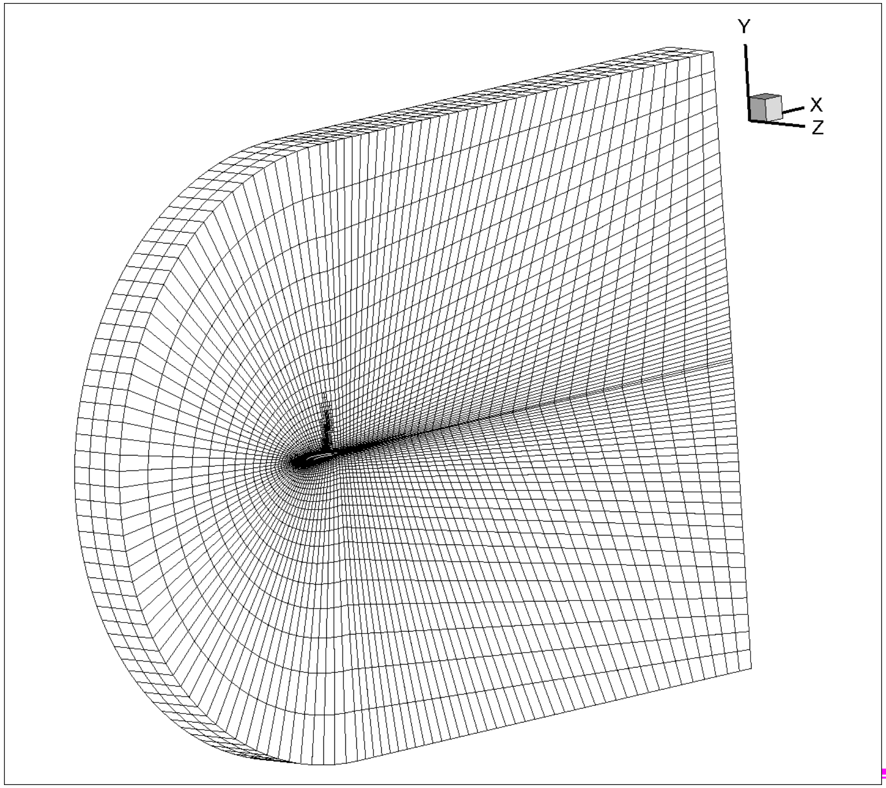
Supersonic Flow Simulation

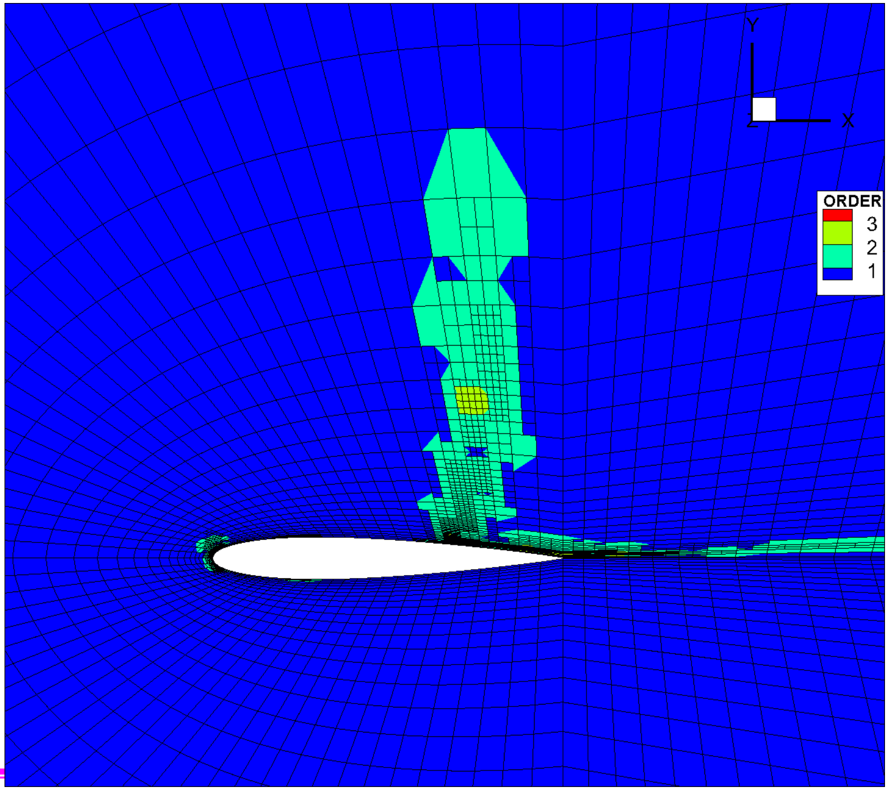
- Comparison with experimental data



3-D Simulation Results

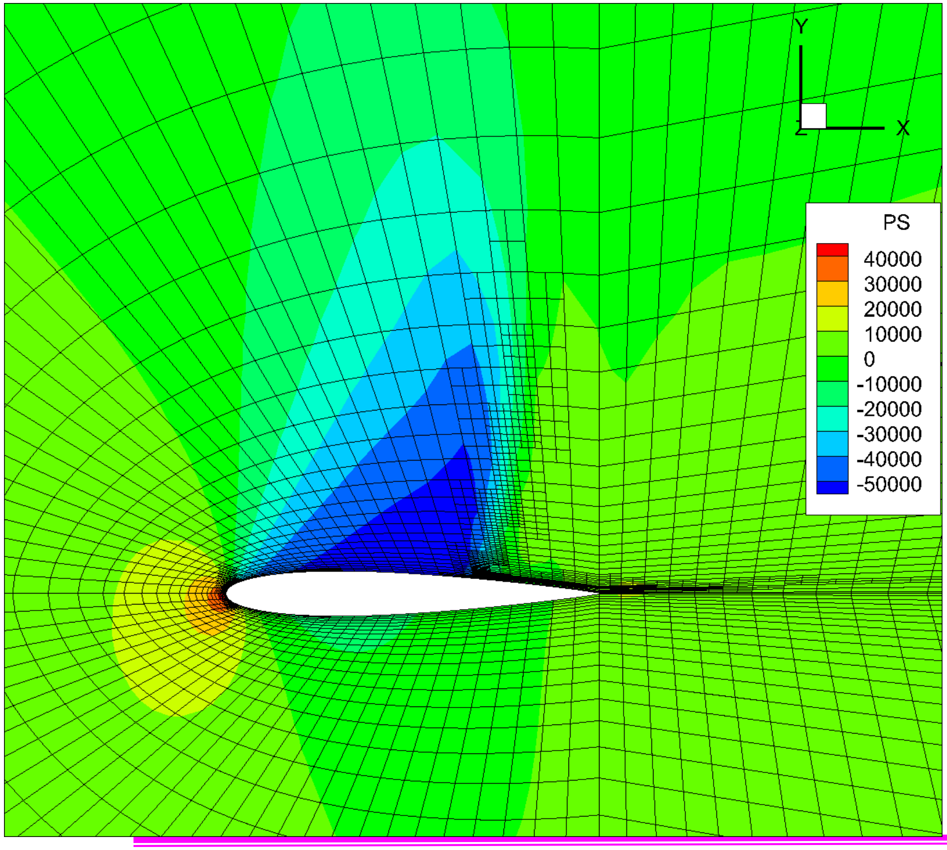
- **Supersonic flow over 3-D NACA0012 airfoil** **Mach = 0.008, attack angle α = 4.00º**

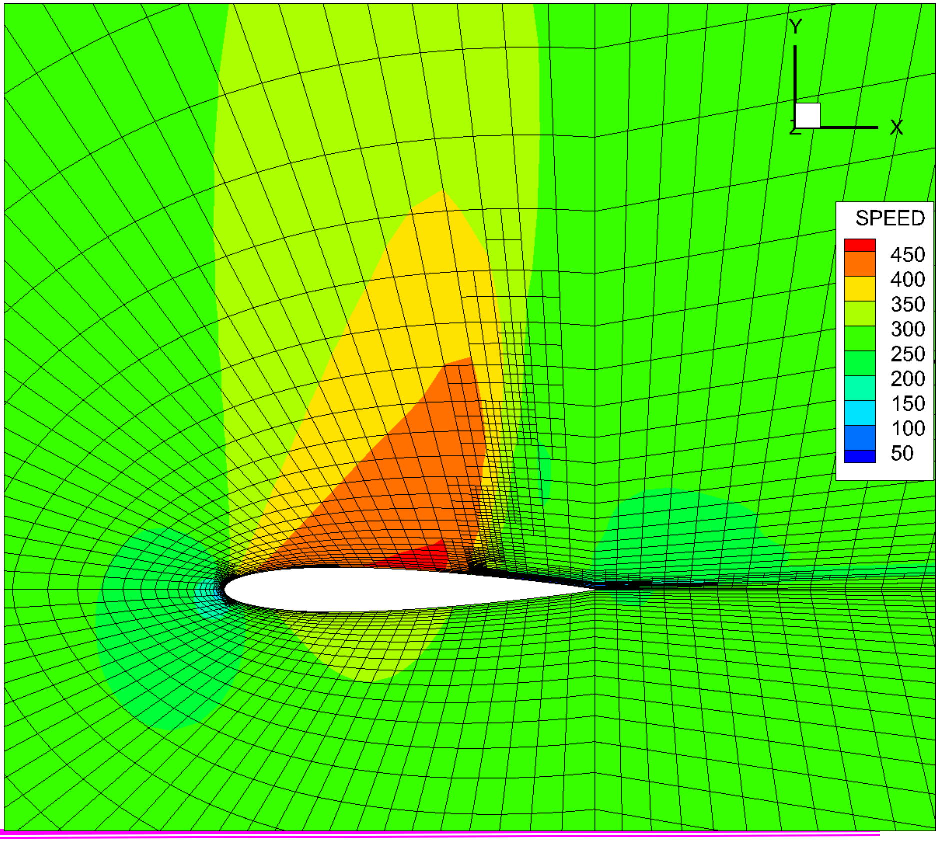




3-D Supersonic Flow Simulation

- Pressure and Velocity Contours





Conclusion

- An *hp*-adaptive PCS FEM method is used to simulate both subsonic, transonic and supersonic flow regimes over a set of NACA airfoils.

- The algorithm shows promise in its efficiency, accuracy and robust.

- Combined with KIVA spray and chemistry models and a moving mesh capability, the algorithm is being implemented into a new generation of KIVA software – parallel KIVA-*hp*FE.

Acknowledgment

Financial support for this effort from Los Alamos National Laboratory through the DOE's Office of Energy Efficiency and Renewable Energy (EERE) Advanced Combustion Program is greatly appreciated

Contacts

Dr. Xiuling Wang
Purdue University Calumet
wangx@purduecal.edu

Dr. David B. Carrington
Los Alamos National Laboratory
dcarring@lanl.gov

Dr. Darrell W. Pepper
University of Nevada Las Vegas
dwpepper@nscee.edu