SAND2007-2485C

Design Space Exploration for Robust Global Optimization of Micro Bistable Mechanisms

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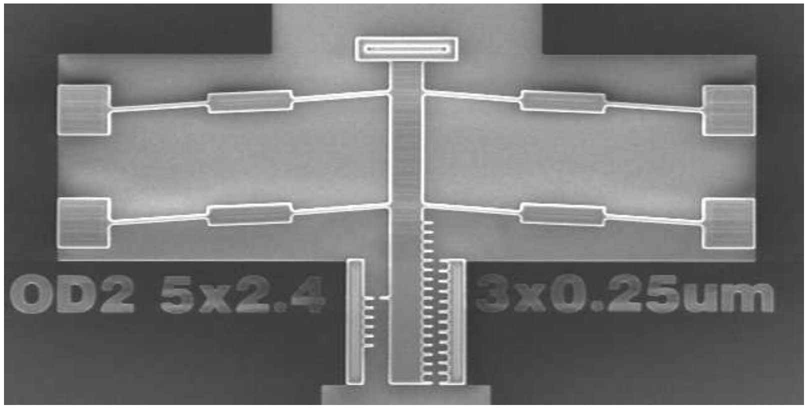
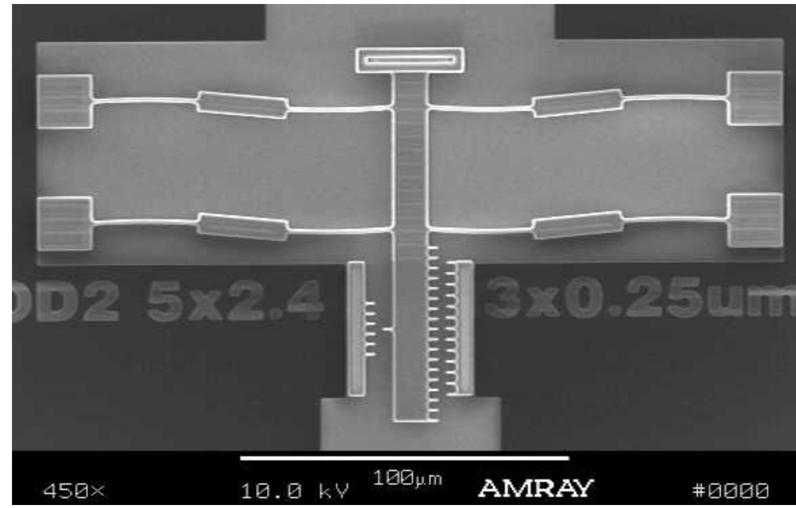
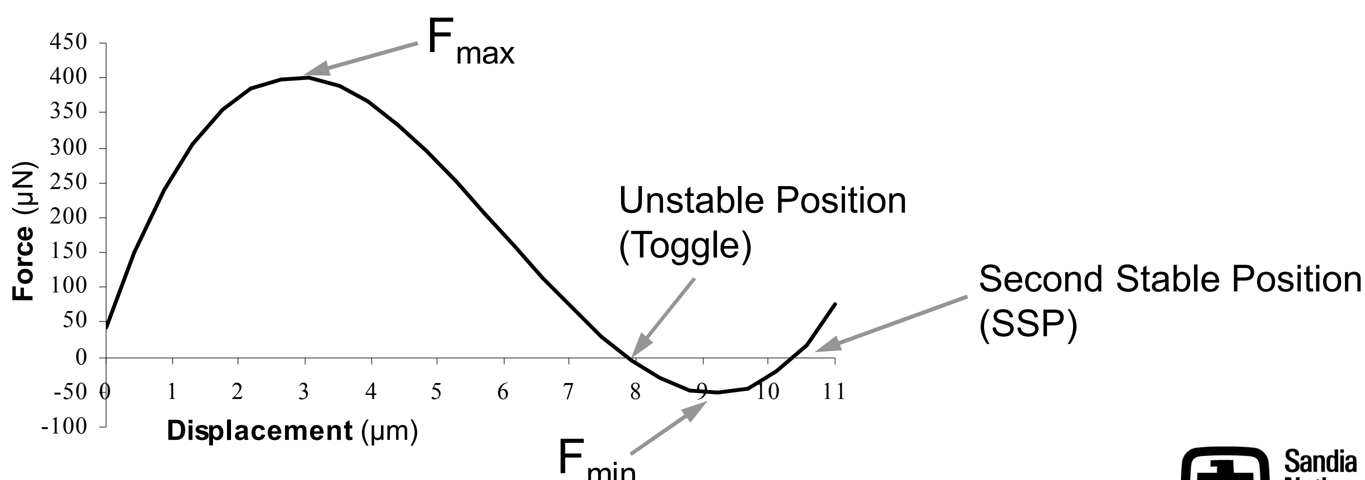
Bistable Micro Mechanism



As Fabricated (First Stable Position)



Second Stable Position (SSP)



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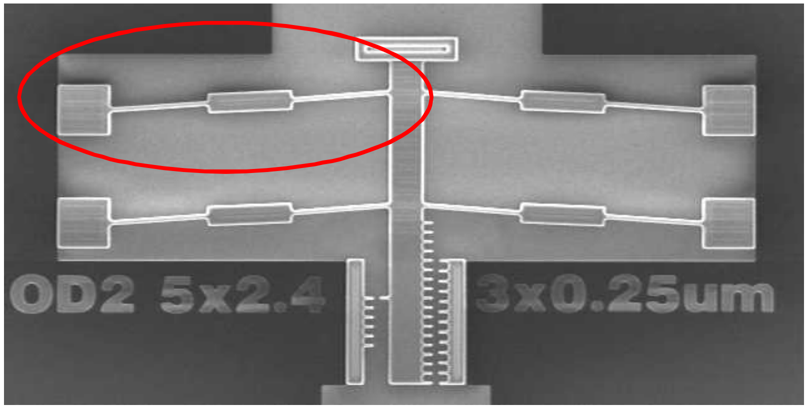
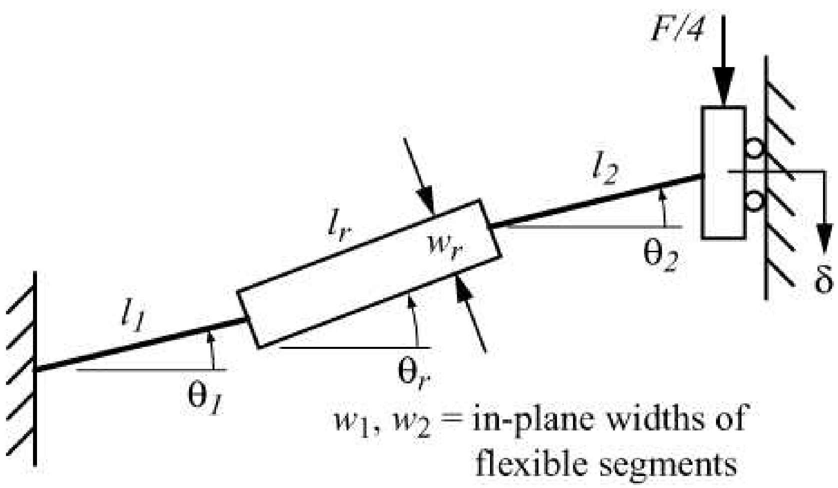
Model and Design Parameters





Quarter Model

Model Details

1. Matlab-Ansys
   - 1D nonlinear, full model
   - 2.25 minutes / eval
2. Dakota-ARIA
   - 2D nonlinear, ¼ model
   - 5.7 minutes / eval

Design Parameters

- Beam lengths: L_1, L_2, L_r
- Beam angles: q1, q2, qr
- Beam widths

$$L_1 = L_2$$
$$\theta_1 = \theta_2$$
$$w_1 = w_2$$

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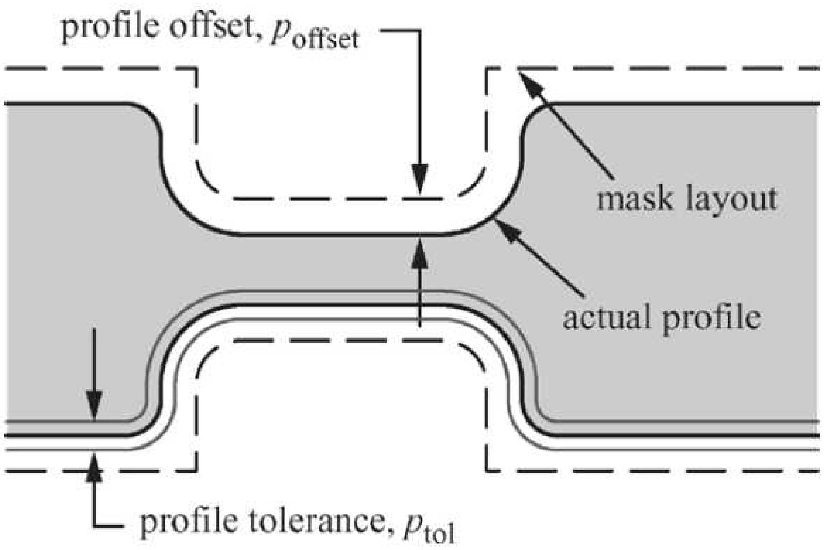
Uncertainties

1. **Residual Stress: -10 ± 6.4 MPa (compressive)**
2. **Edge Bias (due to Lithographic variations)**
   - **Results in a profile offset**
   - **Nominal: 0.1 μm per edge**
   - **Variation across a wafer: 0.02 – 0.18 μm**



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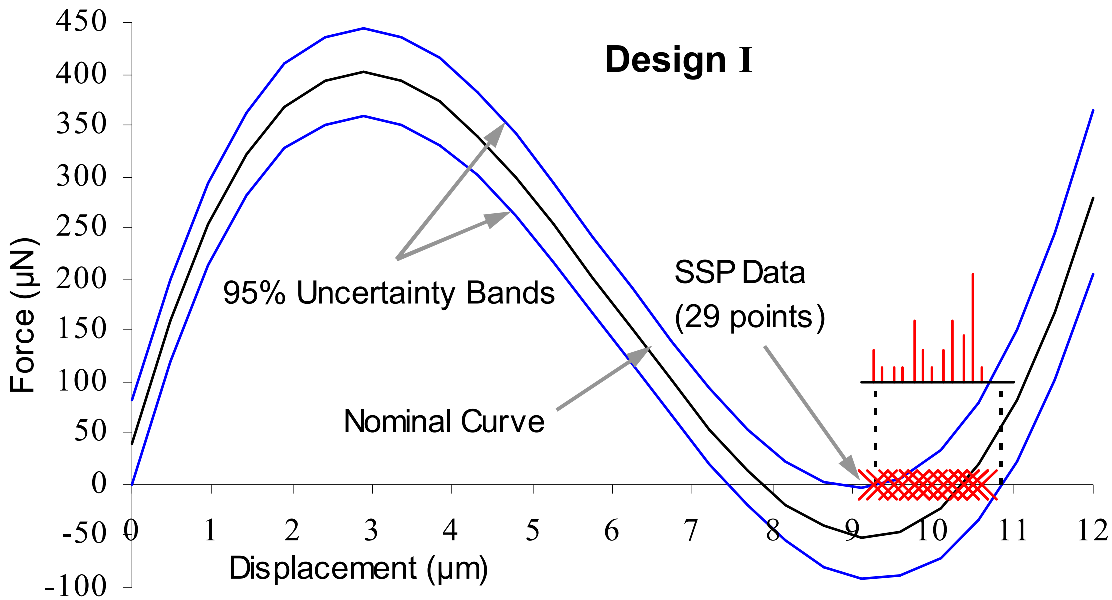
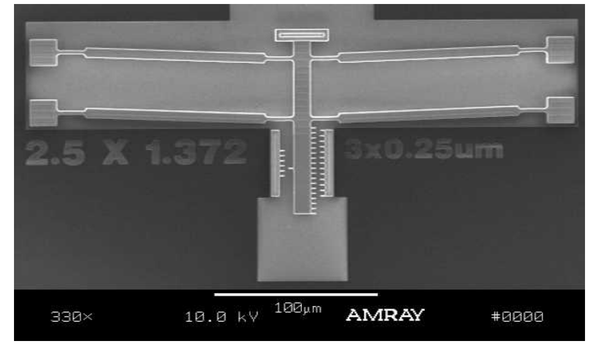
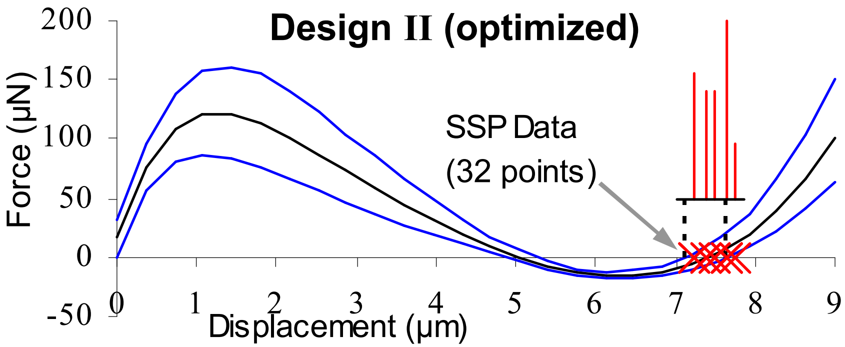
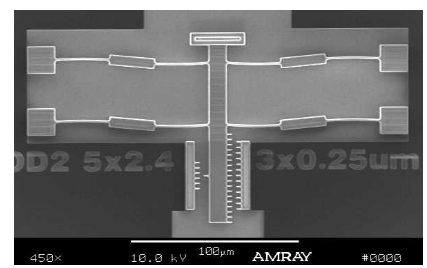
Robust Optimization





F_{min} = -51 ± 47 μN





F_{min} = -16 ± 3 μN

SSP data courtesy of Mike Baker, Sandia National Laboratories
SEM courtesy of Jeremy Walraven, Sandia National Laboratories

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Motivation for Design Space Exploration

- We frequently design new bistable mechanisms
  - Different Forces and Displacements
  - Different Fabrication Processes
- Past design optimization has shown there to be many local minima
- Random starting locations leads to a great deal of wasted analysis time
- Perhaps the design space can be reduced

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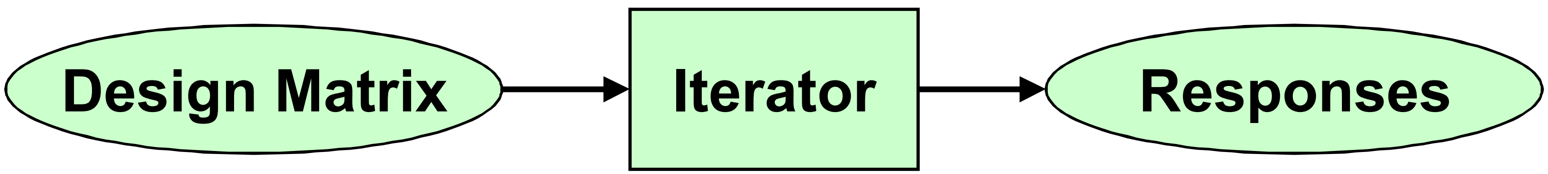
Design Space Exploration (6 Variables)

- **Grid Sampling**
  - Too many points, 10^6 = 1,000,000 points
- **Latin Hypercube Sampling**
  - Can choose the number of points
  - More effective space-filling than Monte Carlo simulation



$$\begin{array}{cccccc} L_1 & \theta_1 & w_1 & L_r & \theta_r & w_r \end{array}$$

$$X = \begin{bmatrix} 21.71 & 4.43 & 1.12 & 47.67 & 3.97 & 6.39 \\ 13.81 & 1.15 & 1.51 & 19.09 & 4.72 & 6.97 \\ 6.06 & 3.54 & 1.89 & 45.89 & 1.52 & 6.99 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \rightarrow Y = \begin{bmatrix} 1308.6 & 156.8 & \ldots & -58.5 & \ldots \\ 1034.3 & 182.5 & \ldots & 64.8 & \ldots \\ 2094.6 & 516.1 & \ldots & 581.5 & \ldots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

$$\begin{array}{ccccc} S_{max} & F_{max} & & F_{min} & \end{array}$$

Iterator: Evaluates 1 row at a time, or parallelized

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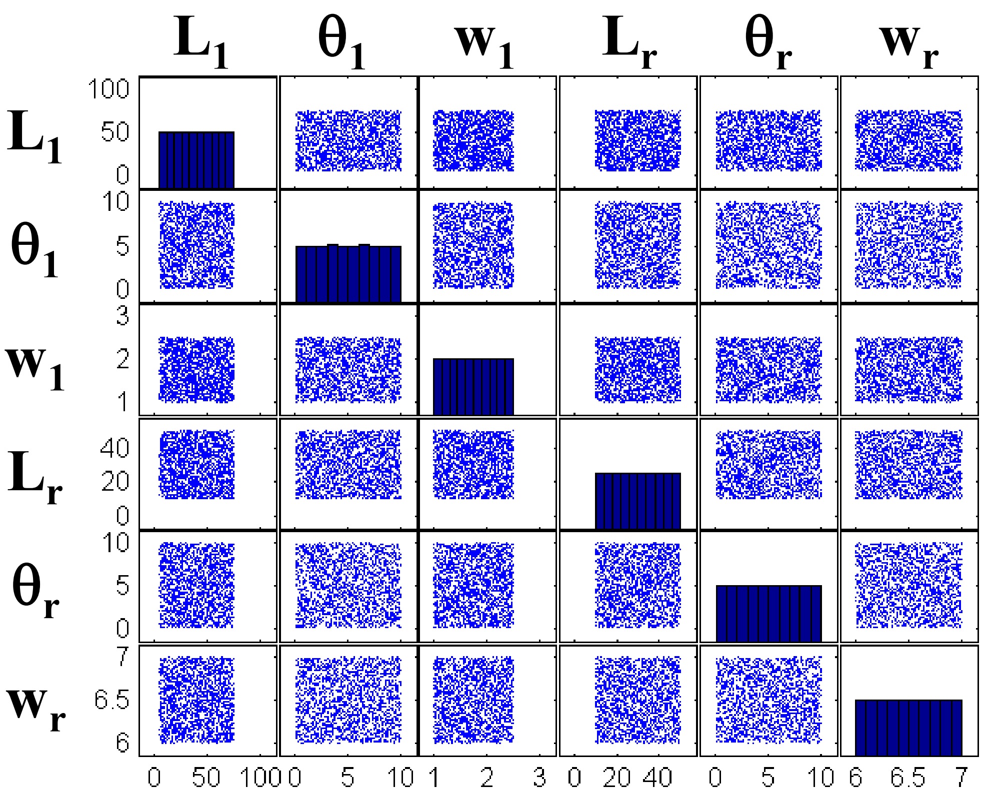
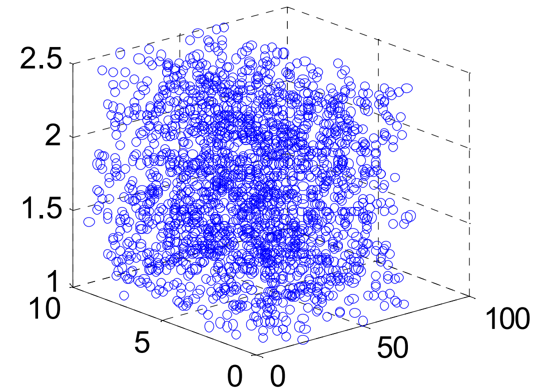
Design Space Visualization – Plot Matrix

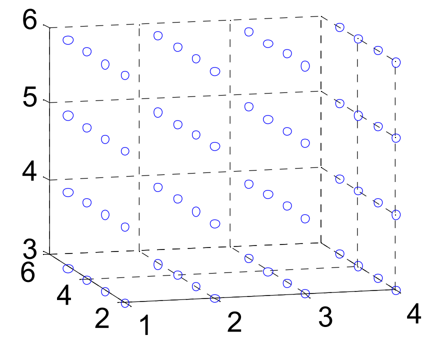
- **plotmatrix(X,X)**
  - **Creates a matrix of 2D scatter plots**
  - **Histograms for each variable along the diagonal**



3D Projection of LHS Design 2000 points



3D Projection of Grid Design 4096 points (4^6)



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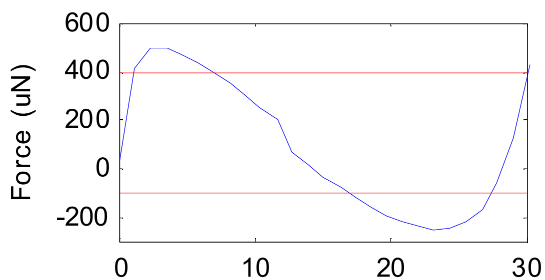
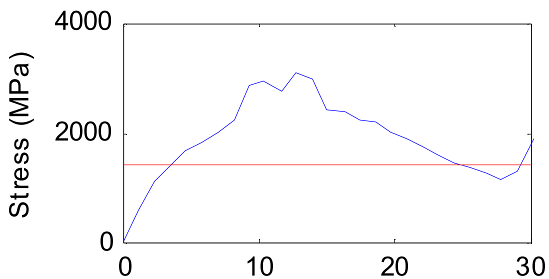
Simulation Run Times

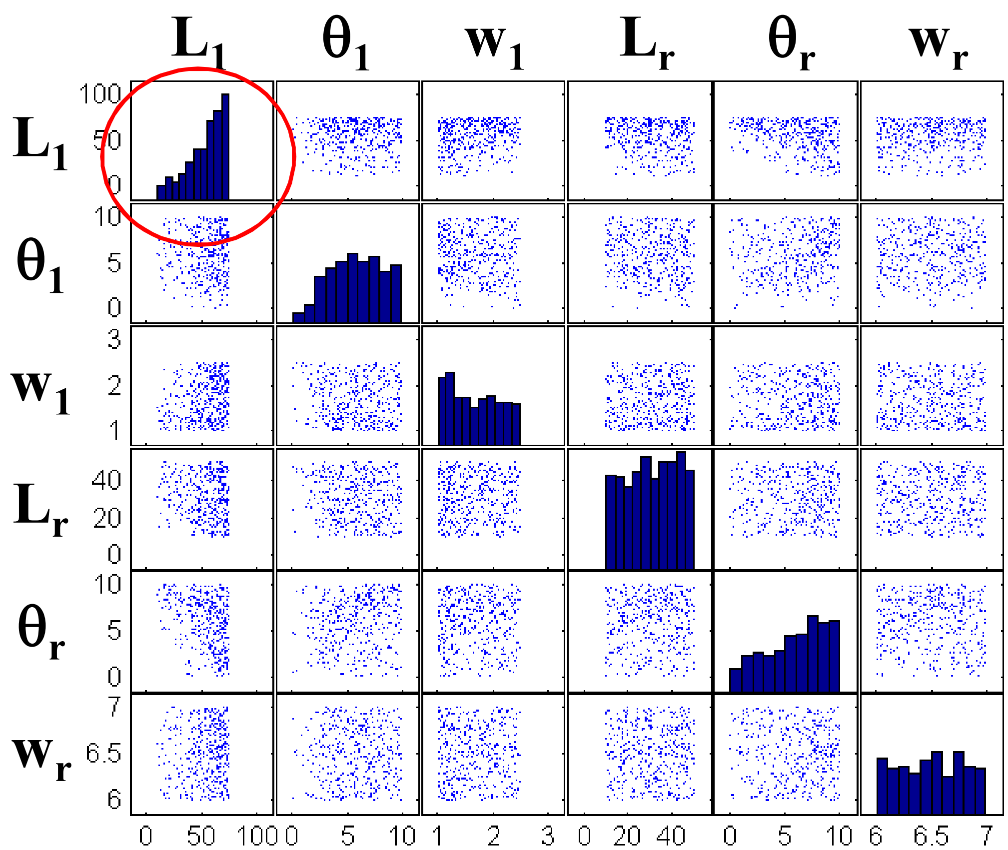
- **Matlab – Ansys**
  - **2000 Evaluations**
  - **1 Processor @ 2.25 minutes / eval ~ 75 hours**
  - **Ran over the weekend, and stopped it when I came into work (ran 1733 Evals in 65 hours)**
  - **411 Failures**
- **DAKOTA – Aria**
  - **10,000 Evaluations**
  - **100 Processors @ 5.75 minutes / eval ~ 10 hours**
  - **Actually ran in two stages (6 nodes then 50 nodes) for a total wall time of about 27 hours**
  - **793 Failures**
- ***All subsequent slides show the Matlab-Ansys results***

Failures

- Only obvious trend is that most failures occurred for the longer beams (L1)
- Main cause for failure was non-convergence due to buckling
- Failures scattered througout the design space







We don't want buckling, so failure to converge is a convenient filter

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Find Set of Starting Points for Optimization

- **Started with 1733 Designs, 411 of these were Failures**
  - **Filtered by Second Stable Position < 15 μm**
    - **445 (445) Total Unqualified (Filtered in Sequence)**
  - **Filtered by Unstable Equilibrium Position > 3 μm**
    - **82 (82)**
  - **Filtered by Maximum Stress < 1400 MPa**
    - **593 (332)**
  - **Filtered by Bistability (Must be Bistable)**
    - **318 (173)**
  - **Filtered by Force Ratio < 0.25 or Fmax > 8*Fmin**
    - **571 (119)**
  - **Filtered by Maximum Force < 400 μN**
    - **454 (5)**
  - **Filtered by Minimum Force > -100 μN**
    - **425 (0)**
- **Ended with 156 Points to Feed to Optimization**

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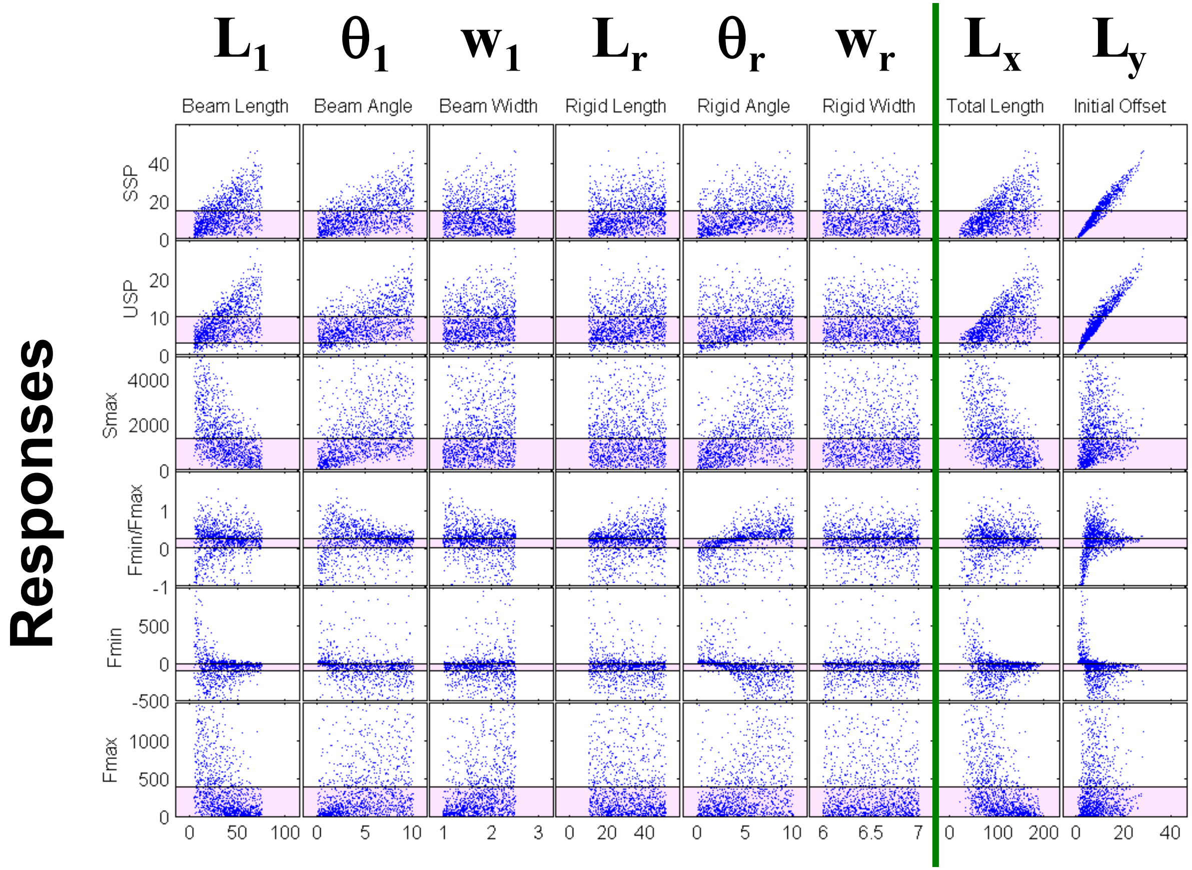
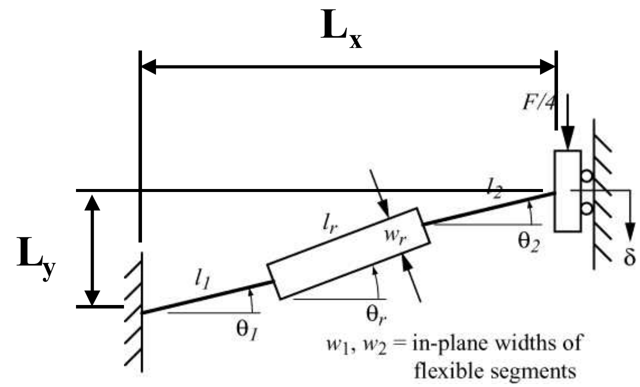
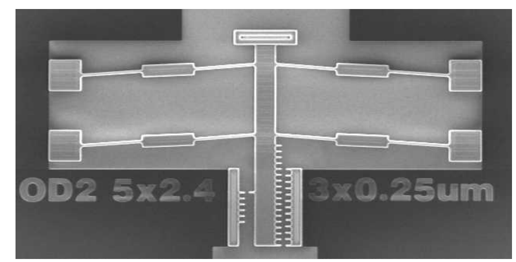
Response Analysis and Constraints







- **Filtered out all points NOT in the pink region**
- **Can pre-filter (no need to analyze) designs with Ly > 15 μm**

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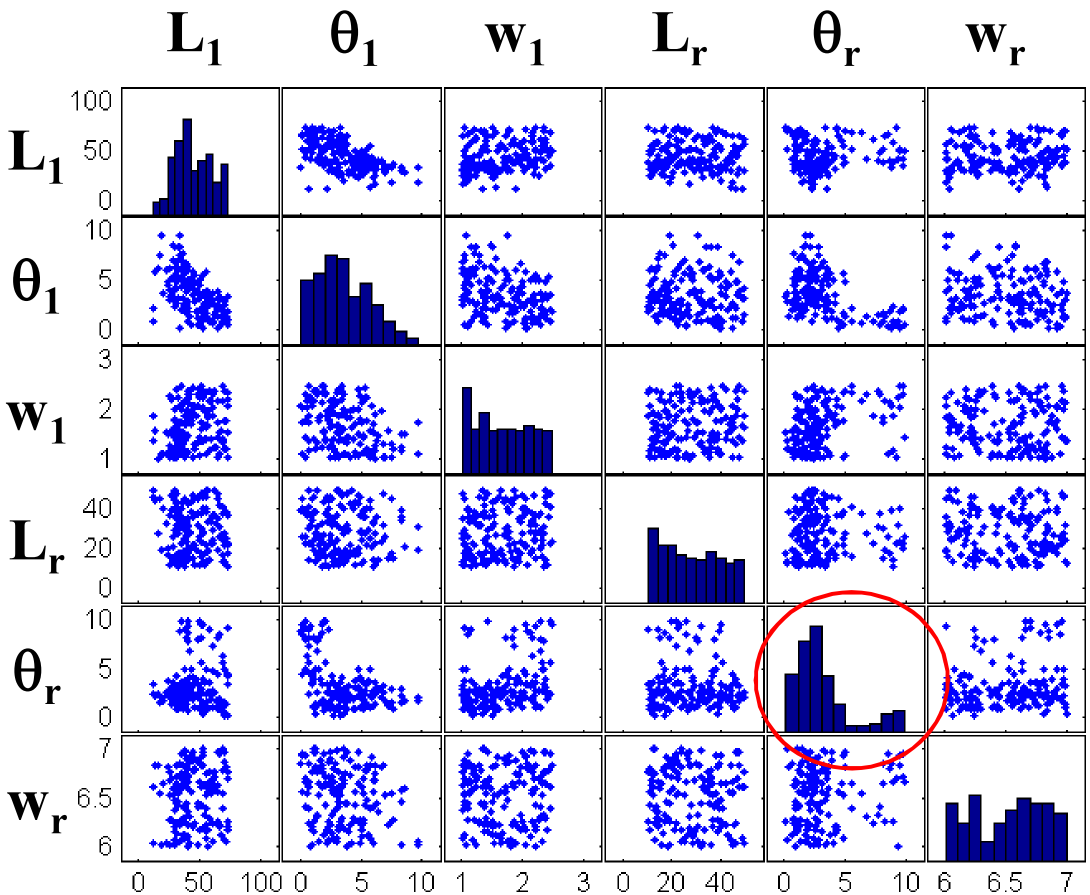
Filtered Design Space (156 points)

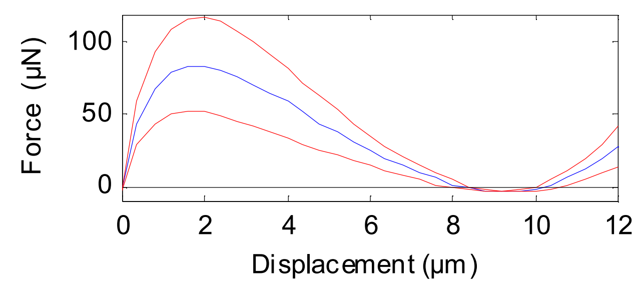
Observations

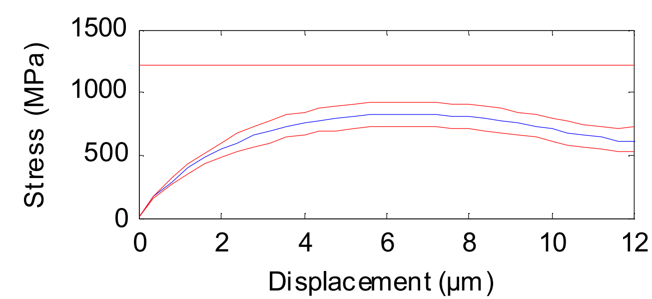
- **Some clustering, but very little except for the angle of the rigid segment**
- **All of the results to this point lead to the conclusion that the design space is highly nonlinear**



Optimization

- Starting Points fed into a Robust Optimization routine
  - Nested Algorithm using SQP for Optimization where each function call runs an uncertainty analysis (MVFOSM)
- Objectives
  - minimize uncertainty in F_{min}, (σ/F_{min}) weight: 3
  - minimize the sensitivity to an off axis load weight: 1
  - hit target by minimizing $(F_{target}-F_{min})^2$ weight: 6
- Constraints
  - Stress (Smax) + 2σ < 1200
  - Force Ratio (Fmin/Fmax) + 2σ < 0.2
  - Second Stable Position + 2σ < 12
  - Toggle Position (USP) + 2σ > 3





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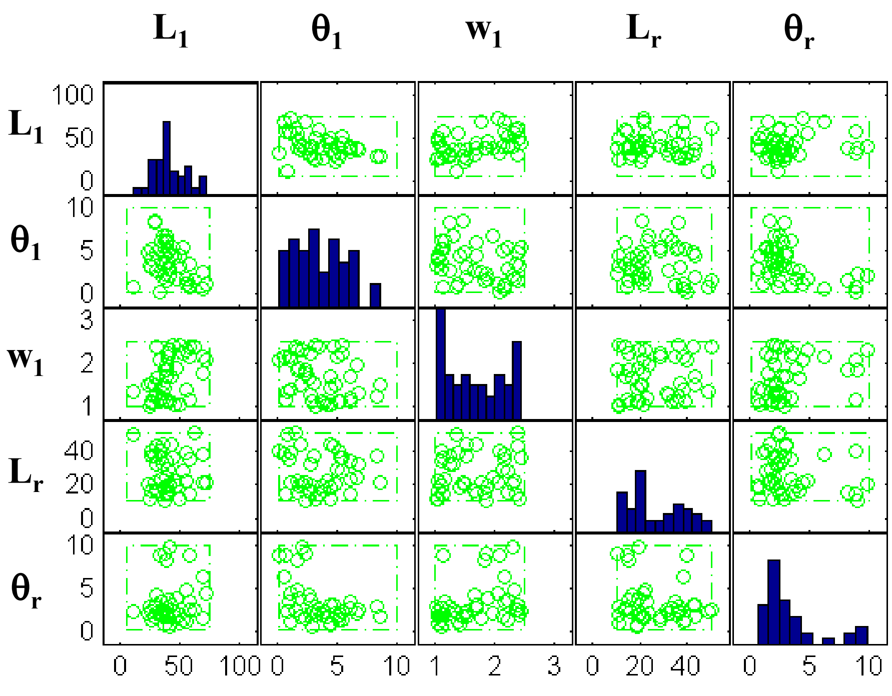
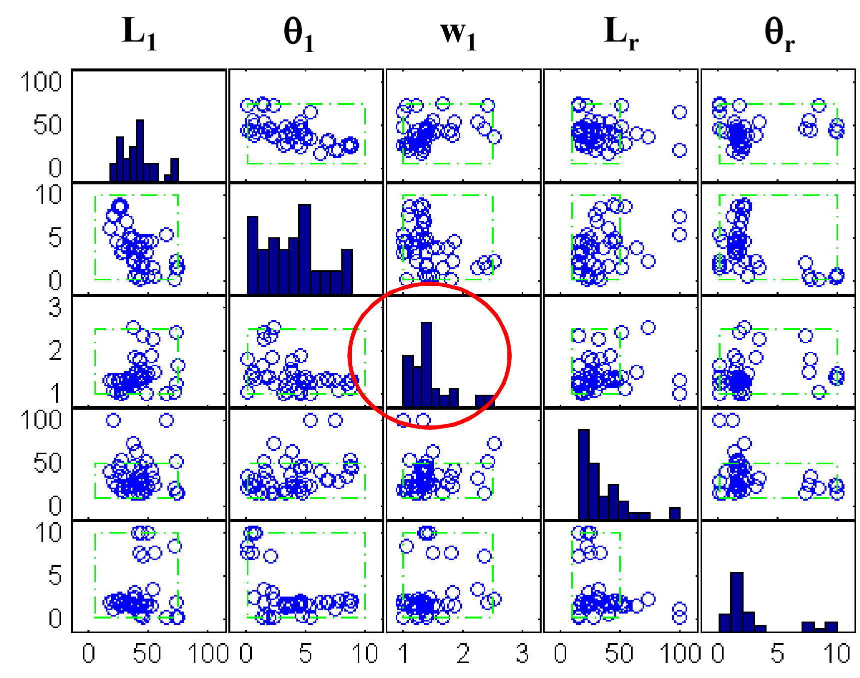
Optimized Designs

- Ran the first 50 points from the filtered set of 156 starting points
- 7 of these failed due to convergence issues
- 11 were very poor optimal designs (trapped in local minima)
- *No clear global optimum*

Starting Values



After Optimization



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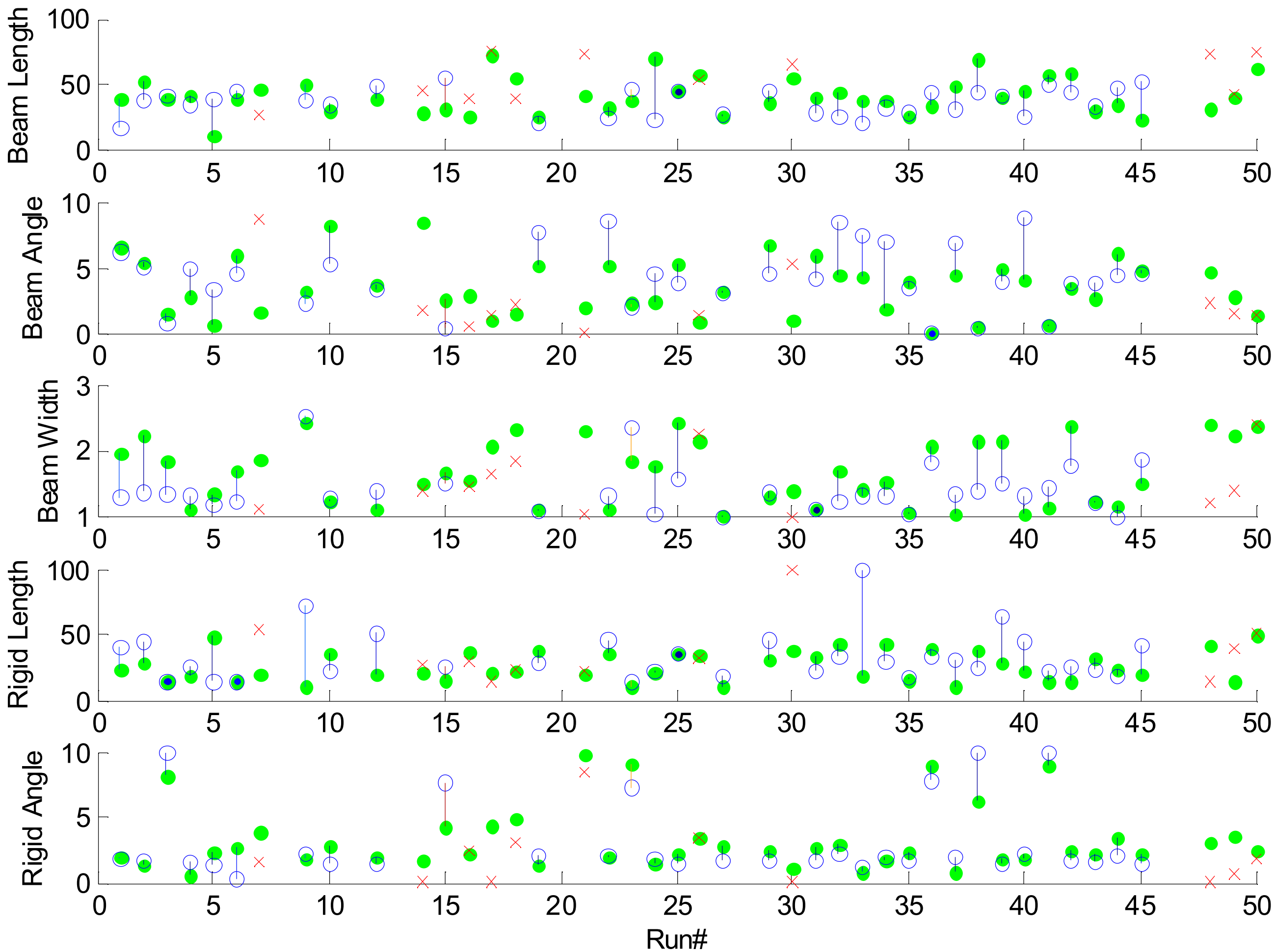
1D Projections (5 Variables = 5 graphs)

- **Green = Start Points, Blue = Optimum, Red = Poor Optimum**



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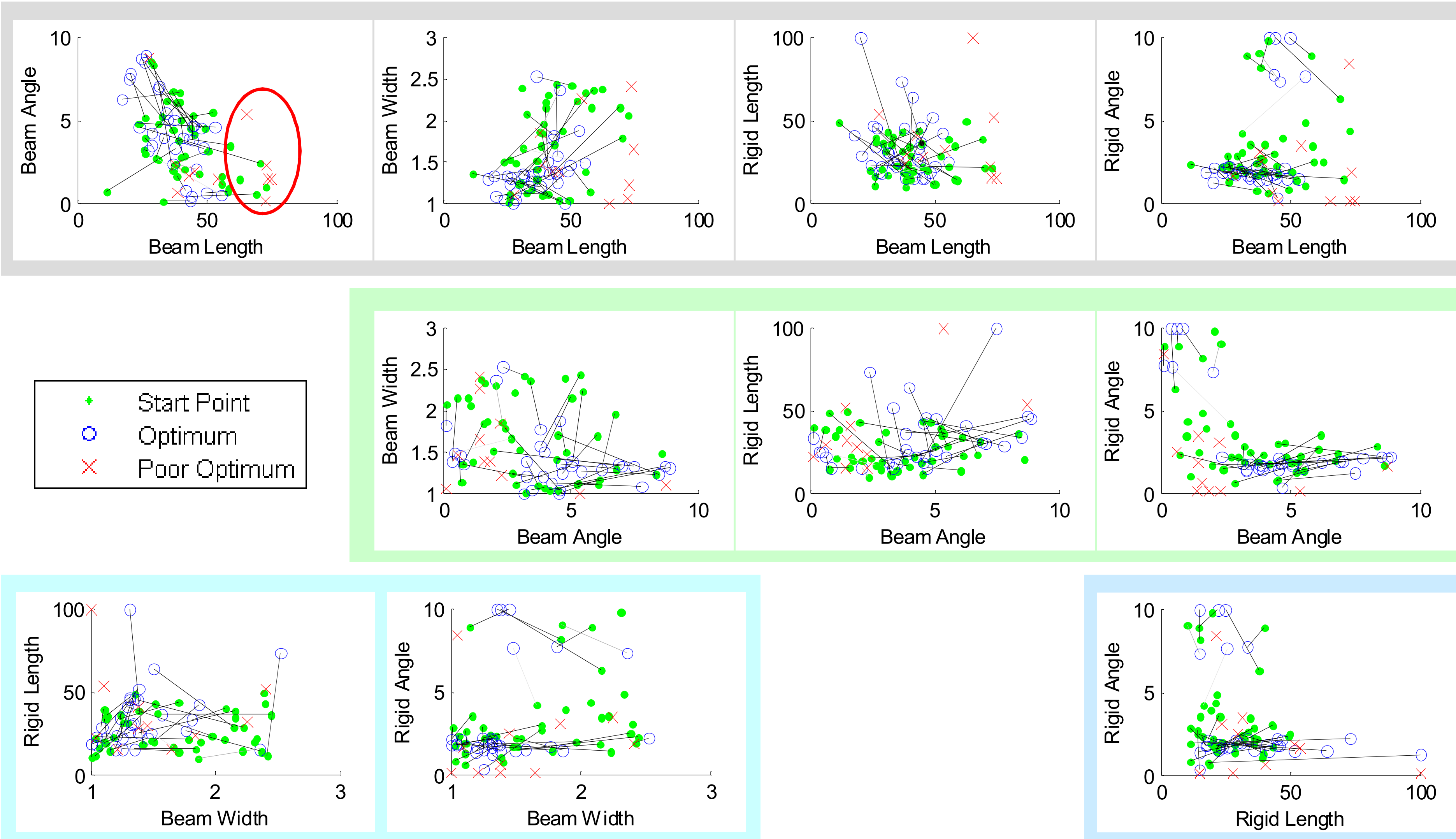
2D Projections (5 Variables = 10 graphs)



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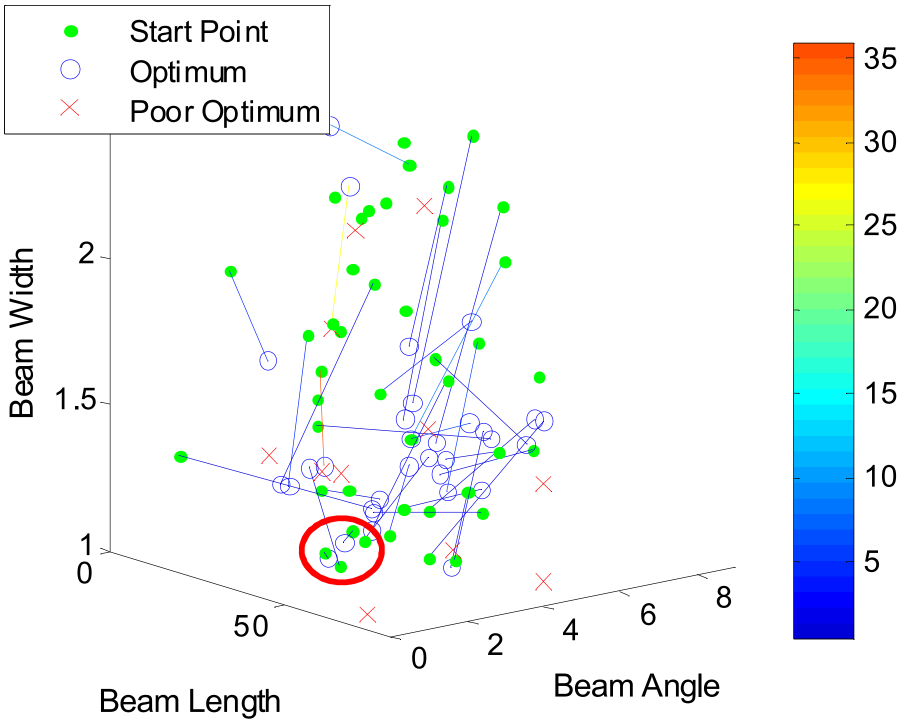
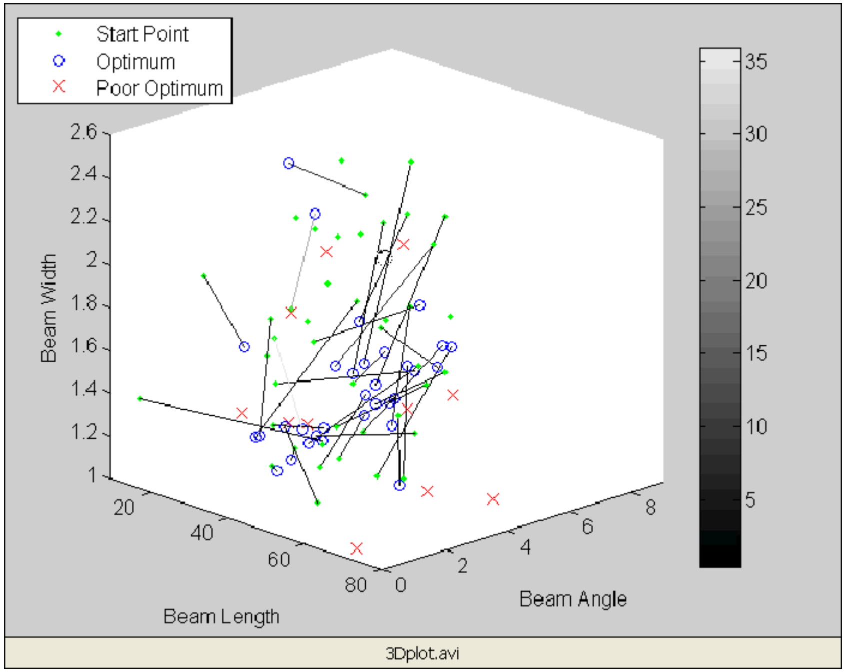
3D Projections (5 Variables = 9 graphs)

- **Rigid Length & Angle (would need 9 to see all combinations)**





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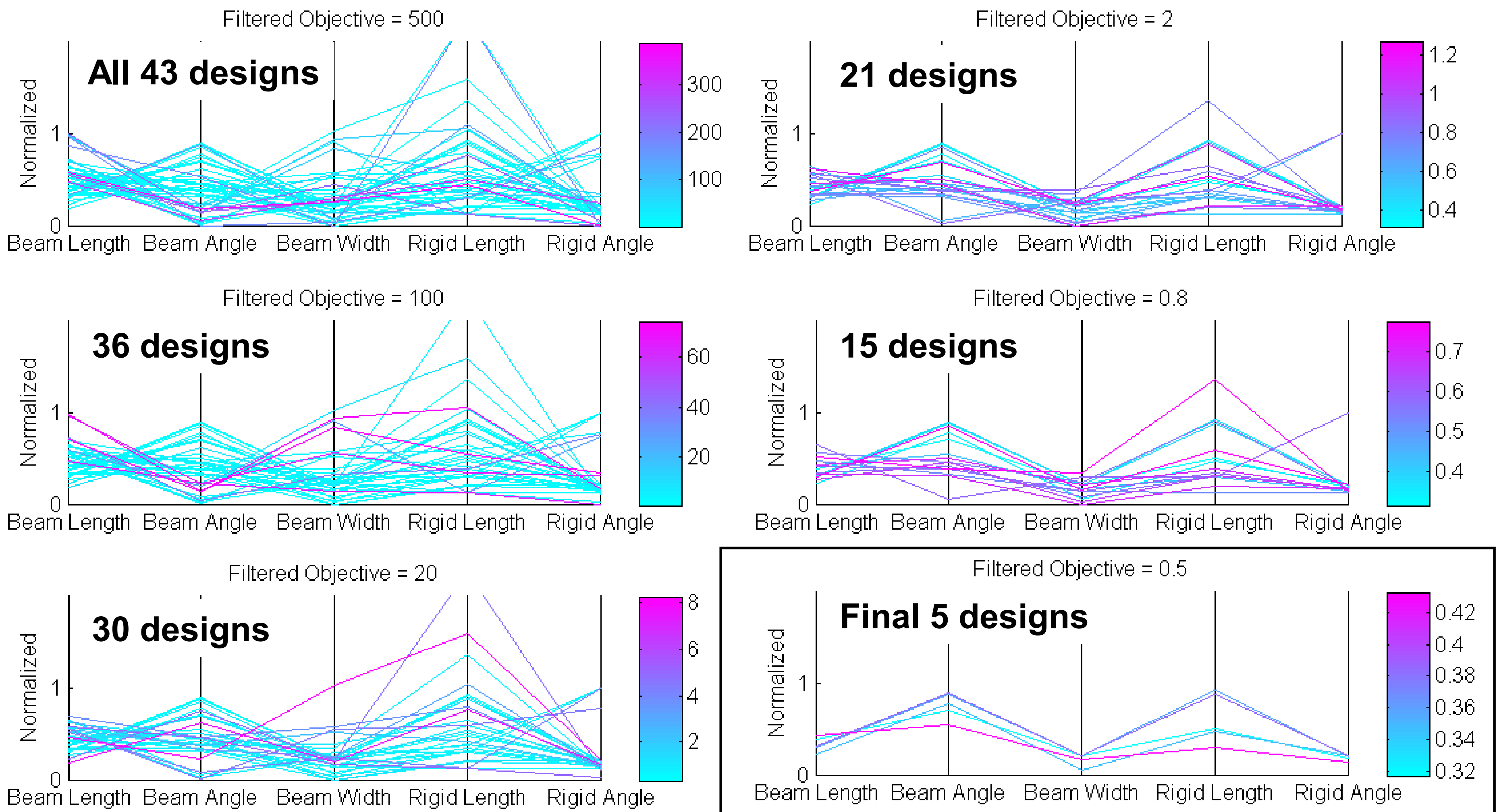
Design Selection

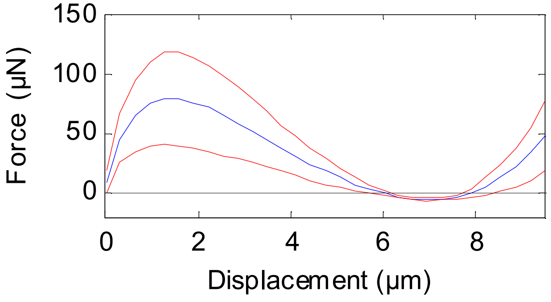
- **How are the best-of-the-best optima clustered?**



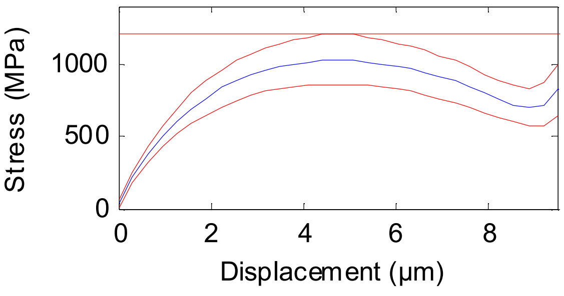
Design Selection & Verification

- Top 5 optimum designs
- Compare performance metrics

Run#	Obj	Size (μm)	Z Stiff	F_{min} (μN)	F_{max}/F_{min}	SSP (μm)	σ/F_{min}	$S_{max}+2\sigma$ (MPa)
34	0.317	93.09	0.90	-4.98	12.90	10.5	10.2%	947.3
19	0.345	69.26	0.96	-4.97	11.00	7.8	11.3%	1197.0
22	0.357	96.50	0.91	-4.97	12.22	10.8	11.5%	1193.2
40	0.384	96.95	0.90	-5.09	12.60	11.4	10.8%	1197.0
10	0.433	91.11	0.88	-4.87	10.03	8.9	10.5%	718.1



- Verify analysis after rounding the design variables
- Verify results with a higher fidelity model



Summary

- Confirmed that the design space is highly nonlinear, with many local minima
- Used filtered set of starting points to come up with 7 different designs
- Recommendations on visualization methods for these types of multi-dimensional analyses would be appreciated
- Future work might include performing other types of global robust optimization algorithms

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