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Pressure Spectra Models for Laminar/Turbulent Transitional Boundary Layer Reentry Flows

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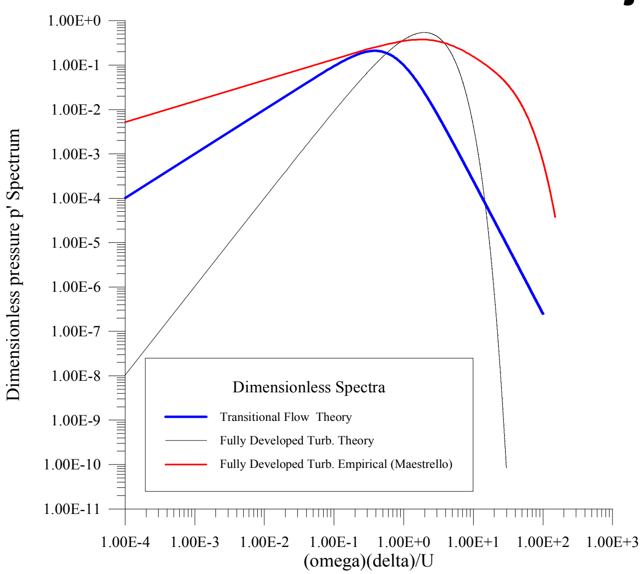

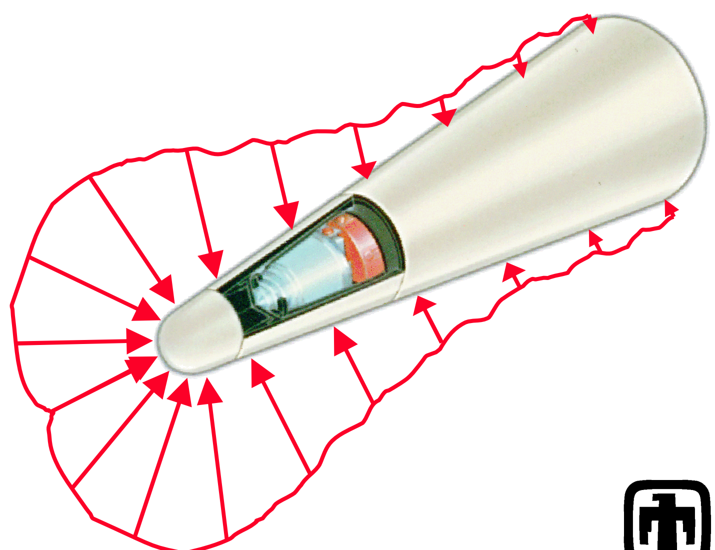

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Modeling Pressure Fluctuation in Laminar/Turbulent Transitional Flow

- **Lam./Turb. transition Pressure fluctuations (magnitude/PSD/spectra):**
  - **Much larger (1-2 orders of magnitude)** large than F.D. turbulence flutucations
  - Full spectral range $0<\omega<<$kHz
  - Power spectral density (PSD) not well modeled using F.D. turbulence PSD.
  - Data (literature) is available for high speed flow: Fujii (2006) hypersonic cones, Howe and Langanelli (1977) hypersonic cones (sharp and blunt).

- **How do we model transitional PSD?**
  - Some (integral) models are known, e.g. Lauchle 1980: Lighthill-based acoustic source analysis, intermittency inducing velocity fluctuations (source) with **empirical** indicator (intermittency) closure.
  - Alternative: phenomenological differential equation (convection reaction diffusion model), extended from Fowler and Howell (2003) combined with current first principle F.D. turb. PSD model (DeChant).

"First Principle Based Models"

- First principle: starting from conservation/transport equations (Reynolds stress transport equation, see Launder et. al. 1975 **)**, to estimate velocity deviation amplitude "A":

$$A_t + u_c A_x + vA_y = r[u - \bar{u}(A)] + (DA_x)_x$$

$$(I) \qquad (II) \qquad (III) \qquad (IV)$$

$$A \propto (C_{f-turb} - C_{f-lam}) = (0.027\left(\frac{u_\infty x}{\nu_w}\right)^{-1/7} - 0.664\left(\frac{u_\infty x}{\nu_w}\right)^{-1/2})$$

- The reaction source term (III), represents the nonlinear bifurcation between laminar drag and turbulent drag at transition. **At transition, the laminar drag is insufficient to balance convective accelerations, while the turbulent drag is overly strong, resulting in local nonlinear "slugging" with attendant turbulent spots.**

Pressure Spectra Models for Transitional B. L. Flows

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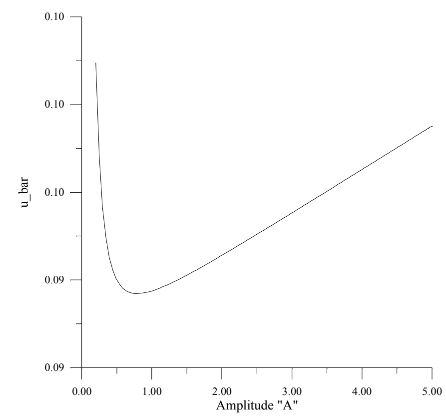
- Source term is **nonlinear/bifurcated polynomial,** solution A=A(u) and u=u(A)

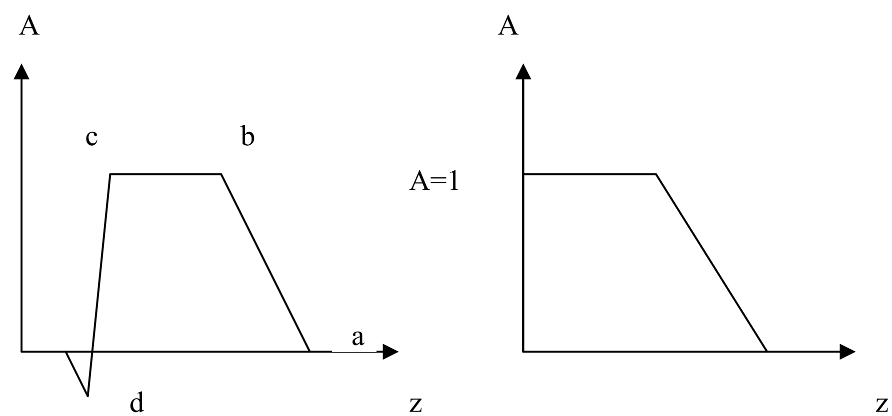
$$r[u-\bar{u}(A)] = A^3(u-0.089) - 0.002A^4 - 0.0005A$$



- Convection-diffusion-reaction equation

$$(A_t + u_c A_x + vA_y) =$$
$$= A^3(u-0.089) - 0.002A^4 - 0.0005A + \varepsilon\left((A+1)A_x\right)_x$$

- Known (Murray 1993) to have "slug" **solution…these are turbulent spots**



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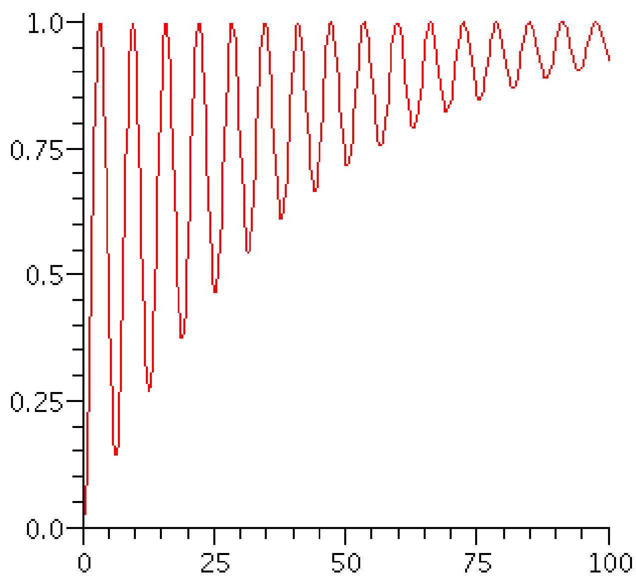
– Rate of appearance of slugs is **intermittency:** Indeed we can show that amplitude equation is approximate non-linear wave equation:

$$A_{tt} + 2A_{xt} + A_{xx} + A^3(A_t + A_x)\varepsilon + u_c^2 A^3 = 0$$

– Envelope of perturbations is intermittency expression:



$$\gamma = 1 - \exp\left(-\frac{\varepsilon}{2}\left(\frac{x}{\delta}\right)\right)$$

$$\mathrm{Re}_{L\_trans} = 20(0.16\,\mathrm{Re}_x^{6/7}) = 3.2\,\mathrm{Re}_x^{6/7}$$

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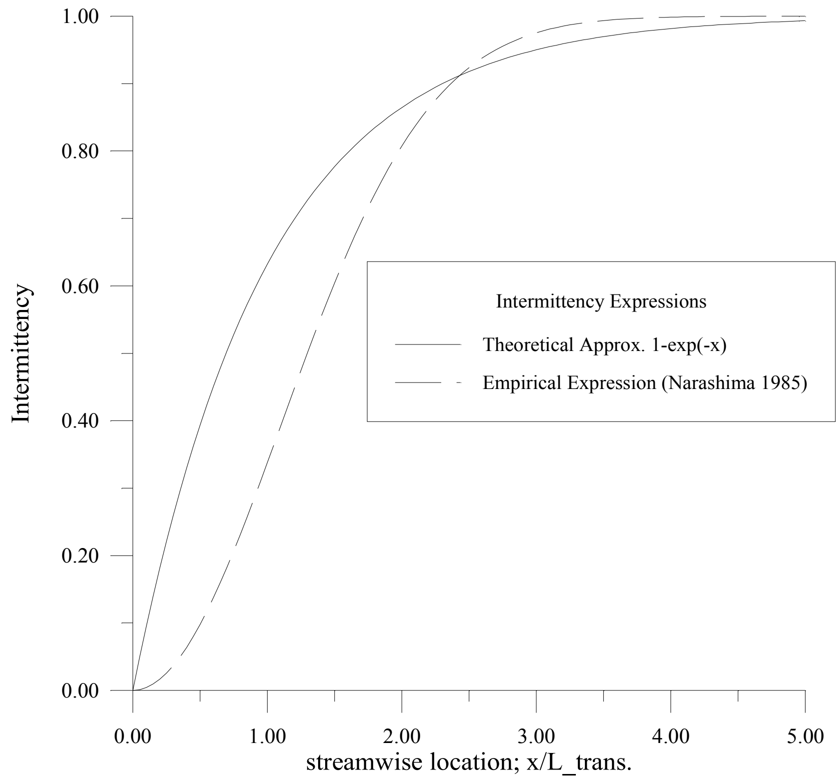
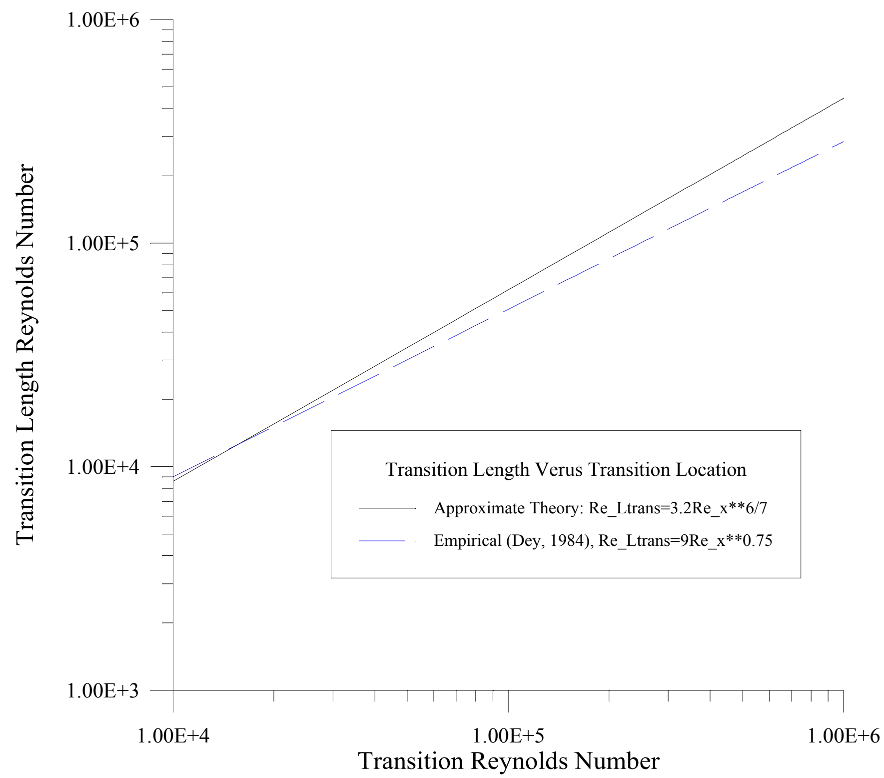
Intermittency we can measure



Transition Zone Length we can measure



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– Want pressure fluctuation field; solve pressure Poisson:

$$\nabla^2 p' = \rho_w \left(\frac{\partial v'}{\partial x}\right)\left(\frac{\partial u'}{\partial y}\right)$$

– Need velocity fluctuation field; decompose momentum into mean and fluctuating

$$u_t + uu_x + vu_y + p'_x = -(u'v')_y$$

$$u'_t + u_x u' + uv'_y + vu'_y = 0$$

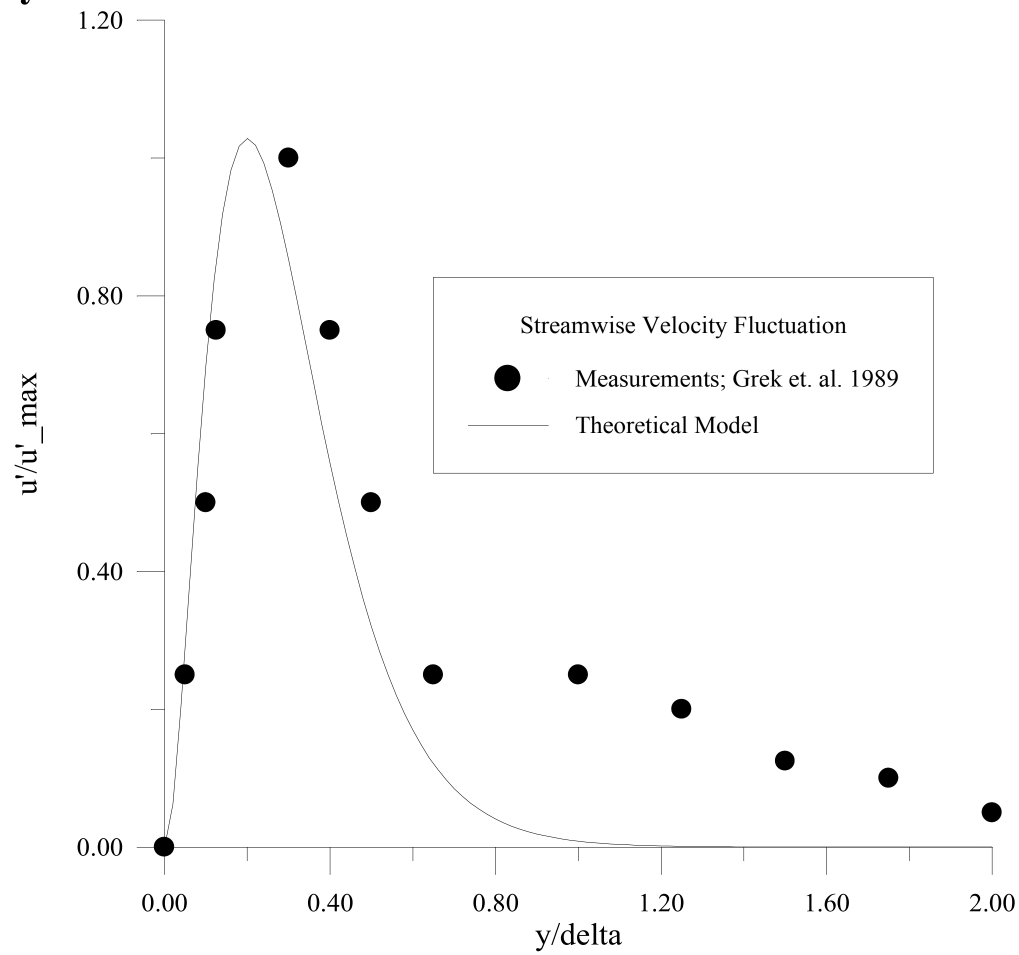
– Relate fluctuations to amplitude "A" expression

$$u' = \frac{dG}{dy} A(\xi) \quad ; \quad v' = -G(y)\frac{dA}{d\xi}$$

$$u' \propto \left[1 - \cos\left(\frac{1}{2}u_c\xi\right)\exp\left(-\frac{1}{4}\varepsilon\xi\right)\right]\left[y^2 \exp(-\frac{y}{\varepsilon})\right]$$

$$v' \propto \left[\frac{1}{2}u_c \sin\left(\frac{1}{2}u_c\xi\right)\exp\left(-\varepsilon\frac{\xi}{4}\right)\right]\left[y^3 \exp(-\frac{y}{\varepsilon})\right]$$

– **Velocity fluctuation u'** in transitional flow we can measure:



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Pressure Spectra Models for Transitional B. L. Flows
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- Solve pressure Poisson using Fourier Sine transform:

$$\frac{d^2P}{dx^2}-\omega^2P=\frac{2}{\pi}A(x)\int_0^\infty y\exp(-y)\cos(\omega y)dy=-\frac{2}{\pi}\exp(-x)\frac{(1-\omega^2)}{(1+\omega^2)^2}$$

- Solution and approximate convolution inversion give wall pressure

$$p'(x,0)=p'_1(x,0)-p'_2(x,0)=\frac{1}{(1+x)}-\exp(-x)$$

- Compute spectrum, i.e. PSD:

$$\Phi_{pp,trans}=\int_0^\infty[\exp(-\frac{1}{2}x)-\exp(-x)]\sin(\omega x)dx\propto\frac{\omega}{(1+4\omega^2)(1+\omega^2)}$$

Lam.-Turb. Transition Pressure Fluctuations
First-Principles-Based PSD Model
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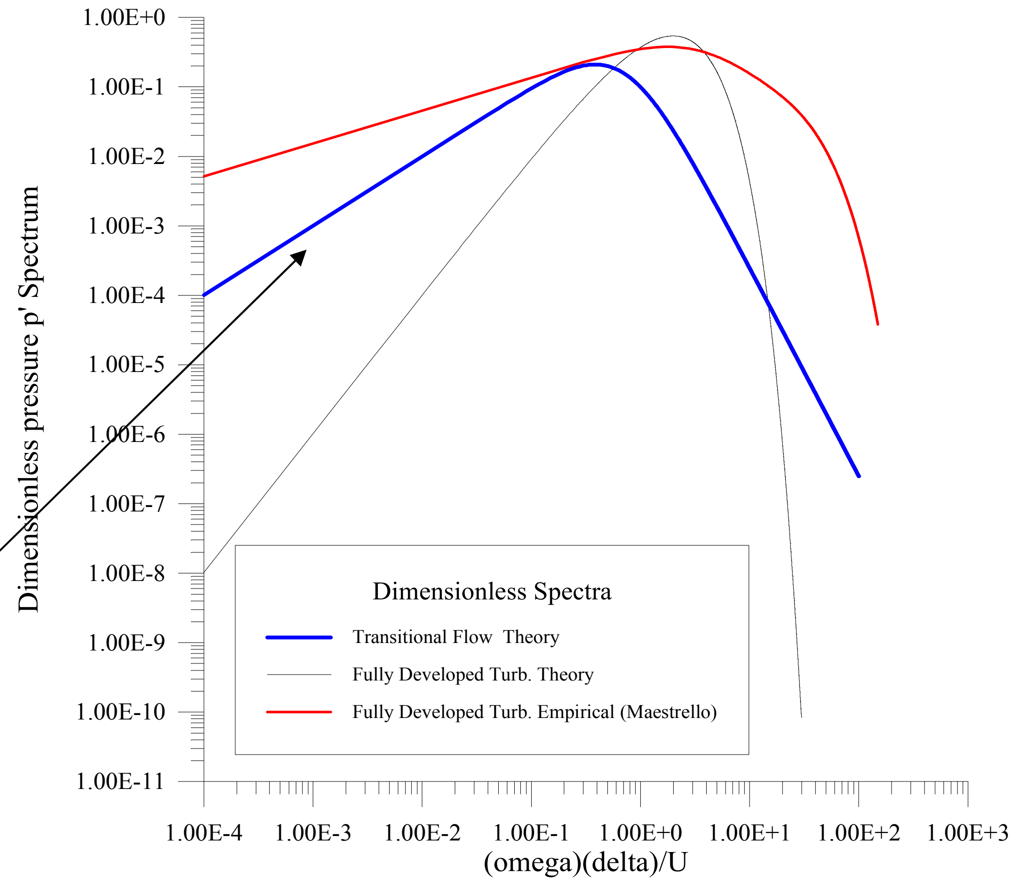
- Can estimate velocity fluctuation field → RHS for fluctuating pressure Poisson equation → PSD.

$$p'_{xx} + p'_{yy} = -\left(\frac{\partial v'}{\partial x}\right)\left(\frac{\partial u'}{\partial y}\right)$$

$$\left\|\Phi_{pp}\right\|_{F.D.} \approx (16)(100)\left(\frac{\delta}{U}\right)\tau_w^2 \rightarrow \frac{\left\|\Phi_{pp}\right\|_{trans}}{\left\|\Phi_{pp}\right\|_{F.D.}} = O(100)$$

$$\Phi_{pp,trans} \propto \frac{\omega}{(1+4\omega^2)(1+\omega^2)}$$



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Pressure Spectra Models for Transitional B. L. Flows

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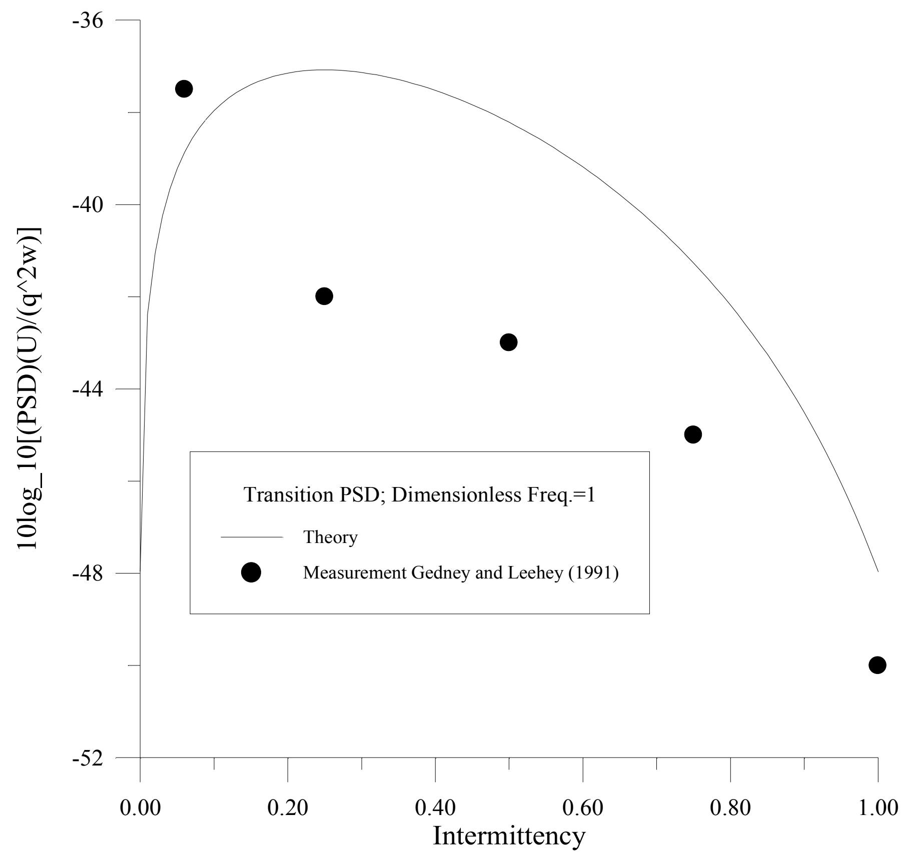
– **Transitional Pressure Fluctuation Magnitude** we can measure:



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Derivation of pressure fluctuation Probability Density Functions (PDF's)

- Derive PDF operator expressions consistent with pressure fluctuation models.
- Apply methods of Pope (2000), Pope and Ching (19930:
- Define fine-grain-PDF, apply Dirac-delta calculus, eliminate terms via conservation equations:
- Fine-grain PDF:

$$f_p(p',\xi) = \delta(P'(\xi) - p')$$

- Averaged derivatives

$$\frac{\partial^2 F_p}{\partial \xi^2} = -\frac{\partial}{\partial p'}\left(F_p\left\langle \frac{d^2 P'}{d\xi^2}\right\rangle\right) + \frac{\partial^2}{\partial p'^2}\left(F_p\left\langle \left(\frac{dP'}{d\xi}\right)^2 \right\rangle\right)$$

- PDF governing equation: (Gaussian approximation…yields Gaussian solution):

$$p'\frac{\partial^2 F_p}{\partial p'^2} + \frac{\partial F_p}{\partial p'} = C(p'^3 - 2p')\exp(-\frac{1}{2}p'^2) \longrightarrow F_p = C_1\exp(-\frac{1}{2}p'^2) + C_2\exp(-\frac{1}{2}p'^2)Ei(1,-p'^2)$$

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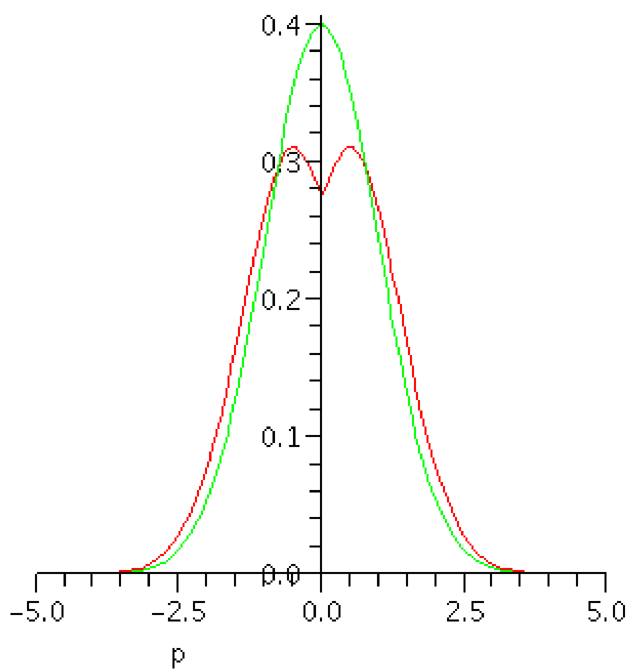
Derivation of pressure fluctuation Probability Density Functions (PDF's)

– Non-Gaussian equation and solution



$$2\xi^2 p'\frac{d^2F_p}{dp'^2}+(1-2\xi^2)\frac{dF_p}{dp'}-(2p'\xi^2+p'-4p'\xi^2)F_p=0$$

$$F_p=C\left(p^{\left(-\frac{1}{4\xi^2}\right)}\right)Whitta\ker W\left[\frac{1}{8}\left(\frac{4\xi^2-1}{\xi^2}\right),\frac{1}{8}\left(\frac{4\xi^2-1}{\xi^2}\right),p^2\right]$$

Preliminary; we can derive physically-based PDF models…

Fully-developed turbulence PDF's are largely Gaussian

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Derivation of pressure fluctuation PDF's for Laminar-Turbulent Transitional Flows

– Derive PDF operator models can extend to laminar-turbulent transition
– Wall pressure model provides PDF equation closure:

$$p'(x,0)=\frac{1}{(1+x)}-\exp(-x)\approx x\exp(-\alpha_0 x)$$

– PDF evolution equation; x>>1 Gaussian

$$p'\frac{\partial^2 F_p}{\partial p'^2}+\frac{\partial F_p}{\partial p'}=C(p'^3-2p')\exp(-\frac{1}{2}p'^2)\left(1+\frac{4}{x}+...O(\frac{1}{x})^2\right)$$

– For x<<1:

$$p'\frac{\partial^2 F_p}{\partial p'^2}+\frac{\partial F_p}{\partial p'}=\left(O(1)\right)p'\exp(-\frac{1}{2}p'^2) \qquad F_p'=E_i\left(1,\frac{1}{2}(c_0+p')^2\right) \quad c_0\approx 2/5$$

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Pressure Spectra Models for Transitional B. L. Flows
PDF Theoretical results

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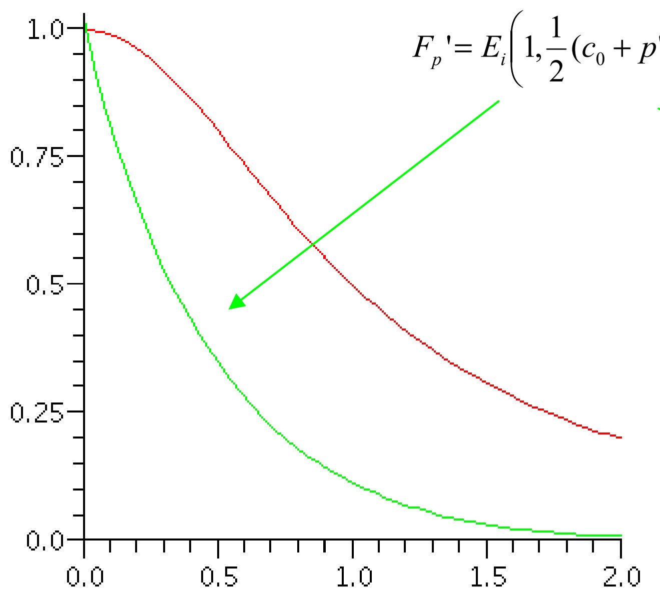
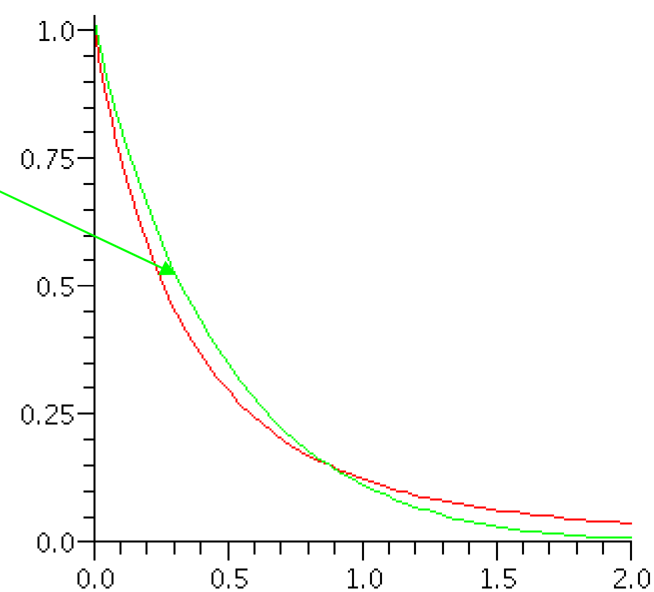
Derivation of pressure fluctuation PDF’s for Laminar-Turbulent Transitional Flows
X<<1; near incipient transition location; PDF is non-Gaussian, e.g.

Cauchy Distribution or Pareto Distribution

$$F_p' \propto \frac{1}{1+p'^2} \qquad F_p' = \frac{k}{p'}\left(\frac{p'_{\min}}{p'}\right)^k$$

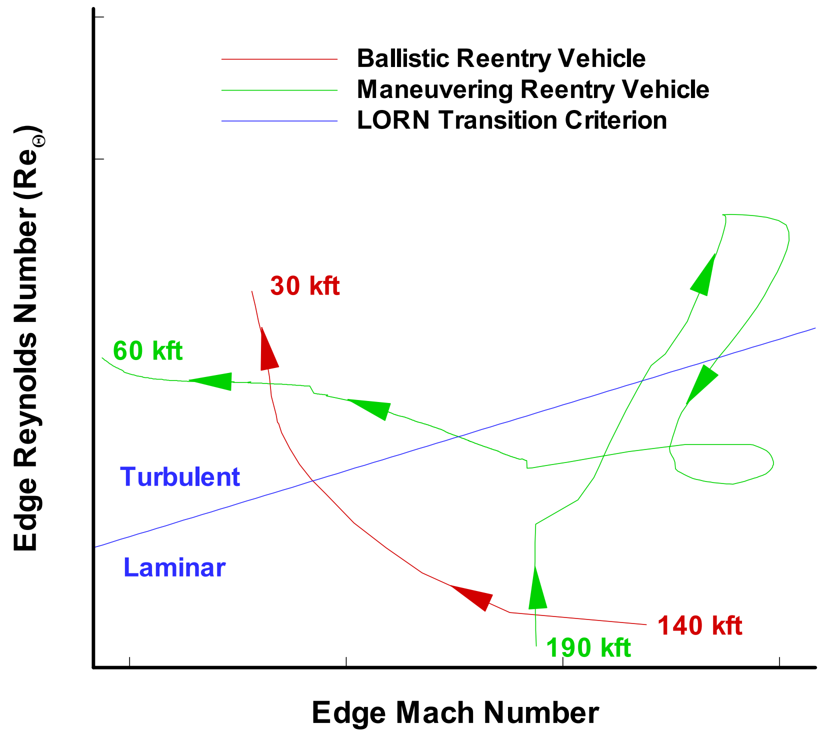
$$F_p' = E_i\left(1, \frac{1}{2}(c_0+p')^2\right) \qquad c_0 \approx 2/5$$





Summary

1. Have a first-principle based laminar-transition pressure fluctuation model…highly approximate…but closed-form.
2. Preliminary results suggest successful…



Next steps

1. Test analysis against additional data, i.e. canonical flat plate/literature, DNS simulations (Pino Martin, Princeton), dedicated data (Schneider, Purdue), Flight test data structural response
2. Apply models to integrated RV aero/structural model

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