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Removing Undesired Periodic Data from Random Vibration Data

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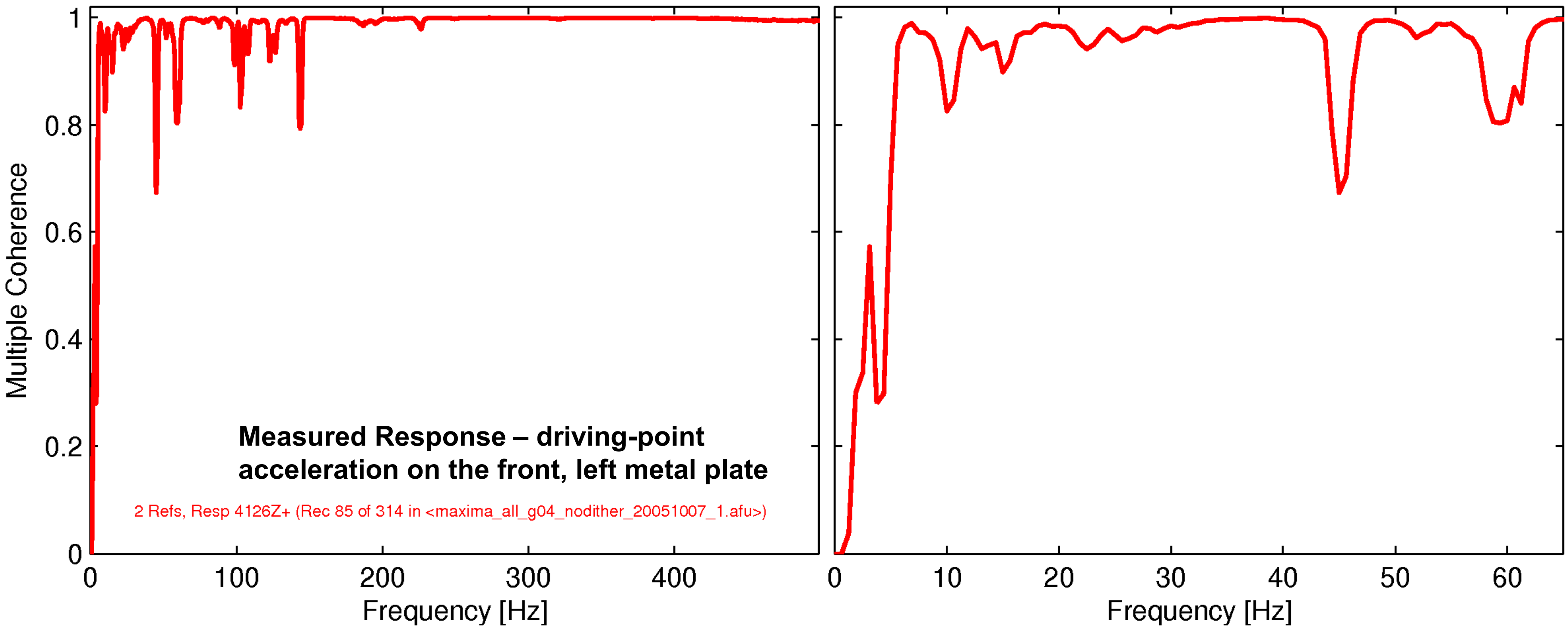
Introduction and Motivation

Vehicle Modal Testing

- Replaced wheels/tires with square metal plates
- Softly support vehicle by putting air spring under each metal plate
- Use shaker to apply continuous, random, force excitations to the metal plates
- Measure resulting acceleration responses on the plates and various locations in and on the vehicle

Typical Coherence Data



- **Excitation Force – continuous random force of 17.8 N (4 lb) RMS to both front metal plates**
- **Looks OK, but there is a problem…**

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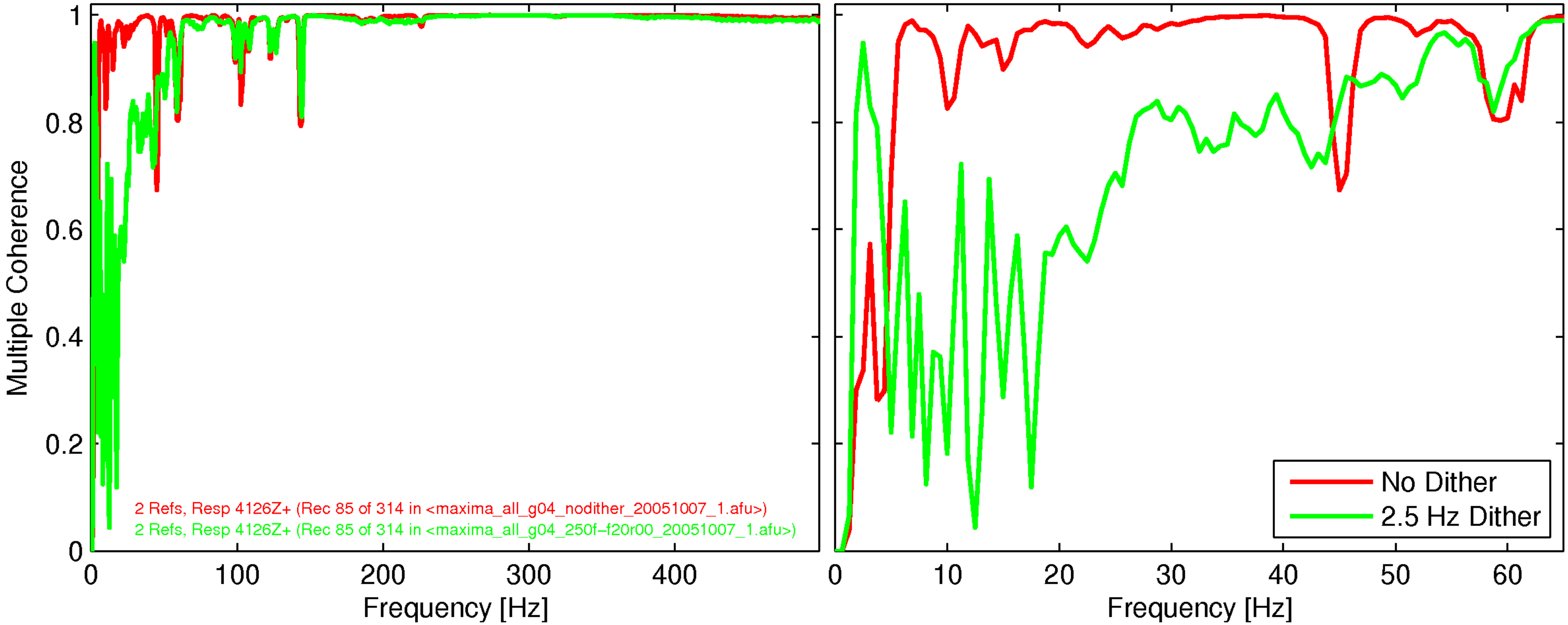
Introduction and Motivation

- For small force levels used:
  - shocks act as rigid members
  - sometimes "break free" for portions of the excitation
  - poor FRF results
    - dependent on force level
    - not repeatable
- "Dither" to ensure that shocks are always broken free
  - extra shaker under vehicle that provides single-freq, sinusoidal forcing
  - choose dither freq to minimize required dither force
  - dither freq is well below first elastic mode of the vehicle

Coherence With and Without Dither



- **Same measurement as before**
- **Dither – 2.5 Hz, 89.0 N (20 lb) RMS under front of vehicle**
- **Looks like expected at high freq, but something's wrong below 50 Hz**

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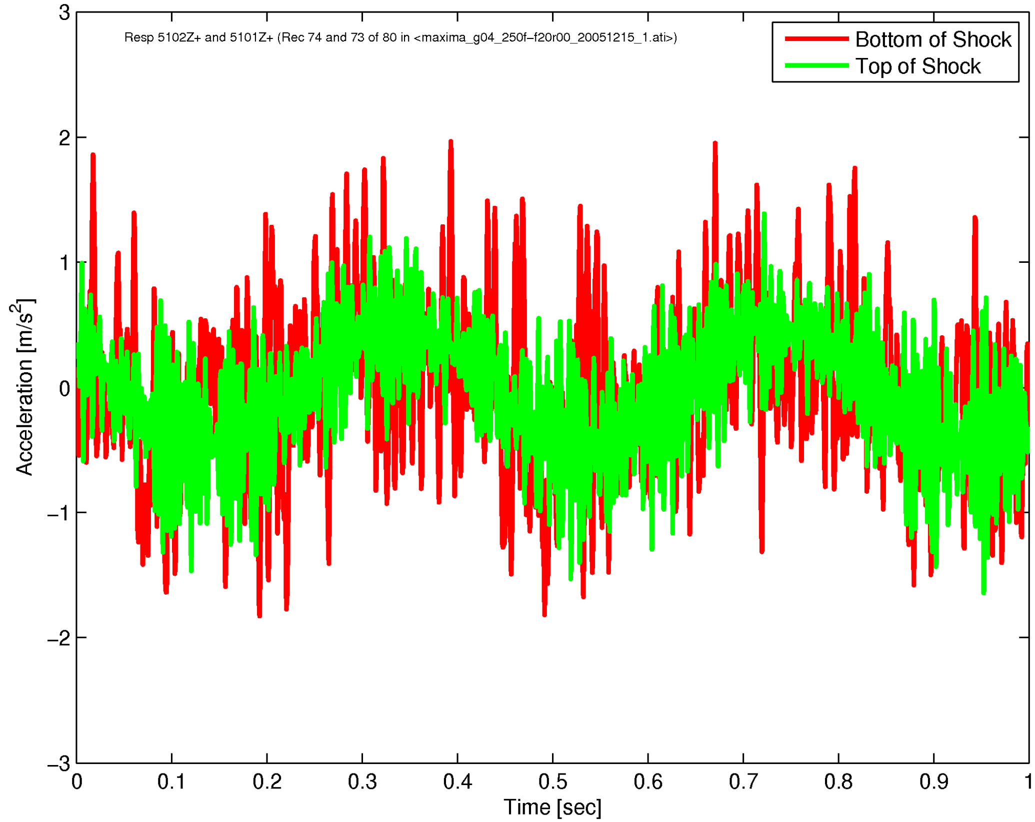
Motion of Shocks

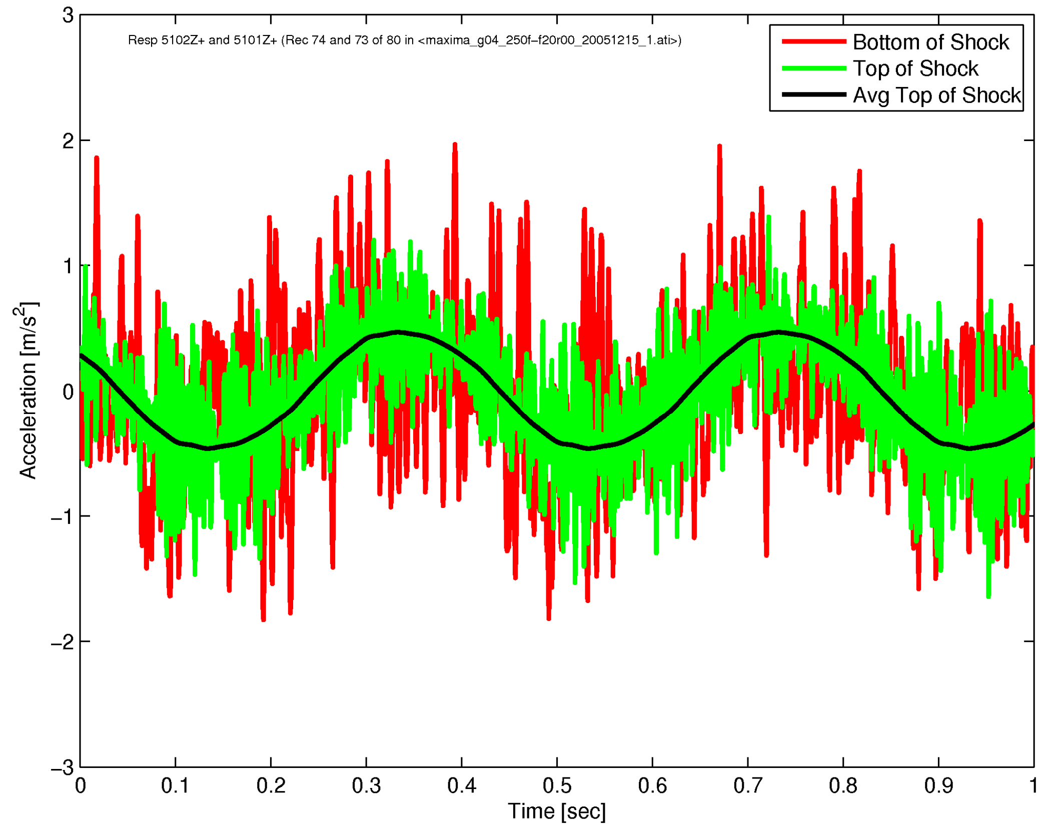
- **Accel at the top and bottom of shocks**
  - **Shock absorber is broken free as its top and bottom do not move identically**
  - **Random motion of shock (due to shaker on plate) is combined with periodic motion due to dithering**



Shock nearest the plate on which random force is applied
- **top of shock is attached to body**
- **bottom of shock is attached to wheel spindle and metal plate**

Motion of Shocks

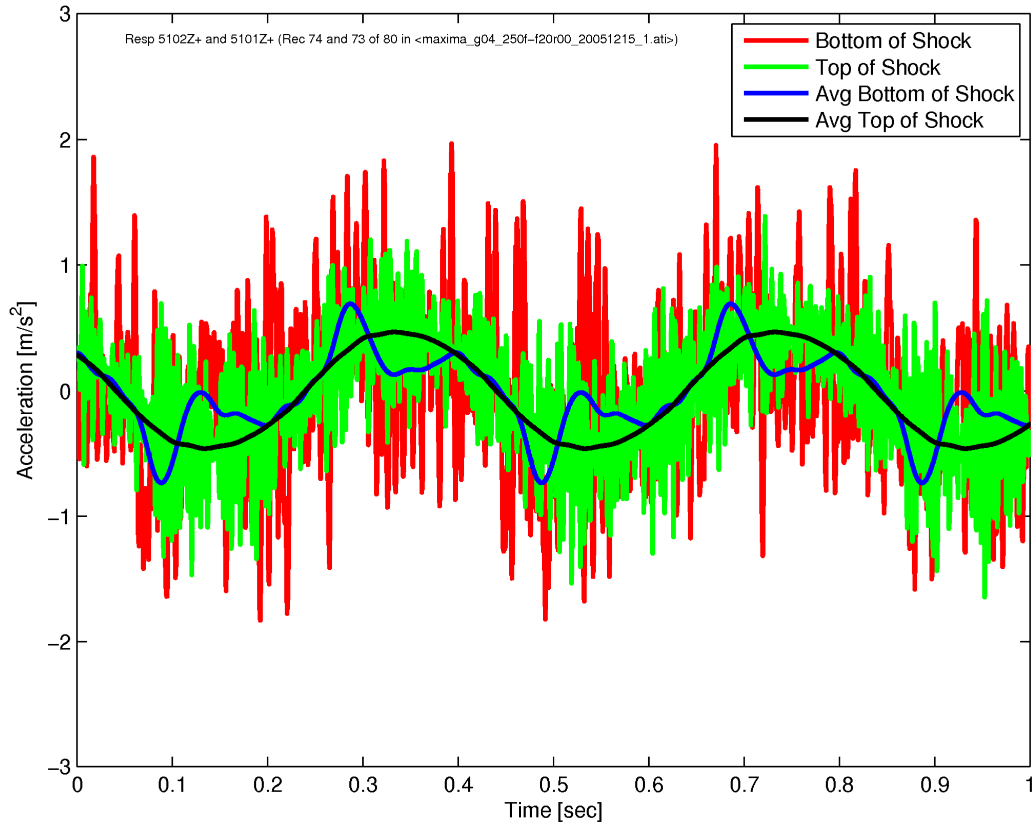
- **Look at average periodic motion of the shock**
  - **Top looks like 2.5 Hz sine (just like the applied dither force)**



Shock nearest the plate on which random force is applied
- **top of shock is attached to body**
- **bottom of shock is attached to wheel spindle and metal plate**

Motion of Shocks

- **Look at average periodic motion of the shock**
  - **Top looks like 2.5 Hz sine**
  - **Bottom is periodic, but has lots of higher harmonics**



Shock is not linear
- **Harmonics due to dithering cause drop in coherence**
- **Effect extends up into freq range of interest even though dither is at low freq**

Shock nearest the plate on which random force is applied
- **top of shock is attached to body**
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What Should We Do?

- Remove periodic dither responses from measured data to improve FRFs/coherences
- This is a problem if we only have averaged spectral data
  - Can't just throw out freq lines at dither harmonics
    - saved data had resolution of 0.625 Hz
    - harmonics of 2.5 Hz dither occur every 4 lines
- Solution is to save time-history data instead of averaged FRF data
  - Can later post-process the data
    - calculate averaged spectral results
      - change freq resolutions/windowing
    - investigate glitches/overranges
    - remove periodic noise present in the data
    - investigate nonstationary or nonlinear behavior

Lobbying for Saving More Data

- **Computers have vastly-increased capabilities in processing power, RAM memory, and hard disk storage space**
- **Advocate always recording time-history data**
  - **Modal testers are conditioned to acquire averaged spectral data**
    - **Never question why**
  - **Brandt, et. al. (*Sound and Vibration*, Apr.2006)**
    - **Only saving averaged spectral data and discarding time data is a remnant held over from the past when computers were extremely limited compared to what is available today**

Lobbying for Saving More Data

- For these tests the time-history data file size was 20 times larger than the averaged spectral data
  - 48.6 Mbytes for time-history data
  - 2.4 Mbytes for averaged spectral data (FRF, autospectra, coherence)

80 channels at 1280 Hz for 192 sec (245,760 samples/chan)

averaged into segments of 2048 samples (801 freq lines 0-500 Hz, 0.625 Hz resolution)

- Extra storage cost is minimal compared to the advantages of having the raw data
  - days/months to plan, set up, and perform test
  - one-of-a-kind test or hardware

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These hard drives can store > 5000 / 8000 time-history data files

Removing Periodic Noise

- **All this increased computer power is also what makes this technique possible**
  - **Removing periodic noise from random data has been done before – adaptive filtering, etc.**
  - **We don't need to be quite so clever since we have lots of power available**
    - **Directly fit sinusoids to the data**
      - **very simple to understand**
    - **Technique can be implemented in only a few lines of computer code**
    - **Can process large data sets very quickly**
      - **80 channels with 245,760 samples/channel in less than 1 minute on my laptop**

Mathematical Development

- **Fit continuous function *y(t)* with *N* harmonics of a sinusoid with fundamental frequency (ω)**

$$y \approx c_0 + c_1 \sin(\omega t + \theta_1) + c_2 \sin(2\omega t + \theta_2) + \cdots + c_N \sin(N\omega t + \theta_N)$$

- **Can also be written as**

$$y \approx c_0 + [a_1 \cos(\omega t) + b_1 \sin(\omega t)] + [a_2 \cos(2\omega t) + b_2 \sin(2\omega t)] + \cdots + [a_N \cos(N\omega t) + b_N \sin(N\omega t)]$$

- **Second equation is preferable for optimization as all (2*N* + 1) coefficients have similar sensitivities**

Mathematical Development

- **For sampled data, write equation in matrix form**

$$\{y\} = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{\mathrm{nPts}} \end{Bmatrix} \approx \begin{bmatrix} 1 & \cos(\omega t_1) & \cdots & \cos(N\omega t_1) & \sin(\omega t_1) & \cdots & \sin(N\omega t_1) \\ 1 & \cos(\omega t_2) & \cdots & \cos(N\omega t_2) & \sin(\omega t_2) & \cdots & \sin(N\omega t_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \cos(\omega t_{\mathrm{nPts}}) & \cdots & \cos(N\omega t_{\mathrm{nPts}}) & \sin(\omega t_{\mathrm{nPts}}) & \cdots & \sin(N\omega t_{\mathrm{nPts}}) \end{bmatrix} \begin{Bmatrix} c_0 \\ a_1 \\ \vdots \\ a_N \\ b_1 \\ \vdots \\ b_N \end{Bmatrix}$$

- **For time-history with *nPts* samples**
  - ***y* is *nPts* × 1 column vector**
  - **coefficients are (2*N* + 1) × 1 column vector**
  - **sinusoidal harmonics are *nPts* × (2*N* + 1) matrix**
    - **time values (t_i) come from data sampling freq.**
    - **if fundamental freq (ω) is known, the harmonics matrix and its pseudo-inverse can be calculated**

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Mathematical Development

The $(2N + 1)$ coefficients can now be calculated in a least-squares sense by

pseudo-inverse only needs to be calculated once, then it can be used for all data records

$$\left\{ \begin{array}{c} c_0 \\ a_1 \\ \vdots \\ a_N \\ b_1 \\ \vdots \\ b_N \end{array} \right\} = \begin{bmatrix} 1 & \cos(\omega t_1) & \cdots & \cos(N\omega t_1) & \sin(\omega t_1) & \cdots & \sin(N\omega t_1) \\ 1 & \cos(\omega t_2) & \cdots & \cos(N\omega t_2) & \sin(\omega t_2) & \cdots & \sin(N\omega t_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \cos(\omega t_{\mathrm{nPts}}) & \cdots & \cos(N\omega t_{\mathrm{nPts}}) & \sin(\omega t_{\mathrm{nPts}}) & \cdots & \sin(N\omega t_{\mathrm{nPts}}) \end{bmatrix}^{+} \left\{ \begin{array}{c} y_1 \\ y_2 \\ \vdots \\ y_{\mathrm{nPts}} \end{array} \right\}$$

Mathematical Development

- **The best fit to *y* using the *N* harmonics is**

$$\{y_{\text{periodic}}\} = \begin{bmatrix} 1 & \cos(\omega t_1) & \cdots & \cos(N\omega t_1) & \sin(\omega t_1) & \cdots & \sin(N\omega t_1) \\ 1 & \cos(\omega t_2) & \cdots & \cos(N\omega t_2) & \sin(\omega t_2) & \cdots & \sin(N\omega t_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \cos(\omega t_{\text{nPts}}) & \cdots & \cos(N\omega t_{\text{nPts}}) & \sin(\omega t_{\text{nPts}}) & \cdots & \sin(N\omega t_{\text{nPts}}) \end{bmatrix} \begin{Bmatrix} c_0 \\ a_1 \\ \vdots \\ a_N \\ b_1 \\ \vdots \\ b_N \end{Bmatrix}$$

- **The time-history with the harmonics removed is**

$$\{y_{\text{random}}\} = \{y\} - \{y_{\text{periodic}}\}$$

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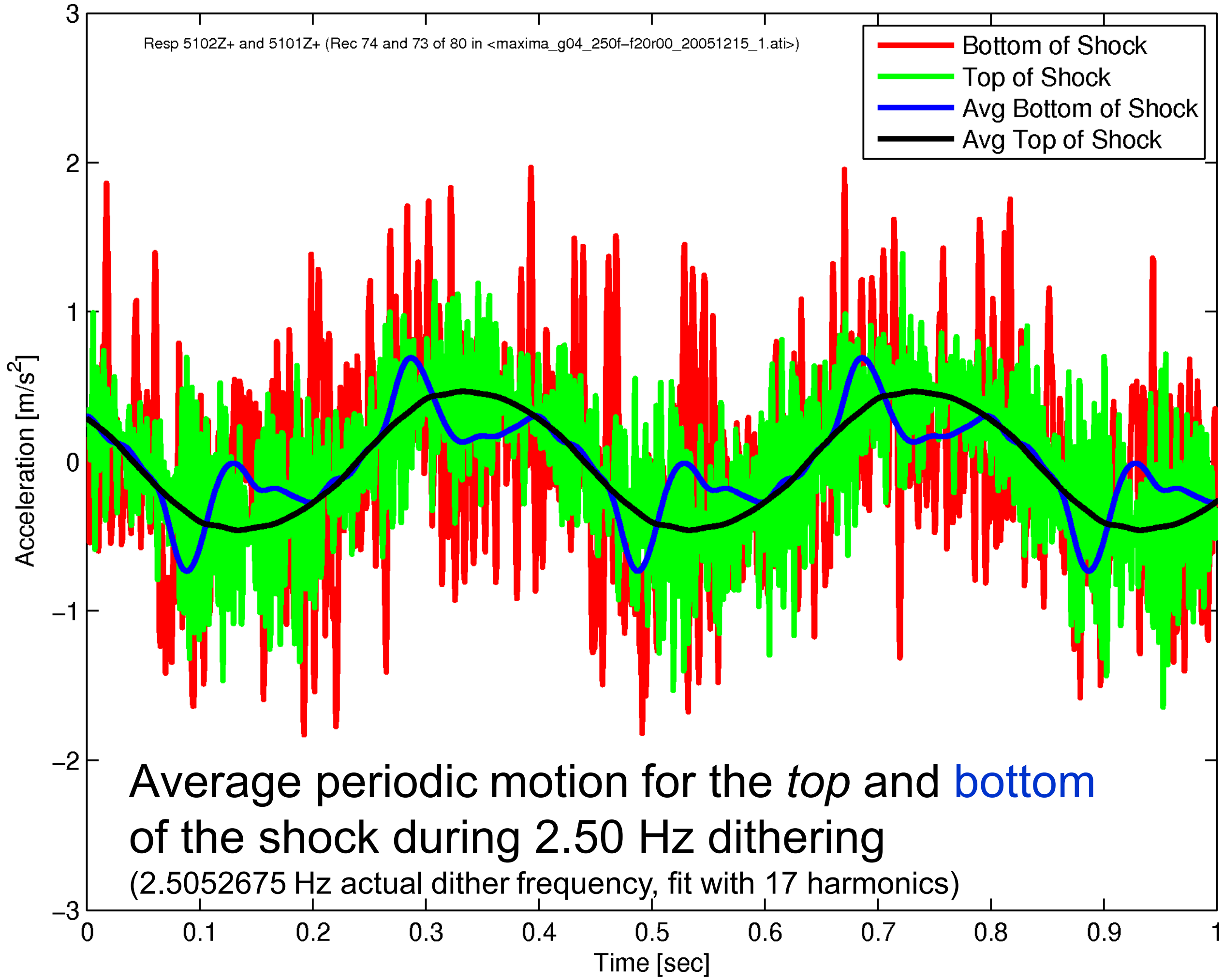
Implementation of Technique

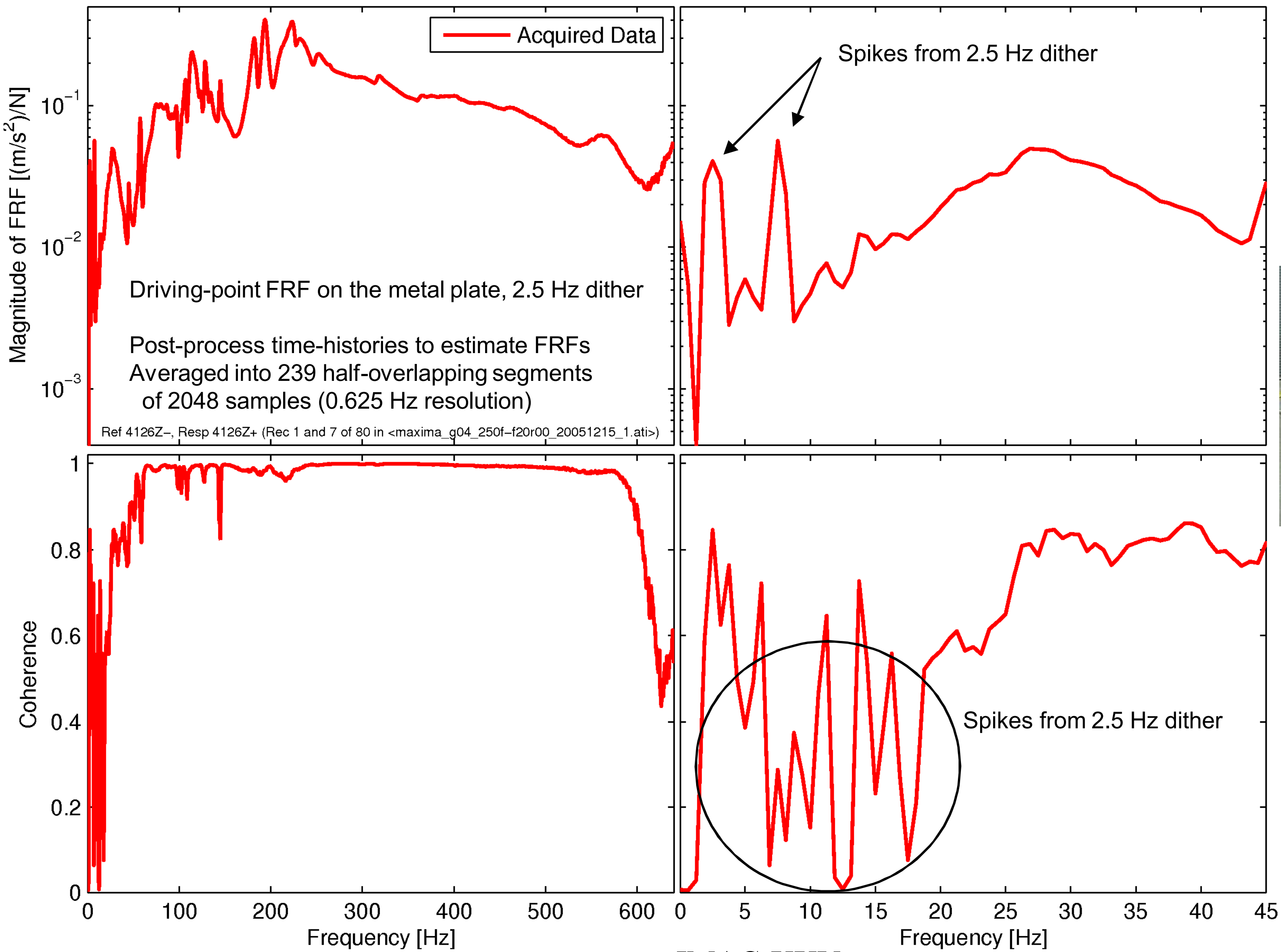
- Technique has been implemented in MATLAB
  - requires less than 30 lines of MATLAB code
  - complete code listing is included in the paper
- Must measure and save data as CONTINUOUS time-history records
  - do NOT use option to automatically discard records when an overrange is detected
- Automated optimization finds fundamental freq.
  - user must supply initial guess
  - need extremely accurate fundamental frequency
    - after *T* seconds, phase error $\Delta\theta = 360\ T\ \Delta f$ degrees
- Can select how many harmonics (*N*) to use in fit

Example of Technique

- **Have already shown results from the technique**



Example — Dither Removal



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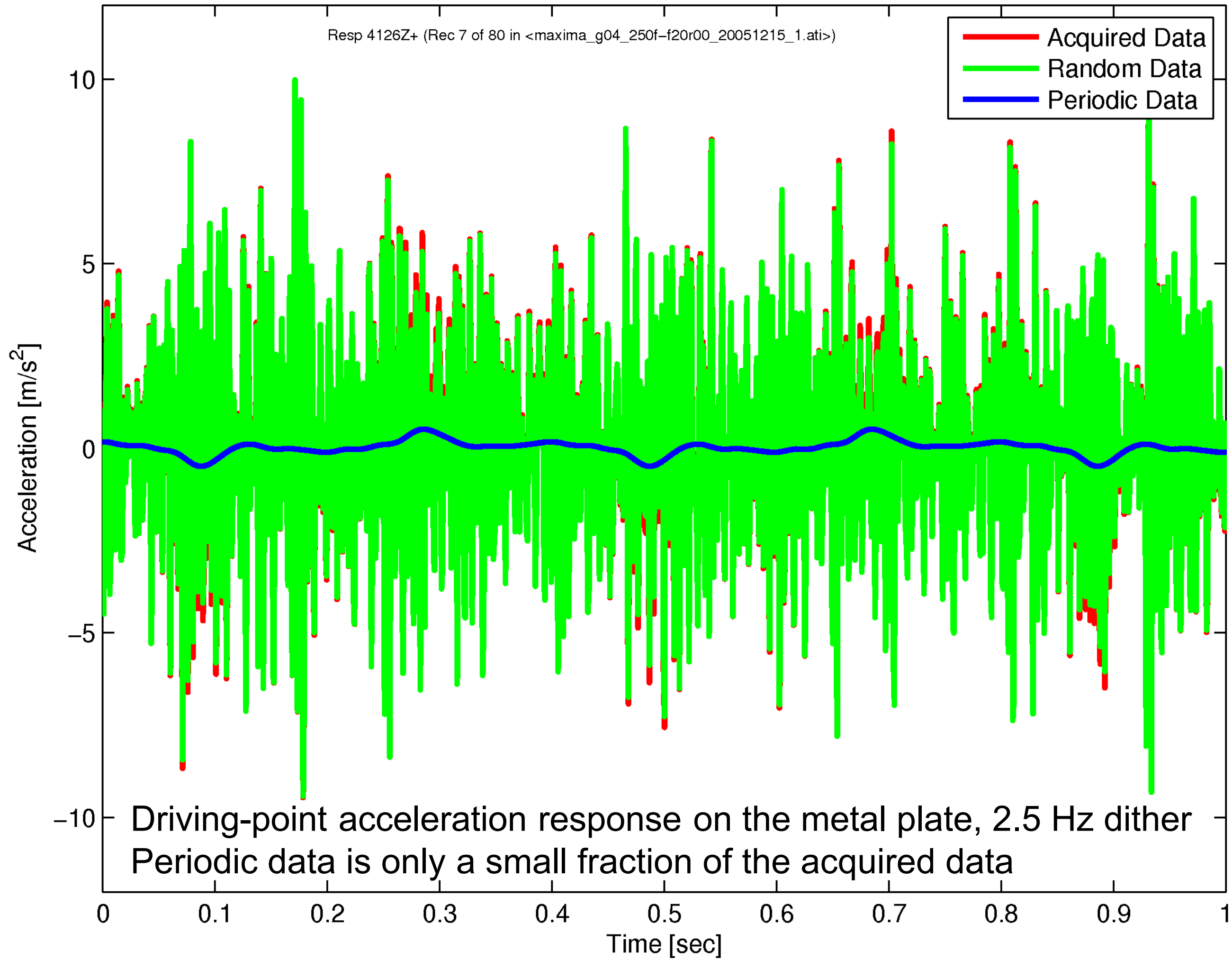

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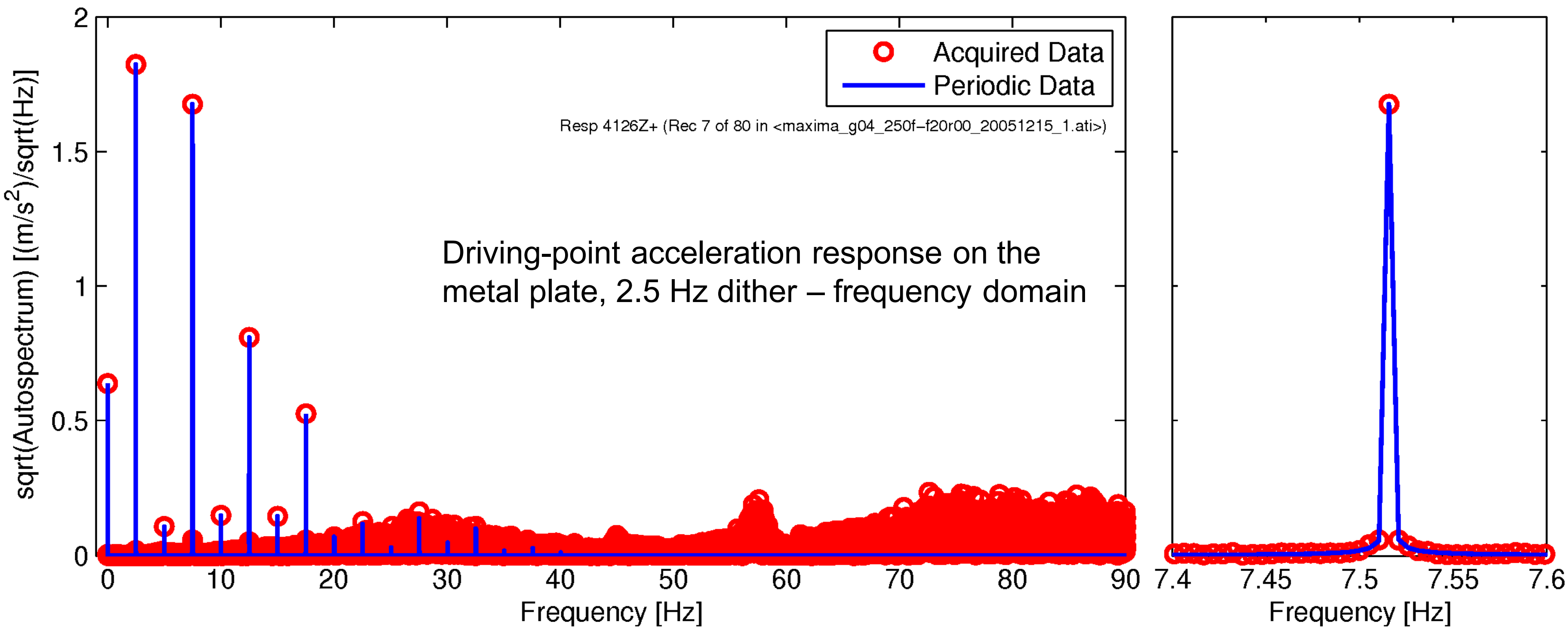
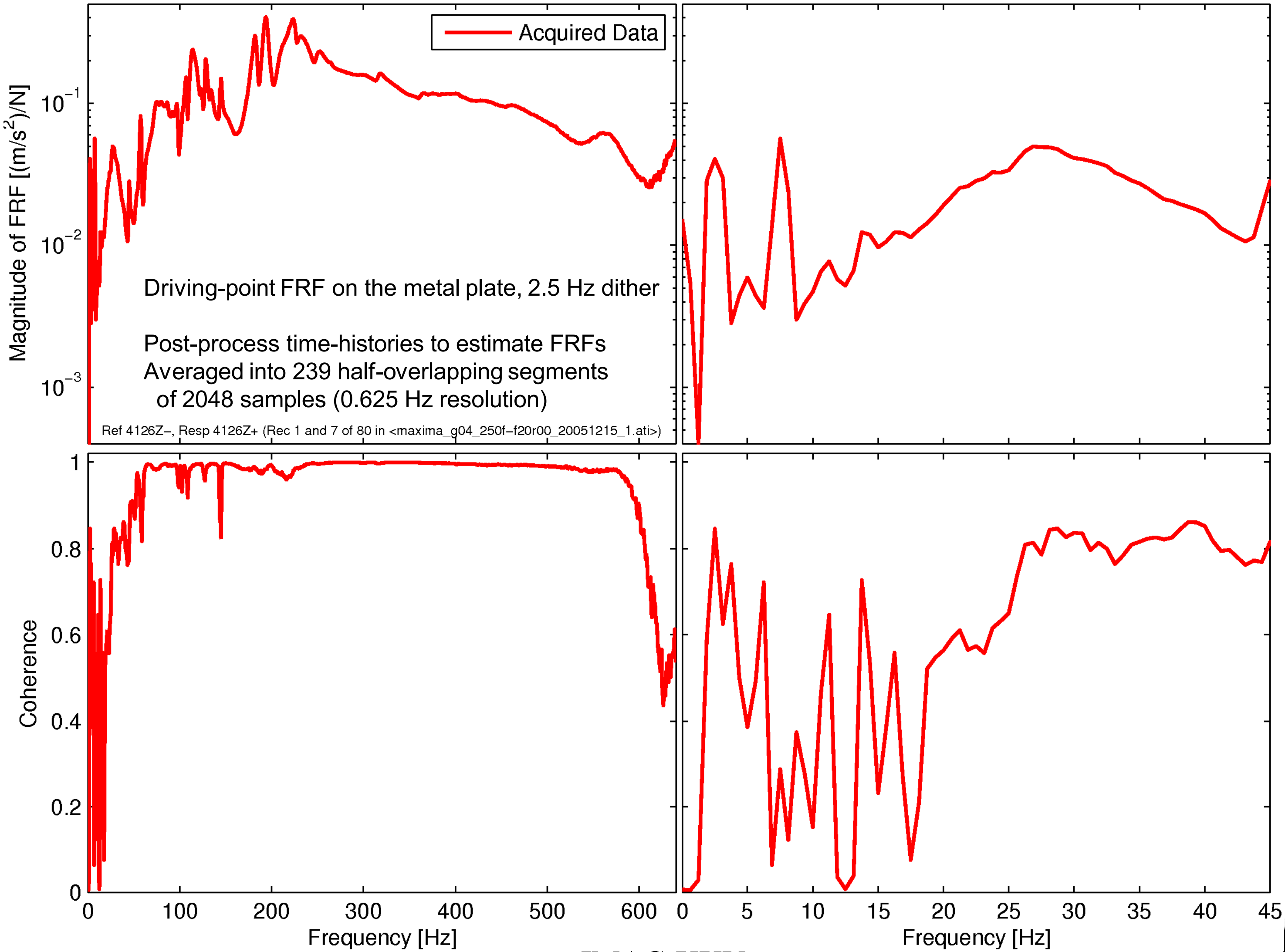


Processed time-history as a single, 245,760-sample record (0.00521 Hz resolution)

Dither harmonics visually dominate the acquired freq-domain data (especially the odd harmonics)…

…but only account for 0.5% of mean-squared acceleration

Example — Dither Removal



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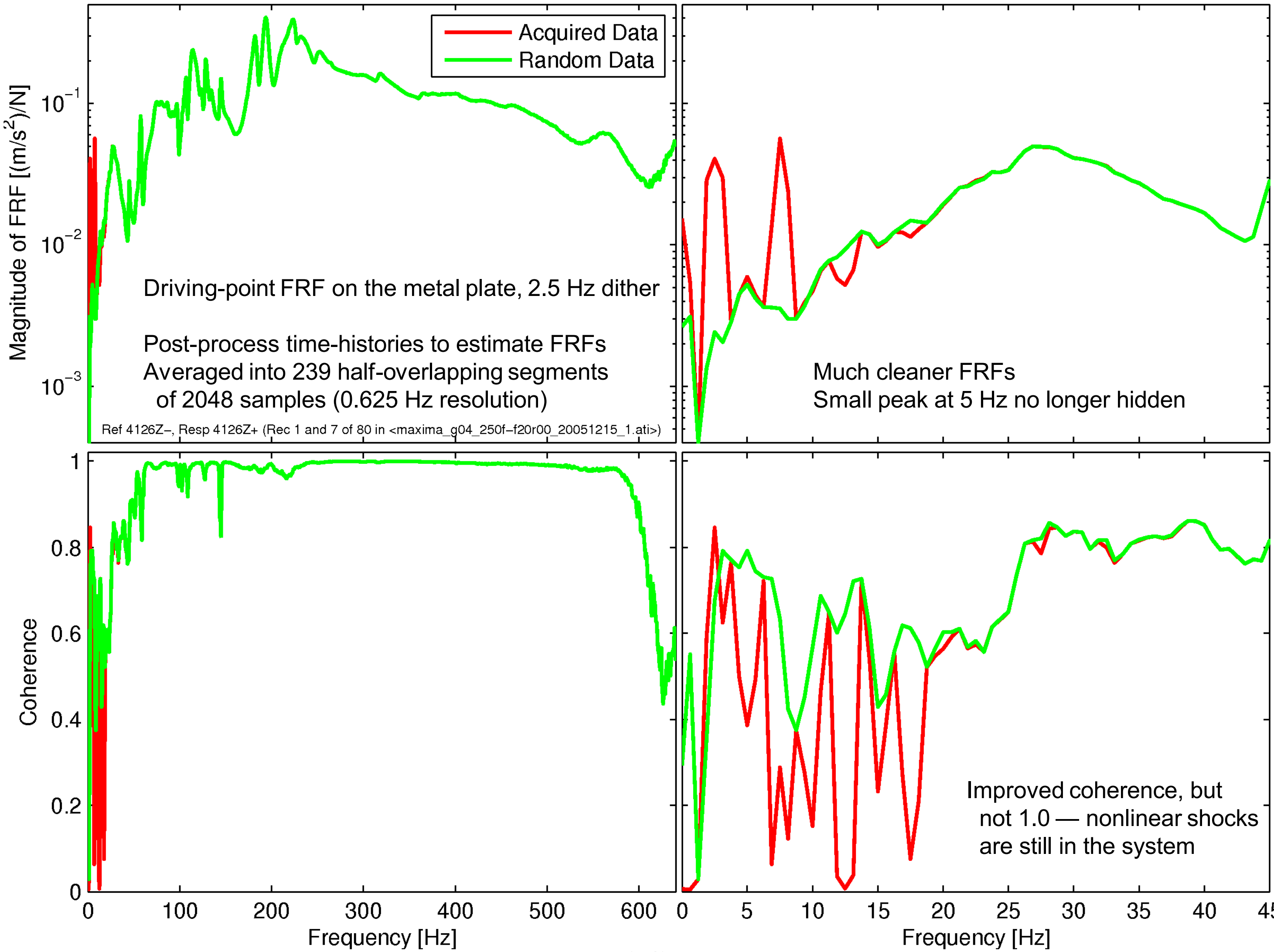

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Conclusions

- **Have developed/demonstrated a simple technique for identifying a stationary, periodic signal in what is otherwise random data**
  - **The periodic data can be removed from the acquired data, resulting in random data that can be post-processed in the usual fashion**
  - **The acquired data must be saved as *continuous* time-history data, not averaged frequency-domain data**
- **Advocate always recording time-history data**

Extra Information

- **Have also used technique to remove 60 Hz powerline noise**
  - **Power frequency is not very constant**
- **Why not just filter?**
  - **Filtering gets rid of a range of frequencies**
    - **can’t just “pluck out” a single frequency in the midst of data you want to keep**
    - **filter rolls off – does not give a sharp cutoff**
  - **Filtering doesn’t give a truly stationary signal**
    - **has some slight cycle-to-cycle variation**
    - **initial transient at start of signal**