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Comparison of Flux-Corrected-Transport and High Resolution Godunov Methods on Overlapping Grids*

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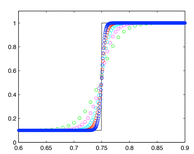
Many high resolution numerical techniques for inviscid compressible fluid flow have origin in ideas originally developed in connection with flux-corrected-transport (FCT) from the 1970's (Boris & Book). Over the years FCT has been applied to a wide range of challenging applications, however the underlying mathematical developments have not been as extensive as for other modern high resolution schemes. Recently the development of FCT finite element methods coupled with algebraic flux correction and implicit time integration (Kuzmin et. al.) has produced a resurgence of interest. Our current interest is in evaluating these methods for shock-hydro simulations in difficult strong shock regimes (e.g. Z-pinch and NIF). In this work we present initial results of a comparison of a high-resolution Godunov method and an FCT implementation within the Overture overset grid framework. These comparisons focus on test problems typically encountered when working in high energy flows as well as a new test problem designed to mimic many elements of a Z-pinch magnetic implosion.

Simple 1-D Solutions

Linear wave (contact):

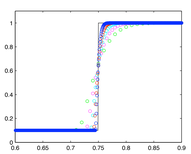
Linear discontinuities provide difficulty for capturing schemes which is manifest in the form of sub-linear convergence. Here a contact jump convergence study is performed and demonstrates the advantage of higher order methods.

$t = 0.5$



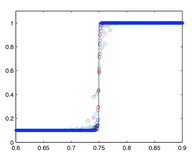
second order Godunov

$t = 0.5$



second order FCT

$t = 0.5$



fourth order FCT

m	ρ
100	$2.2e{-2}$
200	$1.4e{-2}$
400	$8.8e{-3}$
800	$5.5e{-3}$
1600	$3.5e{-3}$
α	0.66

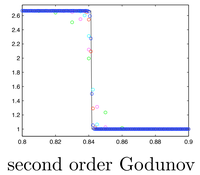
m	$e_m(\rho)$
100	$1.7e{-2}$
200	$1.1e{-2}$
400	$6.6e{-3}$
800	$4.2e{-3}$
1600	$2.6e{-3}$
α	0.67

m	$e_m(\rho)$
100	$1.0e{-2}$
200	$5.9e{-3}$
400	$3.4e{-3}$
800	$2.0e{-3}$
1600	$1.2e{-3}$
α	0.78

Non-linear wave (shock):

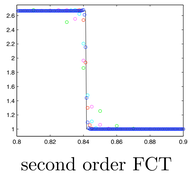
Nonlinear discontinuities have characteristics entering back into the jump and so are esier than the linear case. Here a convergence study for a Mach 2 shock is performed showing linear convergence for all fields.

$t = 0.25$



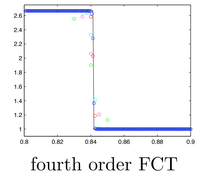
second order Godunov

$t = 0.25$



second order FCT

$t = 0.25$



fourth order FCT

m	$e_m(\rho)$	$e_m(u)$	$e_m(p)$
100	$1.5e{-2}$	$9.8e{-3}$	$2.7e{-2}$
200	$7.1e{-3}$	$4.8e{-3}$	$1.3e{-2}$
400	$3.6e{-3}$	$2.8e{-3}$	$6.3e{-3}$
800	$1.8e{-3}$	$1.2e{-3}$	$3.3e{-3}$
1600	$9.2e{-4}$	$6.7e{-4}$	$1.6e{-3}$
α	1.00	0.97	1.01

m	$e_(\rho)$	$e_m(u)$	$e_m(p)$
100	$1.7e{-2}$	$1.1e{-2}$	$3.0e{-2}$
200	$8.1e{-3}$	$5.5e{-3}$	$1.4e{-2}$
400	$3.9e{-3}$	$2.9e{-3}$	$6.4e{-3}$
800	$2.1e{-3}$	$1.4e{-3}$	$3.6e{-3}$
1600	$9.9e{-4}$	$7.1e{-4}$	$1.6e{-3}$
α	1.01	1.00	1.04

m	$e_m(\rho)$	$e_m(u)$	$e_m(p)$
100	$1.3e{-2}$	$8.3e{-3}$	$2.4e{-2}$
200	$5.8e{-3}$	$4.0e{-3}$	$1.1e{-2}$
400	$2.6e{-3}$	$2.1e{-3}$	$4.7e{-3}$
800	$1.5e{-3}$	$9.1e{-4}$	$2.7e{-3}$
1600	$6.7e{-4}$	$5.1e{-4}$	$1.2e{-3}$
α	1.05	1.02	1.06

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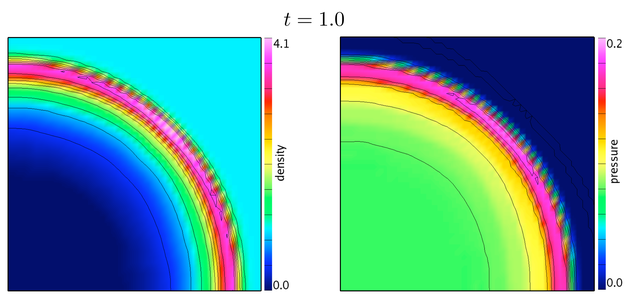
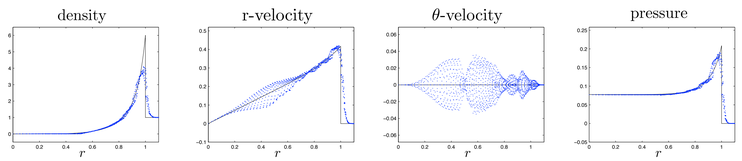
Standard Test Problems

Sedov blast wave:

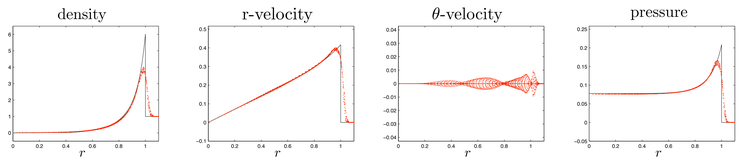
Here the classic Sedov blast wave shows the disparate amounts of numerical diffusion between the two schemes.

$t = 1.0$





FCT second order

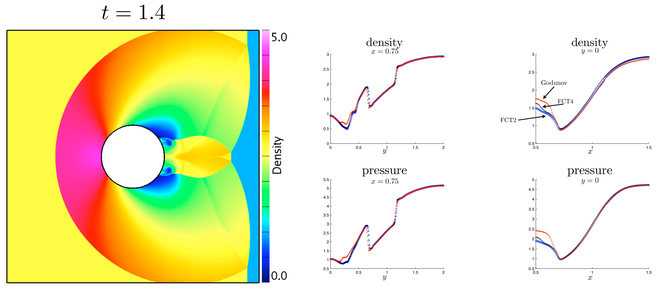


Godunov second order

Shock cylinder impact:

Here a study of a Mach 2 shock impacting a rigid cylinder is performed using an overset mesh to capture geometry.

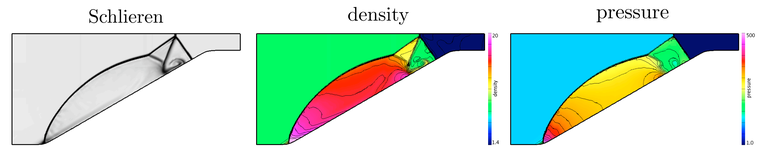
$t = 1.4$



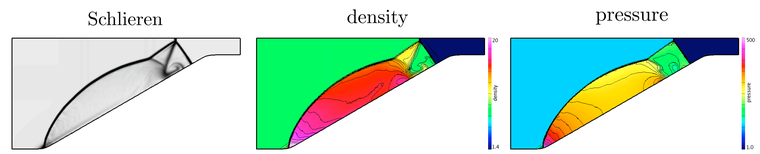
Irregular Mach reflection:

Reflection of a Mach 10 shock produces the so-called irregular Mach reflection which both schemes capture but for which FCT exhibits less diffusion.

$t = 0.225$



FCT second order



Godunov second order

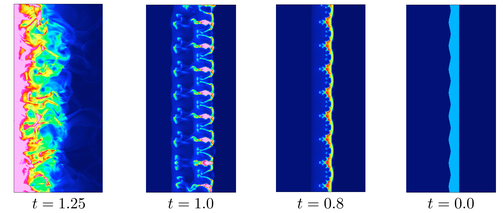
Conclusions

The FCT algorithm has been shown to be an effective algorithm giving comparable results to a high resolution Godunov method for a variety of test problems ranging from typical overset tests, such as shock cylinder interaction, to more classical high energy density physics tests, such as Sedov and irregular Mach reflection. There is indication that the FCT method has less numerical diffusion and thus higher resolving power. FCT also has an additional advantage that the obtainable spatial order of accuracy for a given stencil size is higher resulting in further accuracy gains.

Future Work

The extension of our improved FCT method (sonic fix, upwinded high-order, etc) to finite elements and prototype Z-pinch problems is ongoing and progress is being made.

density



$t = 1.25$ $t = 1.0$ $t = 0.8$ $t = 0.0$