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Peridynamic Computational Model for Damage and Fracture

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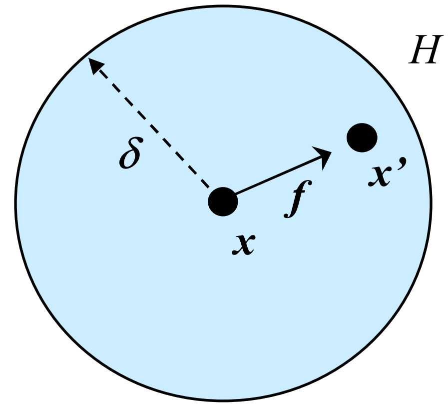
Background:
A better way to model cracks

- Problem
  - Develop a general tool for modeling material and structural failure due to cracks.
- Motivation
  - Fracture is a major mode of material failure.
  - Standard mathematical theory for modeling deformation cannot handle cracks.
    - PDE's break down if a crack is present.
    - Finite elements and similar methods inherit this problem.
- Approach
  - Develop a mathematical theory in which:
    - The same equations apply on or off of a crack.
    - Cracks are treated like any other type of deformation.
    - Cracks are self-guided: no need for supplemental equations.
  - Implement the theory in a meshless Lagrangian code called **EMU**.

Peridynamics – the basic idea



- PDEs are replaced by the following integral equation:

$$\rho \ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \int_H \boldsymbol{f}(\boldsymbol{u}' - \boldsymbol{u}, \boldsymbol{x}' - \boldsymbol{x})\, dV' - \boldsymbol{b}(\boldsymbol{x},t) \qquad (1)$$

- Compare classical PDE:

$$\rho \ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \nabla \bullet \boldsymbol{\sigma}(\boldsymbol{x},t) - \boldsymbol{b}(\boldsymbol{x},t) \qquad (2)$$

where
$\boldsymbol{u}$ = displacement; $\boldsymbol{f}$ = force density that $\boldsymbol{x}'$ exerts on $\boldsymbol{x}$;
$\boldsymbol{b}$ = prescribed external force density; H = neighborhood of $\boldsymbol{x}$ with fixed radius δ;
$\boldsymbol{\sigma}$ = stress tensor field.

- New approach is more general than the classical model in the following ways:
  - (1) is not derivable from (2).
  - (1) converges to (2) in the limit $\delta \to 0$ (proof to be published).
  - (1) holds regardless of discontinuities.
  - (1) includes interaction between particles through a finite distance.
    - (1) is a "**Continuum version of molecular dynamics**"

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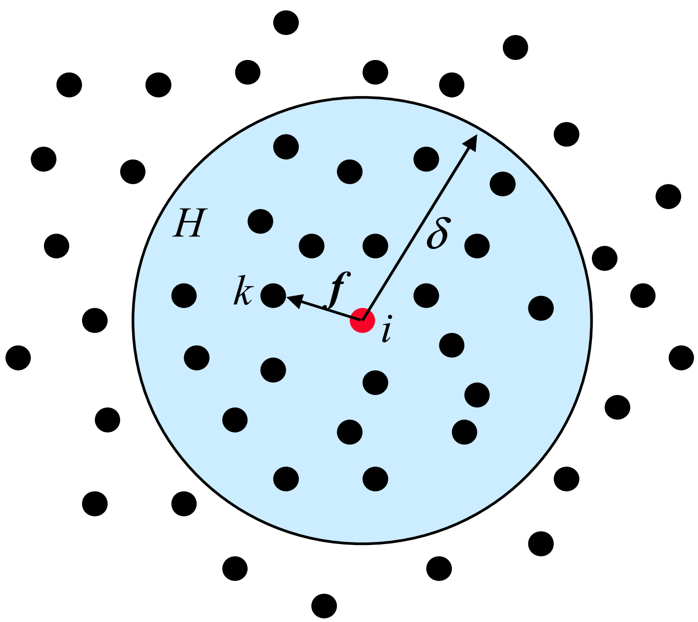
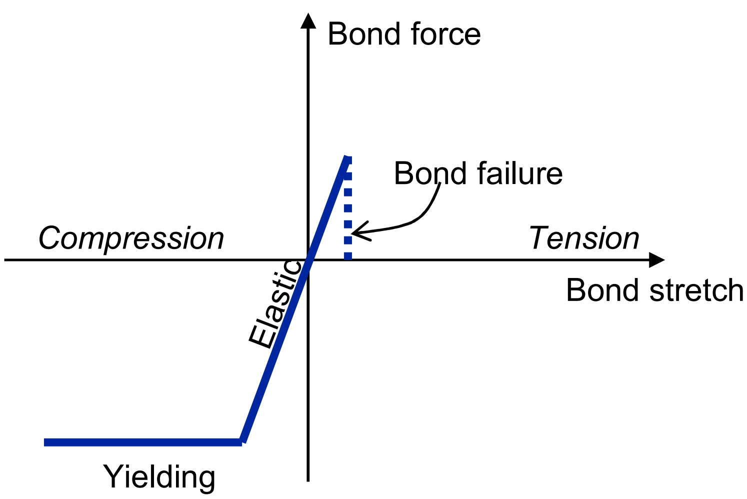
EMU numerical method and material model incorporate damage at the bond level

- Integral is replaced by a finite sum: resulting method is meshless and Lagrangian.
- Parameters come from measurable elastic-plastic and fracture data for materials.
- Sandia's Trilinos solver package is being applied to this method.

$$\rho \ddot{\boldsymbol{u}}_i^n = \sum_{k \in H} \boldsymbol{f}(\boldsymbol{u}_k^n - \boldsymbol{u}_i^n, \boldsymbol{x}_k - \boldsymbol{x}_i) \Delta V_i + \boldsymbol{b}(\boldsymbol{x}_i, t)$$

All material-specific behavior is contained in the function ***f***.





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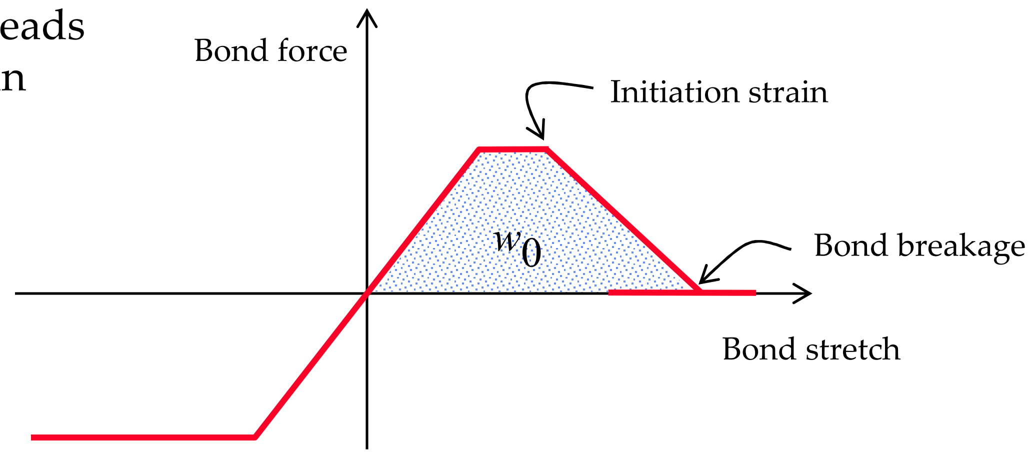
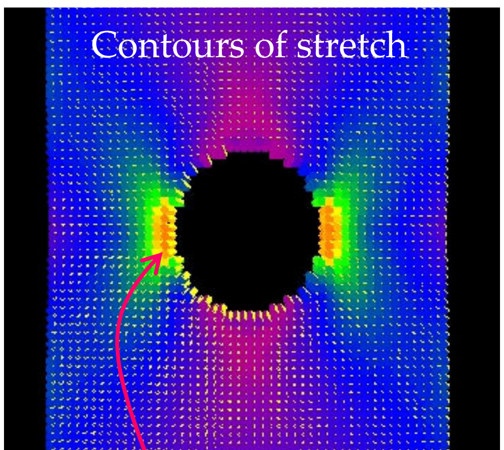
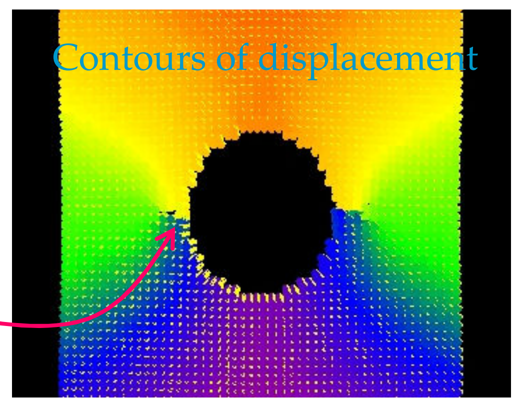
Initiation

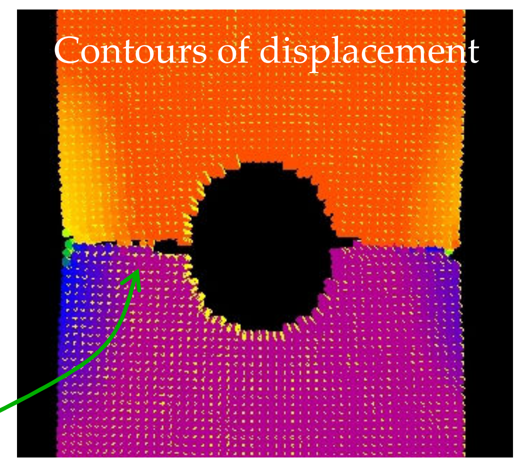
- A maximum in the bond force curve leads to crack initiation at a prescribed strain independent of the breakage stretch.





Stretch > initiation strain leads to crack initiation





Crack initiation leads to dynamic fracture

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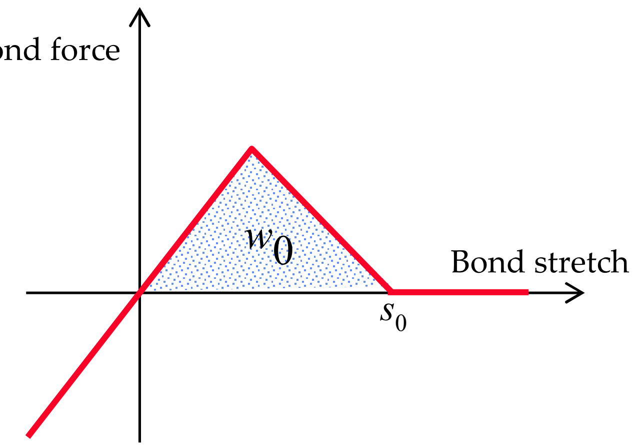
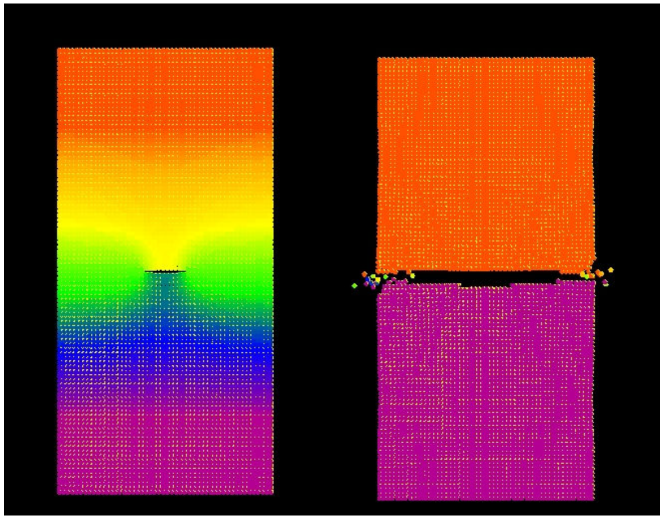
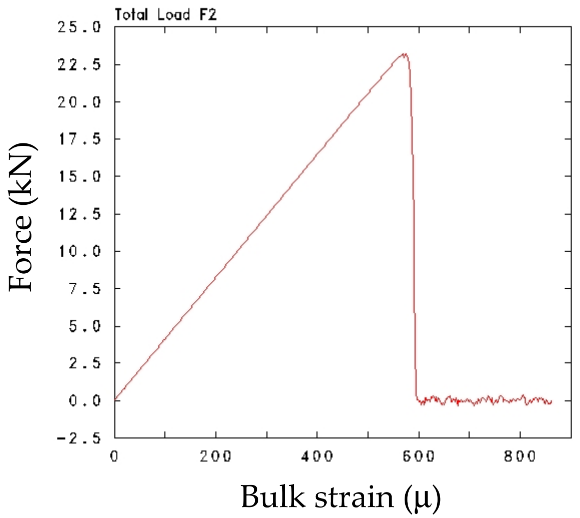
A validation problem:
Center crack in a brittle panel (3D)



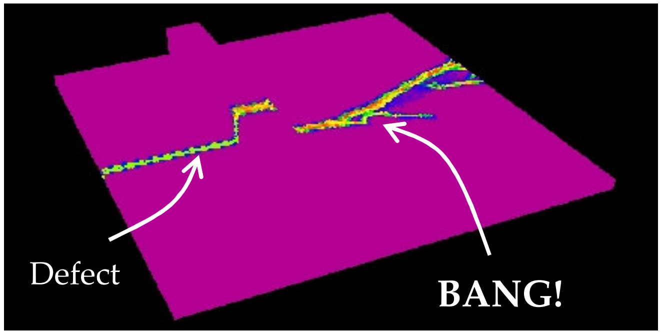




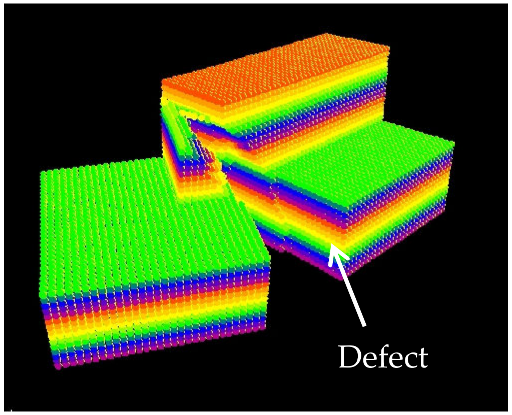
Based on s_0=0.002, find **G=384 J/m^2**.
Full 3D calculation shows crack growth when σ=46.4 MPa. Use this in

$$G = \frac{\pi\sigma^2 a}{E} = 371\,\mathrm{J/m^2}$$

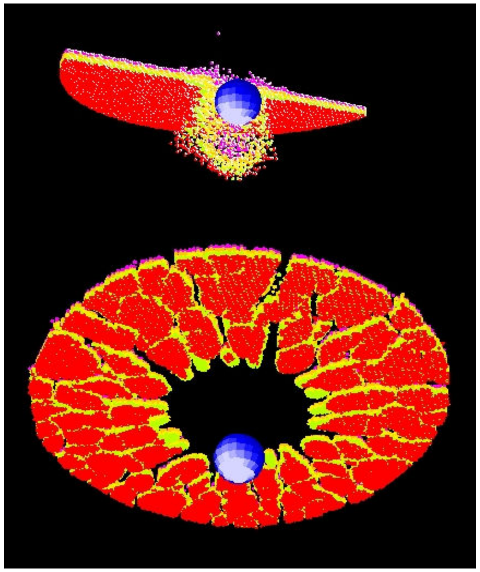
Isotropic materials:
Other examples



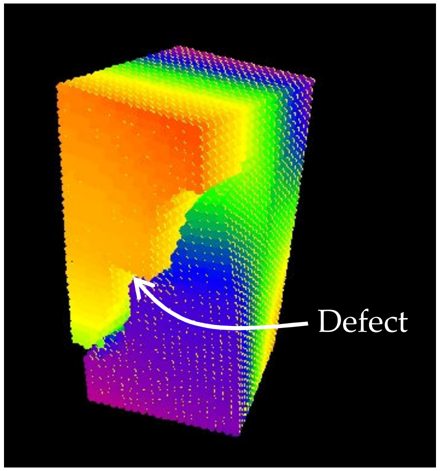
Transition to unstable crack growth



Crack turning in a 3D feature

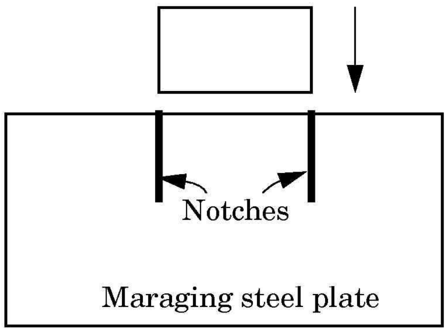


Impact and fragmentation

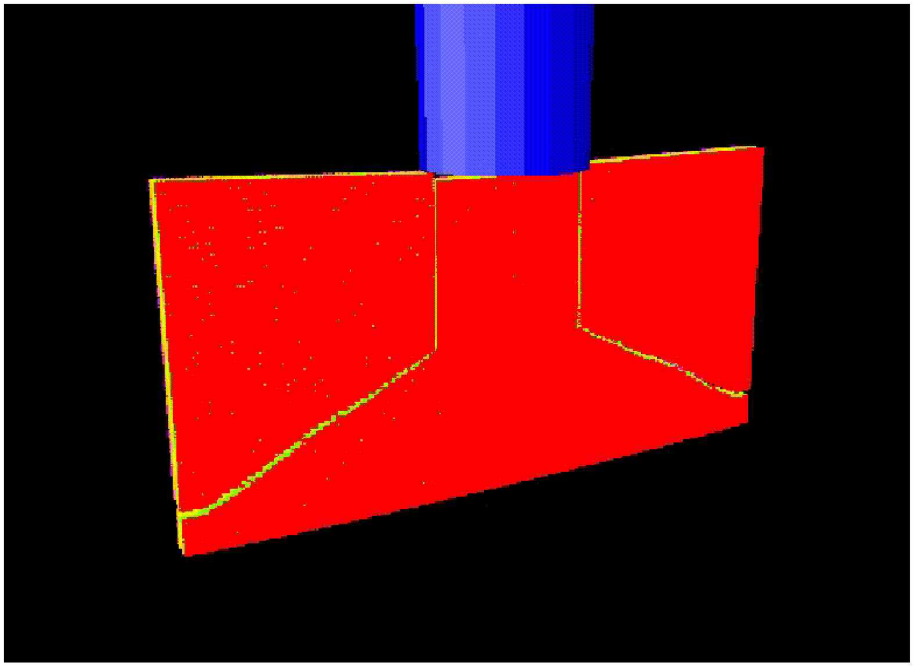


Spiral crack due to torsion

Example: dynamic fracture in steel



- Code predicts correct crack angles*.
- Crack velocity ~ 900 m/s.



*J. F. Kalthoff & S. Winkler, in *Impact Loading and Dynamic Behavior of Materials*, C. Y. Chiem, ed. (1988)

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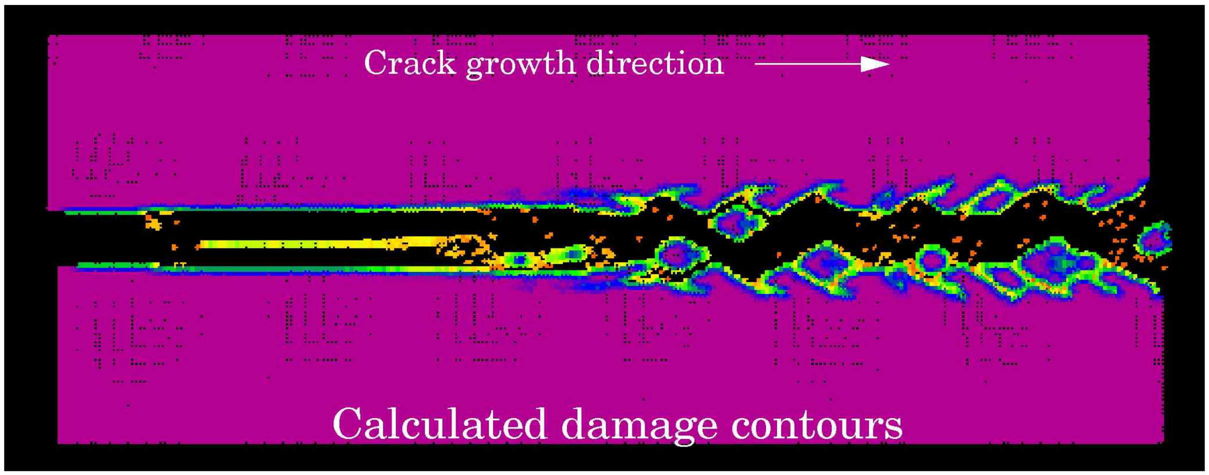
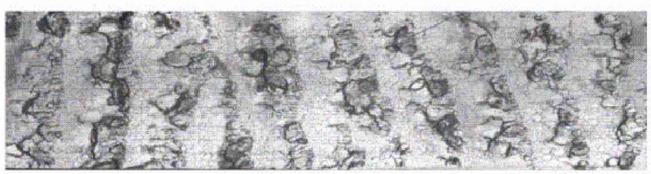
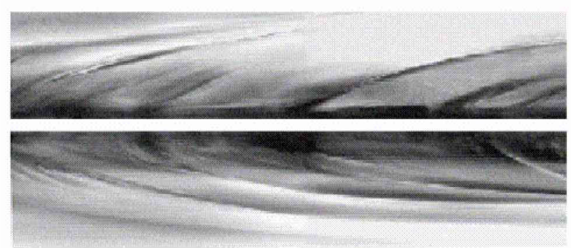
Example: dynamic fracture in PMMA

- Plate is stretched vertically.
- Code predicts stable-unstable transition.







Experiment*

*J. Fineberg & M. Marder, *Physics Reports* **313** (1999) 1-108

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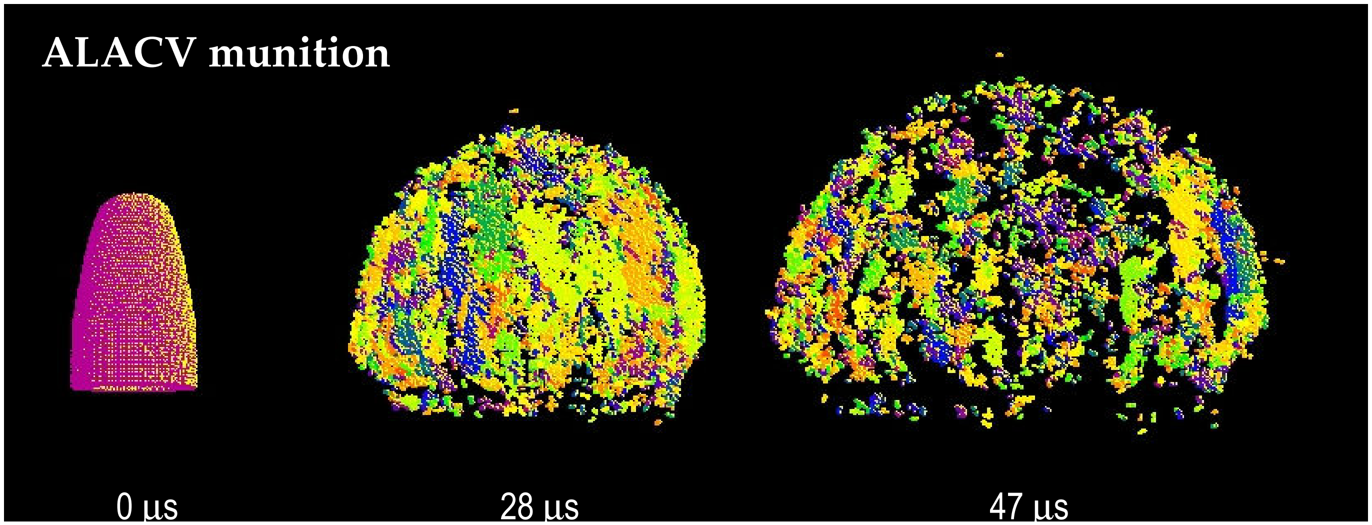
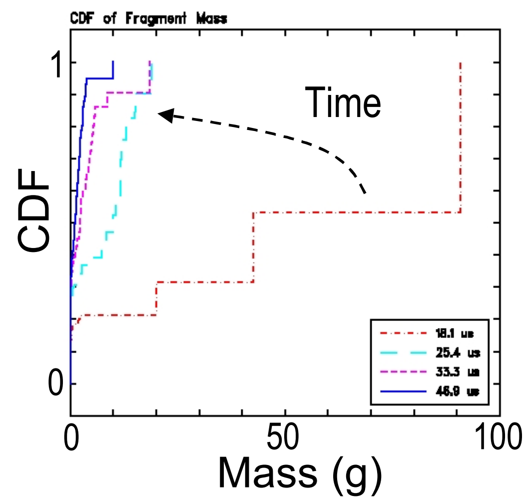
Application (DoD/DOE MOU): 3D mechanics-based modeling of fragmentation



- CDF of fragment mass evolves over time.
- Method predicts statistical distribution of fragment energy, shape, and direction.
- Important application to NW safety.



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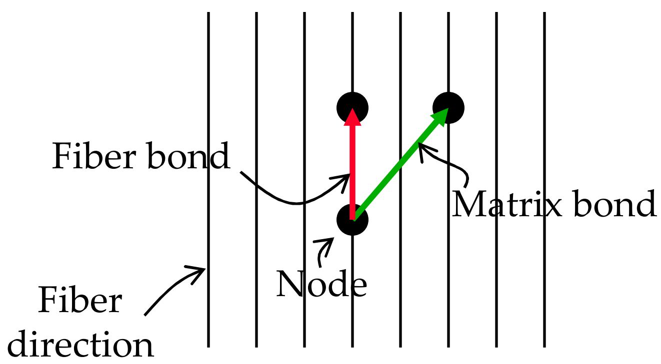
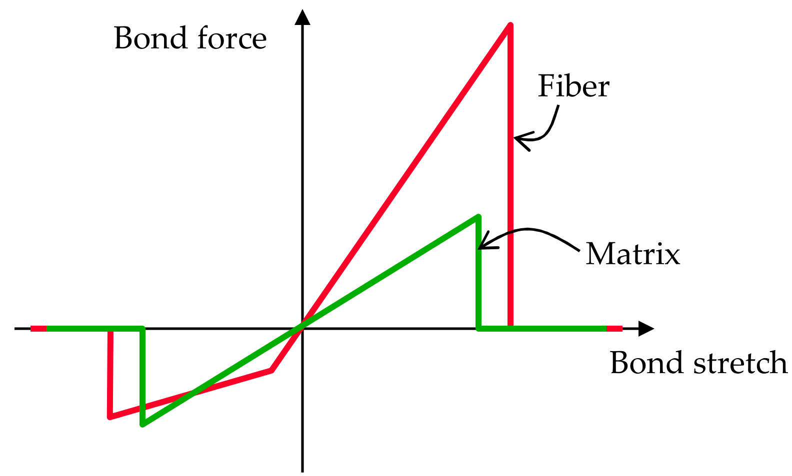
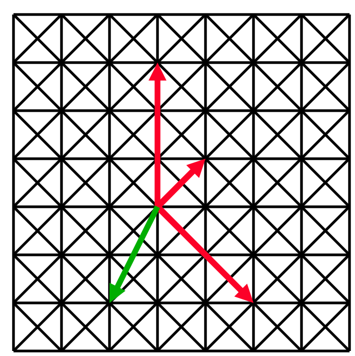
Composites:
Peridynamic model of lamina

- Bonds parallel to fibers (0º, 45º, …) have special properties.





2D **laminate** model is just a lamina with fibers allowed in multiple directions (accounting for different fiber density in each direction)



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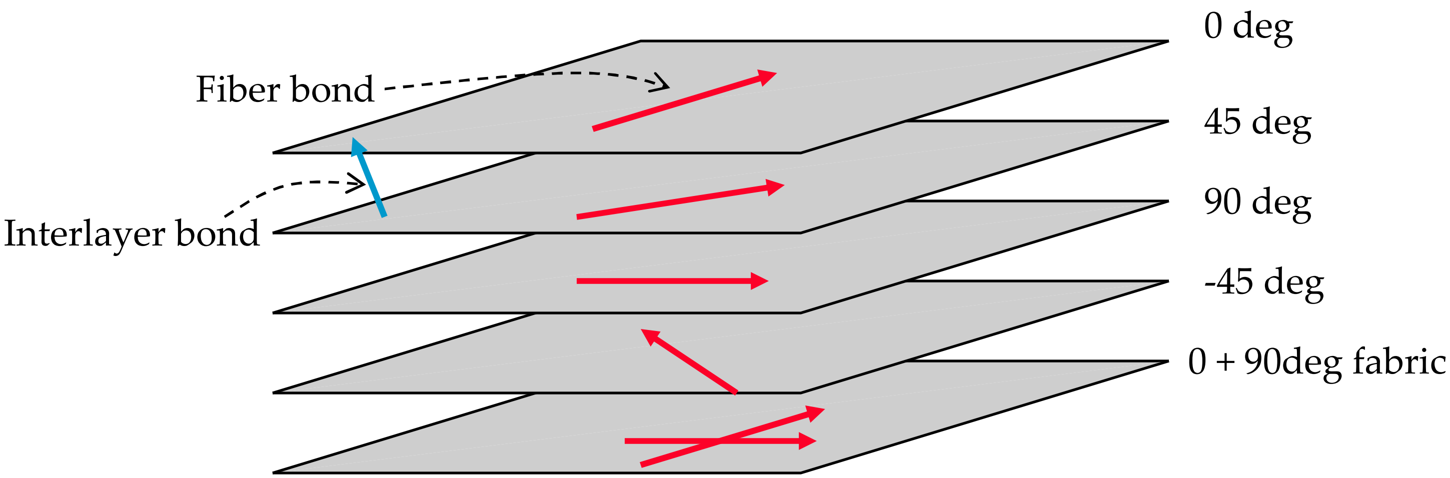
3D peridynamic model of a laminate

- Lamina models are stacked up according to the stacking sequence.
  - Lamina can contain multiple fiber directions (**fabric**).
  - Nodes in different lamina interact through **interlayer** peridynamic bonds.



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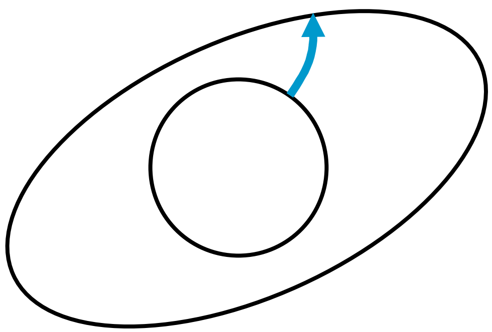
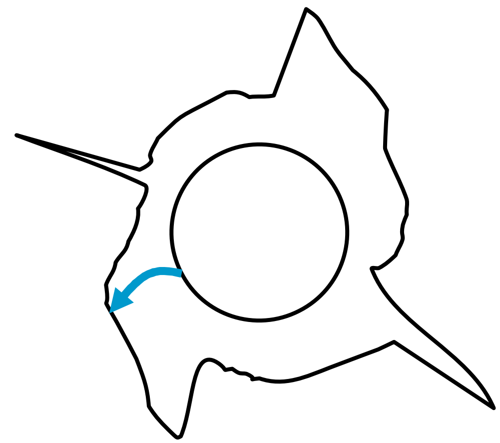
Peridynamic model of composites: Why take this approach?

- Classical ideas of stress and strain tensors break down in a damaged composite.
  - Traditional FE methods don't apply.
  - More "degrees of freedom" in a peridynamic deformation state allow for a greater range of failure phenomena to be captured.
  - Yet we can exploit traditional failure models if applicable.
- Natural correspondence between fibers and peridynamic bonds.
- Integral equations permit discontinuities in the mathematics.



Strain tensor maps any small sphere into some ellipsoid.



Peridynamic **deformation state** maps a sphere into whatever… "infinite degrees of freedom"

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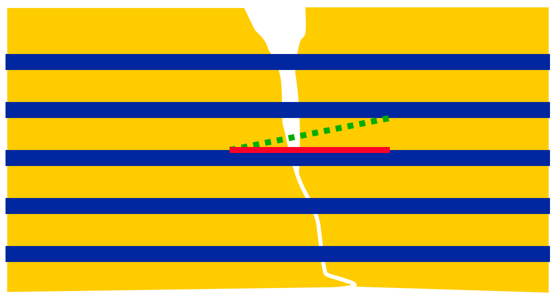
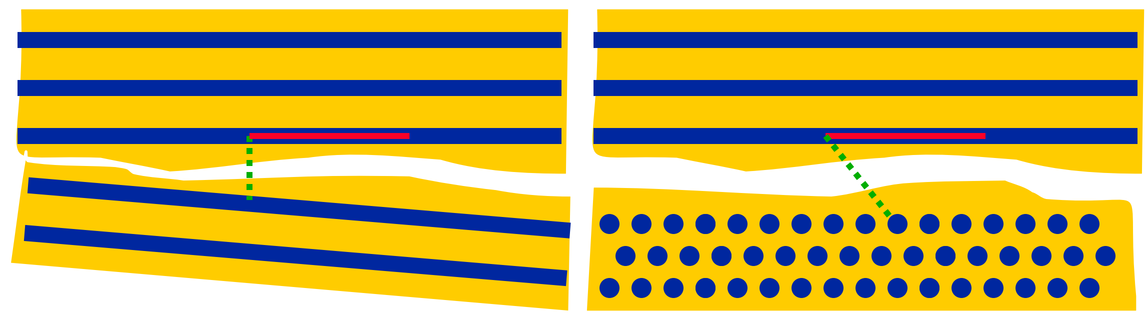
Peridynamic model of composites: Why take this approach?

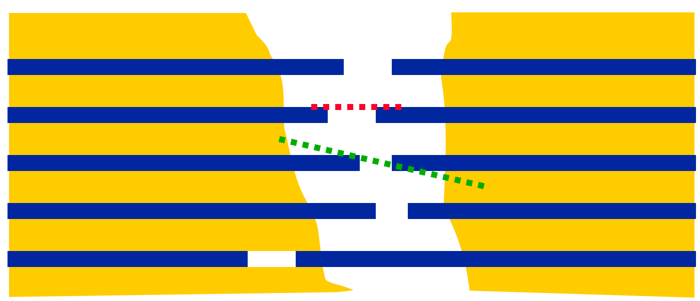
- Network of bonds gives a truer picture of failure modes.



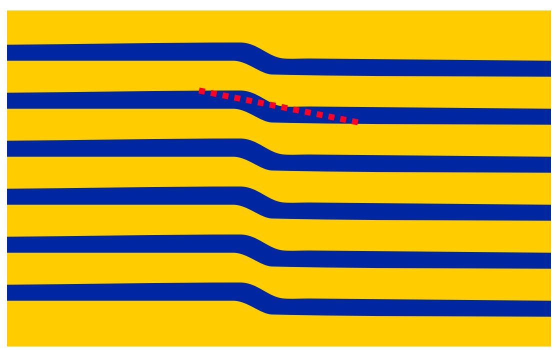
Bridging



Matrix cracking, debonding, delamination



Fiber tensile failure



Fiber compressive failure

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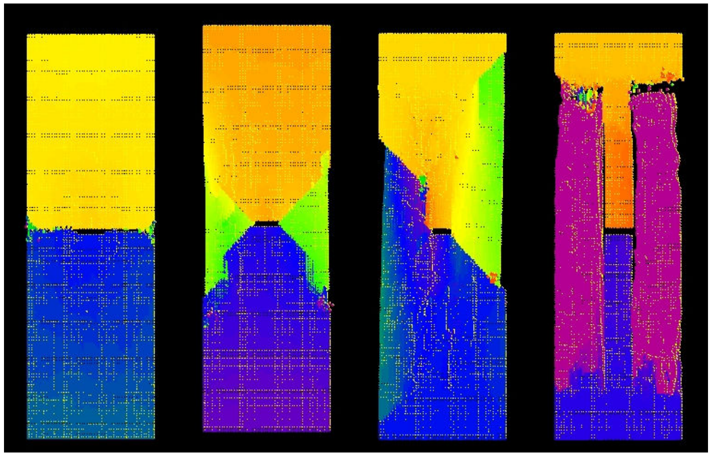
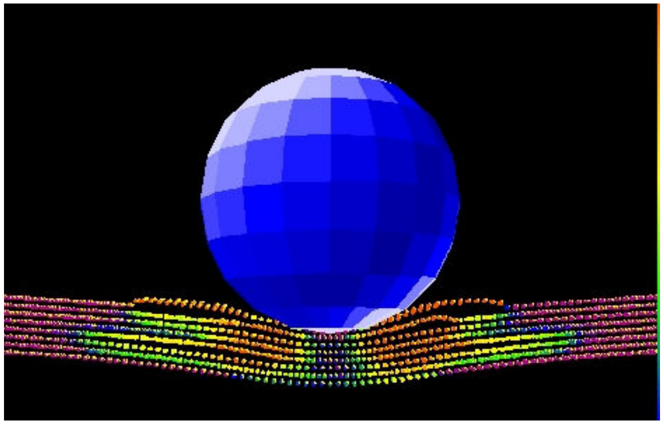
Composite material model applications

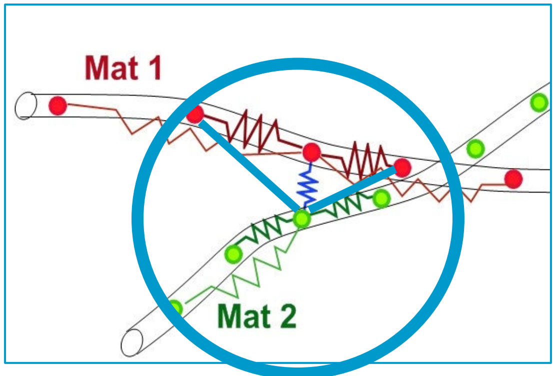
- How does the fraction of fibers in each direction affect the direction of crack growth?
- What damage occurs when a composite panel is struck by hail?


Crack growth in a notched panel

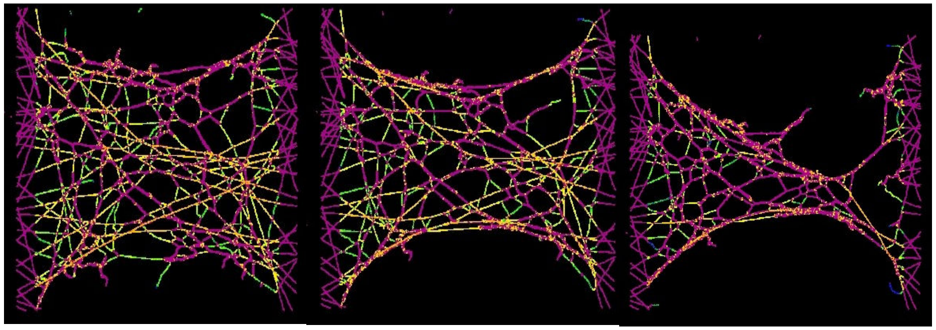

Delamination caused by impact

Nanoscale structural mechanics

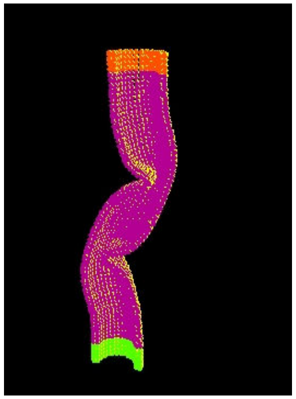
- Continuum version of molecular dynamics.
- All forces are treated as long-range.



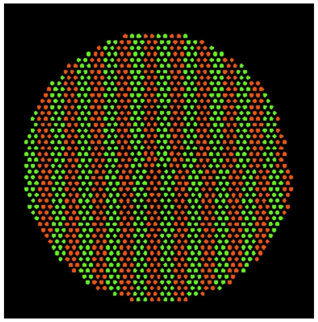
Nanofiber interactions due to long-range forces



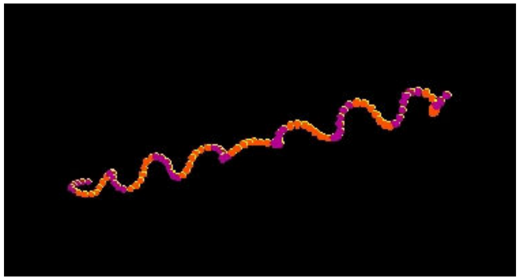
Failure in a nanofiber membrane (F. Bobaru, Univ. of Nebraska)



Carbon nanotube



Dislocation



Nanofiber self-shaping

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Summary: Peridynamic vs. conventional model

- Theory
  - Different assumptions lead to different mathematics.
- Code
  - Discretized equations have different properties from finite elements.
  - Meshless, Lagrangian.
  - Easily implemented on parallel machines.
- Applications
  - New approach addresses the way materials **really** fail.
  - Method can incorporate nanoscale forces.
  - Length scale is variable – possible way to do multiscale modeling.