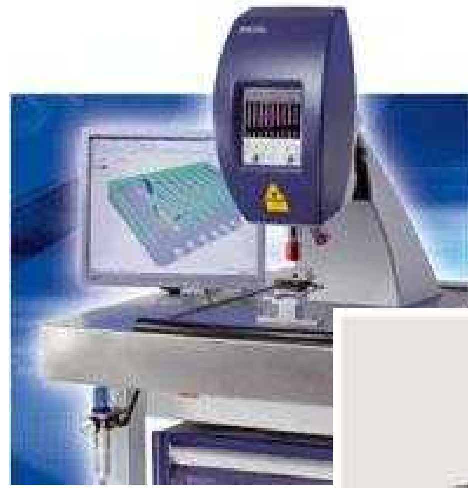
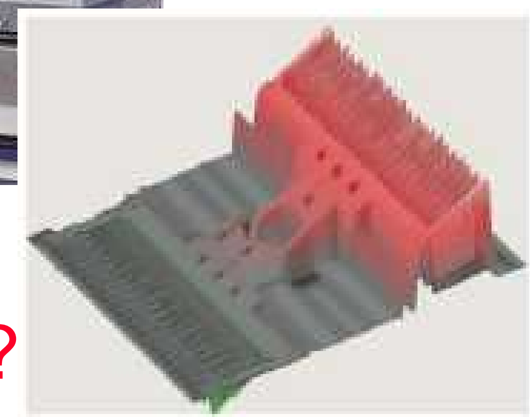
Doppler Electron Velocimeter—a Proposed Nano-scale Dynamic Measurement System

SAND2007-3267C

Can this TEM…



…do this?



SEM Annual Conference 2007

Phillip L. Reu

Senior Member Technical Staff

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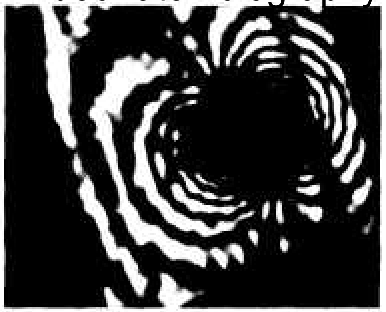
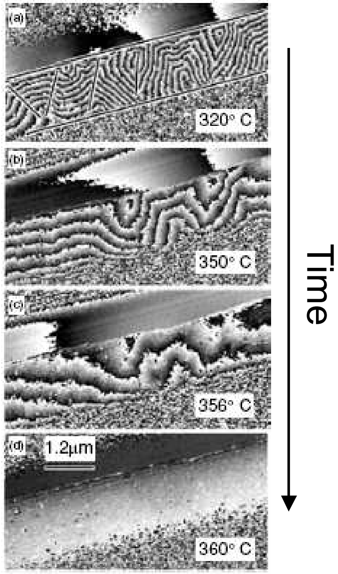
The case for Doppler electron velocimetry

Holography TEM

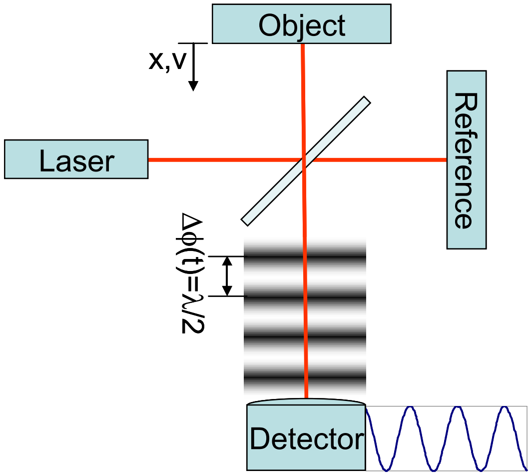
Video rate holography



Moving fringes



Optical analogy



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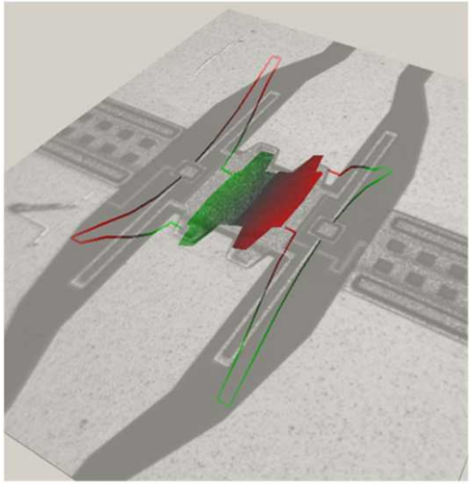
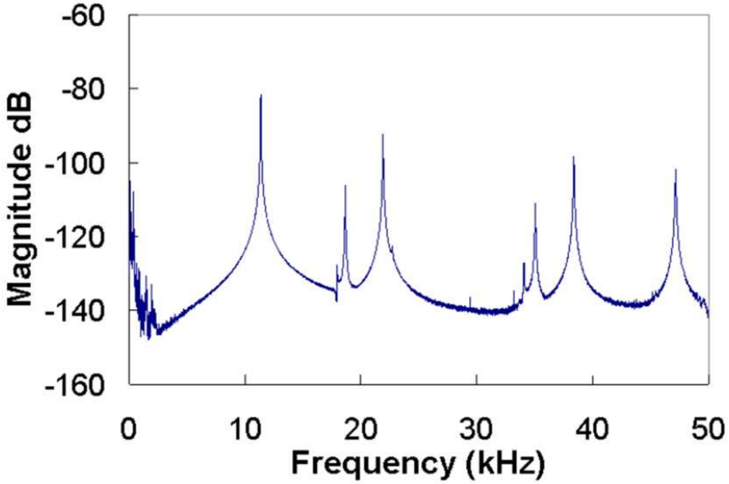
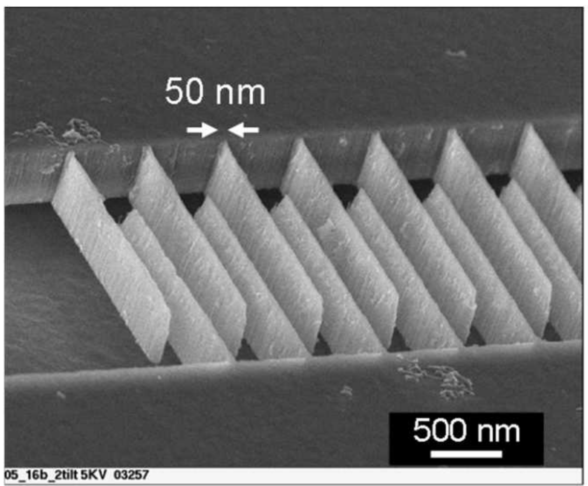
Hitachi: J. Chen et al. / Optics Communications 110 (1994) 33-40: Eur. J. Phys. 26 (2005) 481–489

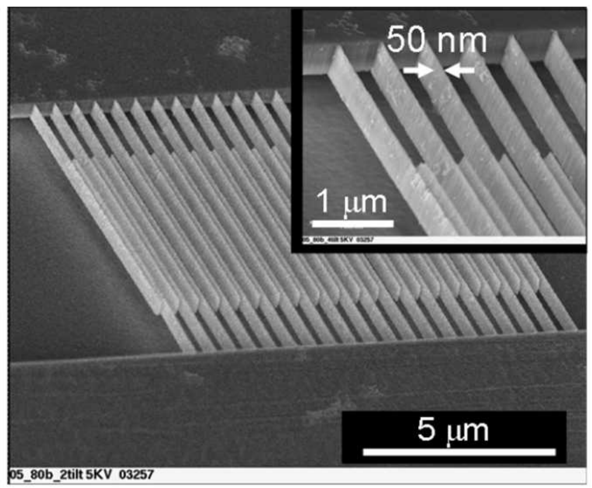
Nanodynamics! A future discipline





Dynamics is important for MEMS





Dynamics is important for Nano

1. Varying magnetic and electric fields
2. Mechanical motion

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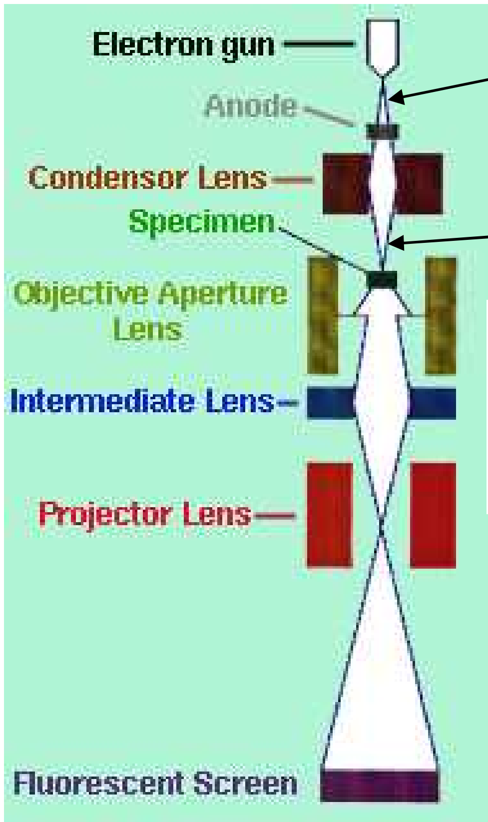
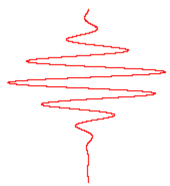
de Broglie waves describe the wave-packet nature of electrons

$$p = \frac{h}{\lambda}$$



U accelerating potential

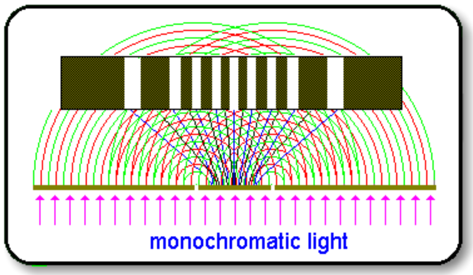
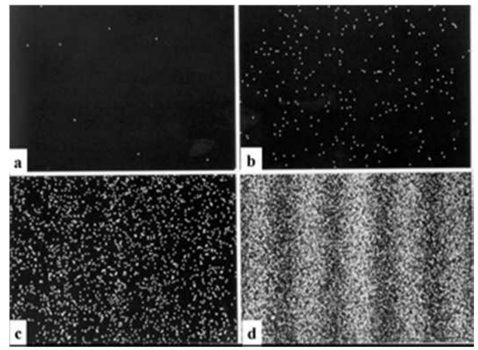
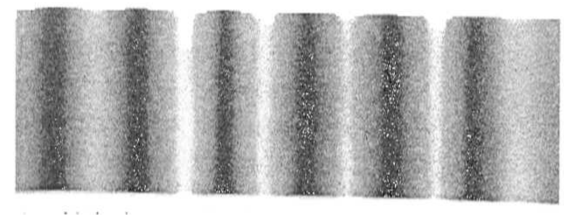
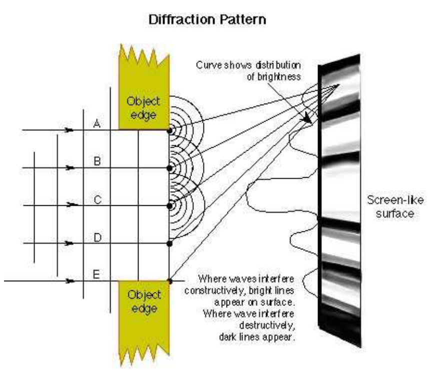
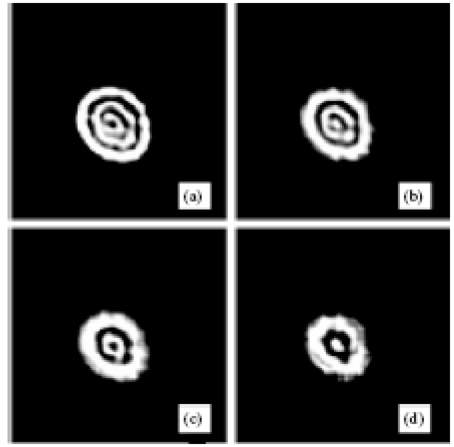
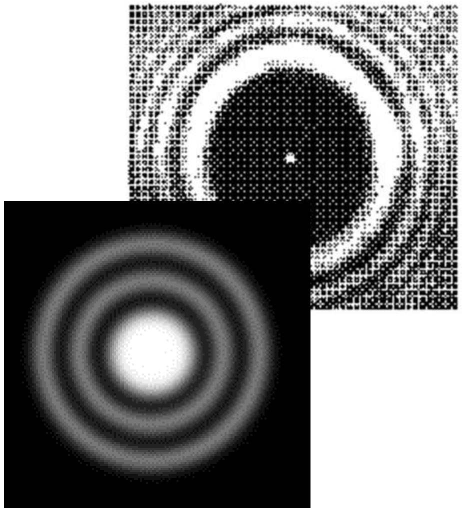
$$\lambda = \frac{1.226}{\sqrt{U\left(1 + 0.9788 \times 10^{-6} U\right)}}$$



$$\psi(x,t) = A\cos\left(2\pi\left[\int_{x_o}^{x} \frac{1}{\lambda(x)} dx - f(x)t\right]\right)$$

http://www.sv.vt.edu/classes/MSE2094_NoteBook/96ClassProj/experimental/electron.html

The electron wave shares many properties with photon waves

	Schematic	Electrons	Photons
Interference			
Diffraction			

http://www.hqrd.hitachi.co.jp/em/emgif/fig2.gif

http://www.physchem.co.za/Light/Diffraction.htm

http://planetquest.jpl.nasa.gov/technology/diffraction.cfm

Eur. J. Phys. **26** (2005) 481–489

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/frestr.html

http://www.atmsite.org/contrib/Poulson/faq/airydisk.JPG

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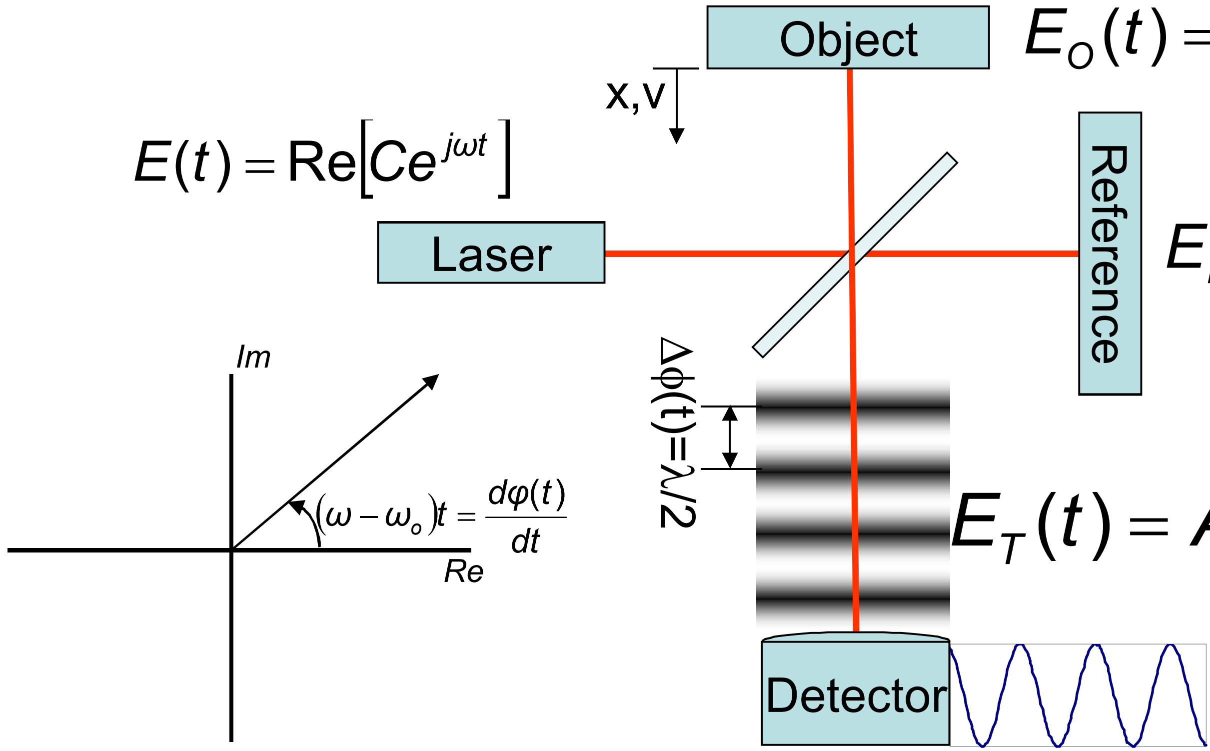
Light waves demonstrate Doppler shifting, so maybe electron waves do also?

$$E_O(t) = Ae^{j(\omega_o t + \varphi_o(t))}$$

$$E(t) = \mathrm{Re}\left[Ce^{j\omega t}\right]$$

$$E_R(t) = Be^{j(\omega t + \varphi_r)}$$



$$E_T(t) = Ae^{j(\omega_o t + \varphi_o(t))} + Be^{j(\omega t + \varphi_r)}$$

$$I(t) = E_T E_T^*$$

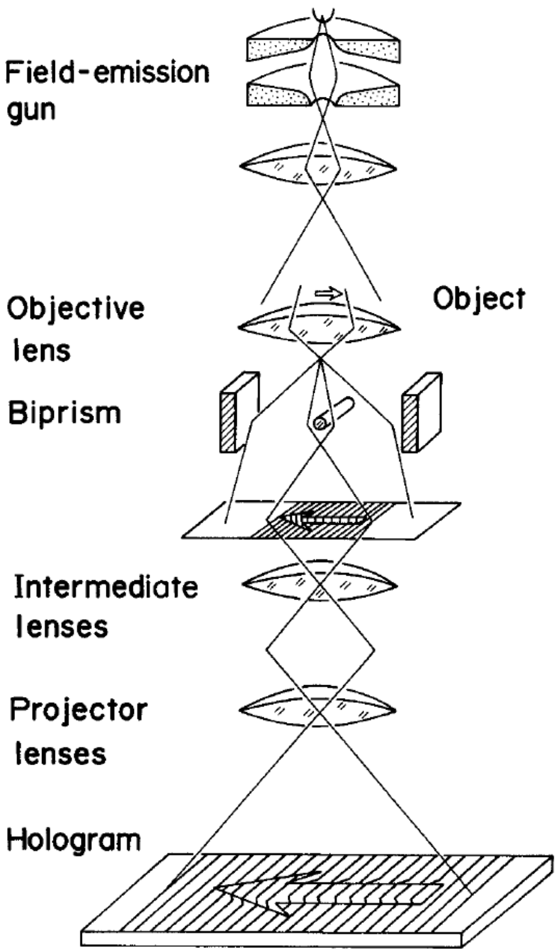
$$I(t) = A^2 + B^2 + 2AB\left[e^{j[(\omega-\omega_o)t + (\varphi_r - \varphi_o(t))]} + e^{-j[(\omega-\omega_o)t + (\varphi_r - \varphi_o(t))]}\right]$$

$$I(t) = I_o + D\cos\left[(\omega - \omega_o)t + (\varphi_r - \varphi_o(t))\right]$$

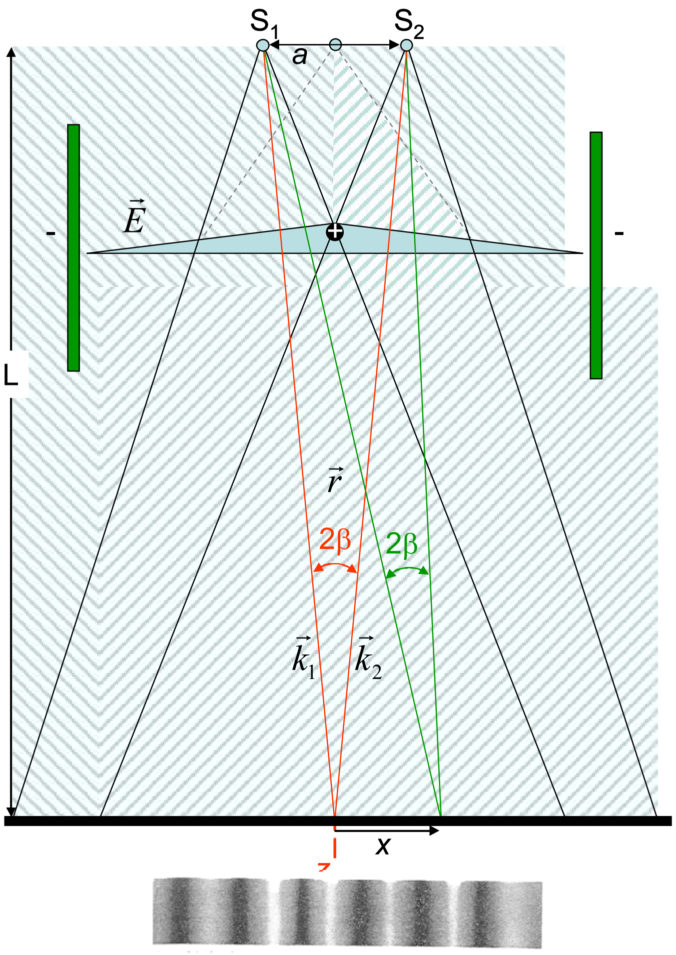
Doppler Term — $(\omega - \omega_o)t$

Phase-Shift Term — $(\varphi_r - \varphi_o(t))$

The Möllenstedt biprism can be used to interfere an electron beam



ADVANCES IN PHYSICS, 1992, VOL. 41, NO. 1, 59-103
Electron-holographic interference microscopy By A. TONOMURA



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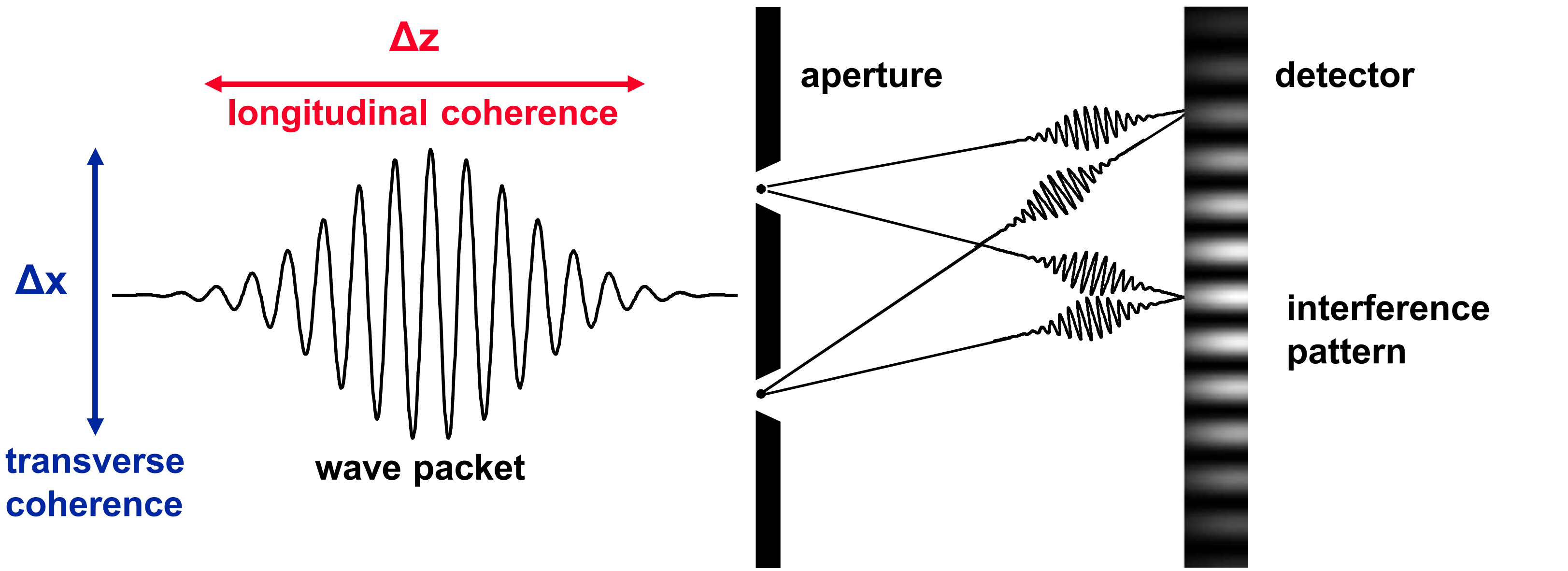
Electron beam coherence is short, but still practical for interferometry

Longitudinal coherence is related to the source energy spread.

Must be long enough to overlap



Transverse coherence is related to the source size.

Must cover both holes

Temporal (Longitudinal) Coherence Length

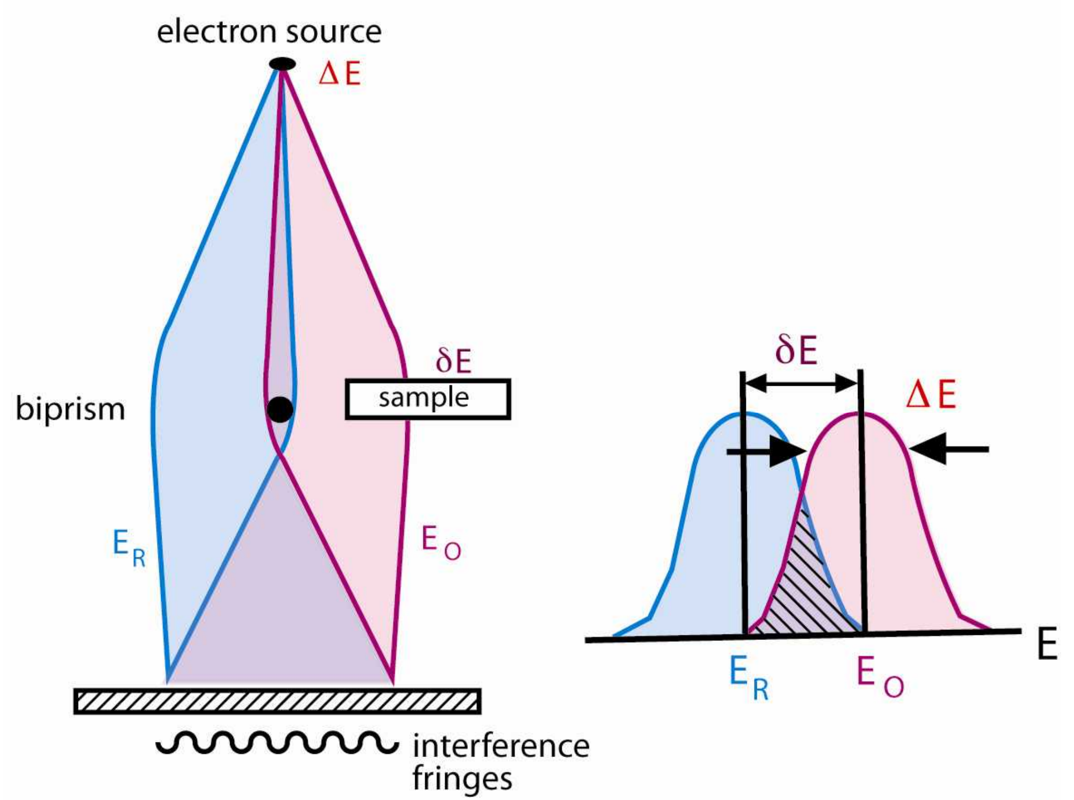
U (kV)	ΔE (eV)	$\Delta\tau$ (s)	v (m/s)	Δz (nm)
100	3	1.38E-15	1.64E+08	227
100	2	2.07E-15	1.64E+08	340
100	1	4.14E-15	1.64E+08	680
100	0.5	8.27E-15	1.64E+08	1359
100	0.1	4.14E-14	1.64E+08	6797

Counter-arguments against interference of electrons

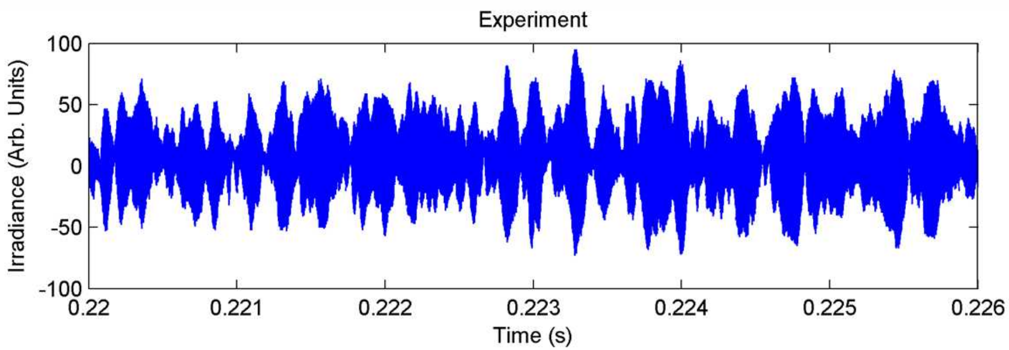
Energy shifted electrons don't interfere

Record t (s)	$\delta E < h/t$ (eV)	f_{sample} (MHz)
→ 1	4.14E-15	0.000001
0.01	4.14E-13	0.0001
0.0001	4.14E-11	0.01
0.000001	4.14E-09	1

$$|\delta E| < \frac{h}{t}$$



Incoherence of electrons after scattering



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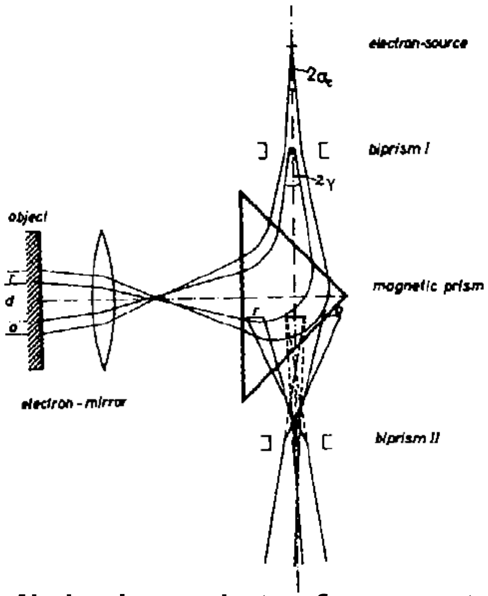
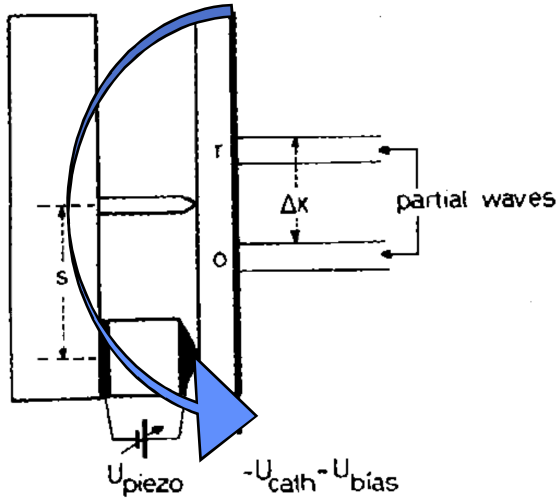
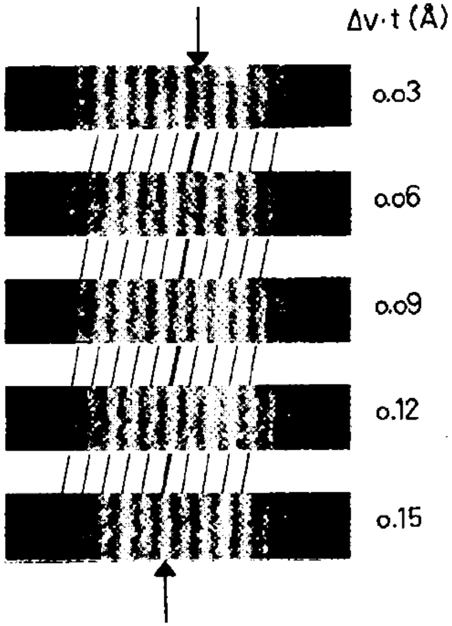
Demonstration of Doppler with electron beams by Möllenstedt and Lichte



Michelson interferometer



Rotating mirror



Moving Fringes

beat frequency $\Delta\nu_b$: $I(t) = 2I_o(1+\cos 2\pi \Delta\nu_b t)$

$$I(t) = I_o + D\cos\left[(\omega - \omega_o)t + (\phi_r - \phi_o(t))\right]$$

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1. Möllenstedt, G., H. Lichte, "Doppler shift of electron waves," 9th International Congress on Electron Microscopy, Toronto, 1978, p178-179.
2. Scherzer, O., "Der electronenoptische Doppler-Effeckt," Optik, 54, 315, 1979.

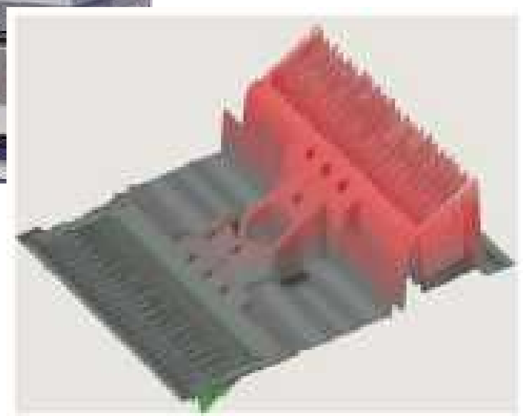
Source brightness and detector sensitivity will define maximum Doppler

Electron Beam Sources	Virtual Source Diameter (μm)	Energy Width (eV)	Acceleration Voltage (kV)	Measured Brightness (A cm^{-2} sr^{-1})
Heated Field Emission	0.1	0.8	100	10^7-10^8
RT Field Emission	0.002	0.28	100	2×10^9
Hair-Pin Cathode	30	0.8	100	5×10^5
Tungsten (W) Cathode	10 – 50	1-2	100	1 to 5×10^5
LaB_6 Cathode	5 – 10	1	75	7×10^6

Coherence limits and detector speeds are the current topic of research

Optical analogs to study coherence issues

Detector bandwidth studies



Can it do it well?

Questions???

Typical wavelength and velocity calculations for an electron beam

Wavelength Calculations

U (kV)	λ (nm)	K (1/nm)	me/mo	v/c
0.1	0.12264	8.154	1.00020	0.01978
1	0.03876	25.797	1.00196	0.06247
5	0.01730	57.796	1.00978	0.13887
10	0.01220	81.935	1.01957	0.19498
50	0.00536	186.729	1.09785	0.41268
100	0.00370	270.163	1.19569	0.54822
200	0.00251	398.732	1.39139	0.69531
300	0.00197	507.933	1.58708	0.77652
400	0.00164	608.289	1.78277	0.82787
500	0.00142	703.594	1.97847	0.86286
600	0.00126	795.666	2.17416	0.88795
700	0.00113	885.514	2.36985	0.90661
800	0.00103	973.753	2.56555	0.92091
900	0.00094	1060.785	2.76124	0.93212
1000	0.00087	1146.886	2.95693	0.94108
2000	0.00050	1982.858	4.91387	0.97907

c	2.99792E+08 m/s
e	1.60219E-19 C
h	6.62620E-34 J-s
m_o	9.10953E-31 kg

$$\lambda = \frac{h}{\sqrt{\left[2m_o eU\left(1+\frac{eU}{2m_o c^2}\right)\right]}} = \frac{1.226}{\sqrt{U(1+0.9788\times10^{-6}U)}}$$

$$v = c\sqrt{1-\frac{1}{\left(1+\frac{eU}{m_o c^2}\right)^2}}$$

Temporal (Longitudinal) Coherence Length

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For lasers:
$\Delta\tau = 5m/c = 1e^{-8}$

$$\Delta z = v\Delta\tau$$

$$\Delta\tau = \frac{h}{\Delta E}$$

Spatial (Transverse) Coherence

HF-2000 TEM – the entire 100 μm aperture is coherently illuminated.

Völkl, E., *Introduction to electron holography*, p. 64

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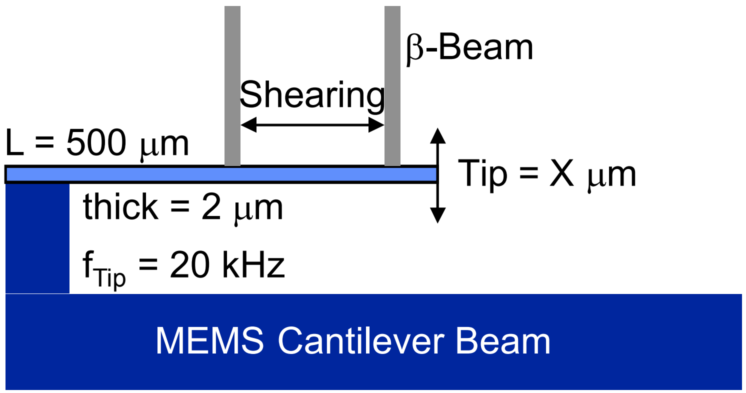
Basic Calculations for a MEMS cantilever beam



$$\Delta E = \frac{4h\Delta v}{3\lambda e}(eV)$$

$$f_{Doppler} = \frac{\Delta E}{h} = \frac{4\Delta v}{3\lambda}$$

Velocity Table

Beam length = 500 μm - Parabolic velocity distribution

λ (nm) 0.0037

Tip (μm)	f_{Tip} (kHz)	Tip v (mm/s)	Shearing (μm)	Δv (mm/s)	Δv (nm/s)	ΔE (eV)	$f_{Doppler}$ (kHz)
2	20	40	1	0.15984	159840	2.382E-07	57600
0.1	20	2	1	0.007992	7992	1.191E-08	2880
0.01	20	0.2	1	0.0007992	799.2	1.191E-09	288
0.001	20	0.02	1	7.992E-05	79.92	1.191E-10	28.8
1.50E-05	20	0.0003	0.01	1.19999E-08	0.012	1.788E-14	0.0043243

1. Möllenstedt, G., H. Lichte, "Doppler shift of electron waves," 9th International Congress on Electron Microscopy, Toronto, 1978, p178-179.
2. Scherzer, O., "Der electronenoptische Doppler-Effeckt," Optik, 54, 315, 1979.

Zhou says it can't be done

Interference effects may be observed if such source is virtually split into two sources sing an optical wave split element like a biprism or a mirror. For electrons, no interference between two different points in the electron source as well as between electrons with different wavelengths can be observed as long as no electron beam with laser properties is invented. The wave functions discussed above describe merely the wave behavior of a single electron. Thus, moving fringes cannot be observed in a contemporary transmission electron microscope.

Moving interference fringes or beat interference may occur in laser optics, electronic signals and mechanical oscillations under the conditions that the corresponding sources produce waves with a fixed phase relation (such as laser), and that these waves will not suffer different inelastic scattering process before they are merged together. The phase relation of the beams may be changed randomly and irreversibly in the inelastic scattering process, and no interference effect can be detected after the inelastic scattering. Moving interference fringes of electrons might be observed provided (1) the electrons are emitted with a fixed phase relation; i.e. the electrons are emitted sequentially and well-ordered, e.g. using a superconducting cathode; (2) the energy difference of the two waves is not caused by any inelastic scattering process; (3) an extremely fast detector (e.g. 10^{-14} s) and an energy filter with very high energy resolution (meV) are available; (4) an electron source with extremely high brightness is available.

From these results we make the following conclusions: (1) Electron holography has perfect energy filtering properties. The inelastically scattered electrons do not contribute to the sidebands; (2) moving fringes cannot be recorded even if an unbelievably fast detector is used as long as the electron waves are emitted with quasi-chaotic phase relations from the source. Moving fringes stemming from scattered waves with different energies do not exist; (3) interference between an inelastically scattered electron wave and a reference wave is not possible even if the energy loss in the object is smaller than the energy spread of the incident beams.

Zhou, Ultramicroscopy 92 (2002) 293–304