SAND2007-5116C

Quality Improvement and Boolean-Like Cutting Operations in Hexahedral Meshes

Jason F. Shepherd[1], Yongjie Zhang[2], Claurissa J. Tuttle[3], Claudio T. Silva[3]

[1]Sandia National Laboratories, Albuquerque, NM 87185, USA
jfsheph@sandia.gov

[2]Carnegie Mellon University, Pittsburgh, PA, USA
jessicaz@andrew.cmu.edu

[3]University of Utah, Salt Lake City, UT 84112, USA

Abstract

In this paper, we propose a solution to improving the quality of hexahedral meshes derived from volumetric data using insights obtained from the structure of the dual of a hexahedral mesh. Our solution is to add a layer of hexes along the boundary, composed of well-shaped elements. The additional flexibility provided by this layer enables the optimization of the original poorly-shaped elements that are no longer on the boundary allowing for creation of a high-quality hexahedral mesh while maintaining a conformal mesh on both sides of the inserted layer. An extra benefit of our technique is being able to capture sharp features using a Boolean-like cutting operation and inserting multiple layers to maintain the high quality of the hexahedral mesh. Our experimental results demonstrate the successful removal of all bad elements (i.e. those elements of the mesh that have a scaled Jacobian measure of less than 0.2) in a number of complex examples.

Introduction

Mesh generation deals with the problem of decomposing complex geometry into discrete elements (meshes), which can be used for modeling, simulation, and visualization. These meshes play a significant role in computationally-based science and engineering.

Our work in this paper demonstrates quality improvement and feature capture in hexahedral meshes of complex geometries. We build on the work of Zhang et al. [1,2] for creating topological hexahedral meshes from volumetric isosurfaces, and the fundamental hexahedral mesh concepts outlined by Shepherd in [3] to improve the quality of the final hexahedral meshes. To create the final hexahedral mesh, sheet insertion [4,5] and mesh

optimization algorithms [6,7] available in CUBIT [8] are utilized. A key result of our paper is to demonstrate high-quality hexahedral mesh generation for complex geometries derived from volumetric images.

Our specific contributions are:
• A method for quality improvement in hexahedral meshes of an arbitrarily complicated domain generated from octree-based algorithms.
• We describe the necessary topologic structures needed within a hexahedral mesh to capture boundary discontinuities (i.e. sharp features).
• Finally, we demonstrate our technique on several models indicating the realized improvement in the overall quality of the final meshes on these models over previous algorithms.

Mesh Acquisition

The base meshes generated in this paper were constructed from volumetric data using the LBIE-Mesher [1,2] The LBIE-Mesher is based on a surface topology preserving octree-based algorithm coupled with a dual contouring method to create a uniform hexahedral mesh boundary-fitted to an isovalue. The uniform mesh is decomposed into finer hexes adaptively using predefined templates without introducing any hanging nodes, and the positions of all boundary vertices are recalculated to approximate the boundary surface more accurately. A final smoothing step is also performed to improve the final surface mesh [2]. The final meshes demonstrated in this paper were built in CUBIT [8] using the original meshes from the LBIE-Mesher as the starting point. In CUBIT, a hexahedral layer insertion was performed along with mesh smoothing, optimization and mesh quality verification. These steps will be discussed in more detail in later sections.

Hexahedral Mesh Structures

A hexahedral mesh can be viewed as a collection of intertwined layers of hexahedra. A single hexahedron will belong to, at most, three separate layers of hexahedra. Each layer of hexahedra can be visualized in a dual representation as a single manifold surface, known as a 'sheet'. We will utilize the dual representation of a hexahedral mesh as defined by Mitchell [9] in his hexahedral mesh existence proof, but also use some concepts from Murdoch [10]. We also utilize the hexahedral constraints for maintaining a hexahedral topology and sufficient quality as outlined in [3].

Surface Capture

The boundary of any hexahedral mesh is a quadrilateral mesh. As shown in Figure 1, the quadrilaterals on the boundary will belong to one or more layers of hexahedra. For a smooth and continuous boundary, a hexahedral mesh consisting of a single layer of hexahedra reduces skew and improves element orthogonality at the boundary, and offers the highest level of topologic regularity in the mesh. Therefore, if we assume no discontinuities in the boundary surface description, meshes which utilize a single layer to capture all of the boundary quadrilaterals will invariably admit elements whose potential

quality is higher than meshes which utilize multiple layers to capture the boundary surface.

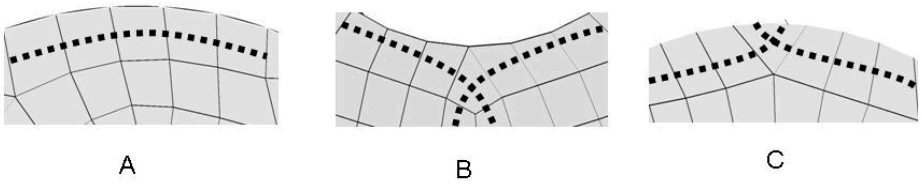

Figure 1: When multiple layers of hexahedra capture boundaries, the quality and regularity of the mesh is affected. Image A shows elements from a single layer capturing the upper boundary of the solid. Image B and C use multiple layers of hexahedra to capture the upper boundary of the solid. In image B and C, note how the regularity of the mesh is affected and the resulting skew in the transition element as the hexahedral layer curves away from the boundary.

Curve Capture
Where discontinuities exist in the geometry, it is often desirable to place a simply connected set of mesh edges that align themselves with the discontinuity. These discontinuities are often called sharp features, and in solid models are often the trimmed boundaries of pairs of surfaces within the solid model (and known as *curves).*.

The intersection of two layers of hexahedral elements results in a column of hexahedra. In the dual representation of a hexahedral mesh, this column of hexahedra is also known as a chord (see Figure 2). For each chord, there are four sets of simply connected edges running along the length of the column of hexahedra. For each sharp discontinuity in the geometry, a conformal hexahedral mesh of that discontinuity will have one or more columns of hexahedra running along the length of the discontinuity. (However, these columns may be disjointed, similar to the layers of hexahedra shown in Figure 1.)

Methods

In octree-based hexahedral meshes, the quadrilaterals on the boundary of the mesh often belong to multiple, disjointed layers of hexahedra. This structure results in diminished element quality near the boundary. In this section we will outline an algorithm for improving the quality of hexahedral meshes near the boundary by inserting a new layer of hexahedra. This new layer effectively aligns the boundary hexahedra with the boundary surface and of provides extra degrees of freedom for subsequent mesh optimization.

Mesh Quality Improvement using Boundary Sheets
In order to add a new layer of hexahedra near the boundary of the geometry, we use a modified version of a pillowing algorithm as described by Mitchell [9].

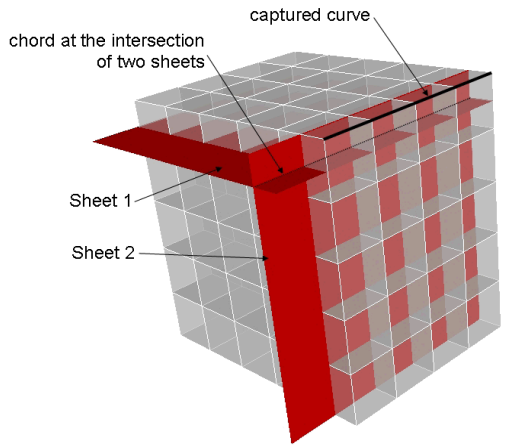

Figure 2: The intersection of two layers of hexahedra results in a column of hexahedra, and a string of simply connected mesh edges sufficient to capture a sharp geometric discontinuity.

The basic layer insertion algorithm is as follows (also refer to Figure 3):
1. *Define a shrink set* - For our purposes, this step involves dividing the existing mesh into two sets of elements: one set for each of the half-spaces defined by the new layer to be inserted. One of these two sets of hexahedral elements comprises the shrink set.
2. *Shrink the shrink set* - Create a gap region between the two previous element sets.
3. *Connect with a new layer of elements* – Because there is a one-to-one mapping between the nodes on the boundaries of the two sets, a new layer of hexes can be created by creating an edge between each node separated during the shrinking operation and generating the connectivity of the new hexahedra in the layer using the quadrilaterals on the boundary of the two sets of hexahedra.

When inserting a single layer next to the boundary, we define our shrink set as all of the elements within the solid. We desire the original surface mesh to be undisturbed by any of our modification operations, so a copy of the surface mesh is made prior to shrinking the layer of elements near the boundary. For each face on the boundary, one new hexahedral element is created. Figure 4 demonstrates a mesh with a newly inserted layer.

Cutting Operations
By inserting multiple layers of hexahedra, we can obtain a simply connected set of mesh edges wherever new or existing layers intersect. Capitalizing on this concept, we can strategically insert layers of hexahedra to perform Boolean-like CG operations in the hexahedral mesh while still maintaining mesh conformity with all of the split-off pieces [5]. At the intersection of the layers, edges can be aligned with any sharp discontinuities in the geometry. In Figure 5 we demonstrate several successive spherical cuts from a

single hexahedral mesh of a cubical geometry. Where two layers intersect, a simply-connected string of mesh edges exists and can be aligned with the cut enabling the sharp features in the resulting model to be recognized.

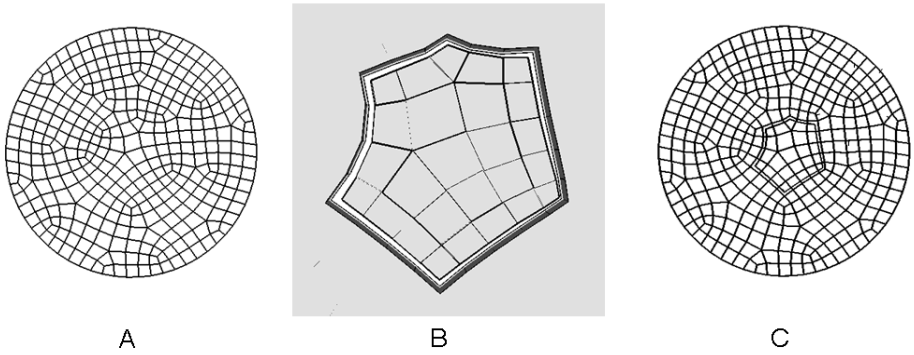

Figure 3: A basic pillowing operation starts with an initial mesh (A). A shrink set is defined and separated by 'shrinking' from the original mesh (B). A new layer of elements (i.e. a sheet) is inserted (C) to fill the void left by the shrinking process.

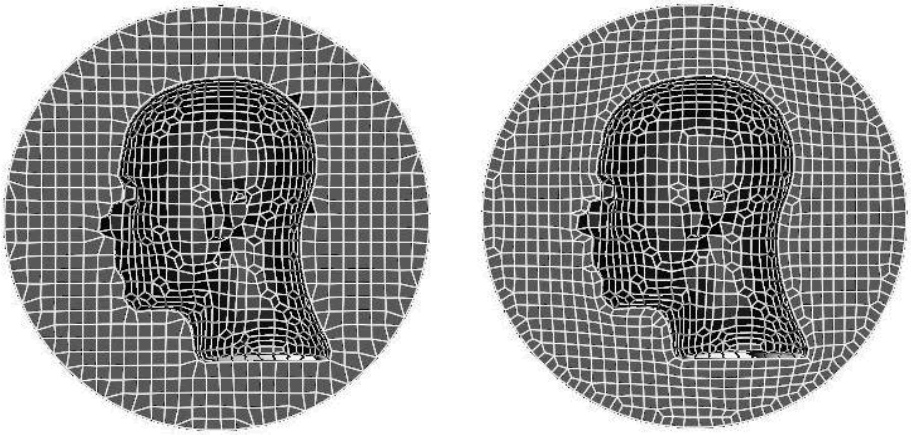

Figure 4: A hexahedral mesh of a sphere with a human head embedded in the center. The mesh on the left is an octree-based mesh that has been refined and oriented capture the geometry of the embedded head. The hexahedral mesh on the right is the same mesh with two additional layers of hexahedra at the interior and exterior boundary. The mesh on the right has a higher quality and flexibility due to the improved mesh topology capturing the geometric boundary.

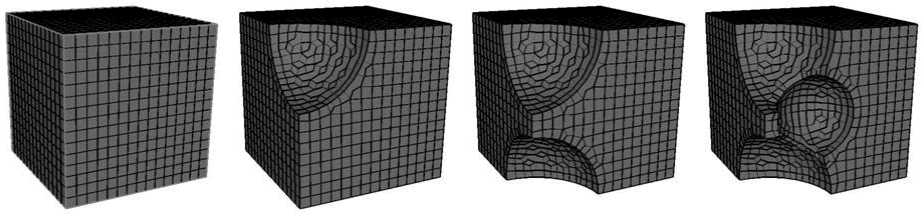

Figure 5: By inserting spherical sheets into the geometry, we can perform Boolean-like cutting operations in the mesh, while maintaining the integrity of the hexahedral mesh.

In Figure 6, a planar layer of hexes is added behind the face of the head model. This layer coupled with the layer of hexes previously added for the face results in string of edges that can be used to capture the sharp discontinuity enabling the cutting operation.

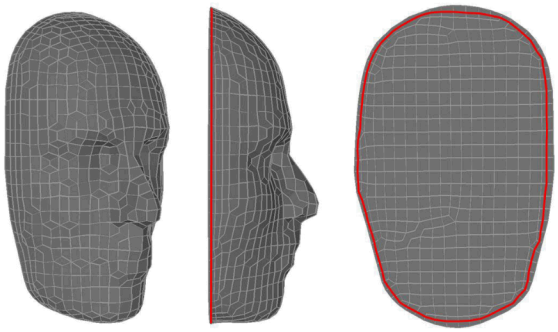

Figure 6: A layer of hexahedra elements is inserted behind the face in the head model enabling the face to be cut cleanly from the rest of the head.

Results

In this section we display results detailing the quality differences after inserting a layer of hexahedra at the boundary for several examples. The scaled Jacobian metric [11], as implemented in VERDICT [12], is used as the definitive measure for hexahedral quality. We will assume that any elements with scaled Jacobian less than zero are unsuitable for analysis and any element with scaled Jacobian less than 0.2 is questionable for use in an analysis.

Because correct and optimal placement of the new layer can be difficult, we also perform a mesh optimization step on both meshes using the TSTT Mesh Quality and Improvement Toolkit (MESQUITE) [7] library of smoothing algorithms as implemented in Cubit [8]. In particular, we have utilized a mean ratio smoother [6], which incorporates an L2-norm template with guarantees that (1) the mesh will remain

untangled if the initial mesh is untangled, and (2) the average value of the mean ratio will either stay the same, or be decreased.

There are four examples given: a human knee model (Figure 7), a human head (Figure 8), a meshed sphere around the human head (Figure 4), and a model of a mAChE biomolecule (Figure 11). Element quality results are given for each of the meshes before and after layer insertion (see Figures 9 and 10). The introduction of the boundary sheet enables the removal of all negative scaled Jacobian values from the displayed meshes.

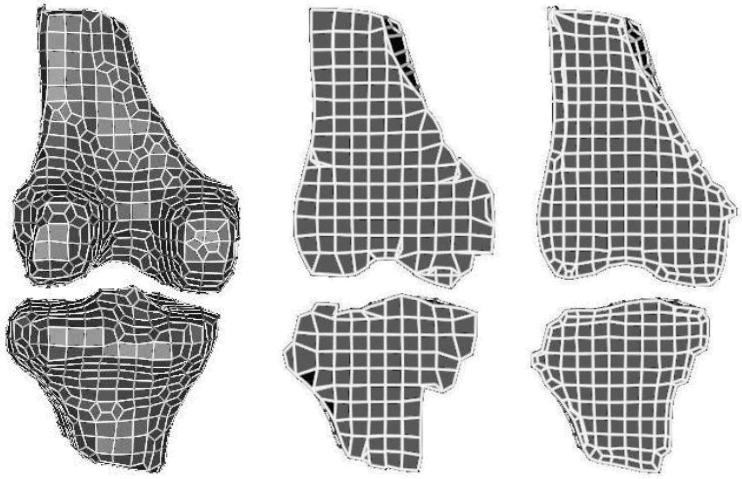

Figure 7: Human knee model. The original mesh contains 1,338 total hexahedra (cut-away in center). An additional layer of elements was added to the boundary to improve quality resulting in a new mesh (right) with 2,682 total elements.

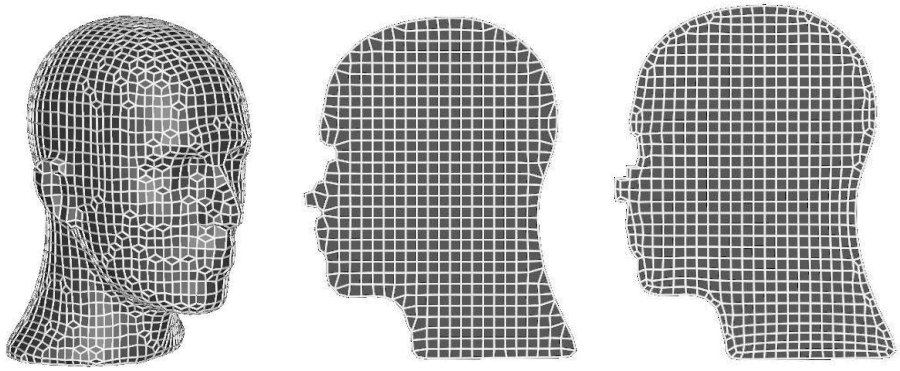

Figure 8: Human head model. The original mesh contains 6,583 total hexes (cut-away view in center). An additional layer of hexahedra was added to the boundary to improve quality resulting in a new mesh (right) with 9,487 total elements.

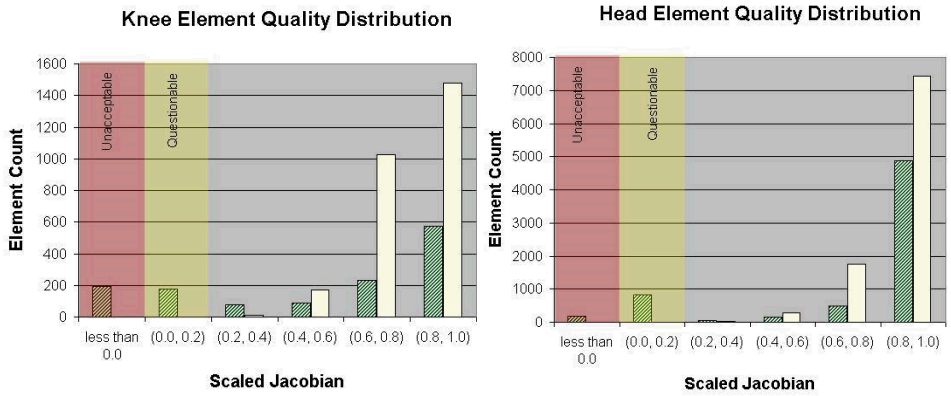

Figure 9: Distribution of scaled Jacobian measure for the knee and the head meshes. The original mesh (hatched, dark) of the knee contains 367 elements of questionable or unacceptable quality, while the new mesh (solid, white) has all elements with scaled Jacobian greater than 0.2. The original mesh (dark, hatched) of the head contains 1025 questionable or unacceptable elements, and the new mesh (solid, white) has one element of questionable quality (related to a poor quality boundary quadrilateral).

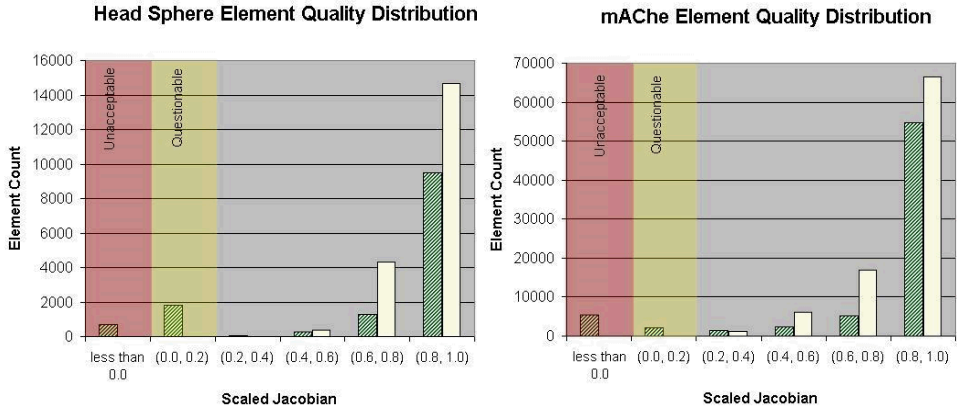

Figure 10: Distribution of scaled Jacobian measures for the head sphere mesh and the mAChE mesh. The original mesh (dark, hatched) of the head sphere contains 2,485 elements of questionable or unacceptable quality, while the new mesh (solid, white) has all elements with scaled Jacobian greater than 0.2. The original mesh (dark, hatched) of the mAChE has 7,298 problematic elements, while the new mesh (solid, white) has only 111 elements of questionable quality (due to poor quality boundary quadrilaterals).

Conclusion

In this paper, we have outlined a method utilizing a simple hexahedral layer insertion process to improve the quality of the elements generated using an octree-based algorithm with methodologies similar to the dual contouring approach that captures isosurfaces within volumetric data. We demonstrate how by strategically inserting hexahedral layers into the octree-produced hexahedral meshes, we can significantly improve the overall mesh quality without changing the boundary mesh. We also show how insertion of multiple sheets can be utilized to capture discontinuities, or sharp features, in the meshes. We have demonstrated the quality improvement on several models. We ensure that the quality comparisons are accurate by utilizing the MESQUITE mesh optimization algorithms on the meshes both before and after the sheet insertion. The VERDICT library is also used to verify and report the final mesh quality.

Acknowledgments

C. Silva is funded by the National Science Foundation (grants CCF-0401498, EIA-0323604, OISE-0405402, IIS-0513692, CCF-0528201), the Department of Energy, Sandia National Laboratories, Lawrence Livermore National Laboratory, an IBM Faculty Award, and the National Institutes of Health.

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

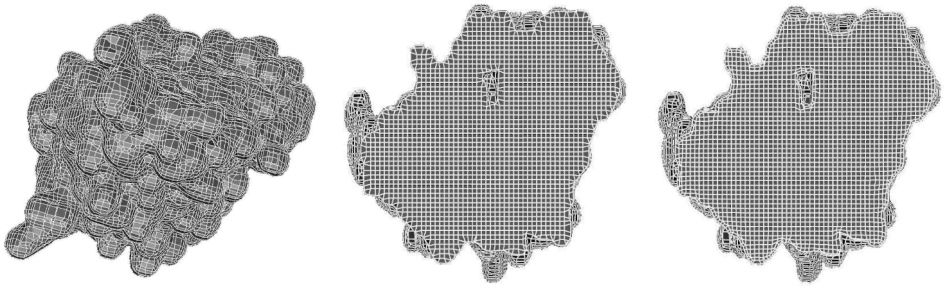

Figure 11: Biomolecule mAChE model. The original mesh contains 70,913 hexahedra (cut-away view in center). An additional layer of elements was added to the boundary to improve quality giving a new mesh (right) with 90,937 elements.

References

[1] Y. Zhang and C. Bajaj. Adaptive and quality quadrilateral/hexahedral meshing from volumetric data. Computer Methods in Applied Mechanics and Engineering (CMAME), 195(9-12):942–960, 2006.

[2] Y. Zhang, C. Bajaj, and G. Xu. Surface smoothing and quality improvement of quadrilateral/hexahedral meshes with geometric flow. In Proceedings, 14th International Meshing Roundtable, pages 449–468. Sandia National Laboratories, September 2005.
[3] J. F. Shepherd. Topologic and Geometric Constraint-Based Hexahedral Mesh Generation. Published Doctoral Dissertation, University of Utah, May 2007.
[4] S. A. Mitchell and T. J. Tautges. Pillowing doublets: Refining a mesh to ensure that faces share at most one edge. In Proceedings, 4th International Meshing Roundtable, pages 231– 240. Sandia National Laboratories, October 1995.
[5] M. J. Borden, J. F. Shepherd, and S. E. Benzley. Mesh cutting: Fitting simple all-hexahedral meshes to complex geometries. In Proceedings, 8th International Society of Grid Generation Conference, 2002.
[6] P. M. Knupp. Hexahedral and tetrahedral mesh shape optimization. International Journal for Numerical Methods in Engineering, 58(1):319–332, 2003.
[7] MESQUITE: The Mesh Quality Improvement Toolkit, Terascale Simulation Tools and Technology Center (TSTT), http://www.tstt-scidac.org/research/mesquite.html, 2005.
[8] The CUBIT Geometry and Mesh Generation Toolkit, Sandia National Laboratories, http://cubit.sandia.gov/, 2005.
[9] S. A. Mitchell. A characterization of the quadrilateral meshes of a surface which admit a compatible hexahedral mesh of the enclosed volumes. In 13th Annual Symposium on Theoretical Aspects of Computer Science, volume Lecture Notes in Computer Science: 1046, pages 465–476, 1996.
[10] P. J. Murdoch. The Spatial Twist Continuum: A Dual Representation of the All Hexahedral Finite Element Mesh. Published Doctoral Dissertation, Brigham Young University, December 1995.
[11] P. M. Knupp. Algebraic mesh quality metrics. SIAM J. Sci. Comput., 23(1):193–218, 2001.
[12] S. N. Laboratories. Verdict mesh verification library (see http://cubit.sandia.gov/verdict.html), October 2005.