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# *Blast and Fragmentation Modeling with Peridynamic Theory*

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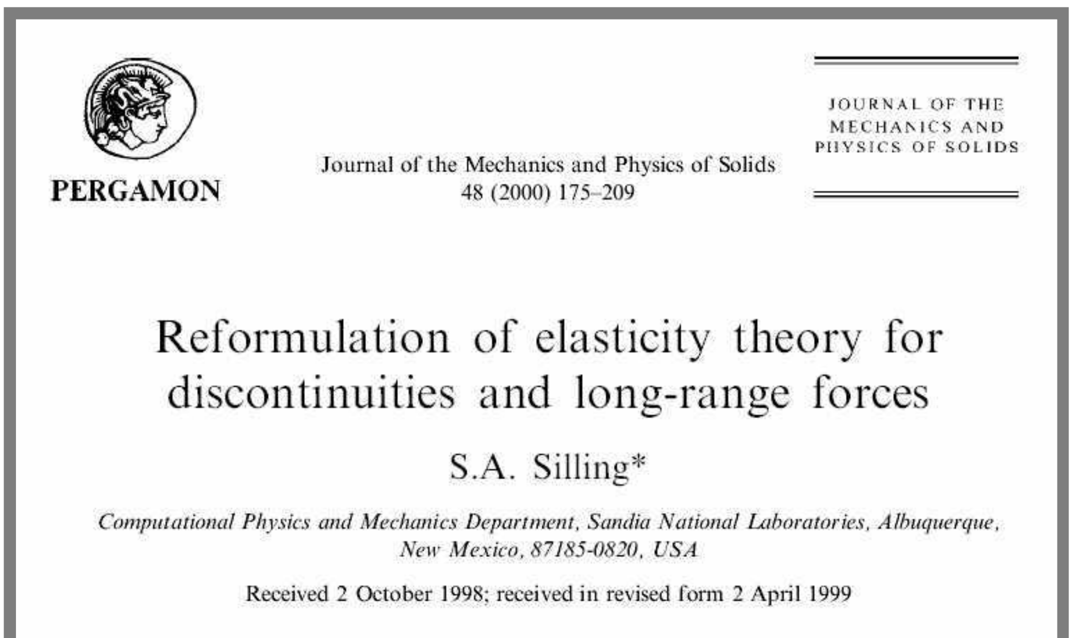
# Outline of Talk

- What is Peridynamic Theory?
- Why Use Peridynamics?
- The Fundamental Peridynamics Equation
- Material Modeling in Peridynamics
- Numerical Method
- Implementation in the EMU Computer Code
- EMU Detonation Model
- Blast Loading of a Structure
- Fragmentation of an Exploding Shell
- Current and Future Work

# What is Peridynamic Theory?

- *Peridynamic theory* is an approach to continuum mechanics that uses differo-integral equations without spatial derivatives rather than partial differential equations.
  - Reformulation of fundamental equations that applies everywhere regardless of discontinuities
  - Theory first published in 2000 by Stewart A. Silling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Reformulation of elasticity theory for discontinuities and long-range forces

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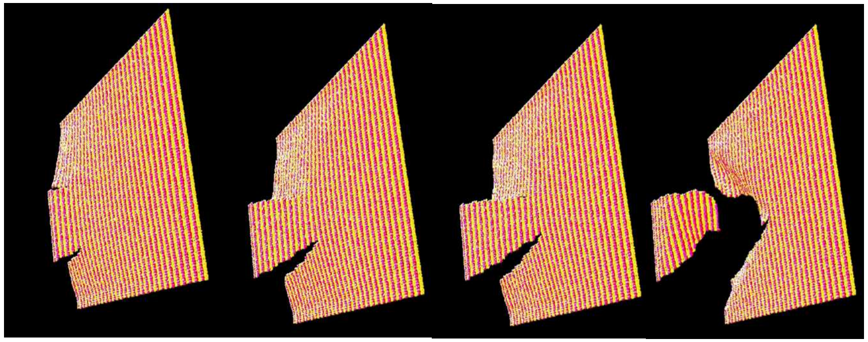
# Why Use Peridynamics?

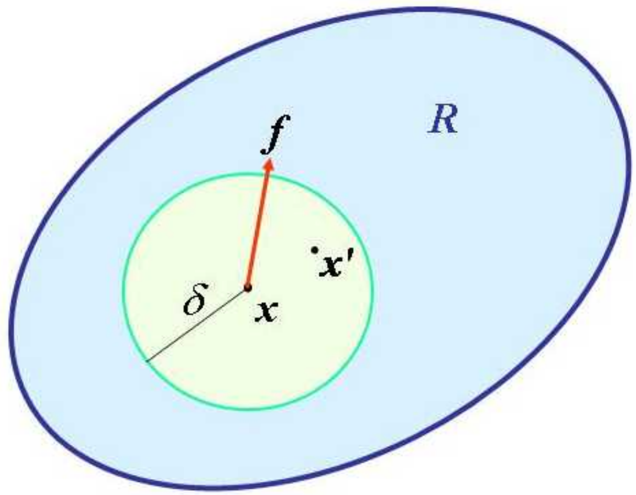
- The fundamental partial differential equations used in conventional finite element codes do not apply at discontinuities such as cracks.



**Real life:**
**Discontinuities can evolve in complex patterns not known in advance.**

- Fragmentation occurs as a result of the initiation and growth of multiple, mutually interacting dynamic fractures. With peridynamics, cracks initiate and grow spontaneously and there is no need for externally supplied "crack growth laws".

# Fundamental Equation of Peridynamic Theory




**Configuration Variables**

$$\boldsymbol{\xi} = \boldsymbol{x}' - \boldsymbol{x}$$

$$\boldsymbol{\eta} = \boldsymbol{u}(\boldsymbol{x}', t) - \boldsymbol{u}(\boldsymbol{x}, t)$$

$$\rho(\boldsymbol{x}) \frac{d^2}{dt^2} \boldsymbol{u}(\boldsymbol{x}, t) = \iiint_R \boldsymbol{f}(\boldsymbol{u}(\boldsymbol{x}', t) - \boldsymbol{u}(\boldsymbol{x}, t), \boldsymbol{x}' - \boldsymbol{x}) dV' + \boldsymbol{b}(\boldsymbol{x}, t)$$

**where**

$\rho$ is the density at $\boldsymbol{x}$,
$t$ is the time,
$R$ is the computational domain,
$\boldsymbol{b}$ is the body force.
$\boldsymbol{x}$ is the position vector,
$\boldsymbol{u}$ is the displacement vector,
$\boldsymbol{f}$ is the pairwise force function, and

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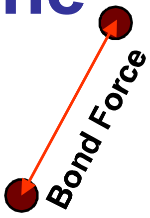
# Material Modeling in Peridynamics

- The force per unit volume squared between particles located a two points is given by the *pairwise force function* (*PFF*) $f$.
  - Peridynamic interaction between two points is called a *bond*.
  - Constitutive properties of materials are given by specifying the *PFF*.
    - Thus, material response, damage, and failure are determined at the bond level.
  - Bond properties are derivable from measured material properties including:
    - elastic modulus, yield properties, and fracture toughness.



# Some Properties of the Pairwise Force Function

- Newton's third law of motion implies that the *PFF* satisfies

$$\boldsymbol{f}(-\boldsymbol{\eta},-\boldsymbol{\xi}) = -\boldsymbol{f}(\boldsymbol{\eta},\boldsymbol{\xi}) \quad \forall\, \boldsymbol{\eta},\boldsymbol{\xi}$$

- Furthermore, conservation of angular momentum implies that the *PFF* satisfies

$$(\boldsymbol{\eta}+\boldsymbol{\xi}) \times \boldsymbol{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = 0$$

- These properties imply that the *PFF* is of the form

$$\boldsymbol{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = F(\boldsymbol{\eta},\boldsymbol{\xi})(\boldsymbol{\eta}+\boldsymbol{\xi}) \quad \text{where} \quad F(-\boldsymbol{\eta},-\boldsymbol{\xi}) = F(\boldsymbol{\eta},\boldsymbol{\xi})$$

- For isotropic materials, $F$ has the form

$$F(\boldsymbol{\eta},\boldsymbol{\xi}) = I(p,q,r) \quad \text{where} \quad p = |\boldsymbol{\eta}+\boldsymbol{\xi}|,\ q = \boldsymbol{\eta}\cdot\boldsymbol{\xi},\ r = |\boldsymbol{\xi}|$$

# Micro-Elastic (Plastic) Materials

- A *PFF* is said to be ***micro-elastic*** (*ME*) ***-plastic*** (*MP*) if and only if there exists a scalar function, $w$, such that

$$\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = \frac{\partial w}{\partial \boldsymbol{\eta}}(\boldsymbol{\eta},\boldsymbol{\xi})$$

- A *ME* material is said to be ***proportional*** if and only if the *PFF* is proportional to the stretch, $s$, where $s = (p - r)/r$.
- Failure occurs when $s$ exceeds a value, $s_0$, called the ***critical stretch***.
- Isotropic, proportional ME materials have

$$F(\boldsymbol{\eta},\boldsymbol{\xi}) = \frac{1}{p} g(s,r) \quad \begin{cases} \text{where } g(s,r) \text{ is a piecewise} \\ \text{linear function of } s. \end{cases}$$

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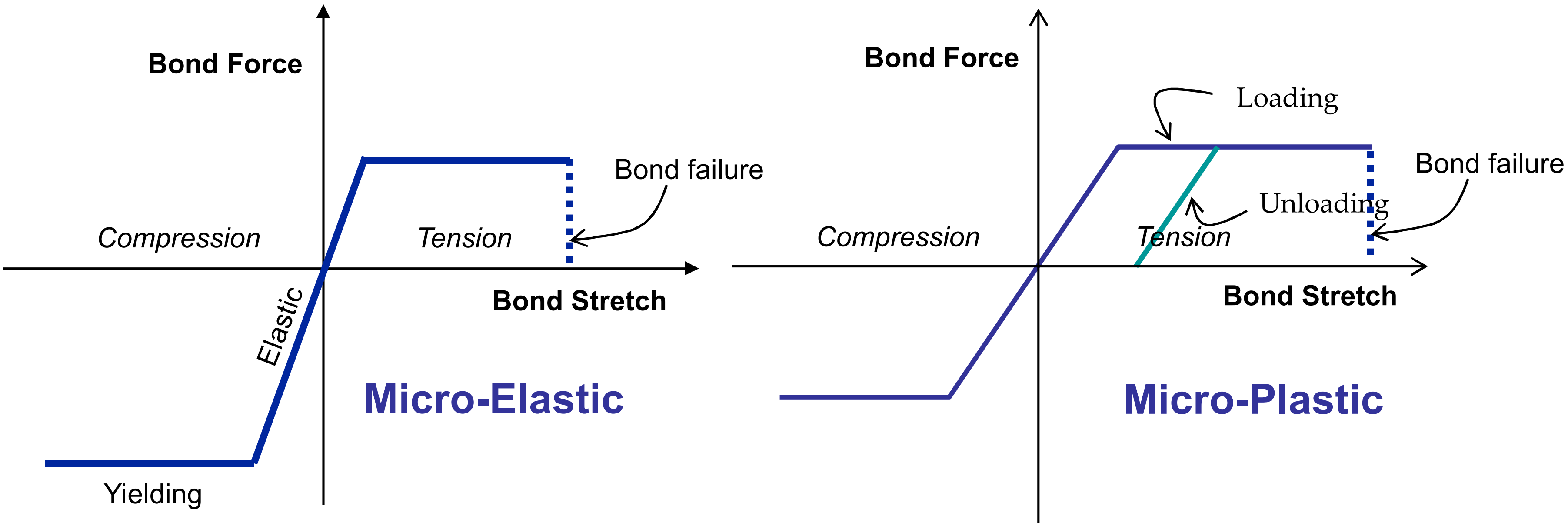
# Micro-Elastic and -Plastic Materials

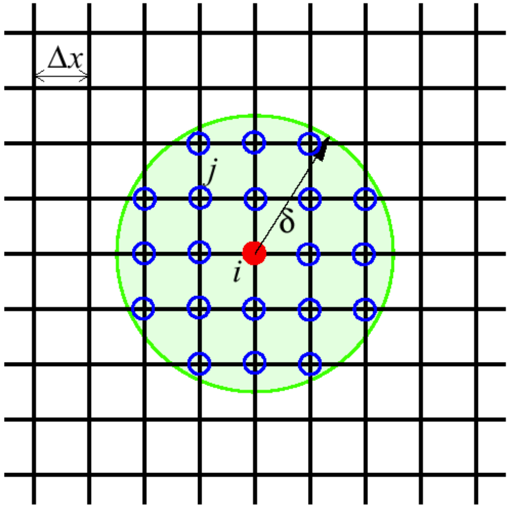
- The difference between isotropic, proportional micro-elastic and micro-plastic materials is their behavior on unloading.




Bond is a spring in these cases.

- For extreme loading analyses, we use isotropic, proportional, micro-elastic (plastic) materials.

# Numerical Method




- The computational region is discretized into nodes with a known volume in the reference configuration, forming a *grid of nodes*.
- The fundamental equation is replaced by a finite sum, which at time $\boldsymbol{t_n}$ is

$$\rho_i \frac{d^2}{dt^2}\boldsymbol{u}_i^n = \sum_p \boldsymbol{f}(\boldsymbol{u}_p^n - \boldsymbol{u}_i^n, \boldsymbol{x}_p - \boldsymbol{x}_i)V_p + \boldsymbol{b}_i^n, \quad \boldsymbol{u}_i^n = \boldsymbol{u}(\boldsymbol{x}_i, t_n)$$

- For each node, the peridynamic interaction is assumed to be zero outside a distance $\delta$ called the *horizon*.

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# Implementation in EMU Computer Code

- Peridynamics is implemented in the EMU computer code.
- EMU is
  - mesh free (no elements, just generate a grid of nodes),
  - Lagrangian (each node represents a fixed amount of material),
  - explicit (simple, reliable time-integration method),
  - parallel (executes on multiple processors).

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# EMU Detonation Model

$$(x_{det}, y_{det}, z_{det}, t_{det}), \rho_{unreacted}, v_{det}$$

- **Detonation model inputs:**
  - Location of detonation point(s) and initial detonation time(s), density of unreacted explosive, and detonation speed.
  - Parameters for equation of state (ideal gas or JWL).
- **Program burn model for detonation times.**
  - Detonation times computed prior to time advancement using Huygen's construction.
  - Detonations can propagate around obstacles.
- **Upon detonation:**
  - Reaction products are treated as ideal or JWL gas undergoing an adiabatic expansion.
  - Energy conserved using volume-burn algorithm.

# Gases as Peridynamic Materials

- Since detonation products are gases, gases must be modeled as peridynamic materials.
- Consideration of the work required to stretch in bond *k* leads to the following *PFF* for a gas:

$$f_k = -\frac{6P_S(X)}{r_k V}\left(\frac{p_k}{r_k}\right)^{-m-1} X^{1+m/3} \quad \text{where}$$

$p = |\boldsymbol{\eta} + \boldsymbol{\xi}|$, $r = |\boldsymbol{\xi}|$, $V$ is volume, $P_S$ is pressure along an isentrope,

$$X = \frac{\rho_0}{\rho} = \left[\frac{1}{V}\sum_j \left(\frac{p_j}{r_j}\right)^{-m} \Delta V_j\right]^{-3/m}, \quad V = \sum_i \Delta V_i$$

$\rho$ is the density in deformed configuration,

$\rho_0$ is the density in undeformed configuration

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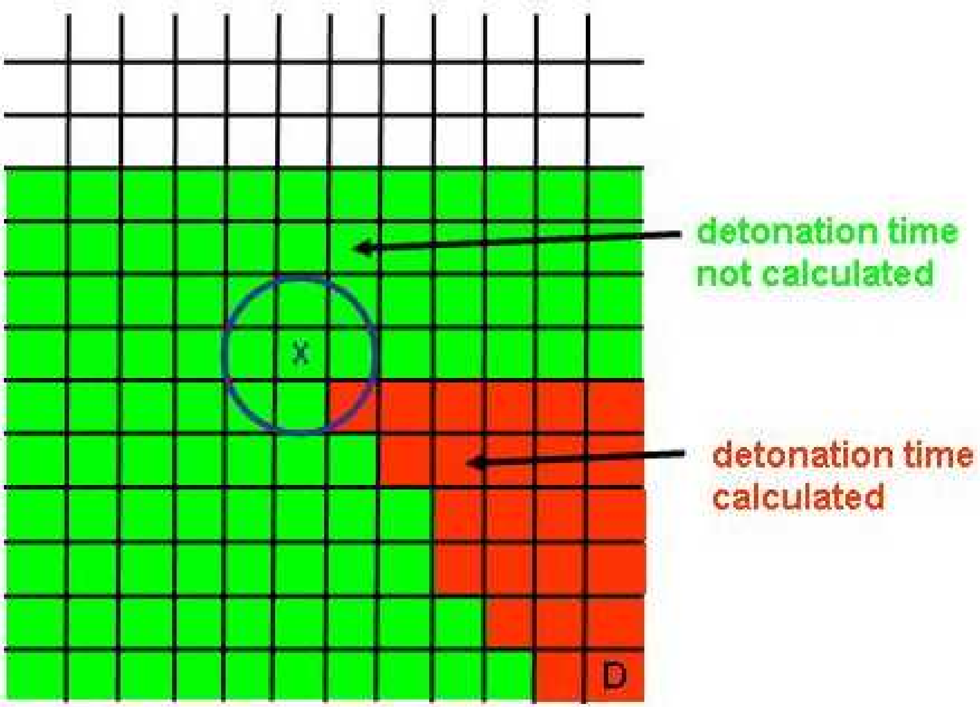
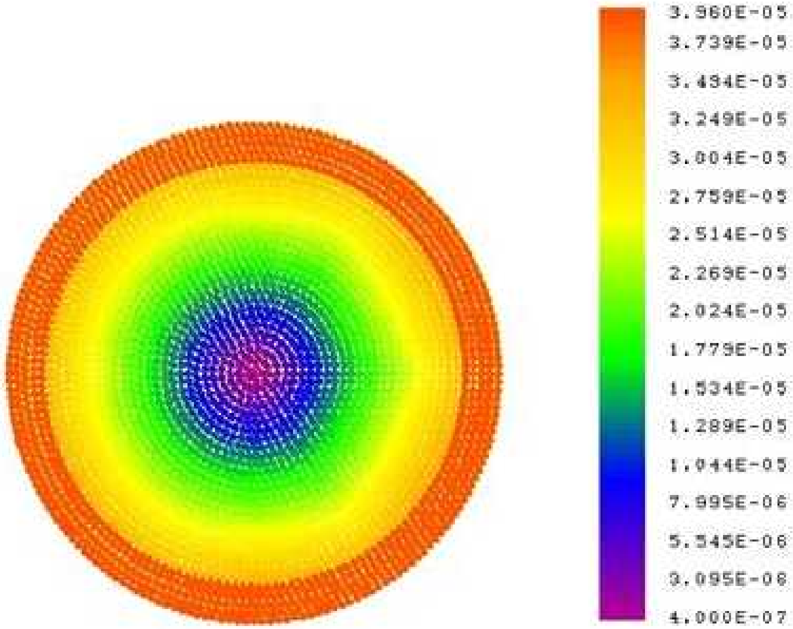
# Program Burn

- Reliable, time-tested method (since 1950's)
- Huygen's Construction (in two dimensions)



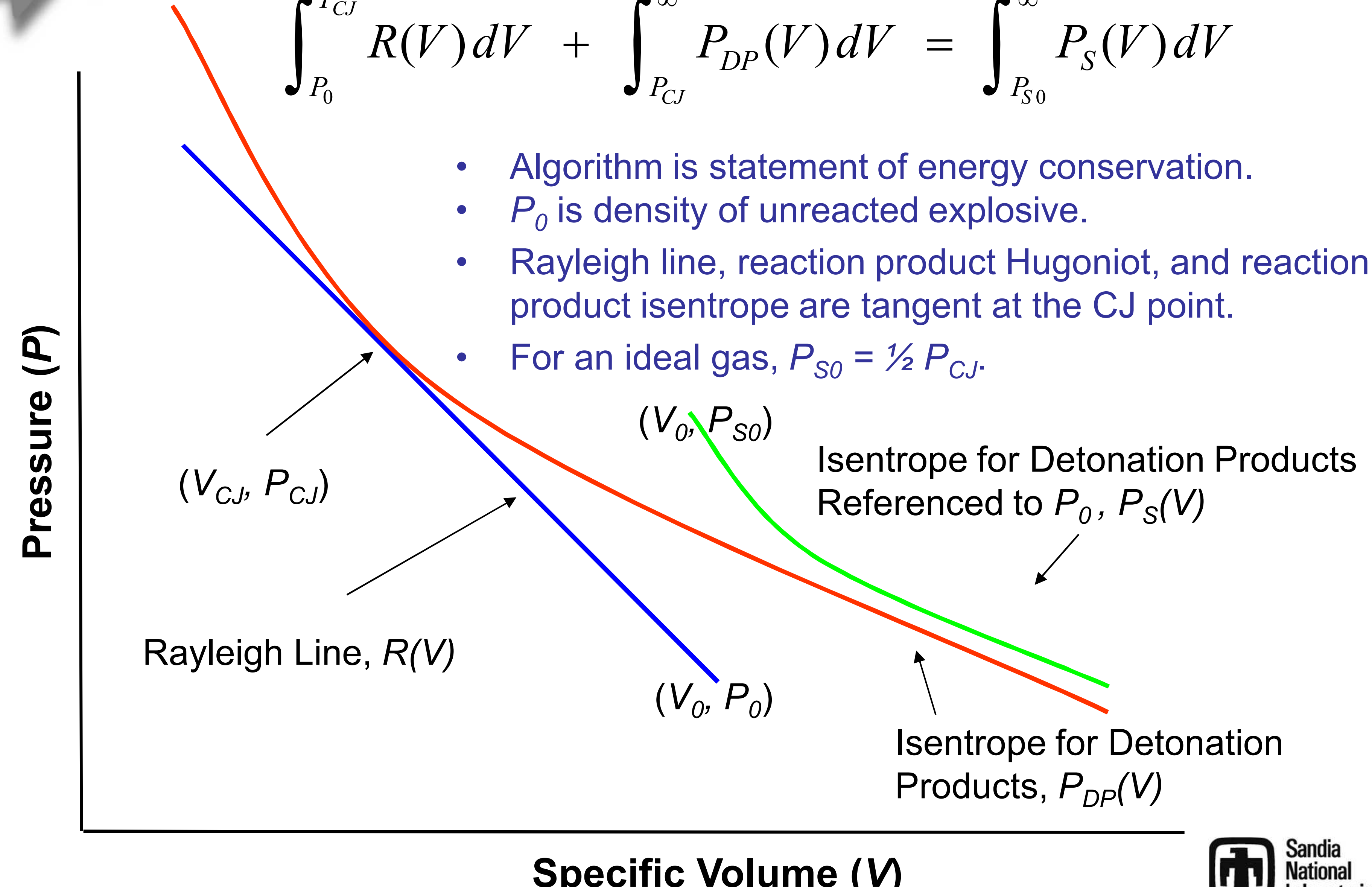




Detonation Times

# Volume Burn Algorithm

$$\int_{P_0}^{P_{CJ}} R(V)\,dV \;+\; \int_{P_{CJ}}^{\infty} P_{DP}(V)\,dV \;=\; \int_{P_{S0}}^{\infty} P_S(V)\,dV$$

- Algorithm is statement of energy conservation.
- $P_0$ is density of unreacted explosive.
- Rayleigh line, reaction product Hugoniot, and reaction product isentrope are tangent at the CJ point.
- For an ideal gas, $P_{S0} = ½\, P_{CJ}$.



# JWL Equation of State

- **JWL Equation of State (EOS), pressure**

$$P(X,E) = \sum_i A_i\left(1-\frac{\omega}{R_i X}\right)e^{-R_i X} + \frac{\omega E}{X}$$

energy

expansion

Remaining quantities are JWL parameters.

- **Expansion Isentrope: pressure**

$$P_S(X) = \sum_i A_i e^{-R_i X} + C X^{-(\omega+1)}$$

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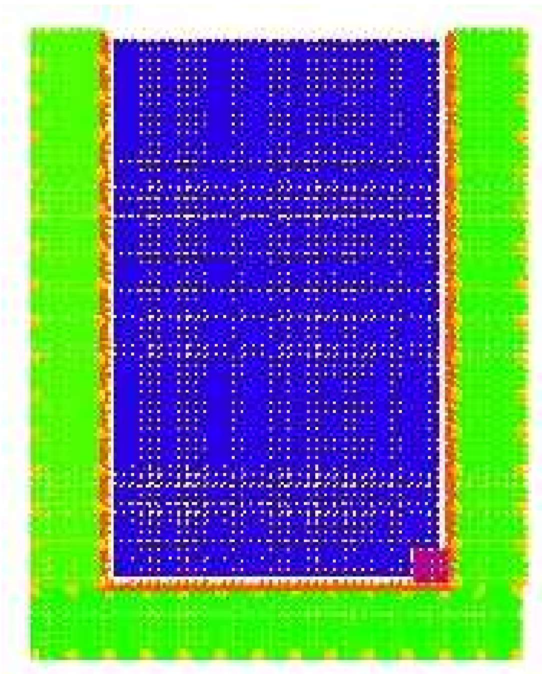
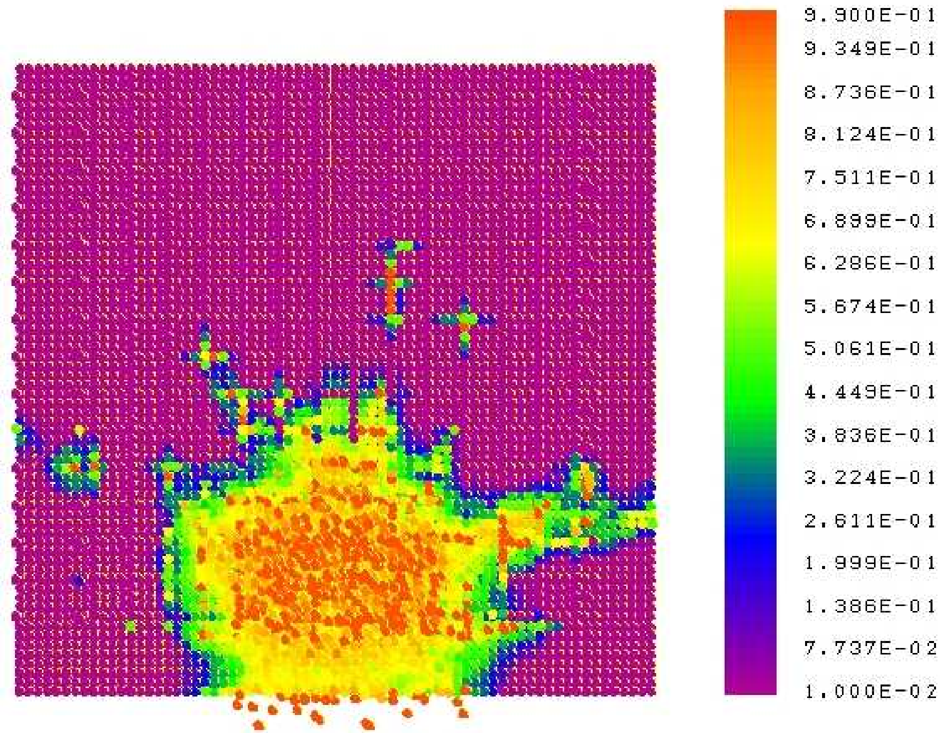
# Blast Loading of a Structure

Structure has 6-ft thick walls and floor slab. The floor slab is 40 ft by 52 ft. The walls are 45 ft above the floor. All concrete is reinforced with #18 rebar at 12-in spacing. A cubic yard of explosive with unreacted density 1785 kg/m$^3$ and detonation speed 8747 m/s is placed on the floor at the center of the wall and detonated at time zero.



**Materials**




**Internal damage**

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# Fragmentation of an Exploding Shell

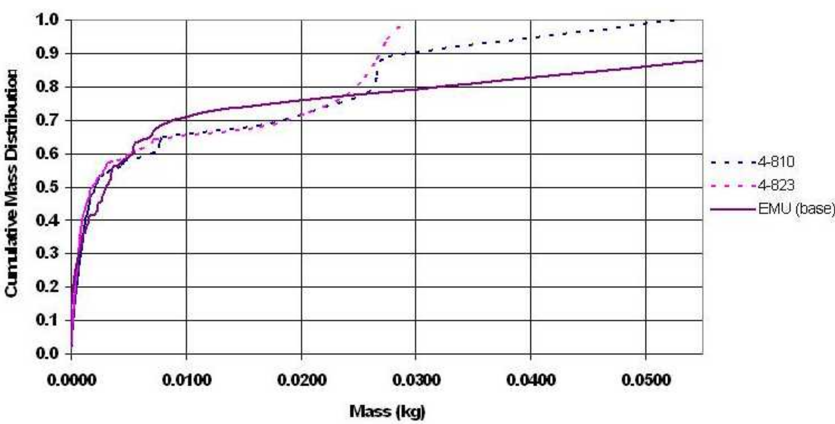
**Shell Loaded with Explosive**

**EMU Model**

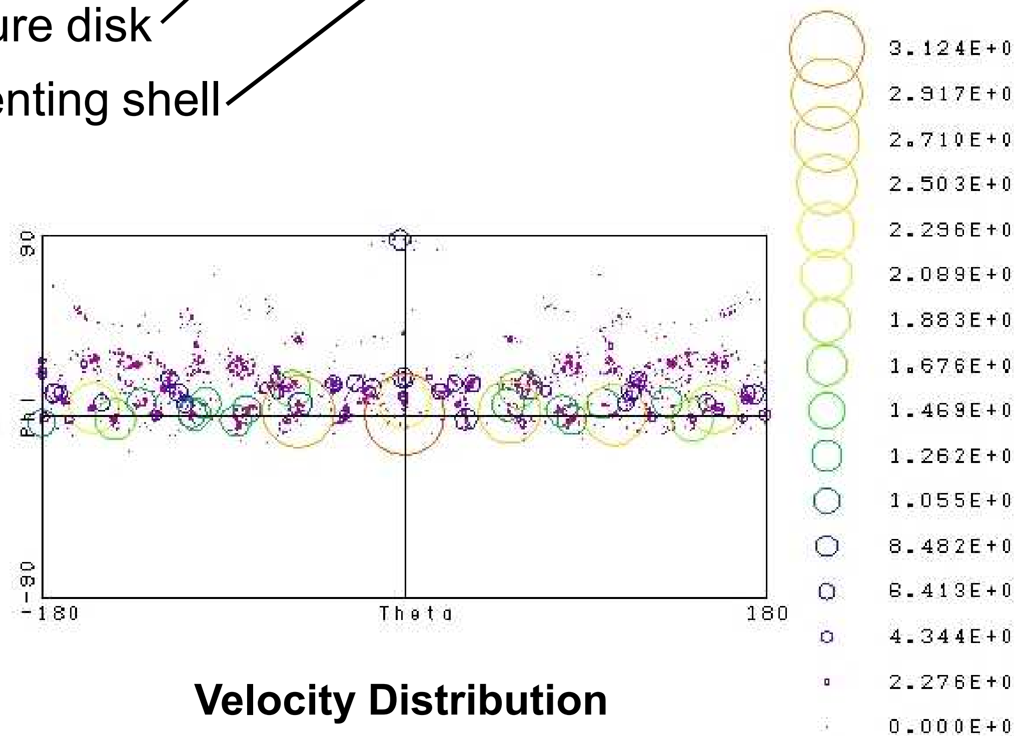
explosive

steel closure disk

steel fragmenting shell

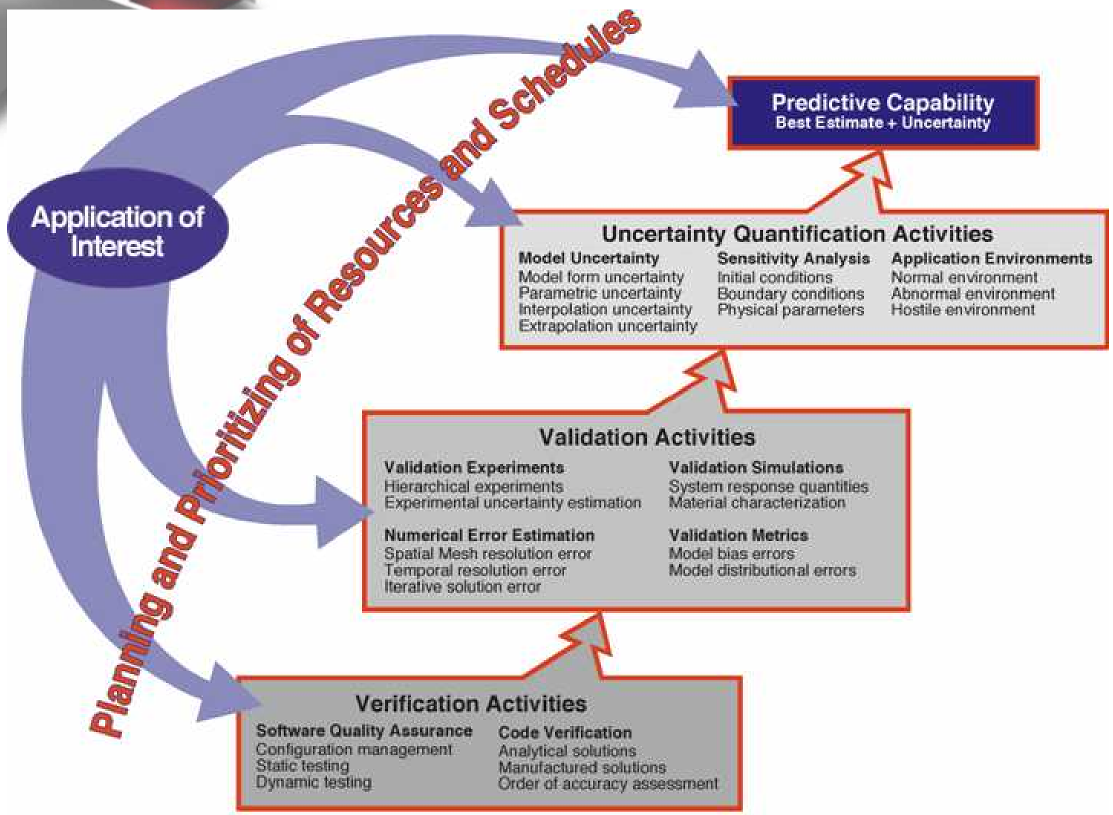


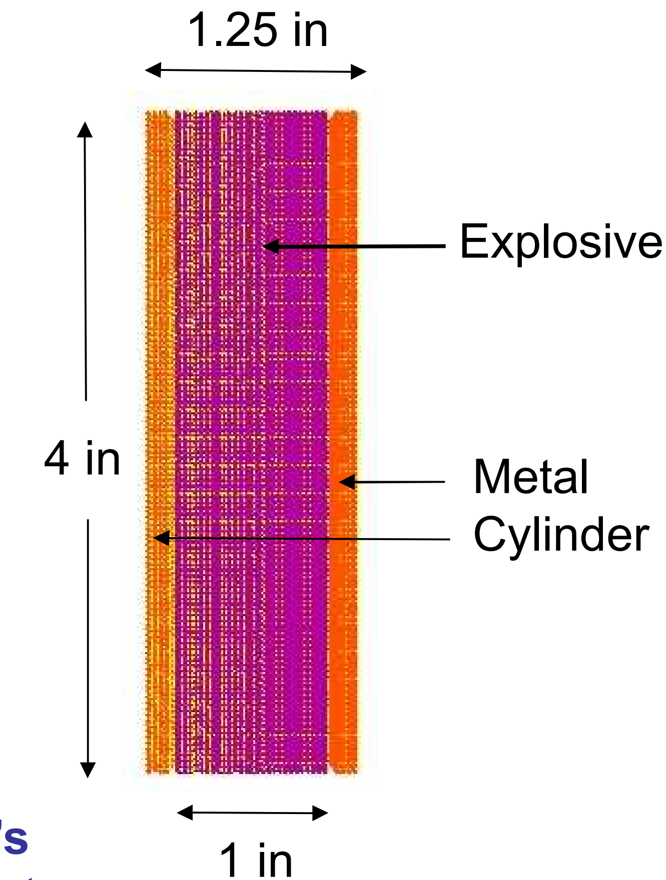

**Cumulative Distribution Functions**




**Velocity Distribution**

# Verification and Validation (V&V)




Cross Section of Cylinder Filled with Explosive

**Verification: Process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.**

**Validation: Process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.**

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# EMU Research and Development

- Current R&D includes
  - modeling fluids, composite materials, and explosive materials and explosive loading
  - verification and validation
  - software engineering
  - investigating use of existing constitutive models in state-based peridynamics to represent stress-strain and yield behavior in EMU
- Future R&D possibilities include
  - nanoscale to continuum coupling using the embedded atom method
  - mechanical failure of nanoscale systems
  - inclusion of phenomenology (fire, heat transfer, material degradation, etc) to provide a comprehensive methodology for vulnerability assessment of critical structures
- URL - http://www.sandia.gov/emu/emu.htm

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