EXPERIMENTAL VALIDATION OF A SQUEEZE-FILM DAMPING MODEL BASED ON THE DIRECT SIMULATION MONTE CARLO METHOD

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Gas damping is important in MEMS.

Motivation:

- Many micro/nano devices need high *Q* factor. Examples abound in
  - MEMS switches need high speed (high *Q*).
  - Resonant cantilever sensors need high responses.
  - MEMS gyroscopes.
  - MEMS accelerometers need controlled damping.
- Damping can reduce *Q* from several hundred thousands to several hundreds.
- Squeeze-film damping determines the dynamics of plates moving a few microns above the substrate.
- Molecular-dynamics-based models for predicting squeezed-film damping give different results.
- A new model based on the Direct Simulation Monte Carlo (DSMC) method shows potential for good accuracy.

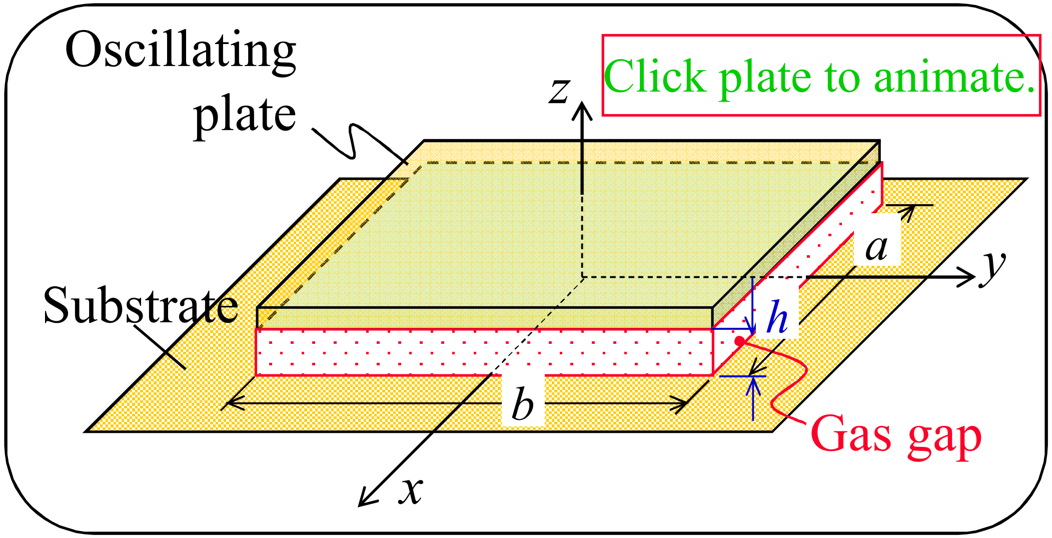
•Needs experimental validation.

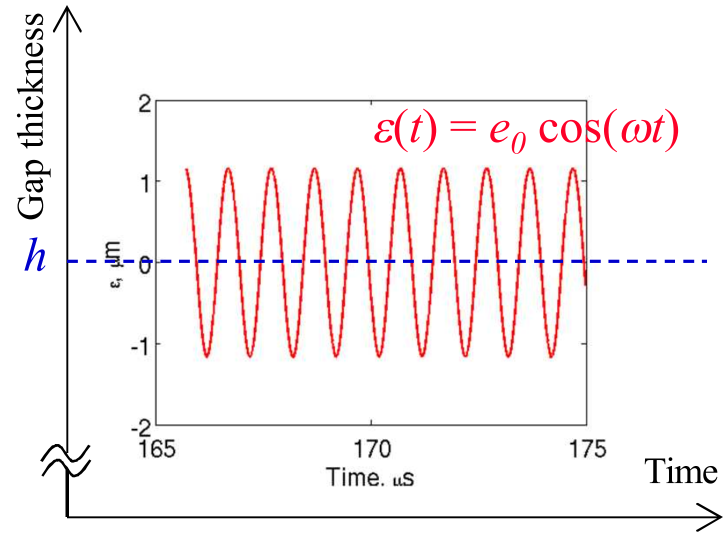
Objective:

- Provide experimental validation of the DSMC-base squeezed-film damping model for rigid plates.

Squeezed fluid damps oscillation.

Plate oscillates at fequency ω.





The squeezed fluid between the plate and the substrate creates damping forces on the plate.

How can squeeze film damping be predicted?

- Forces on moving plate from gas layer can be obtained from the linearized **Reynolds equation**

$$\frac{Ph^2}{12\mu}\nabla^2\left(\frac{p}{P}\right)-\frac{\partial}{\partial t}\left(\frac{p}{P}\right)=\frac{\partial}{\partial t}\left(\frac{z}{h}\right)$$

P = ambient pressure, Pa
h = gap size, m
μ = viscosity, Pa s
p = pressure at (x,y), Pa
t = time, s

Assumptions:

1. Rigid plate
2. Small gap
3. Small displacement
4. Small pressure variation
5. Isothermal process

The model to be validated is a direct Simulation Monte Carlo (DSMC) method.

Instead of the trivial boundary conditions at the plate edges, GT introduced

$$P - p = \eta G(\hat{\mathbf{n}} \cdot \nabla p) + \zeta \left(\frac{12\mu U}{G} \right) \left(1 + \chi \frac{6\Lambda}{G} \right)^{-1}$$

DSMC simulations were used to determine correlations for the gas-damping parameters

$$\eta = \frac{0.634 + 1.572(\Lambda/G)}{1 + 0.537(\Lambda/G)} \qquad \chi = \frac{1 + 8.834(\Lambda/G)}{1 + 5.118(\Lambda/G)} \qquad \zeta = \frac{0.445 + 11.20(\Lambda/G)}{1 + 5.510(\Lambda/G)}$$

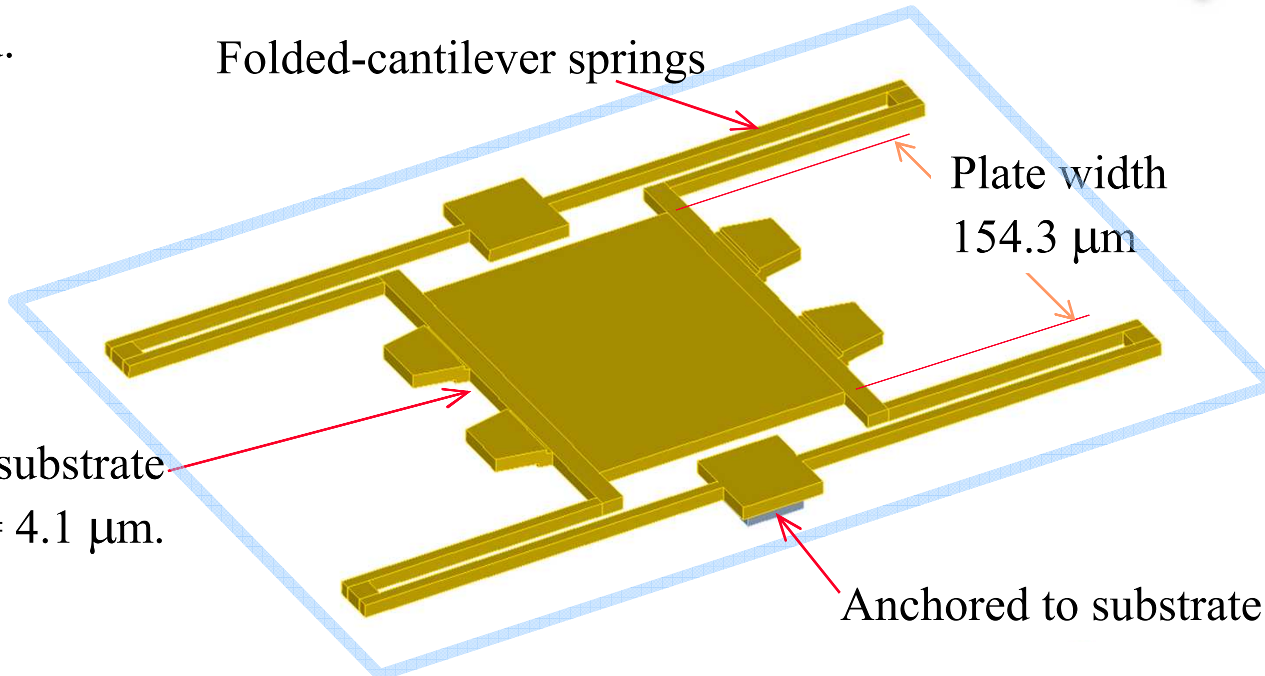
G = gas film (gap) thickness. Λ/G is modified Knudsen number $\Lambda = \frac{2-\alpha}{\alpha}\lambda$ $\quad 0 \le \Lambda/G \le 1$

α = accommodation coefficient. (For this test device a = 1).

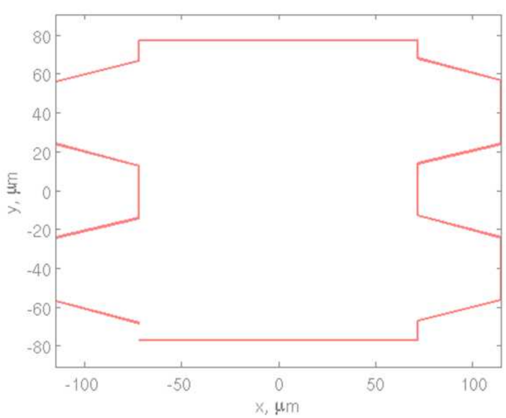
Present measurement was done on an oscillating plate.

- Structure is electro-plated Au.
- Thickness around 5.7 µm.
- Substrate is alumina.

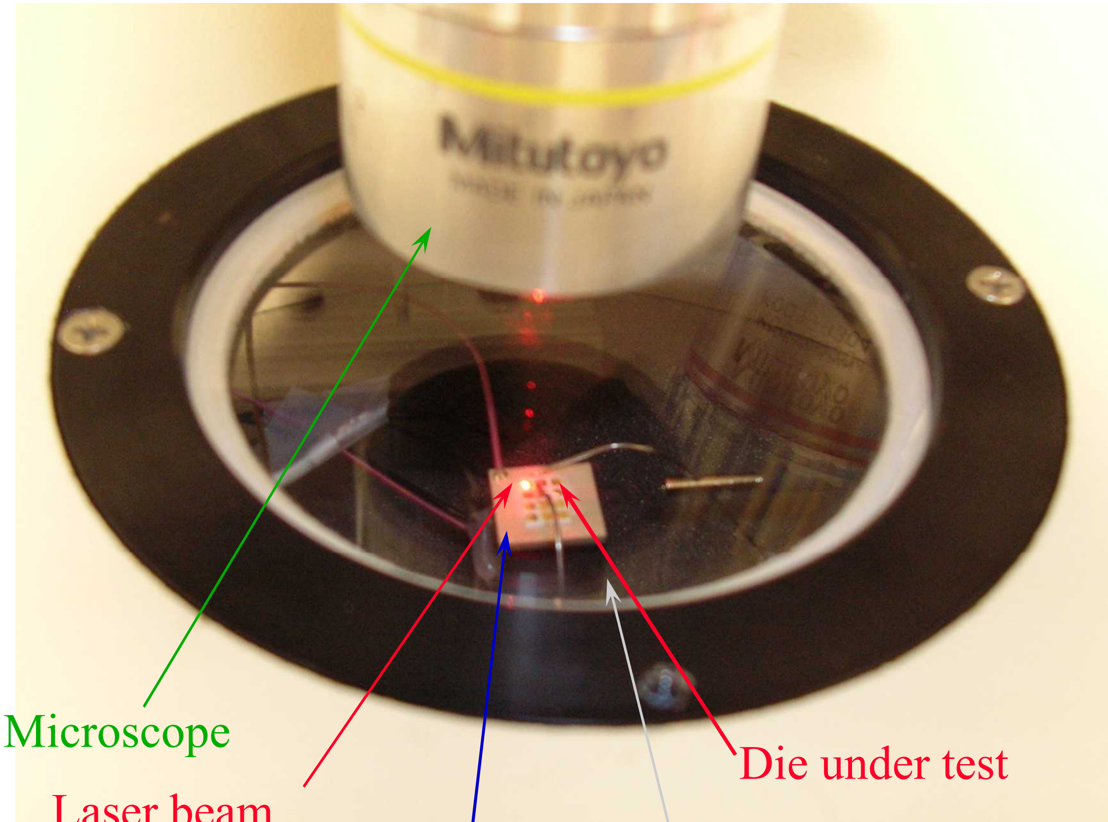
$A = 29717(\mu m)^2$
$a = 154.3\ \mu m$



Air gap between plate and substrate
Mean thickness = 4.1 µm.

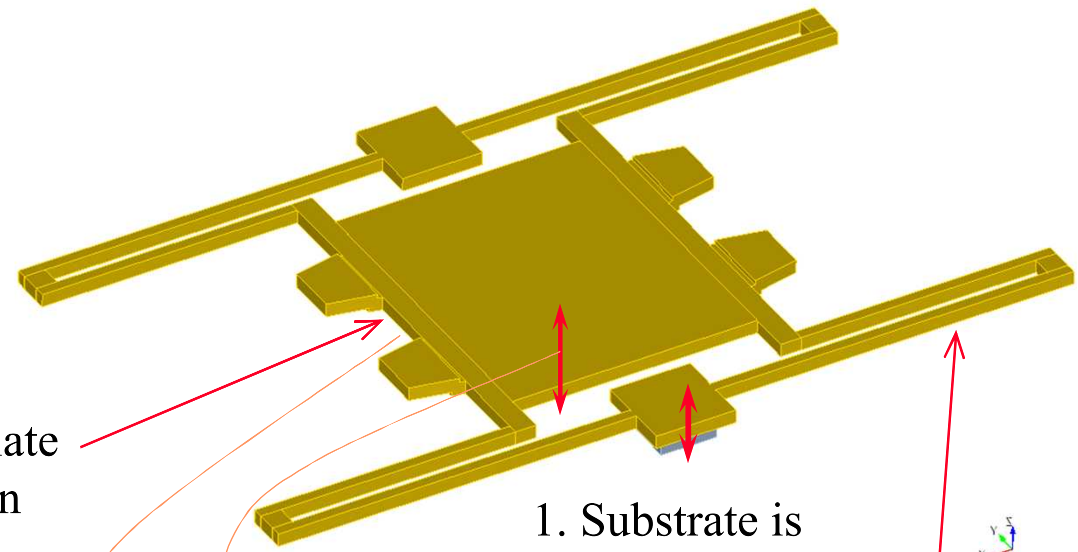


Measurement uses LDV and vacuum chamber.

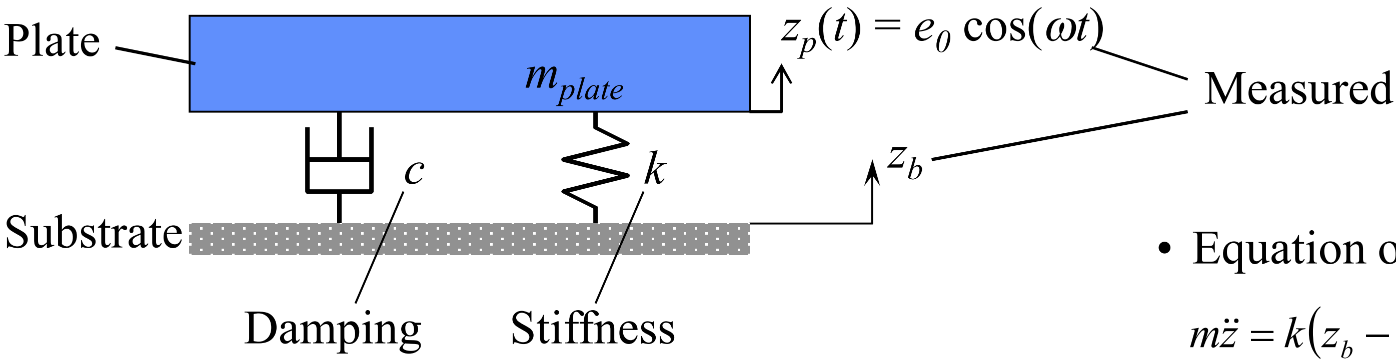


- Substrate (base) was shaken with piezoelectric actuator.
- Scanning Laser Doppler Vibrometer (LDV) measures velocities at base and at several points on MEMS under test.

Oscillating plate was shaken through its support.



The rigid plate test structure can be modeled as SDOF.



- Equation of motion

$$m\ddot{z} = k(z_b - z) + c(\dot{z}_b - \dot{z})$$

- Frequency response function (FRF) from base displacement to plate displacement:

$$\frac{Z_p(\omega)}{Z_b(\omega)} = \frac{m\omega^2}{-m\omega^2 + j\omega c + k_s} + 1 \qquad \text{(Measured)}$$

- Frequency response function (FRF) from base displacement to gap expansion:

$$\frac{Z(\omega)}{Z_b(\omega)} = \frac{Z_p(\omega) - Z_b(\omega)}{Z_b(\omega)} = \frac{m\omega^2}{-m\omega^2 + j\omega(\beta + c_s) + k_s}$$

Squeeze-film damping

Non-SF damping

- obtained from measurement at vacuum.
- will be subtracted from measured data.

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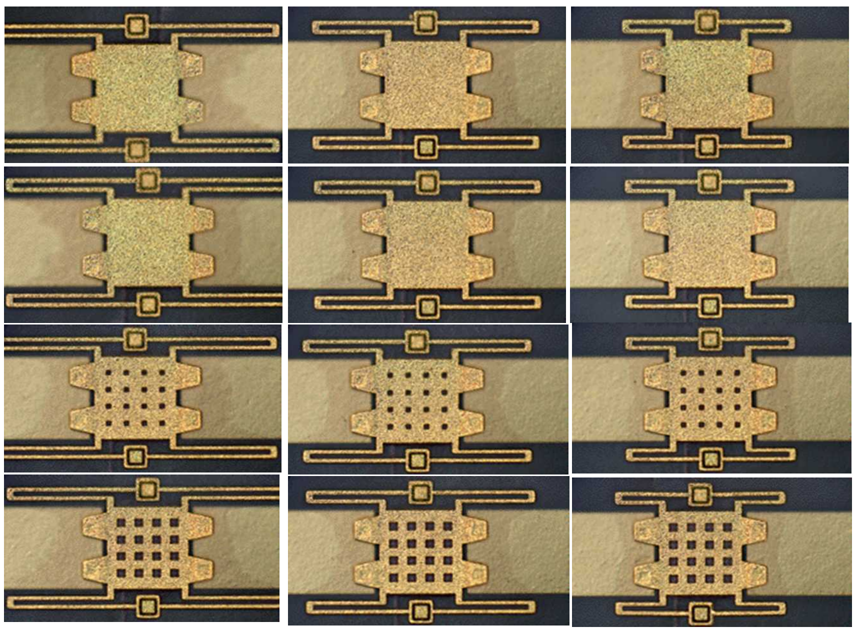
Measurement on array of plates gave natural frequency and damping.

Numbers in tables correspond to position in array

Des. and Fab. by Chris Dyck, SNL
Photograph by Carl Diegert, SNL



Undamped Natural Frequency f_1, Hz		
10509	17350	21721
12003	18507	29071
11922	17902	30405
12951	20561	31300

- The first two rows were designed to be identical.
- Differences were due to fabrication tolerance.

ζ_1 for $P = 83.3$ kPa, % of Critical		
10.05	7.61	5.35
10.36	7.72	5.93
9.52	8.19	5.06
9.51	8.60	5.99

ζ_1 for $P = 8$ kPa		
5.58	3.57	2.26
4.93	3.56	2.58
5.28	4.03	2.13
5.32	3.84	2.63

ζ_1 for $P = 0.8$ kPa		
1.03	0.59	0.49
1.00	0.79	0.41
1.01	0.64	0.38
0.81	1.16	0.38

ζ_1 for $P = 8$ Pa		
0.49	0.12	0.20
0.30	0.34	0.09
0.13	0.11	0.12
0.20	0.32	0.13

ζ_1 for $P = 0.8$ Pa		
0.54	0.07	0.10
0.12	0.28	0.09
0.15	0.12	0.12
0.50	0.41	0.19

Need to compare computed damping factor c with measured damping ratio ζ.

- Models predict ***damping factor*** c in the equation of motion

$$m_{eff}\ddot{z} + \beta\dot{z} + kz = f_{ext}(t)$$

- Measurement method gives ***damping ratio*** ζ in the equation of motion

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n{}^2 z = \hat{f}_{ext}(t)$$

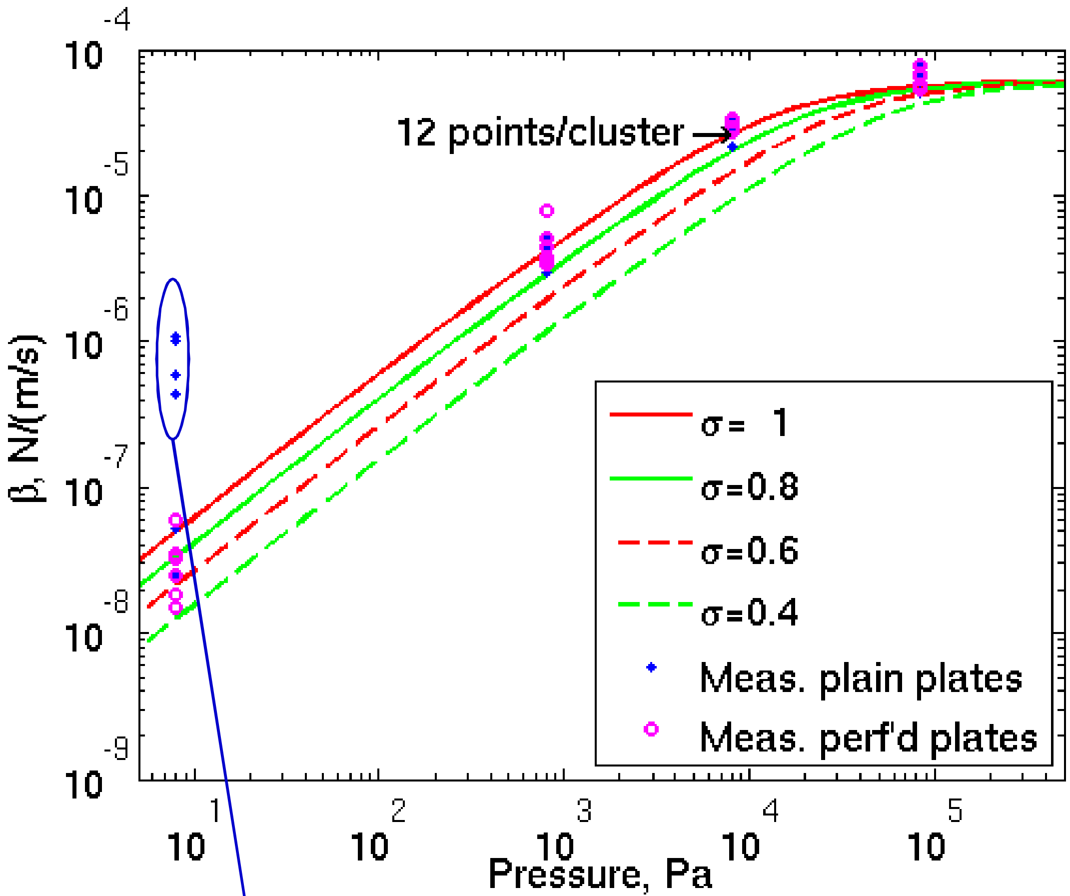
- To compare prediction with measurement, use the relationship between c and ζ

$$\beta = 2m_{eff}\omega_n\zeta$$

$$m_{eff} = m_{plate} + 4\times 0.37 m_{spring}$$

h_{plate} = plate thickness, m
ω_n = natural frequency, rad/s
m_{eff} = effective mass, kg

Accommodation coefficient was determined from test data.

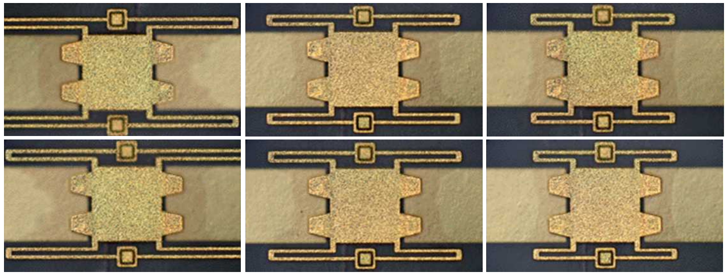


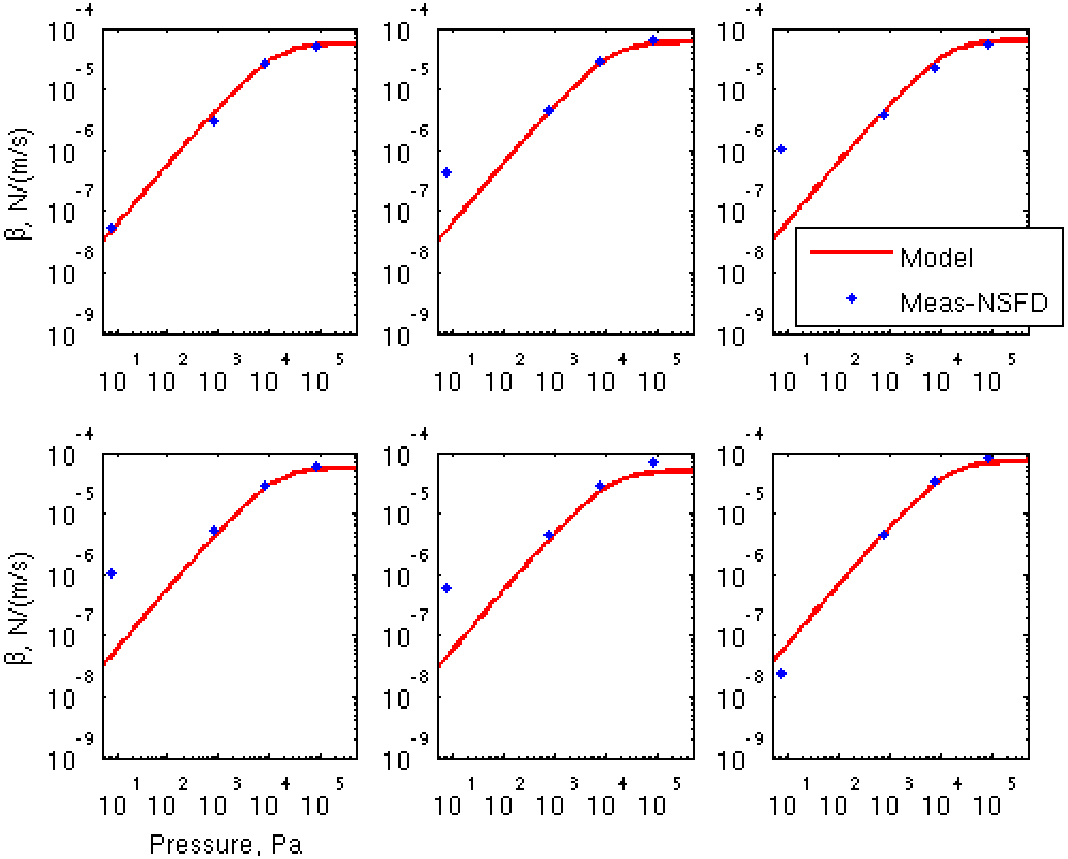
Hanning window, needed to reduce leakage in signal processing, distorted damping measurement.

- Lower pressure results in lower damping, as expected.
- Curve-fitting was not reliable at 6 mT.
  - Very high Q means very few data points around resonance.
- $\sigma = 1$ fits measured data the best.

Model Predictions Agree with Measurement.

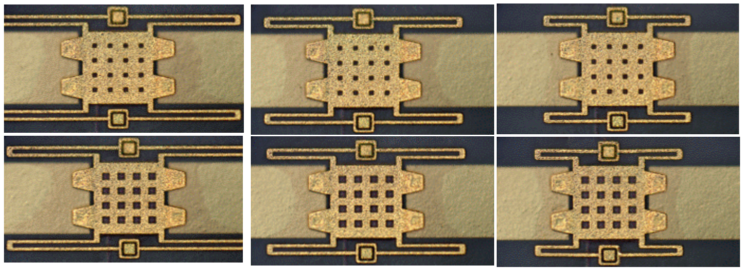
Plates with no holes.

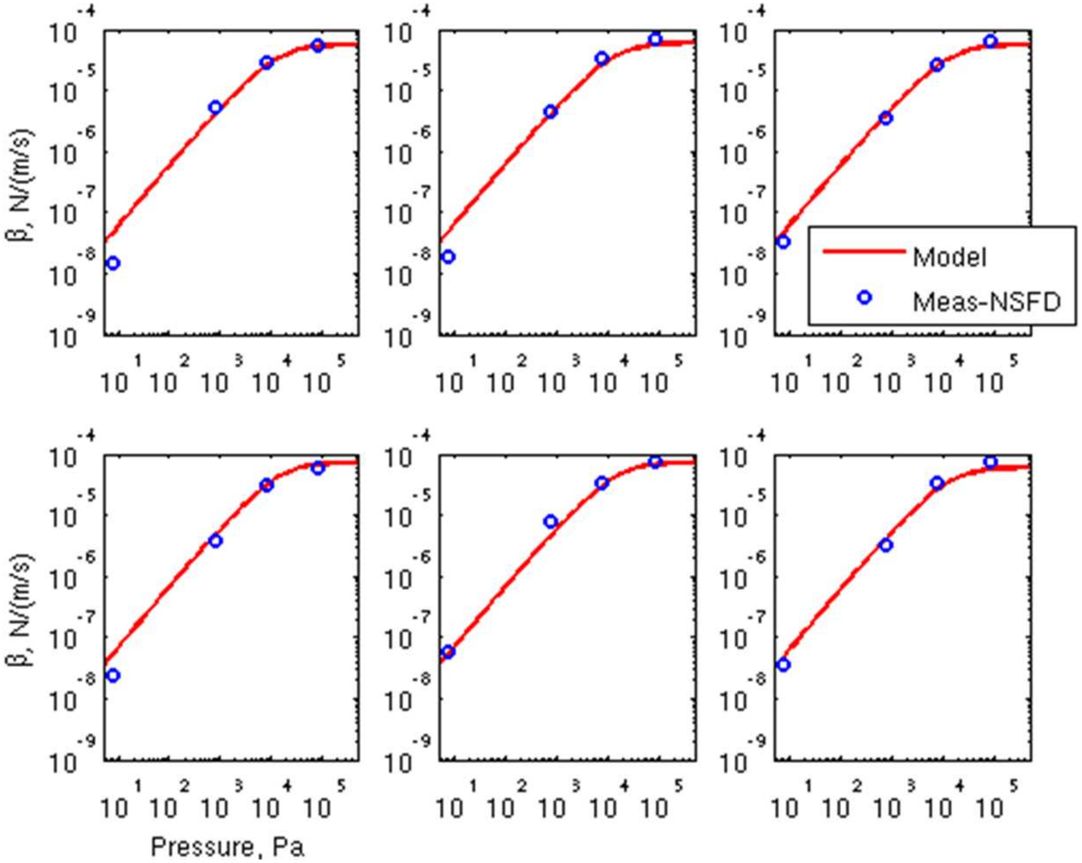




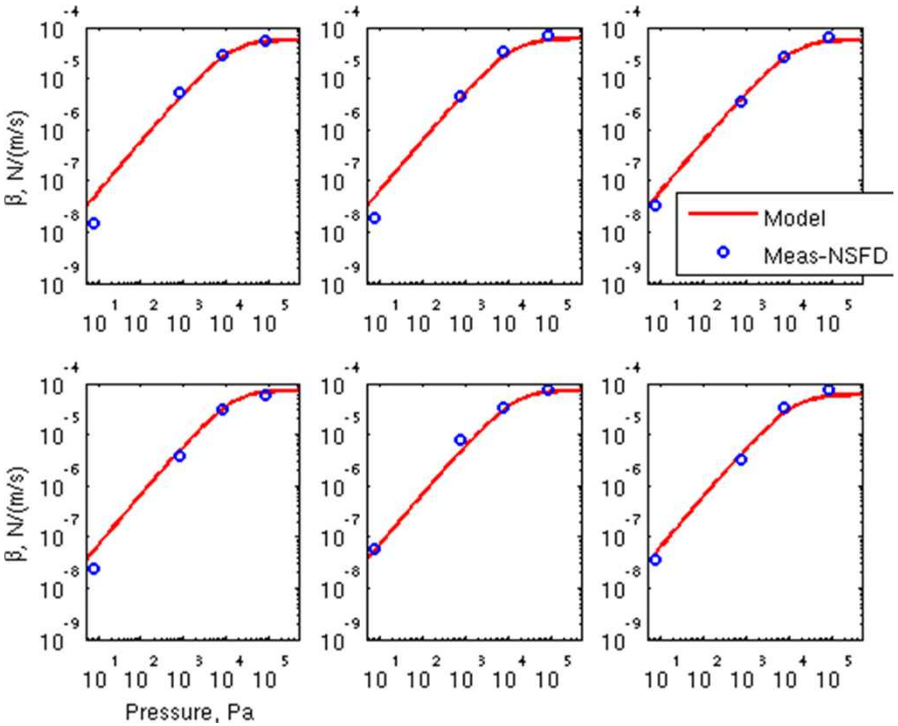
Model Predictions Agree with Measurement.

Perforated plates.





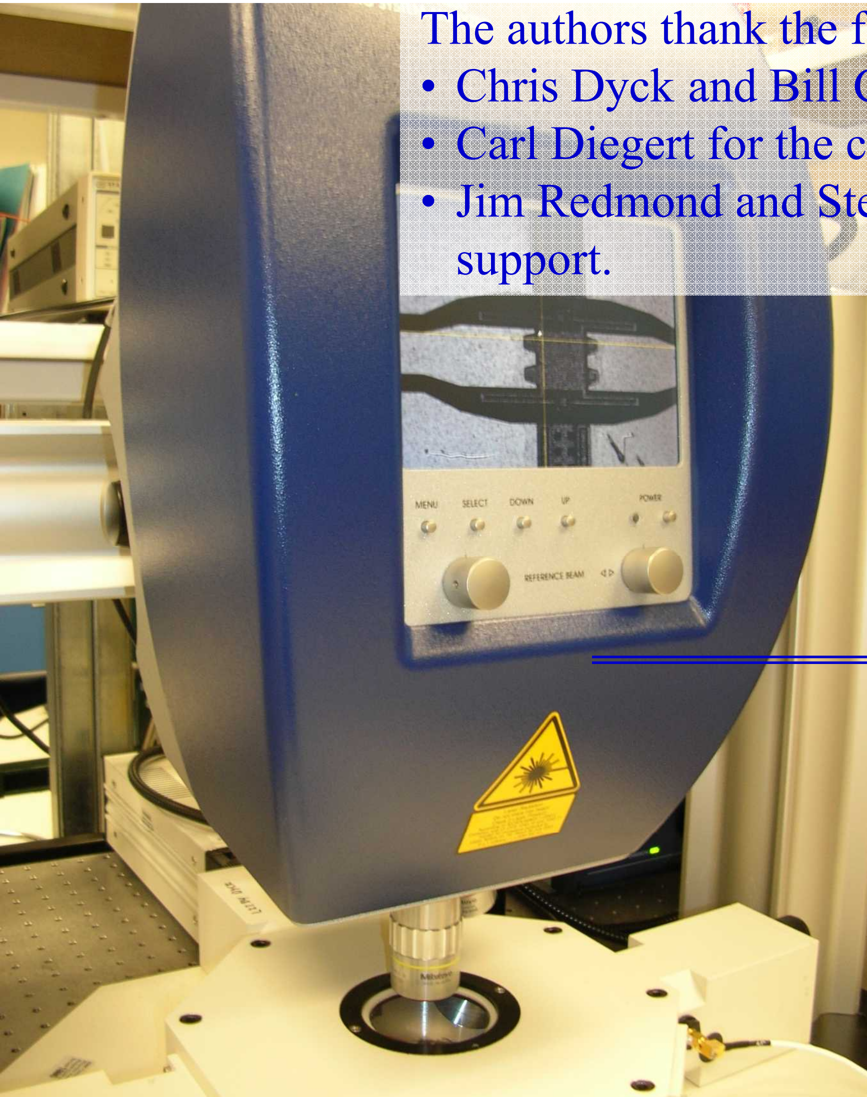
Conclusions:



- Model and experimental results are compared for a MEMS plate oscillating above an adjacent substrate with a gas-filled gap between.
- The results indicate that the damping coefficient decreases almost linearly with pressure on the log-log scale below 8 kPa.
- The model and experiment agree to within the experimental uncertainty if an accommodation coefficient of unity is employed, which is a reasonable value based on previously reported values.
- Small perforation holes do not result in deviation from prediction without holes.

- Better experiment control is needed to reduce the data scatter at low pressure (rarefied gas regime).

Acknowledgment



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Questions ??

Thank you!

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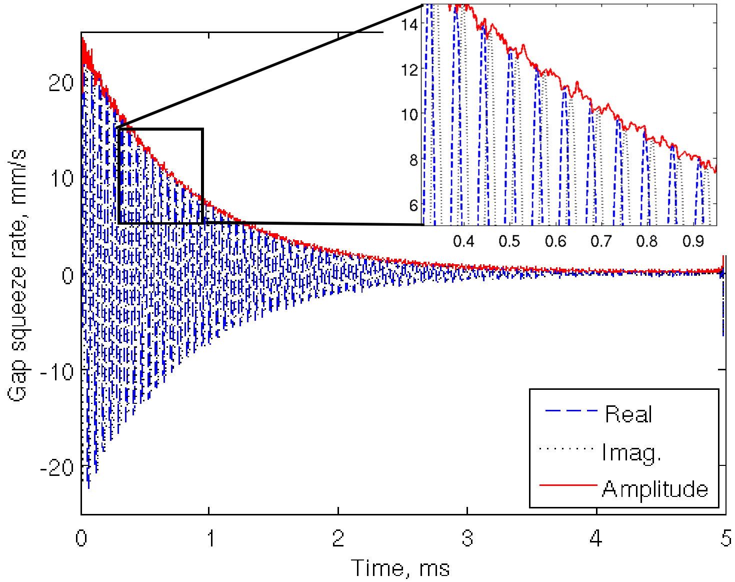
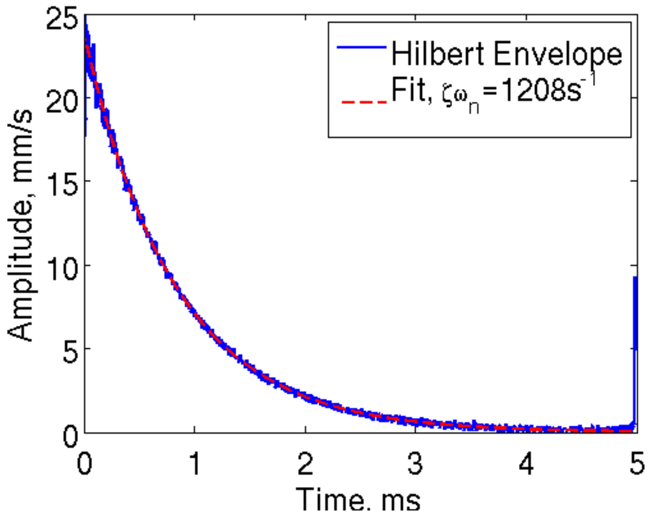
Damping from EMA agrees well with damping from free decay.

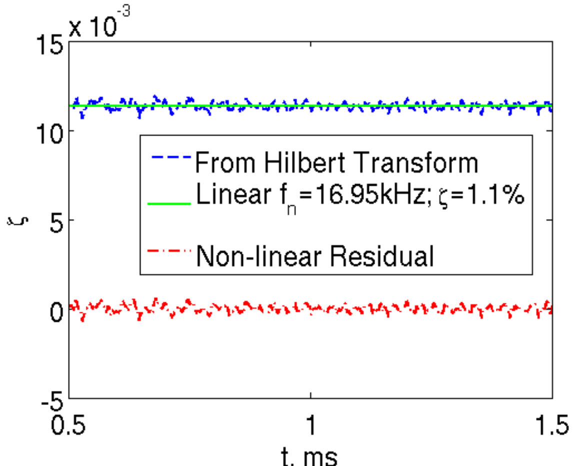
1. Hilbert transform gives decay envelope.



2. Exponential fit gives damping times natural frequency.



3. Damping is constant with time.



4. For this case (P=3830milliTorr), both experimental modal analysis (frequency domain fit) and free decay curve-fitting (time domain fit) give damping ratio $\zeta = 0.0011$

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