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A Computational Methodology For Simulating The Pervasive Failure Of Quasi-Brittle Materials And Structures

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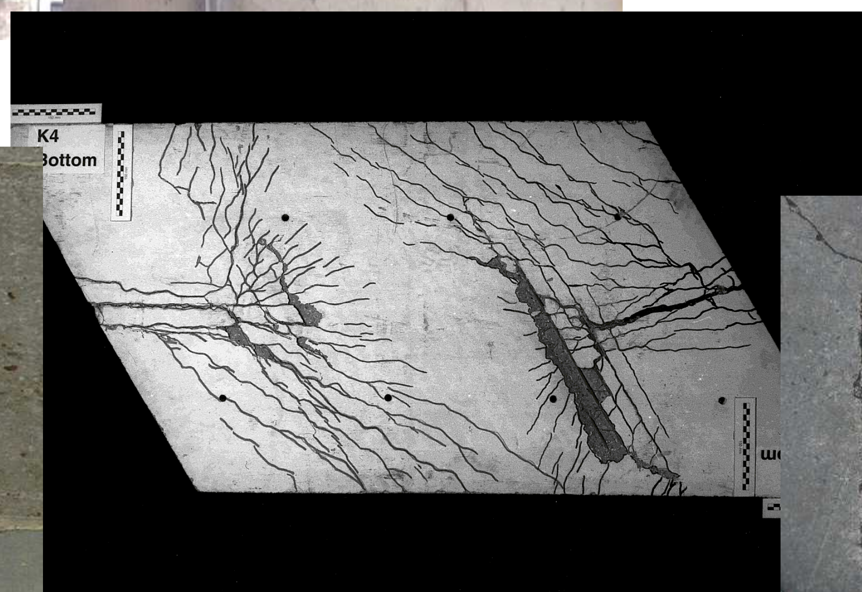
Target Problems

- weapons effects
- vulnerability assessments
- blast effects on structures
- arbitrary dynamic fracture
- penetration, perforation, fragmentation
- pervasive failure

rupture of containment vessel

Remains of the Murrah building after blast induced progressive collapse

Crack Branching, Coalescence, Tortuous Crack Paths



Goals

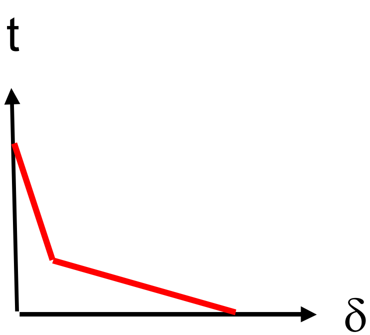
- Mesh independent modeling of the pervasive failure of structures (objectivity, convergence)
- (almost) arbitrary crack growth, nucleation, bifurcation, coalescence
- *a posteriori* fragment sizes (output of analysis instead of input)
- Continuum analysis with new surface generation
- Macroscopic analysis (homogenized continuum, nonlocal)
- Usable for ‘real world’ problems in a production environment

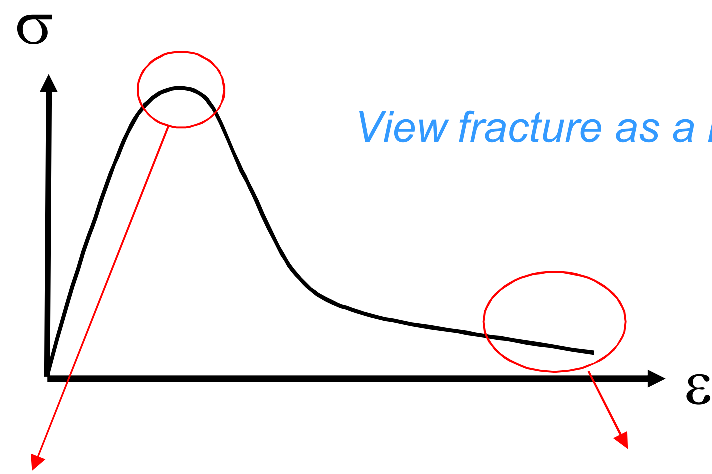
PDE Regularization

Planas, J. et al, (2003) ‘Generalizations and specializations of cohesive crack models,’ *Engineering Fracture Mechanics*, 70, 1759-1776

Jirasek, M. (1998) ‘Nonlocal models for damage and fracture: comparison of approaches,’ International Journal of Solids and Structures, 35, 4133-4145.

strain softening material response: consider two types of regularization





View fracture as a localization of damage.

$$\overline{w}(x) = \frac{1}{V_r}\int_{V_r} \alpha(|x-\xi|)\, w(\xi)\, dV$$

α= window function
w = state variable

cohesive approach

- cohesive crack inserted into mesh at inception of softening/localization
- additional material law
- potential ‘mismatch’ with continuum material model
- difficult to handle mixed mode
- really only applicable for diffuse cracking

nonlocal (integral form)

- nonlocal continuum model handles entire range of material response through softening
- can provide a localization ‘limiter’
- discontinuity inserted into mesh only upon completion of softening
- can handle nondiffuse cracking, fragmentation?
- can explain macroscopic size effects

Computational Approach

How to allow a continuum to transform into a discontinuum?

- Need a constraint on minimum feature size/angles to control time step and robustness.
- Want volume continuity in time (max sphere packing is only 74%)
- Want to be able to recover original continuum behavior under consolidation.

Methodology

1. Random Voronoi tessellation (mesh)
2. Polyhedral finite-elements (shape functions generated by RKPM)
3. Fracture only allowed at element edges (dynamic change in mesh connnectivity)
4. Dynamic insertion of cohesive tractions at limit surface
5. Penalty contact (discrete element paradigm)
6. Explicit dynamics solution

dynamic insertion of cohesive tractions based on . . .

Pandolfi, A. and Ortiz, M. (2002) ‘An efficient adaptive procedure for three-dimensional fragmentation simulations,’ *Engineering with computers*, 18, 148-159.

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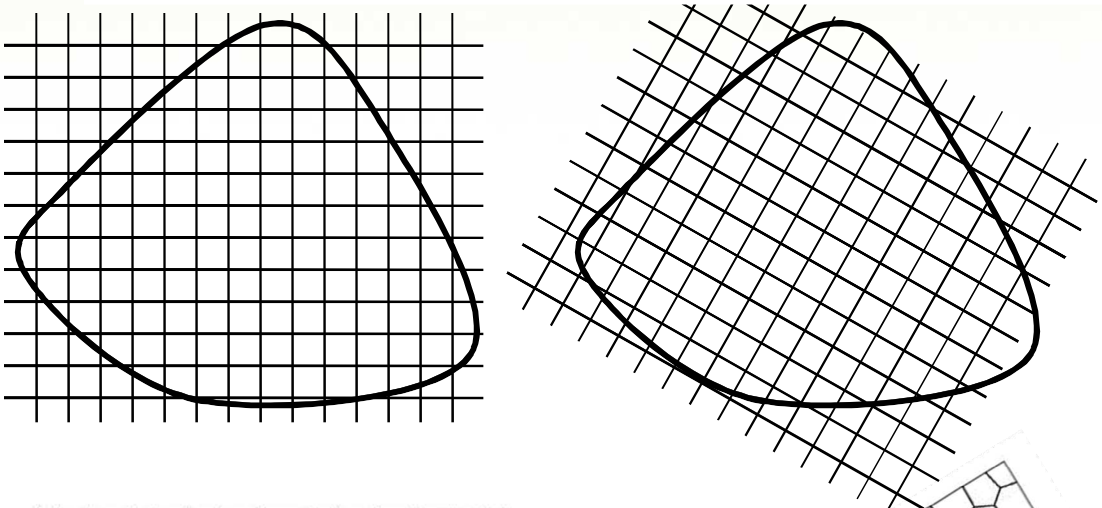
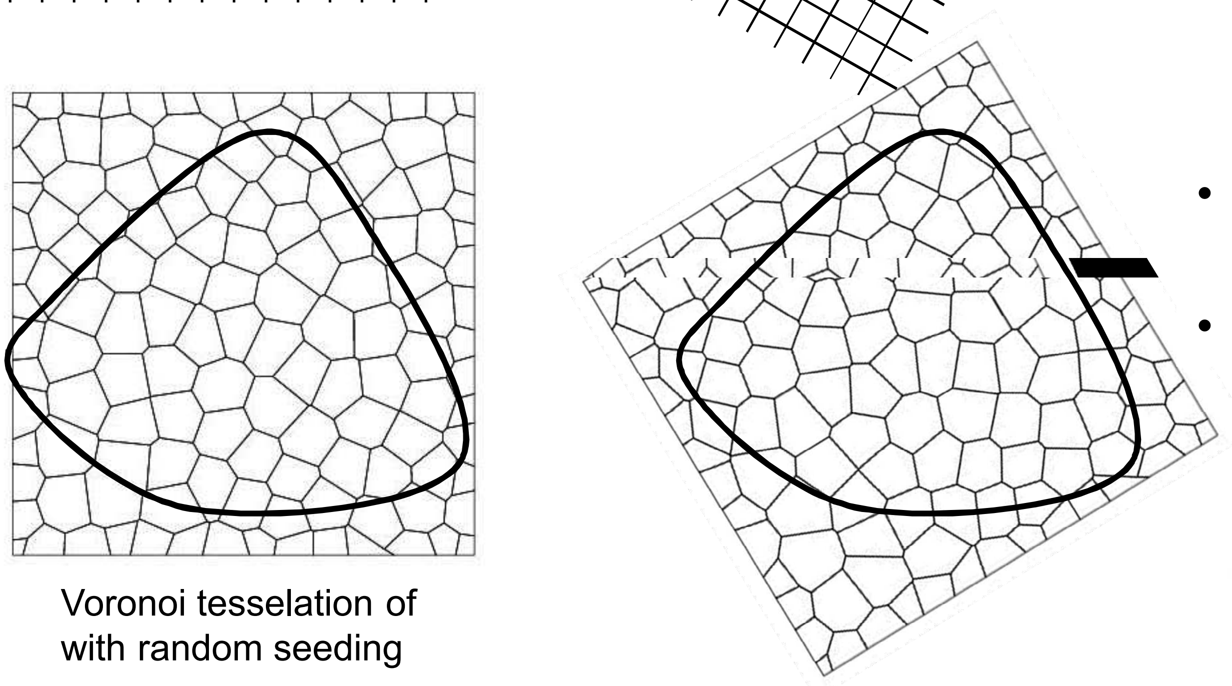
Eliminating Mesh Induced Crack Bias

If cracks can grow only at element edges, then need to eliminate any directional bias in crack growth (well known in 'lattice' methods).



Structured grids can result in strong mesh induced bias (potentially nonobjective).



Voronoi tesselation of with random seeding

- need to use 'random' discretizations
- statistically isotropic

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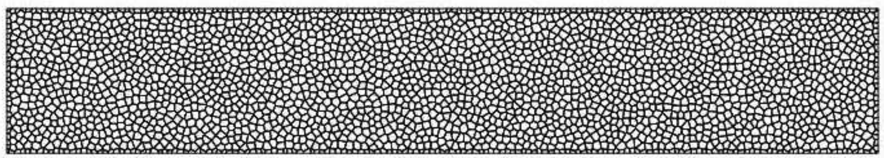
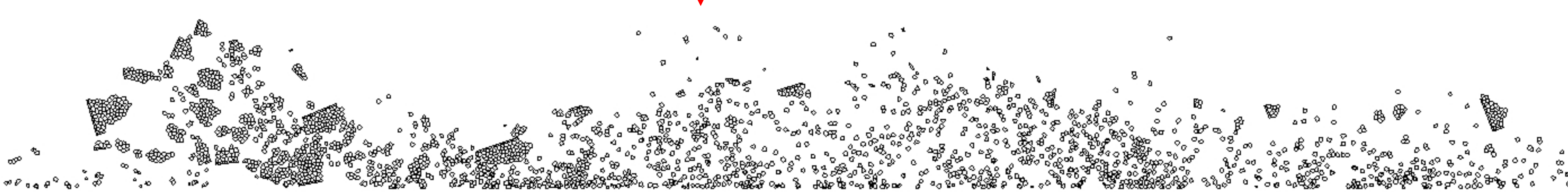
Dynamic Connectivity

- In the simulation of pervasive failure, can generate multiple new crack surfaces per time step.
- Need to have an efficient algorithm for modifying element connectivity.



Need to be able to handle arbitrary changes in connectivity (multiple new crack faces per time step).

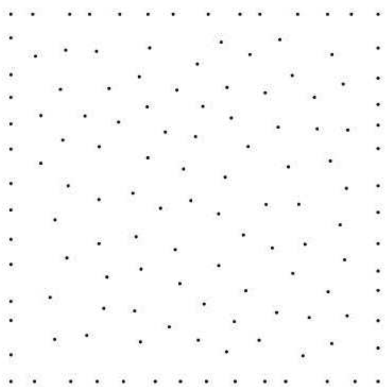


- bottom-up approach to reform connectivity
- loop over all faces, partition nodes based on equivalence relation of a shared intact face
- map equivalence classes to new node defs.
- use `C++` STL `set` and `map` storage classes

Voronoi Mesh Generation

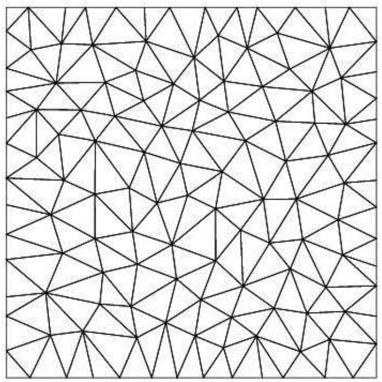
adapted from . . .

Bolander, J., Saito, S., 1998, ‘Fracture Analyses using Spring Networks with Random Geometry,’ *Engineering Fracture Mechanics*, 61, 569-591



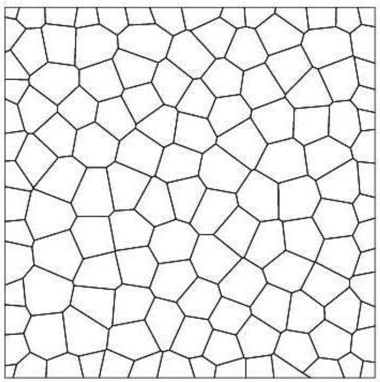
sequentially random seeding

- constraint on min. dist.
- seed until ‘max’ packing

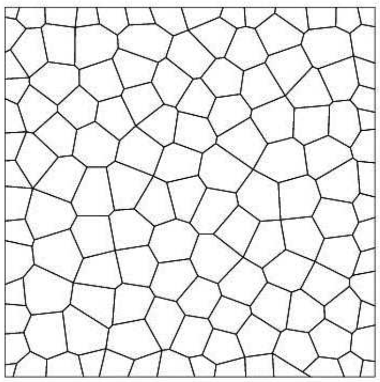


Delaunay triangulation

(Bowyer-Watson insertion)



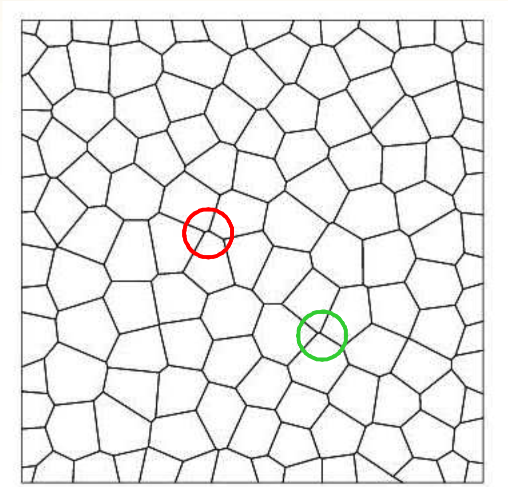
dual Voronoi



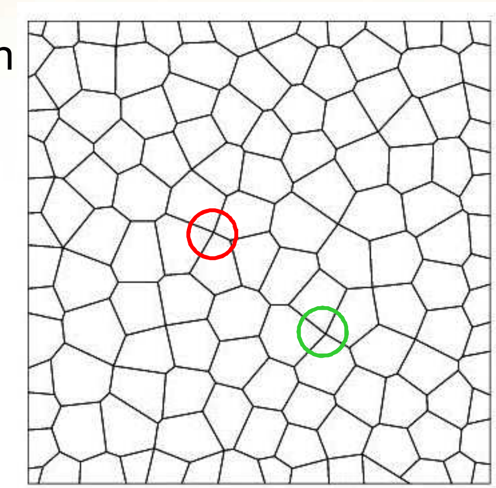
small edge regularization for use in explicit dynamics (21 small edges eliminated)

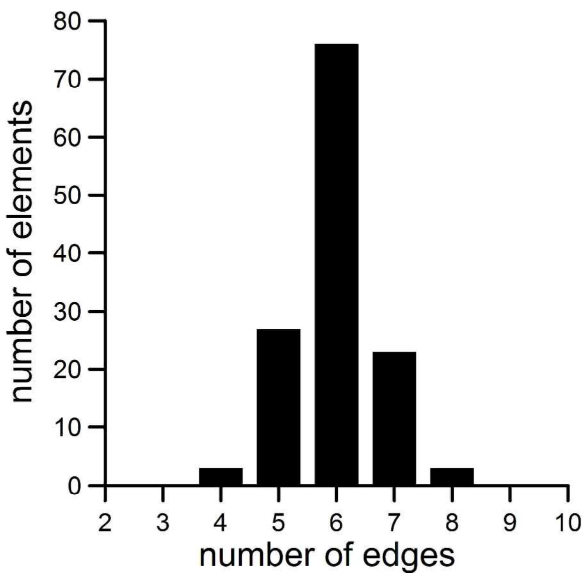
- Note that each Voronoi junction is randomly oriented.
- Most Voronoi junctions are triples with interior angles of 120°.
- Expect robust behavior in large strain gradients compared to a triangulation.

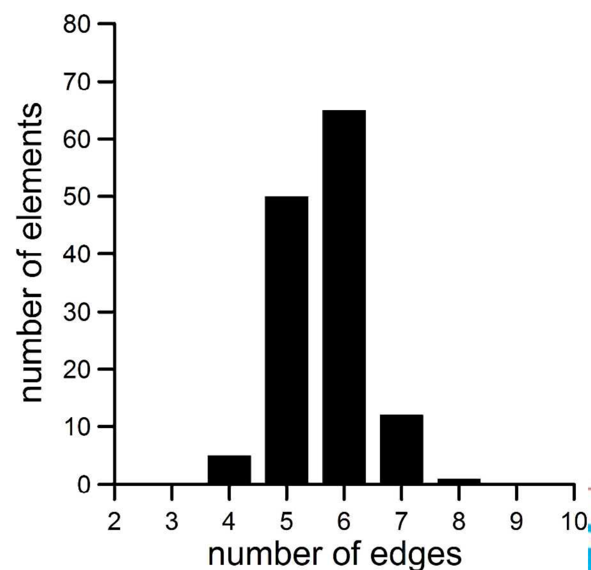
Small Edge Regularization



small edge regularization for use in explicit dynamics
(21 small edges eliminated)

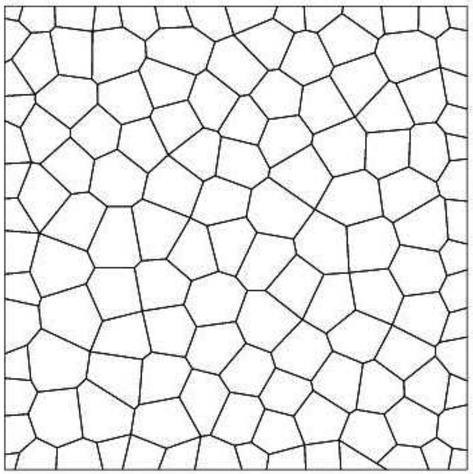




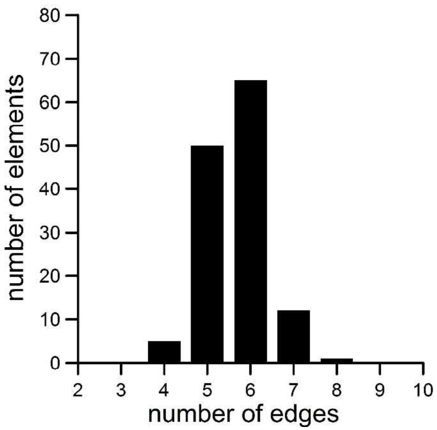


Multiple Random Mesh Realizations

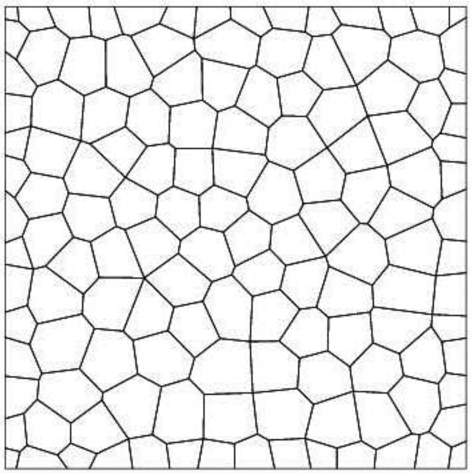
realization 1



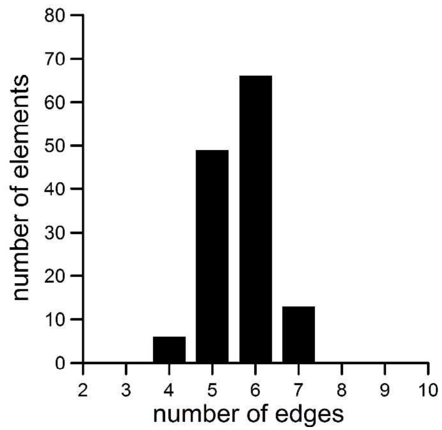
133 elements



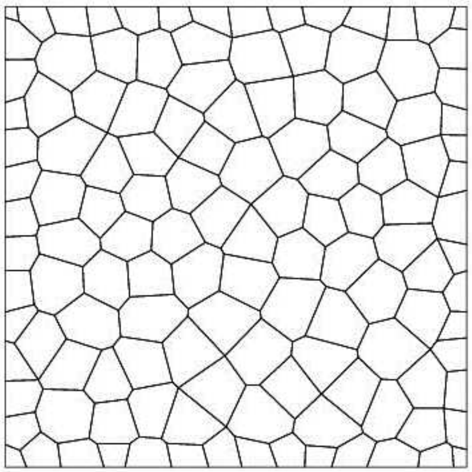
realization 2



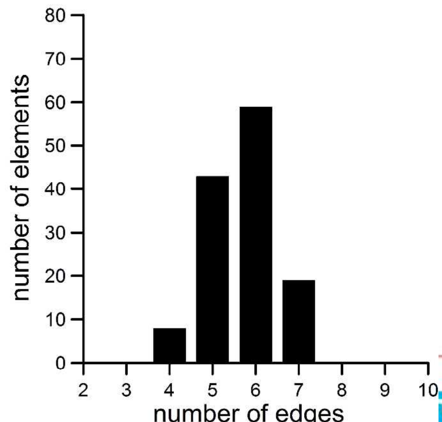
134 elements



realization 3

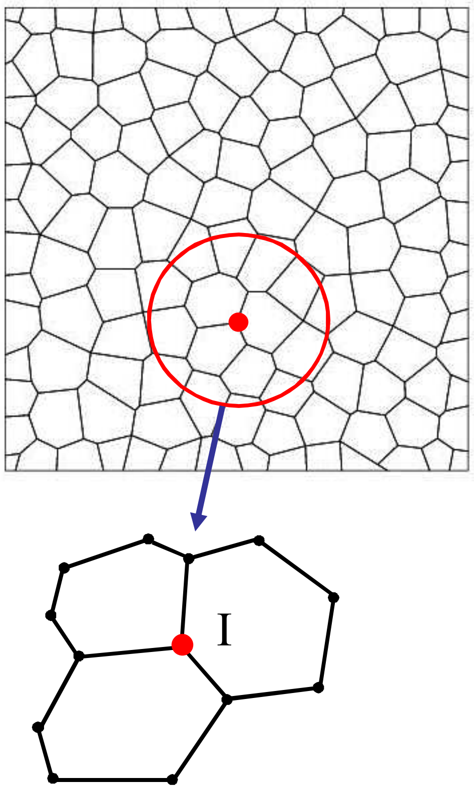


129 elements



Polyhedral Element Formulation

Use EFG/RKPM methodology to generate shape functions.

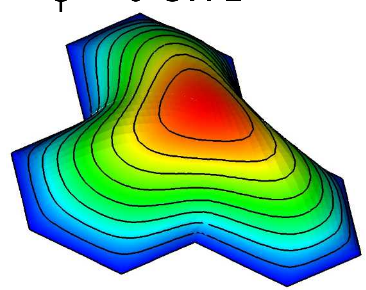


local support for node I

1. Generate nodal *weight* function ϕ by solving Poisson equation on compact support.
2. Generate nodal *shape* function ψ at each integration point using reproducing kernel method.
3. Correct shape function derivatives to satisfy integration consistency (Gauss's theorem).

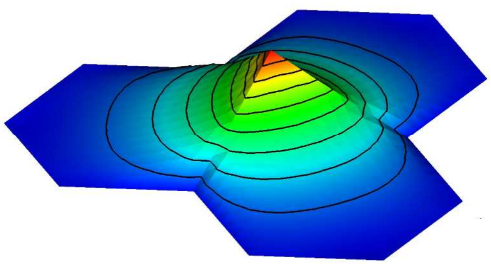
$$\nabla^2\phi + 1 = 0$$

$$\phi = 0 \text{ on } \Gamma$$



weight function ϕ

RKPM methodology



shape function ψ

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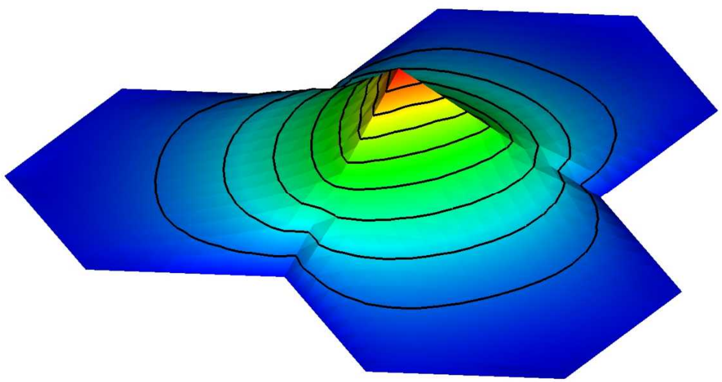
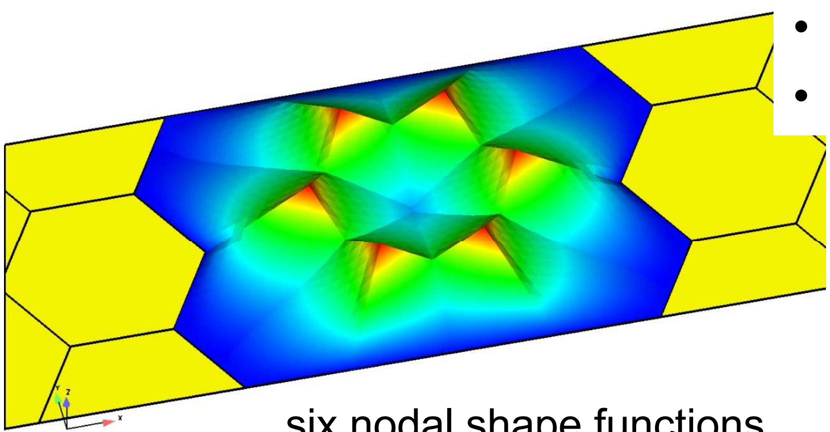
Shape Function and Element Properties

- partition of unity and ***x***
- Kronecker delta property at nodes
- linear on edges
- fully compatible with existing finite elements
- works for non-convex elements
- shape functions defined on original configuration
- no isoparametric mapping to ‘parent’ shape
- need to use total-Lagrangian formulation
- mean dilation formulation for incompressibility
- can use conventional material models
- ‘special’ mass-lumping

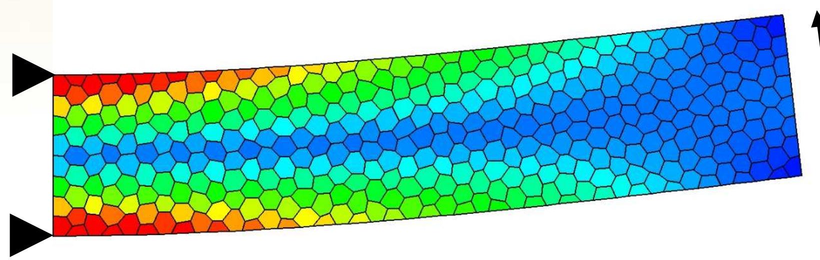




six nodal shape functions
for a regular hexagon

Shape Function Integration Consistency

Chen, J.S. et al (2001) 'A stabalized conforming nodal integration for Galerkin mesh-free methods,' *International Journal for Numerical Methods in Engineering,* 50, 435-466.



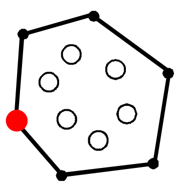
Gauss's theorem

$$\int_{\Omega_e} \psi_{,i} = \int_{\Gamma_e} \psi\, n_i$$

$$\sum_j w_j \psi^j_{,i} = \sum_j w^\Gamma_j \psi^j n_i$$

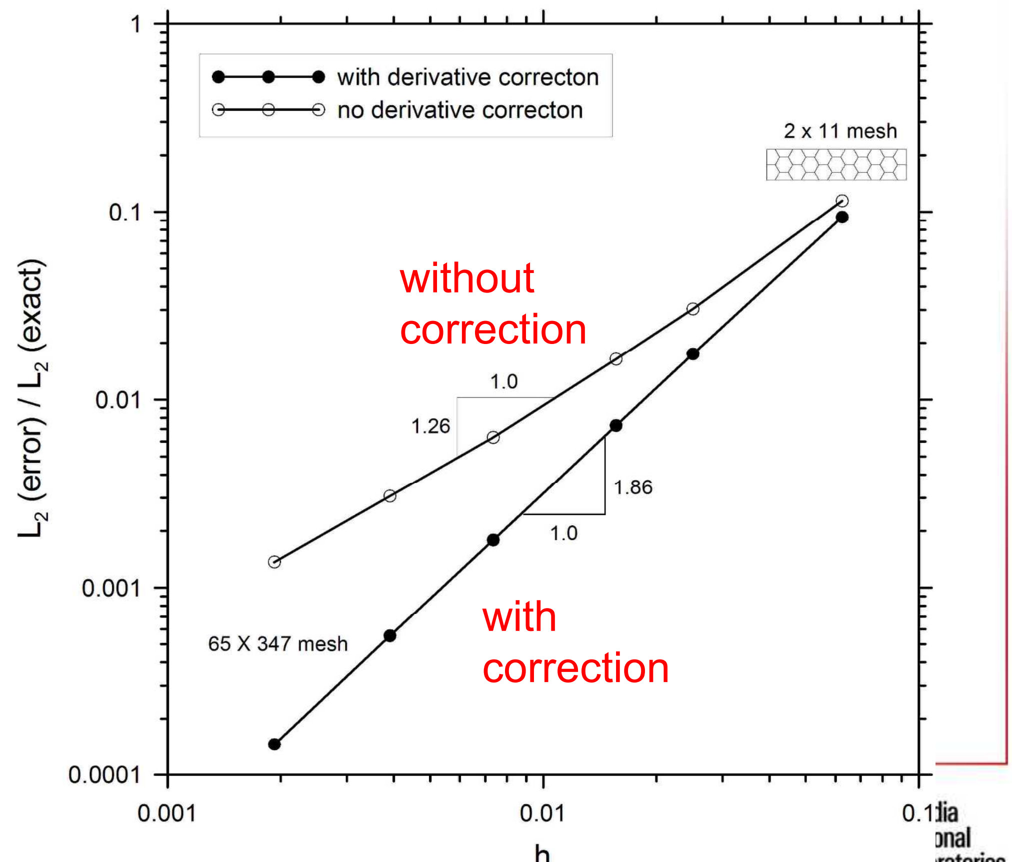
ψ = shape function

w_j = integration weight,



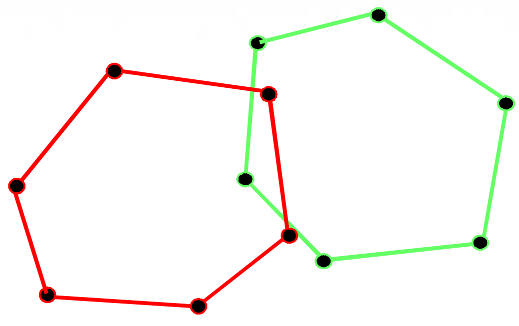
○ integration point

'Tweak' $\psi_{,i}$ to satisfy this constraint while maintaining previous properties.



Contact Formulation

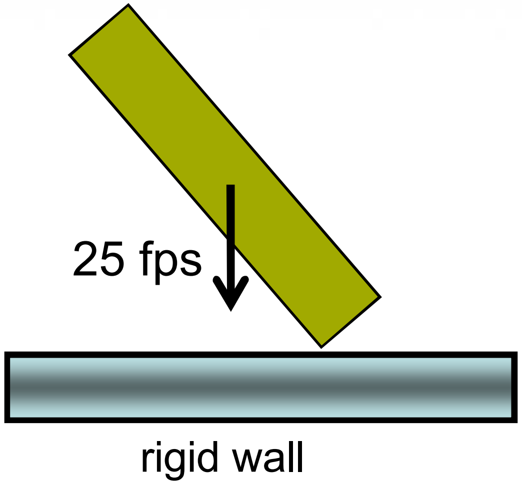
Heinstein, M. et al (2000) ‘Contact-impact modeling in explicit transient dynamics,’ *Computer Methods in Applied Mechanics and Engineering,* 187, 621-640.

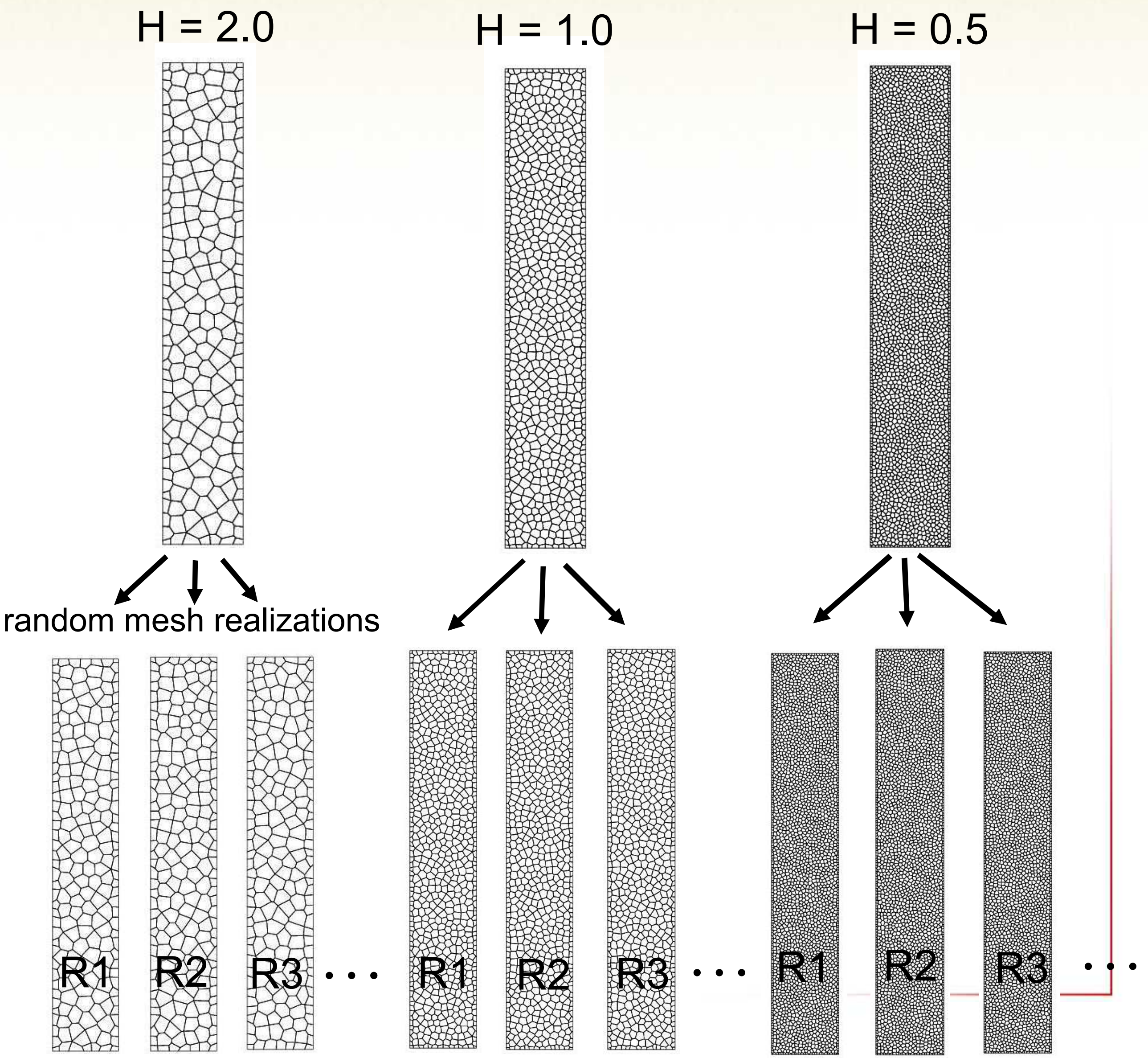


- each element is treated discretely, no overall surface structure
- element is included in search if any edge is ‘cracked’
- penalty formulation (velocity and displacement)
- velocity penalty (plastic contact)

Demonstration Problem

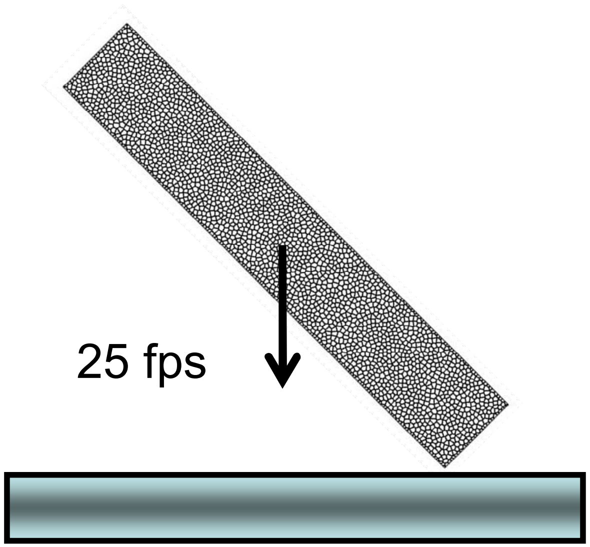
1' x 6' unreinforced concrete column (low strength)

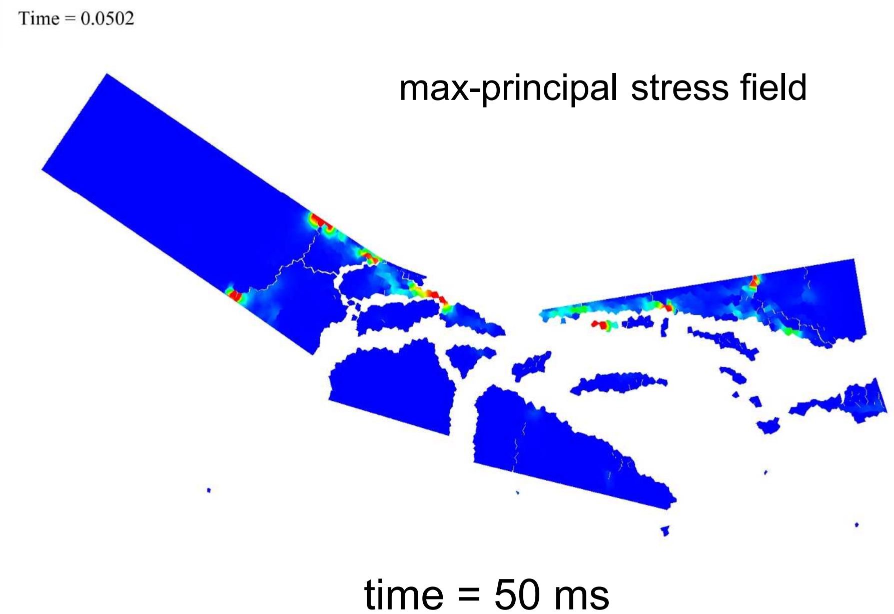




Demonstration Problem

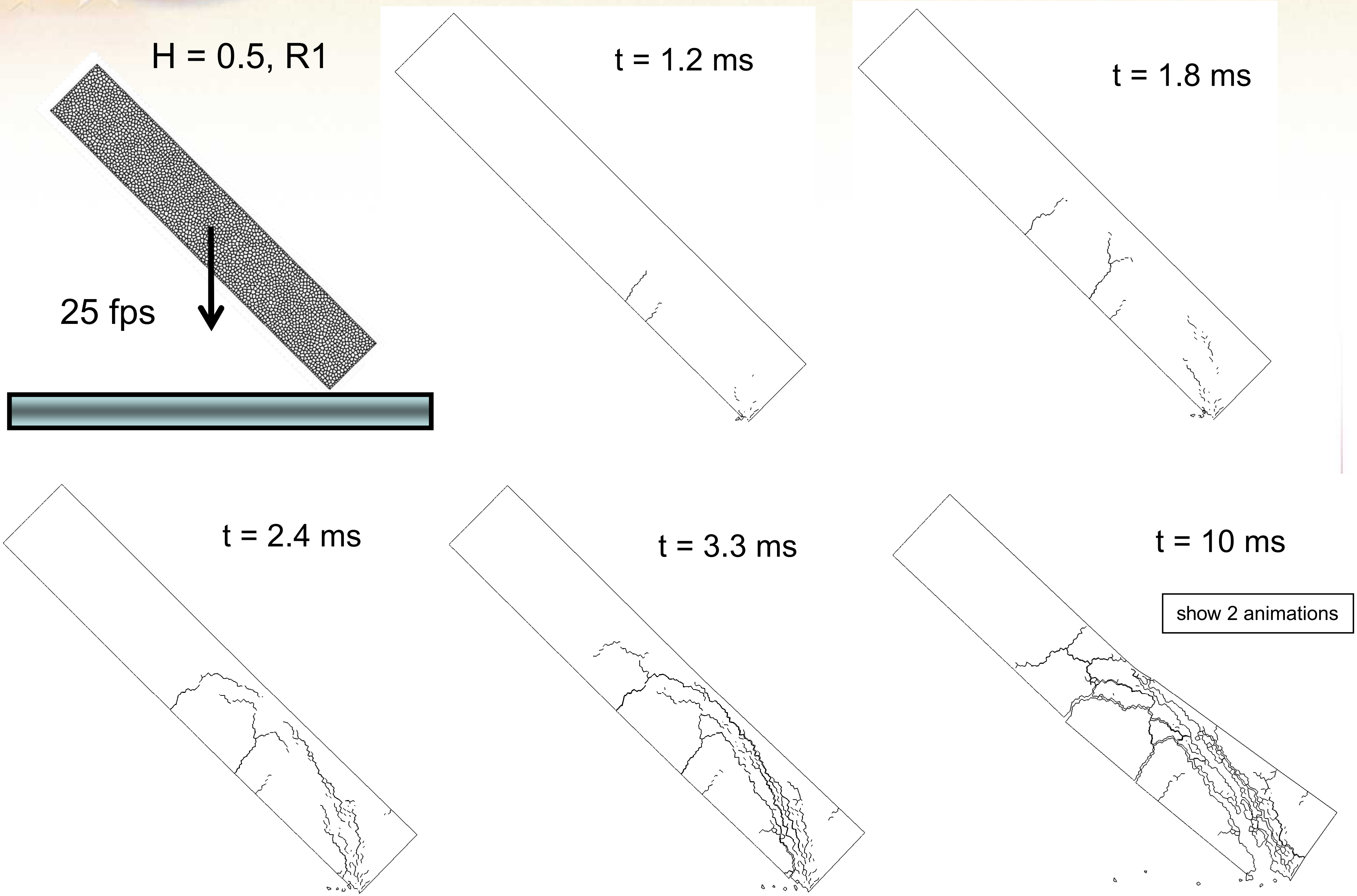
H = 0.5, R1



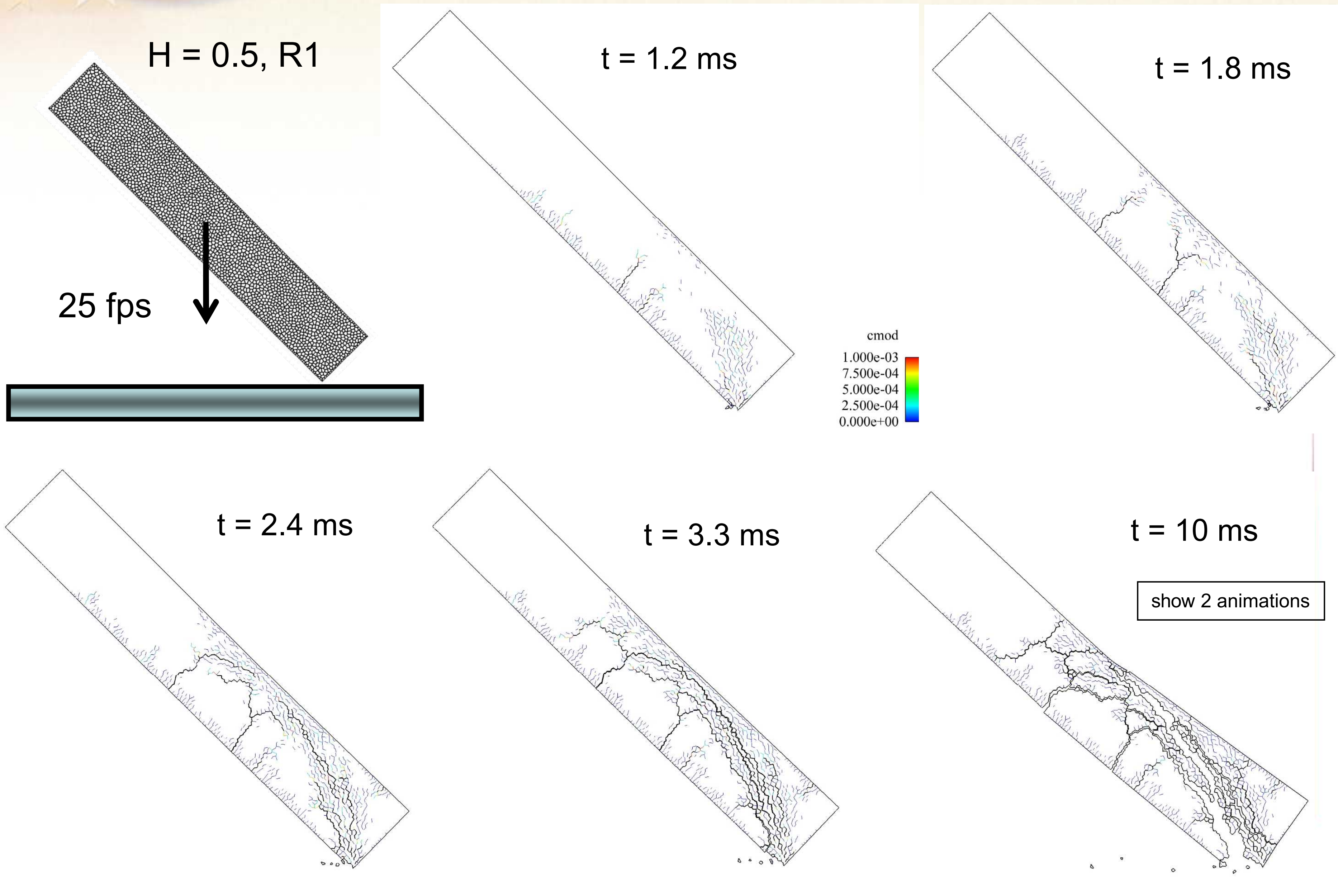


show 2 animations

Crack-Boundary View, H=0.5, R1

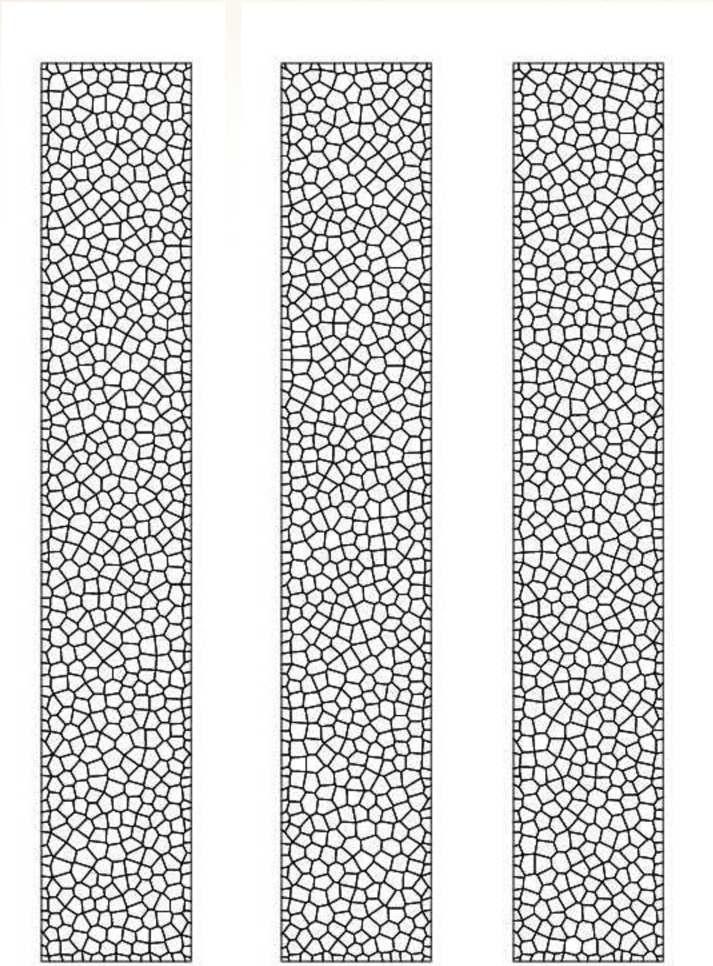


Cohesive-Crack View, H = 0.5, R1



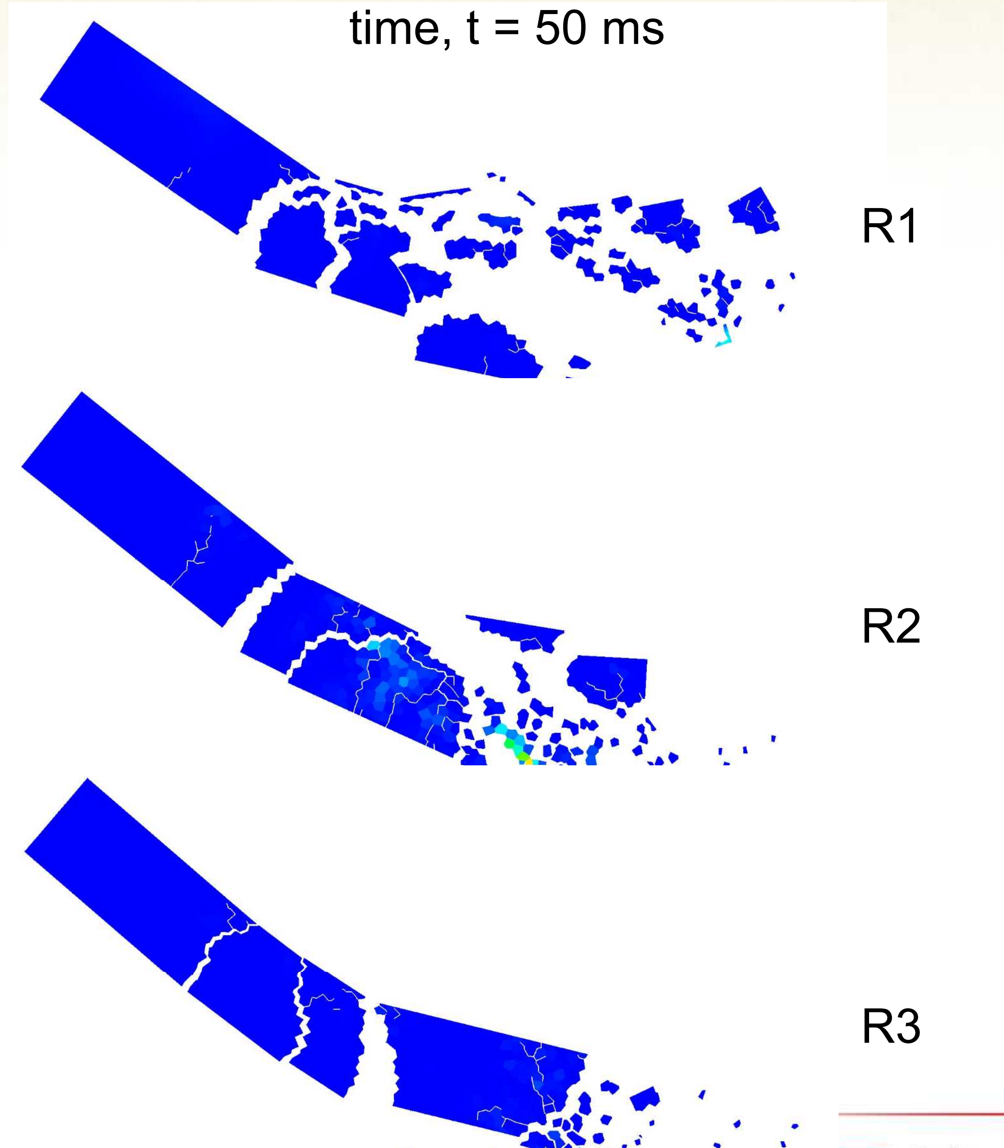
show 2 animations

Multiple Random Realizations, H = 1.0



R1 R2 R3

random mesh realizations



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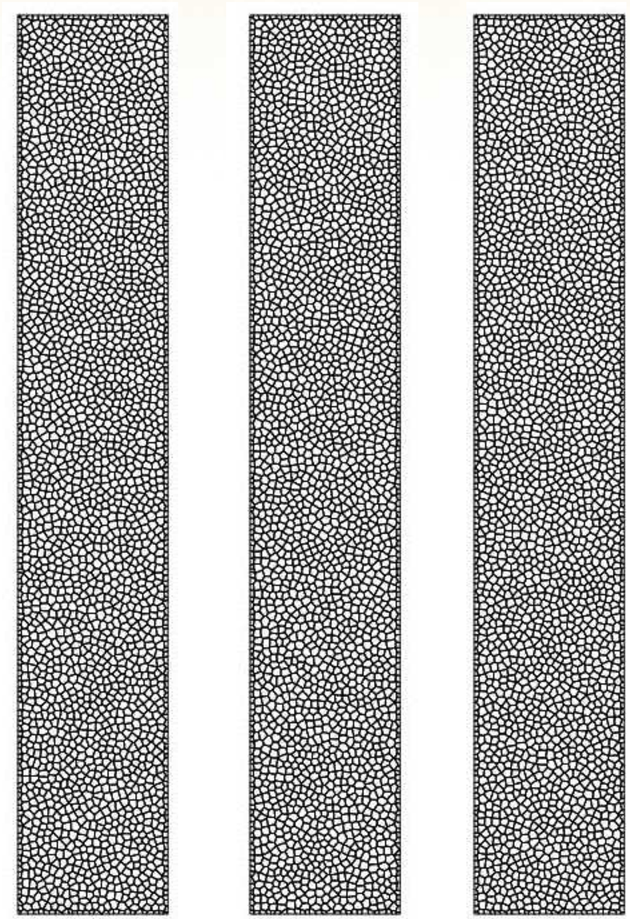
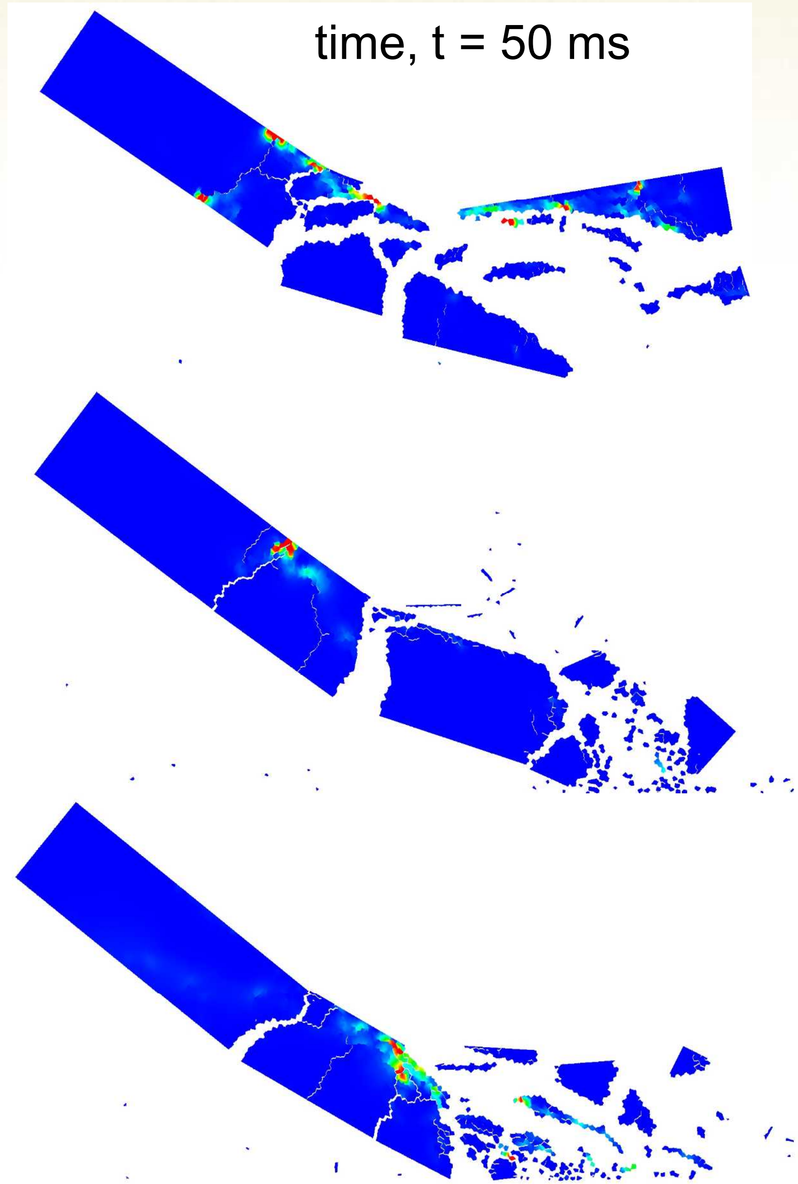
Multiple Random Realizations, H = 0.5



R1 R2 R3

random mesh realizations



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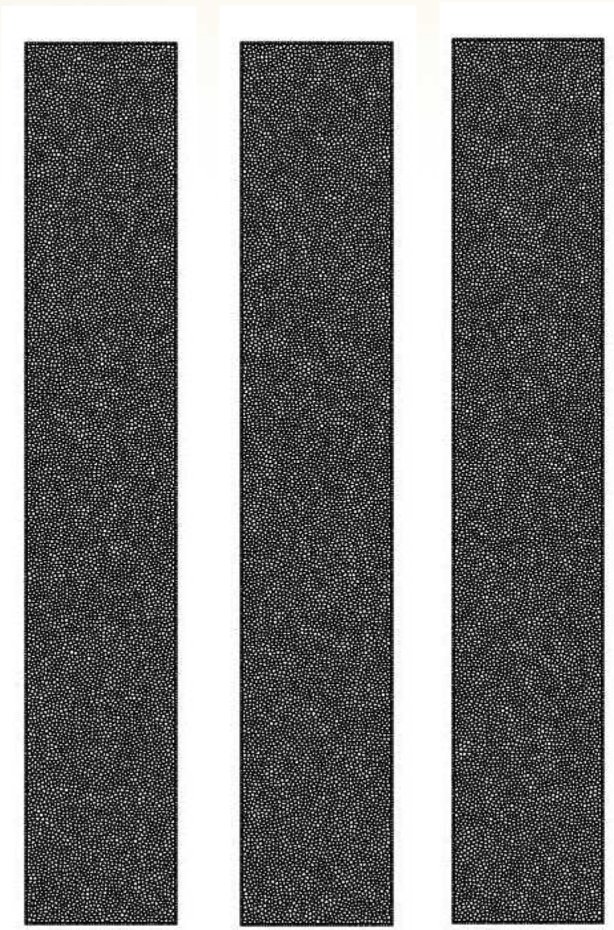
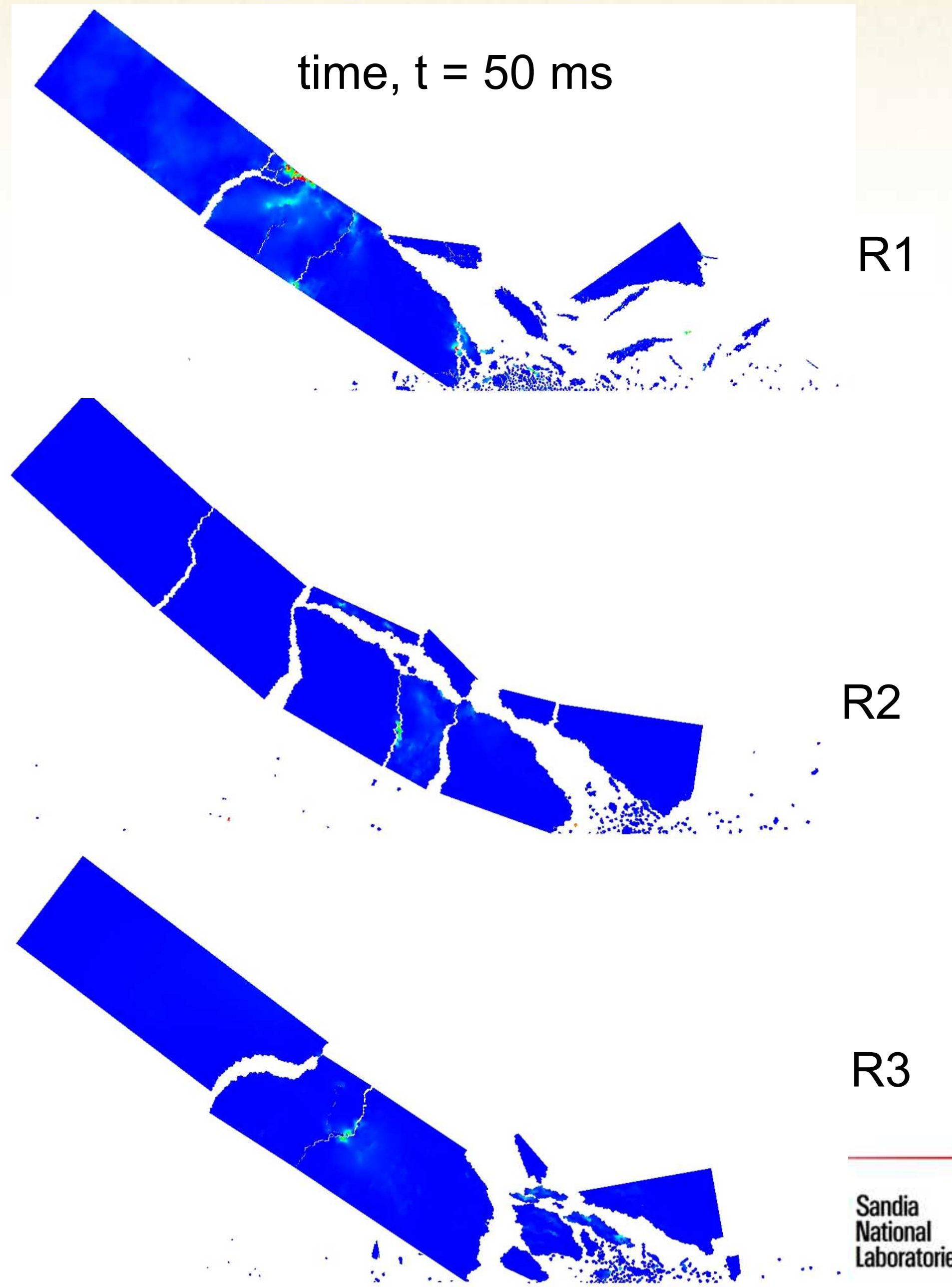
Multiple Random Realizations, H = 0.25



R1 R2 R3

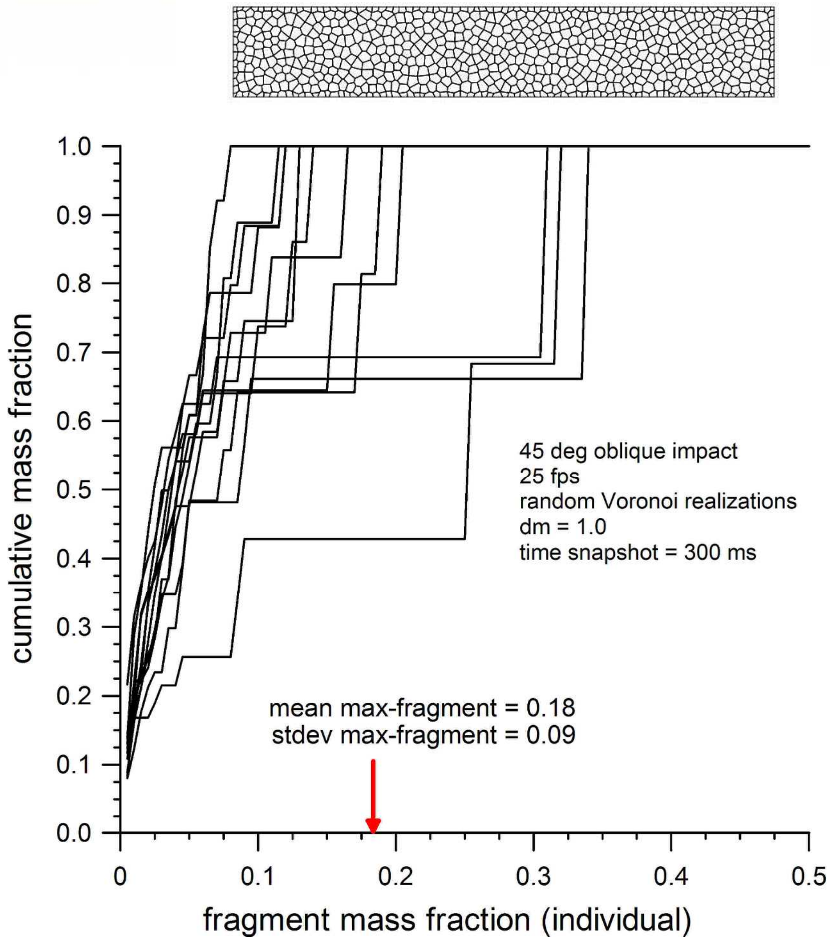
random mesh realizations



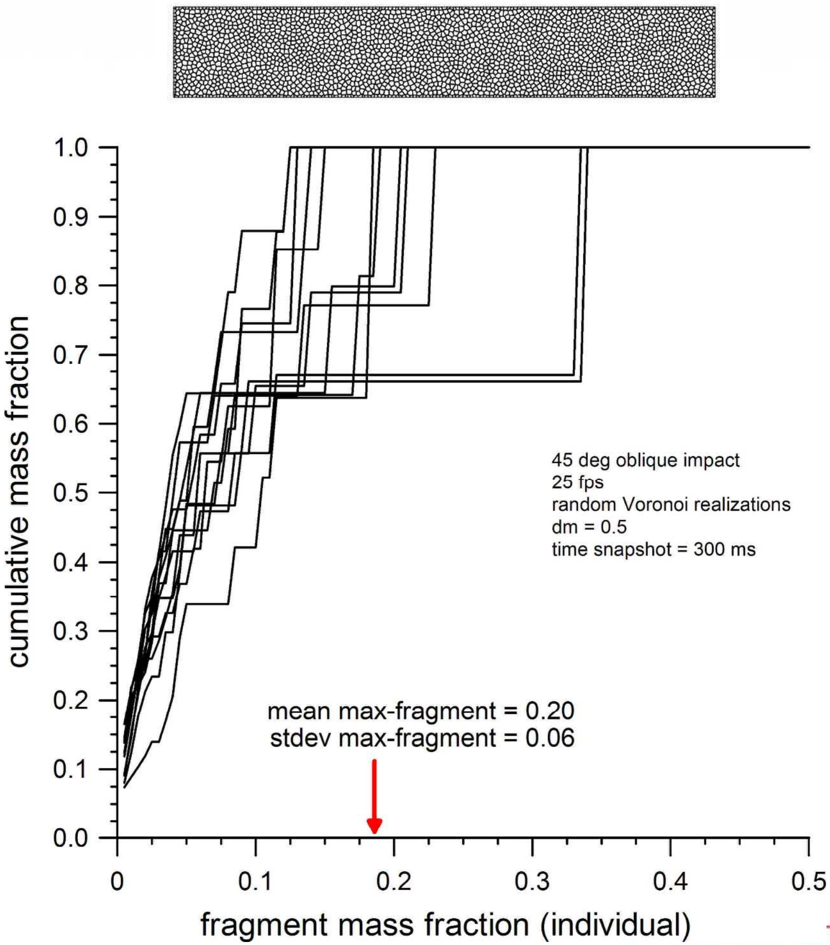
Fragmentation Statistics

12 random mesh realizations

element size ~ 1.0



element size ~ 0.5



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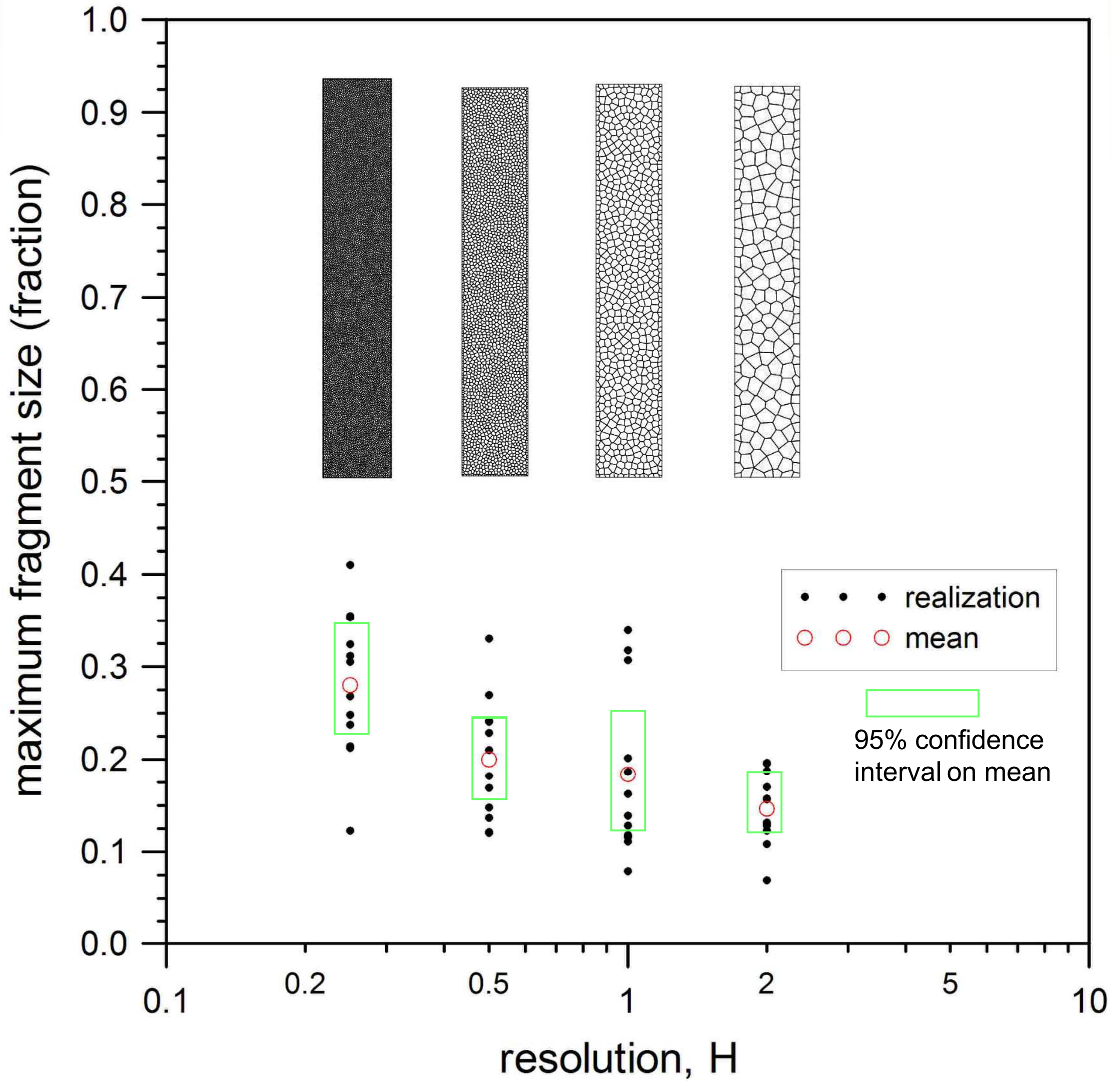
Maximum Fragment Size Statistics

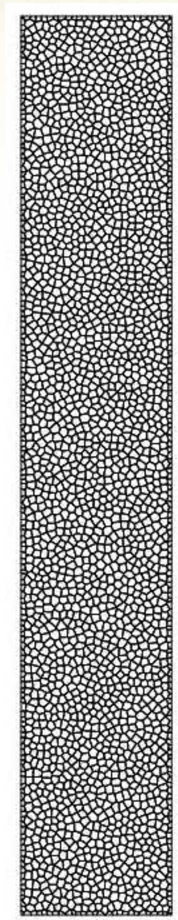
4 mesh sizes, 12 random mesh realizations, homogenous material

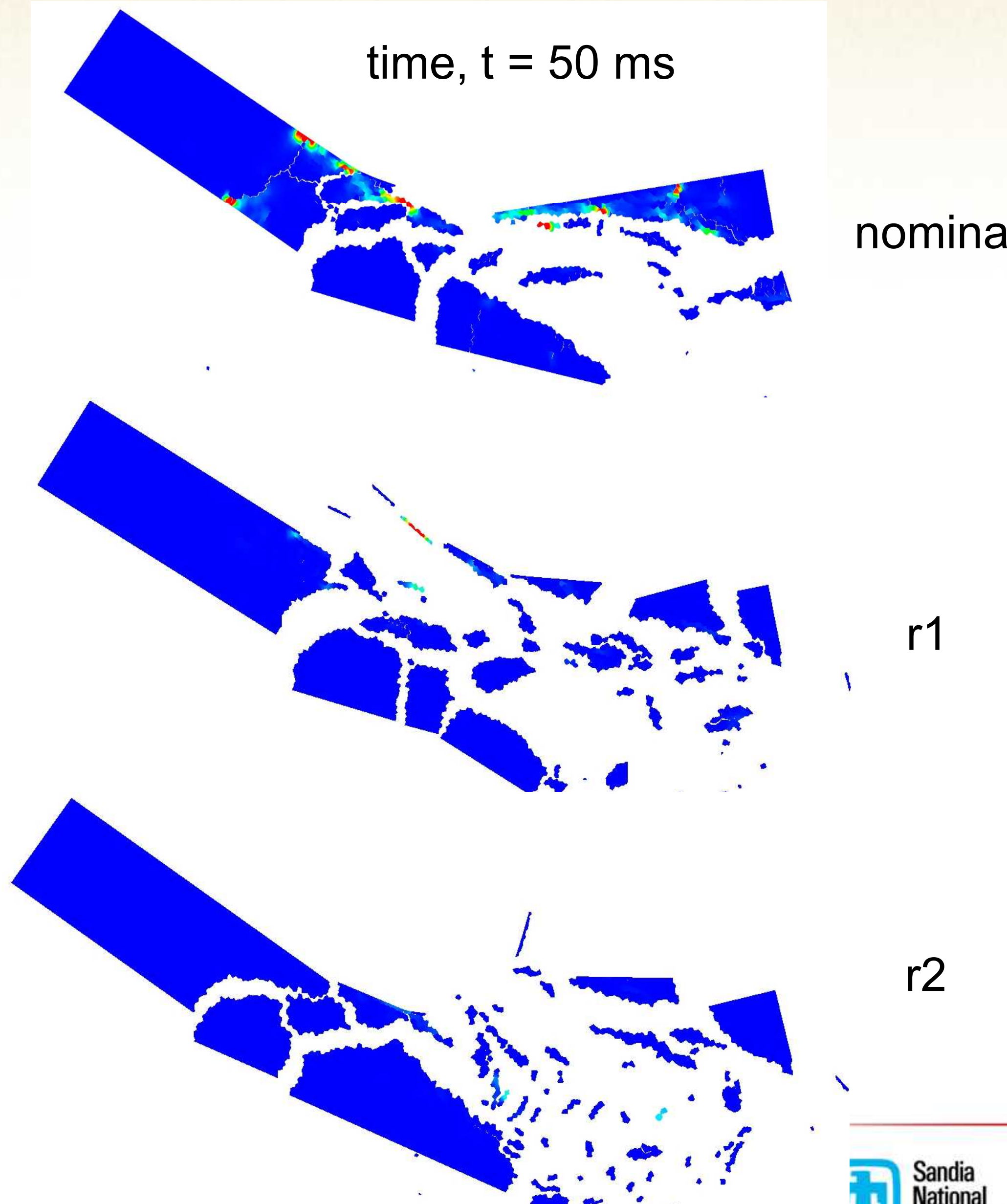


convergence in distribution?
sample size?

Random Material Realizations, H = 0.5

one mesh, R1



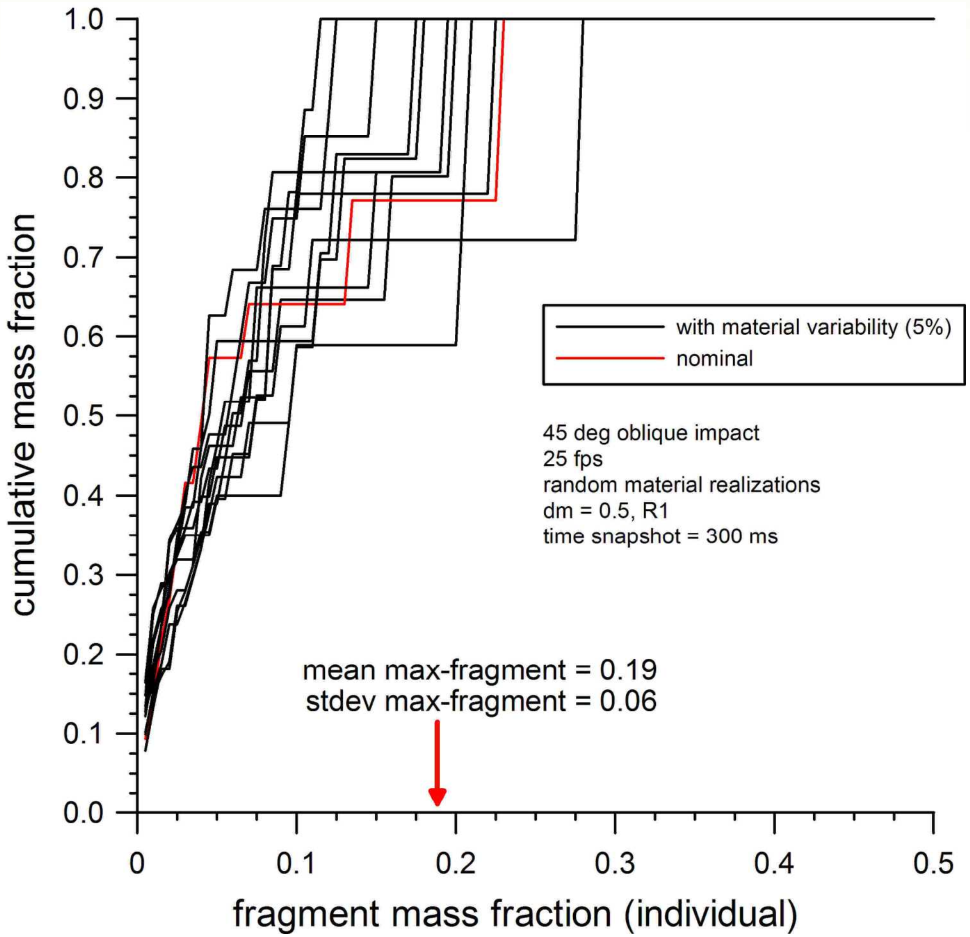


random material realizations

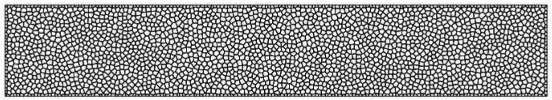
- ±5% variation on E
- ±5% variation on failure surface

Fragmentation Statistics

12 random material realizations



element size ~ 0.5



random material realizations

- ±5% variation on E
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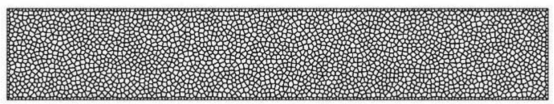
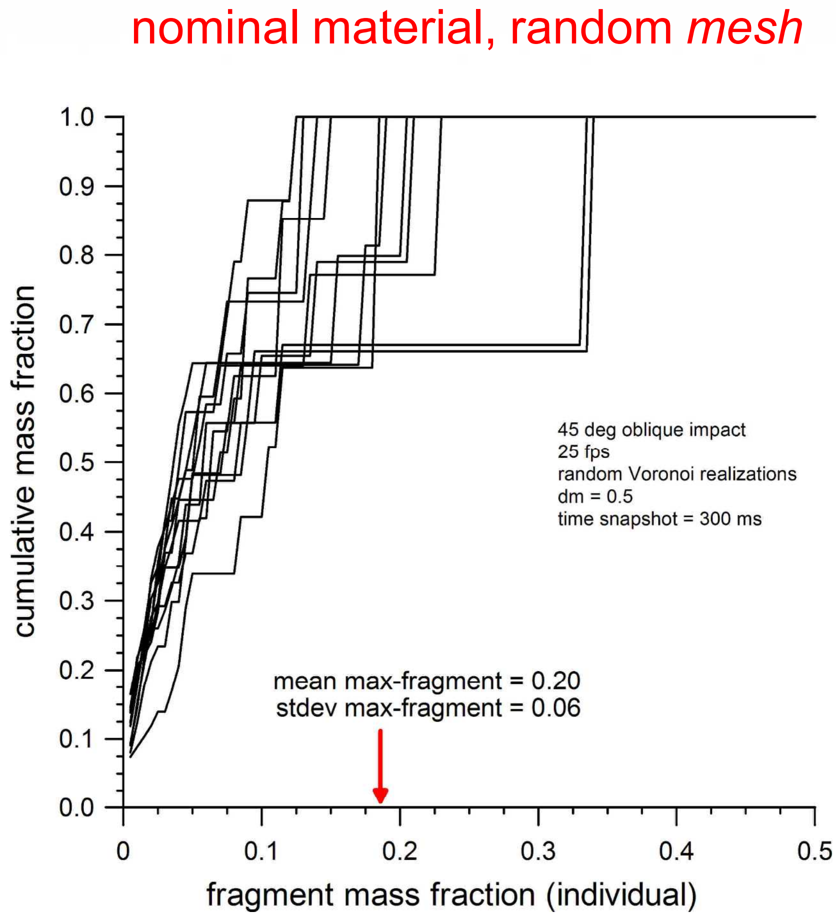
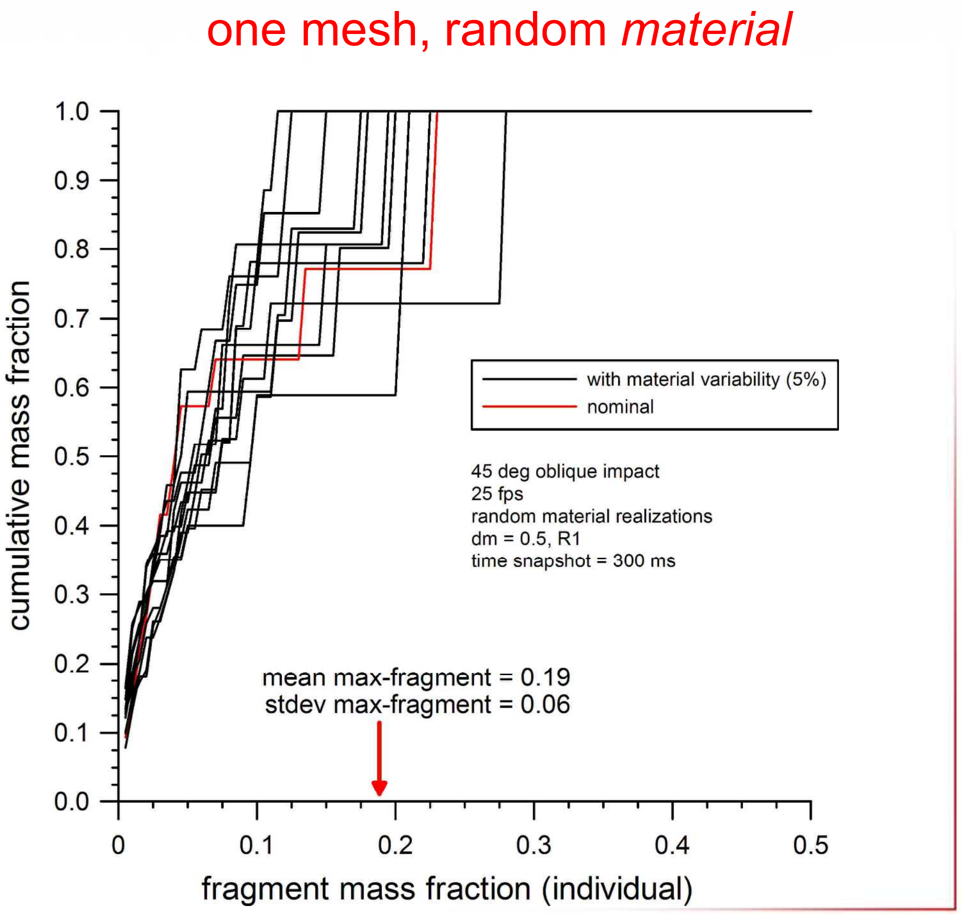
Fragmentation Statistics

element size ~ 0.5







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How to describe mesh convergence?

<u>Given:</u>

- Deterministic governing equations are highly nonlinear and have a multitude of bifurcations.
- Material randomness (Brannon and Strack)
- Random Voronoi mesh realizations

Use the probabilistic concept of *convergence in variation* (convergence in distribution)?

Next Step, Challenges

1. Validation examples (3-point bend, Brazilian, . . .)
2. Nonlocal material (integral form) instead of cohesive approach.
3. Objectivity: How to define convergence?
4. How to maintain consistency between subscale representation of damage (CDM) and explicit representation of damage by interelement cohesive cracks?
5. Application to ductile materials, brittle materials