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# Simulation of Cathode Plasma Expansion in Magnetically Insulated Transmission Lines (MITLs)*

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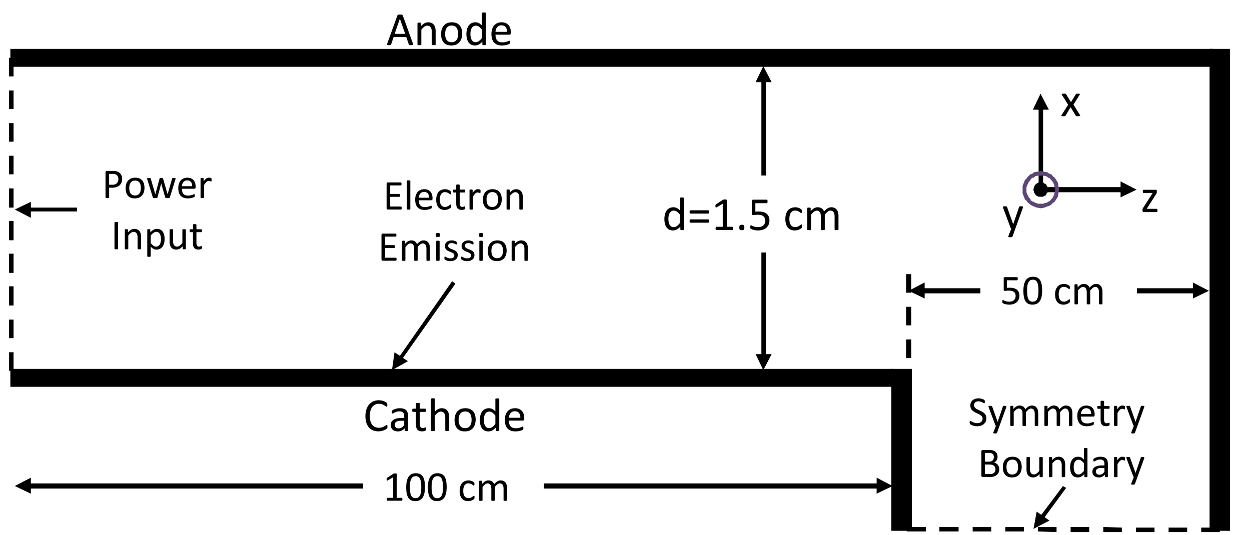
# Outline/Overview

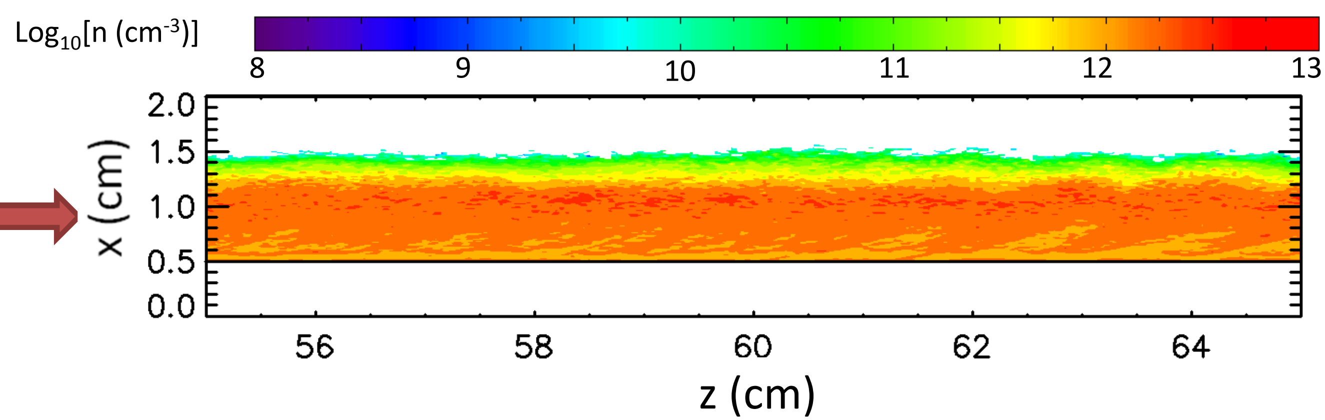
- We have a developed a cathode plasma model for the PIC code Lsp and applied it 1D and 2D simulations magnetically insulated transmission lines (MITLs).
- The numerical implementation of the algorithm is detailed.
- The algorithm is tested by using pre-calculated MITL equilibria into which we introduce a thin cathode plasma which is allowed to evolve in time and study
- The Lsp results are qualitatively similar to a simple model in which a cathode plasma diffusion equation is coupled to laminar flow theory.
- 1D simulations exhibit simple scaling of cathode plasma thickness, $x_S \sim t^{1/2}$.
- In 2D, with cyclotron orbits in the simulation plane, a flute instability can develop from transverse charge separation. In this case the plasma thickness can grow much more rapidly than for a purely diffusive process.

# High-Power 2D Parallel-Plate MITL Simulations*

- Explicit EM, kinetic particles, 1 – 10 MV
- High resolution across the AK gap (150 cells)
- Electron flow initiated by SCL emission from cathode, electron flow develops a quasi-equilibrium sheath profile along the MITL.




Electron density along a 10-cm section of the sheath upstream of the self-limited front at 3.22 MV.

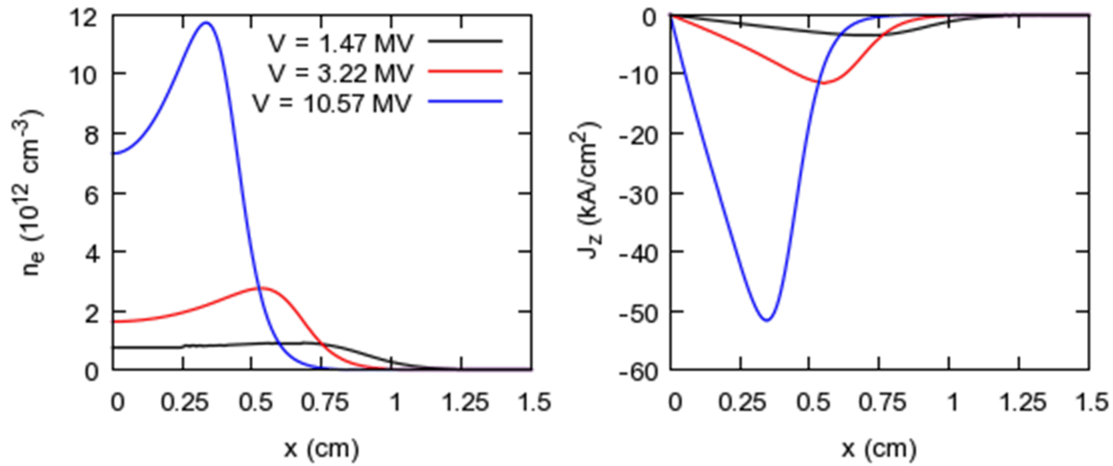



*D.V. Rose, *et al*. "Electron Flow Stability in Magnetically Insulated vacuum transmission lines", Phys. of Plasmas **18**, 033108 (2011).

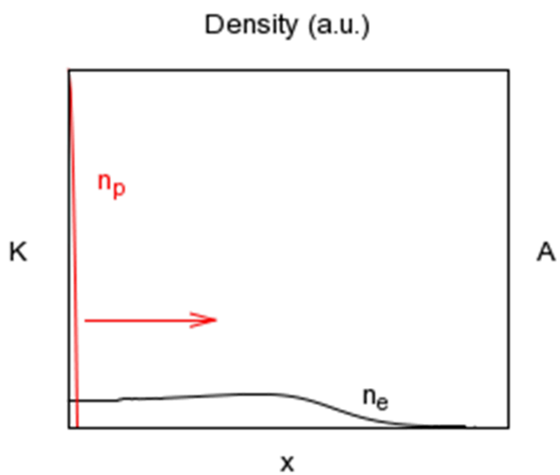
# Simulated MITL Equilibria for Voltages from 1-10 MV

- Equilibrium electron numberand current density profiles for Voltages of 1.47, 3.22, 10.57 MV.

- Averaged over 10-cm section of MITL in 2D simulation



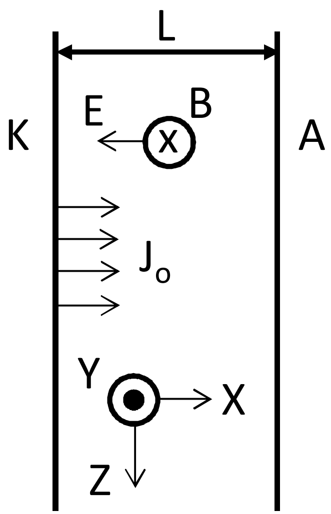

**Schematic illustration of 1D Cartesian Cathode Plasma Simulation**




- Load in an initial MITL equilibrium.

- Introduce a thin cathode plasma and allow it to diffuse into the AK gap.

# Quasi-Laminar MITL Equilibria

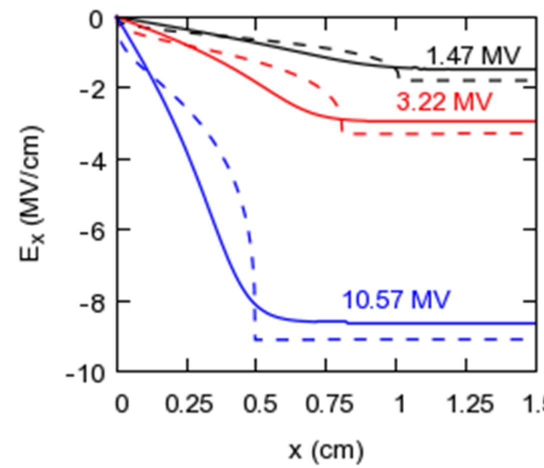



Compare LSP with Quasi-Laminar theory of Ron, et al.*

Theory provides equilibrium solutions for all fields in normalized coordinates, e.g.,

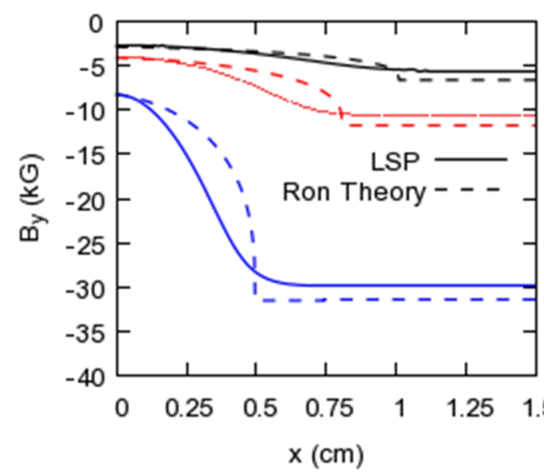
$$\eta = x\sqrt{\frac{2e\mu_o J_o}{m_e c}} \qquad \beta = B\sqrt{\frac{\varepsilon_o}{2m_e c\mu_o J_o}}$$

For a specified voltage and AK gap (L = 1.5 cm), compare Ron equilibria and LSP for same cathode current, $I_K$.




| Voltage (MV) | $\beta_K$ | $\eta_A$ | $I_K$ (kA/cm) | PIC $I_A$ (kA/cm) | RON $I_A$ (kV/cm) |
|---|---|---|---|---|---|
| 1.47 | 2.33 | 1.04 | 2.19 | 4.48 | 4.99 |
| 3.22 | 2.233 | 1.635 | 3.30 | 8.47 | 9.35 |
| 10.57 | 2.1512 | 3.399 | 6.61 | 23.76 | 25.04 |

Anode currents differ by ~10-20 %




Note that Lsp fields lack discontinuities of Ron model at sheath edge

*A. Ron, *et al*. "Equilibria for Magnetic Insulation", IEEE Trans. Plasma Sci **PS-1**, 85 (1973).

# Cathode Plasma Diffusion Equation

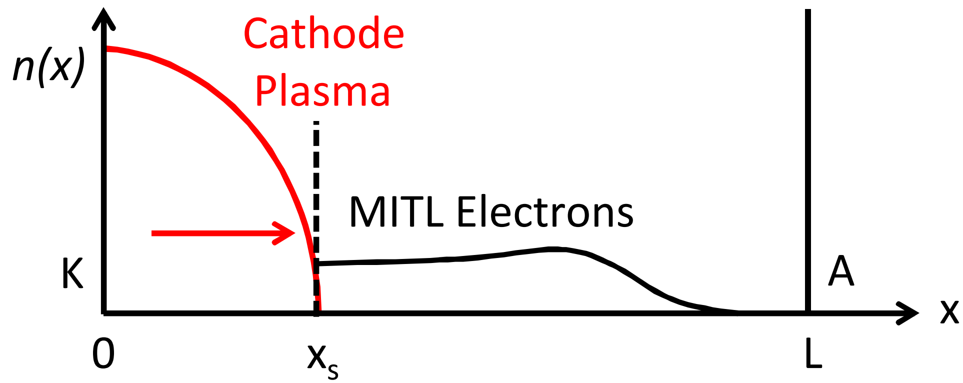
**Cathode plasma: Fully ionized, Isothermal, Quasi-neutral plasma** $n = n_i = n_e/Z$

$$\frac{\partial n}{\partial t} = -\frac{\partial \Gamma}{\partial x}$$

$$\Gamma_e \square \Gamma_i = \Gamma = -\tilde{D}\frac{\partial n}{\partial x}$$

$$\tilde{D} = \frac{\nu_{ei}}{Z m_e \omega_{ce}^{\;2}}(T_i + Z T_e)$$

$$\frac{\partial B_y}{\partial x} = \mu_o Z e \left(\frac{\omega_{ce}}{\nu_{ei}}\right)\tilde{D}\frac{\partial n}{\partial x}$$




**Plasma is coupled to Ron equilibrium for MITL electrons at $x = x_S$.**

**Numerical Implementation**

Initial flat-top density profile, width $x_S$.

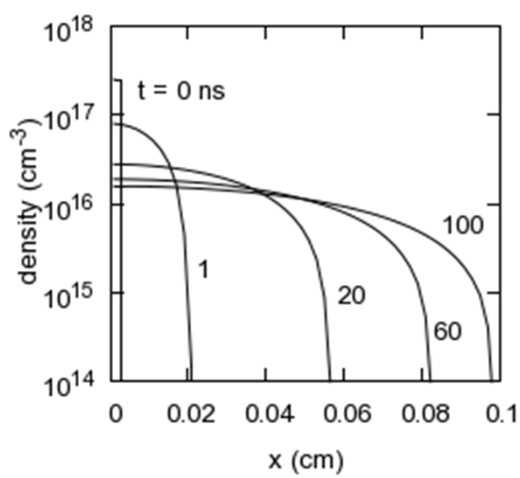
Diffusion equation is advanced.

$$\text{If: } \quad n(x_S)\sqrt{T_e/m_e} \geq \left(\frac{m_e c}{2 e \mu_o}\right)\left(\frac{\eta_A}{L - x_S}\right)^2$$

then $x_S$ is incremented by $\Delta x$.

Integrate from $x_S$ to get magnetic field in plasma, where

$$B_y(x_S) = -\left(\frac{m_e c}{e}\right)\frac{\beta_K \eta_A}{L - x_S}$$




L = 1.5 cm

$m_i$ = 1

Z = 1

$T_e = T_i$ = 5 eV

Initial plasma width: 40 μm

$\Delta x$ = 20 μm, $\Delta t$=0.001 ns

V = 10.57 MV ($\beta_K$ =2.15, $\eta_A$=3.399)

$x_S(t) \sim 0.01$ cm $(t/\text{ns})^{1/3}$

Gap closure time ($x_S \sim L$) ~ 0.1 ms

Roughly independent of voltage.

# Lsp Cathode Plasma Model

- Lay in initial MITL electron kinetic particles from fitted equilibria.

- Initial static and magneto-static solve to get $E_x$ and $B_y$.

- Introduce thin cathode plasma ( ~ 1 ML over 1-2 cells $\Delta x \sim 10\ \mu m$).

- The plasma is modeled by fluid electron and ion species, which are periodically remapped onto the Eulerian grid to control the particle number and avoid noise.[1]

- Any ion flux leaving exiting the cathode is replaced by "repopulation" particles. Reflect normal component of velocity, lose transverse. Repop particles get local fluid temp.[1]

- Energetic fluid electron particles are converted to kinetic electrons to resupply the electron flux at at the plasma edge (the "effective" cathode).[2]

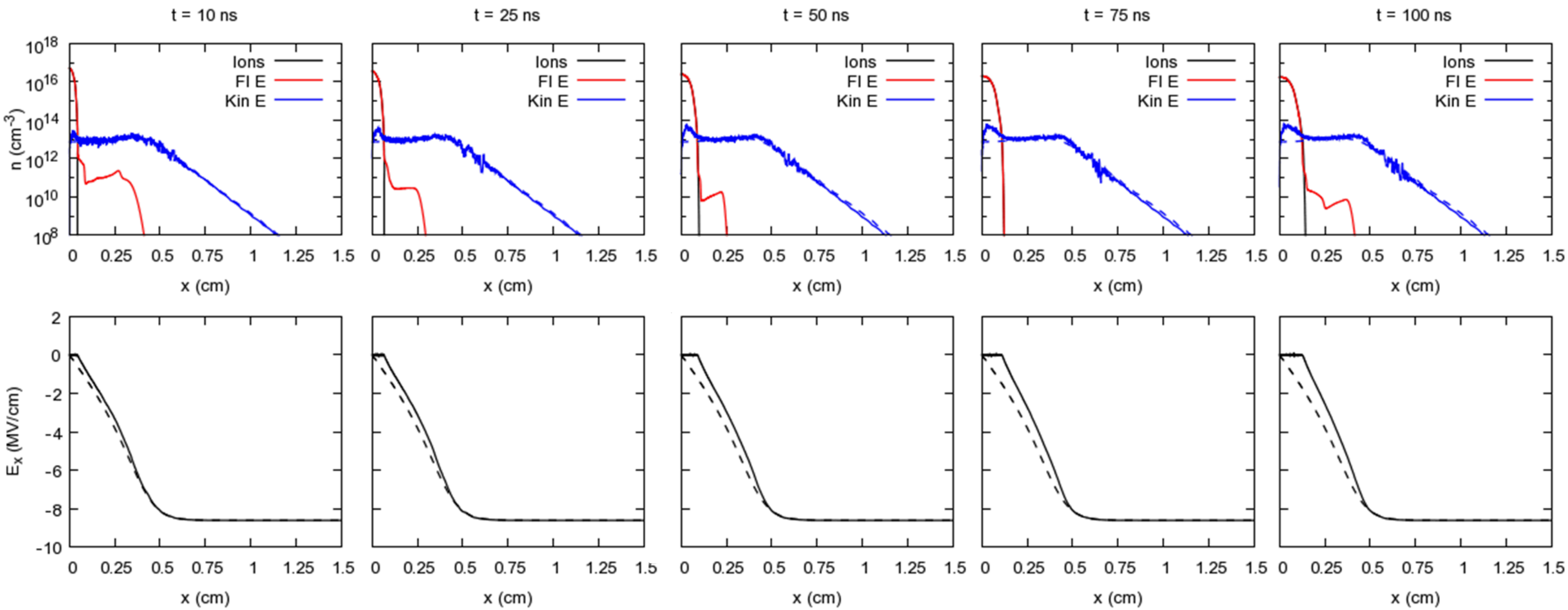
- The simulations shown in the following were performed using direct implicit field solution, due to the high initial cathode plasma density ($\omega_{pe}\ \Delta t \sim 1$).[3]

[1]C. Thoma, *et al*. Phys. Plasmas **18**, 103507 (2011) (1973).
[2] D. R. Welch, et al. Phys. Plasmas **13**, 063105 (2006).
[3] D. R. Welch

# 1D MITL/Cathode Plasma Simulation (10.57 MV)

**V = 10.57 MV**
**L = 1.5 cm**
**$m_i$ =1**
**Z = 1**
**$T_e = T_i$ = 5 eV**
**Initial plasma width: 10 μm (1 ML)**
**Δx = 5 μm, c Δt= 0.25 Δx**

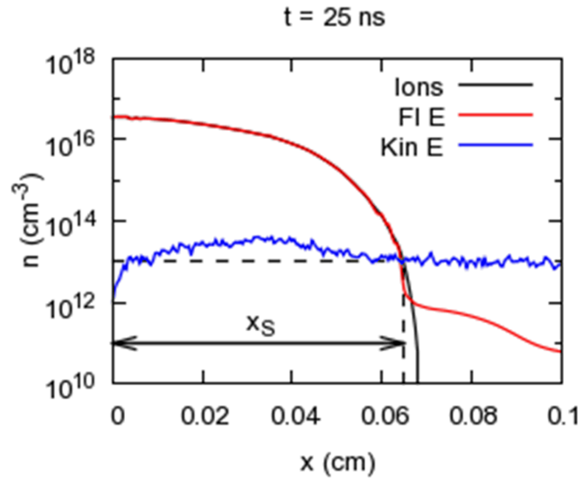
- Cathode plasma diffuses into AK gap, qualitatively similar to diffusion model.
- Electric field is screened out in plasma. Plasma edge becomes "effective" cathode.
- Electron flow in MITL sheath is only slightly perturbed for $x_S/L$ <<1.




Fluid electrons with total KE > 300 eV are converted to kinetic electrons.

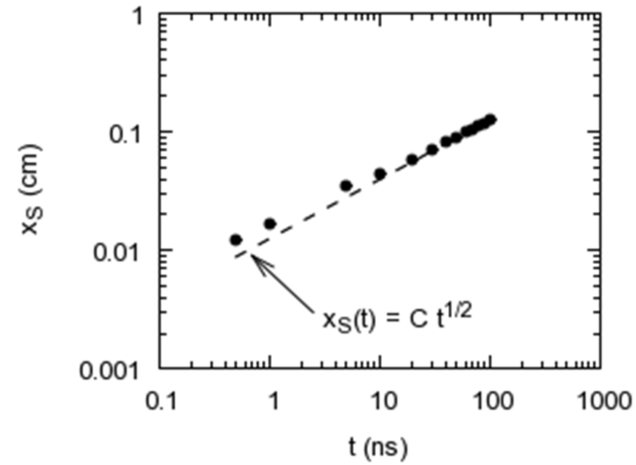
# Sheath edge calculation and Electrical Properties

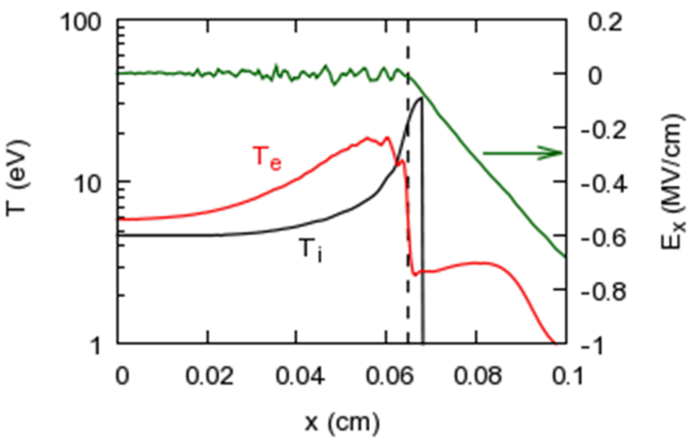
**Procedure used to calculate $x_S(t)$.**



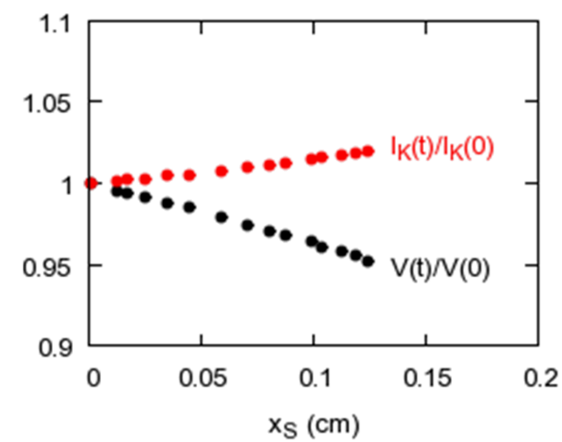

•After a few ns $x_S(t) \sim t^{1/2}$, as in a classical diffusion process.

•Approximate gap closure time ($x_S$ = 1.5 cm) ~ 10 μs



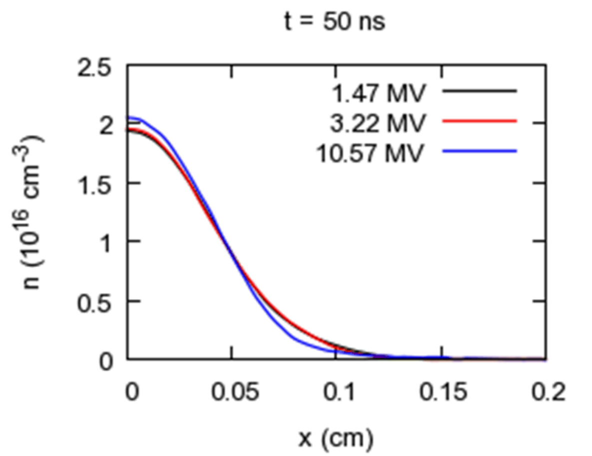

• Both V and $I_K$ vary linearly with $x_S$ for $x_S$ <<L.



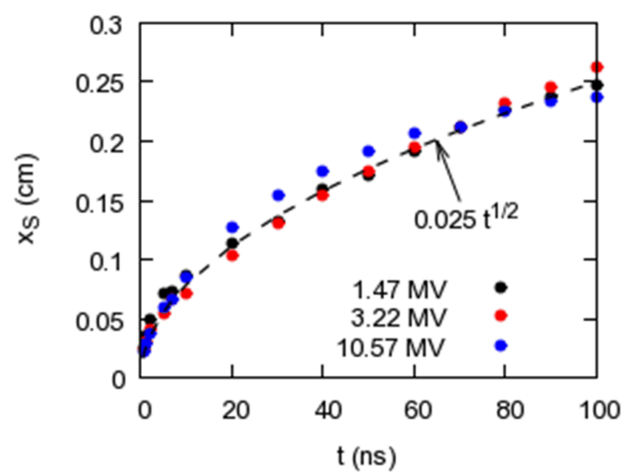

Recall that diffusion model used isothermal approximation.

# 1D MITL/Cathode Plasma Simulations

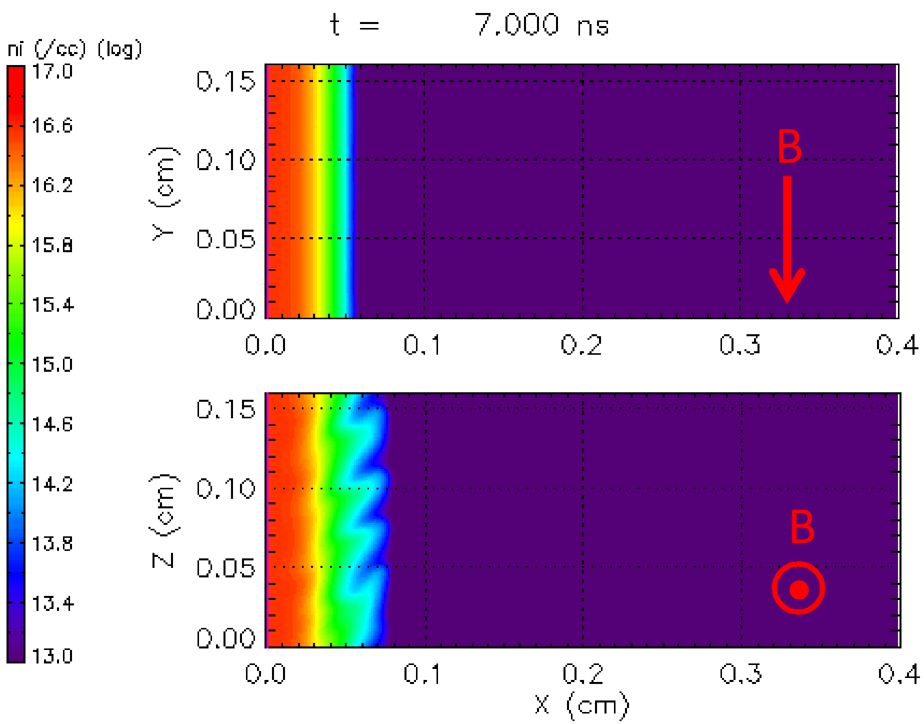
**V = 1.47, 3.22, and 10.57 MV.**
**L = 1.5 cm**
**$m_i$ =1**
**Z = 1**
**$T_e$ = $T_i$ = 8 eV**
**Initial plasma width: 40 μm (1 ML)**
**Δx = 20 μm, c Δt= 0.25 Δx**




- Very little variation in plasma density profile with voltage (in the bulk of the cathode plasma and at edge, E = 0).
- For all three voltages $x_S(t)$~ C $t^{1/2}$.
- Constant C can vary slightly with migration parameters, and begins to increase with cell size for x > 10-20 μm. But $t^{1/2}$ scaling is very robust.



# 2D Cartesian Simulations: XY and XZ



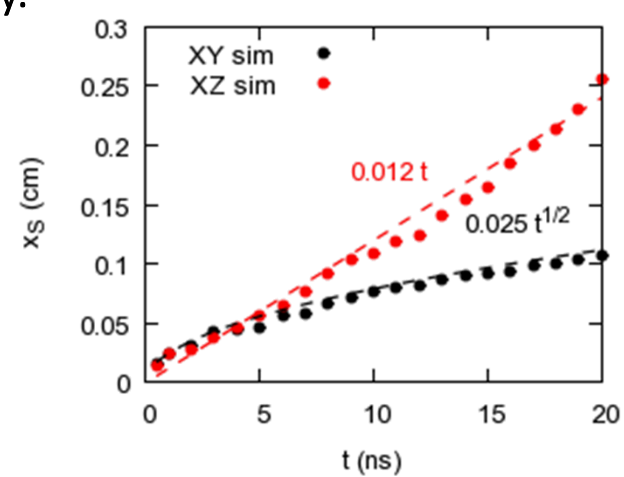

- Repeat 10.57 simulation in 2D Cartesian coordinates: Both XY and XZ.

- Periodic boundaries in Y(Z).

- YZ coordinates look essentially like 1D analog.

- In XZ coordinates, charge separation due to opposite cyclotron motions for electrons and ions is captured in the simulation plane and can seed a flute instability.

- This instability in multiple dimensions allows much faster expansion of the cathode plasma than the 1D diffusive process predicts.



# Conclusions

- We have described the  developed a cathode plasma model for the PIC code Lsp and applied it 1D and 2D simulations magnetically insulated transmission lines (MITLs).
- The detailed PIC algorithm is in qualitative agreement with a simplified diffusion model, and predicts plasma expansion of $t^{1/2}$.
- The algorithm is tested by using pre-calculated MITL equilibria into which we  introduce a thin cathode plasma which is allowed to evolve in time and study
- The Lsp results are qualitatively similar to a simple model in which a cathode plasma diffusion equation is coupled to laminar flow theory.
- 1D simulations exhibit simple scaling of cathode plasma thickness, $x_S \sim t^{1/2}$.
- In 2D simulations, with cyclotron orbits in the simulation plane, a flute instability can develop from transverse charge separation. In this case the plasma thickness can grow much more rapidly than for a purely diffusive process.
- In future we hope to consider larger-scale 2D and 3D simulations with both anode and cathode plasmas modeled self-consistently.