SAND2007-5066C

Push-pull PDV analysis

Sub-fringe data reduction

PDV workshop

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,
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The uncertainty principle

- To date, most PDV applications use time-frequency analysis
  - Sliding FFT, etc.
  - Velocity-time resolution limited by uncertainty principle
- Fractional uncertainty related to the number of fringes within the sliding window (τ)
  - At least eight fringes needed for 1% velocity precision
  - 1 km/s: T=0.775 ns, >6.2 ns window
  - 1 m/s: T=755 ns, >6200 ns window?!
- Sub-fringe analysis is needed for low velocity transients
  - Radiation effects
  - Elastic precursor/phase transitions

Gaussian window, no noise, constant velocity

$$(\delta v)\underbrace{(\delta t)}_{\sim\tau} > \frac{\lambda_0}{8\pi}$$

$$\frac{\delta v}{v} > \frac{1}{4\pi}\frac{T}{\tau} \qquad \left(T = \frac{\lambda_0}{2v}\right)$$

Jensen et al., J. Appl. Phys. **101**, 13523 (2007).

Solution: calculate fringe shift directly

- **Velocity can be calculated directly from the fringe shift**
  - **Fringe shift is proportional to displacement**
  - **Numerical differentiation required...**
  - **Only a single source can be tracked without contrast loss**

$$F(t) = 2\frac{x(t) - x(t_i)}{\lambda_0}$$

- **Method needs to handle:**
  - **Intensity variations**
  - **Incoherent light**
  - **Imperfect contrast**

- **Single channel PDV only works in ideal situations**
  - **Phase ambiguity is still a problem**

- **Like the transition from WAMI to VISAR, multiple signals are required**

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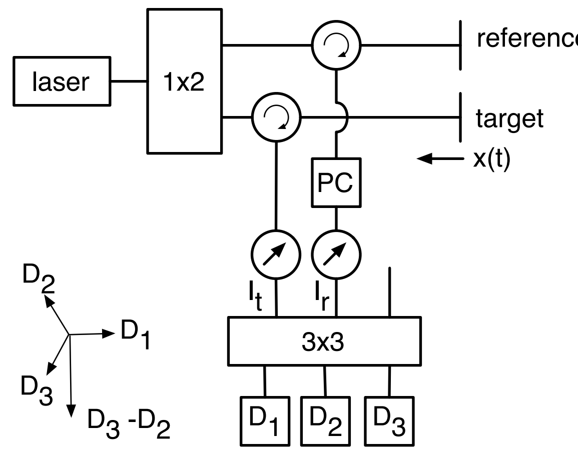
Three-phase measurements

- **3 x 3 fiber coupler provides phase shifted output**
  - **Bruce Marshall discussed this last year**
  - **Signal pairs can be used obtain quadrature**
- **Reference intensity assumed to be completely coherent and constant**
- **Target intensity can be time dependent, and may contain an incoherent contribution**
- **No beam intensity is used--it wouldn't be useful anyway!**
  - **Unlike VISAR, target and reference light do NOT share time dependence.**



Dolan and Jones, Rev. Sci. Instrum. **78**, 76102 (2007).

$$D_i(t) = a_i I_r + b_i I_t(t) + 2\sqrt{a_i b_i I_r I_c(t)} \cos\left(\Phi(t) - \beta_i\right) \qquad i = 1, 2, 3$$

Parameters *a* and *b* include 3 x 3 coupler and detector sensitivity

Push-pull approach

$$D_i(t) = a_i I_r + b_i I_t(t) + 2\sqrt{a_i b_i I_r I_c(t)}\cos\left(\Phi(t) - \beta_i\right) \qquad i = 1, 2, 3$$

- **Goal: Remove offset and amplitude variation**
  - **Step 1: subtract off reference offset**
  - **Step 2: construct signal pairs**
  - **Step 3: take pair ratios to eliminate intensity from the problem**

$$\begin{aligned}\tilde{D}_i(t) &\equiv D_i(t) - D_i^{(t)} \\ &= b_i I_t(t) + 2\sqrt{a_i b_i I_r I_c(t)}\cos\left(\Phi(t) - \beta_i\right)\end{aligned}$$

$$\begin{aligned}\tilde{D}_{ij} \equiv \tilde{D}_i - \frac{b_i}{b_j}\tilde{D}_j &= 2\sqrt{a_i b_i I_r I_c(t)} \\ &\times \left[\cos\left(\Phi(t) - \beta_i\right) - \sqrt{\frac{a_j}{a_i}\frac{b_i}{b_j}}\cos\left(\Phi(t) - \beta_j\right)\right]\end{aligned}$$

- **Conventions:**
  - **Signal i=1 is reference phase**
  - **Signal j=2 leads signal 1**
  - **Signal k=3 lags signal 1**

$$\frac{\tilde{D}_{ij}}{\tilde{D}_{ik}} = \frac{\cos\left(\Phi(t) - \beta_i\right) - \sqrt{\frac{a_j}{a_i}\frac{b_i}{b_j}}\cos\left(\Phi(t) - \beta_j\right)}{\cos\left(\Phi(t) - \beta_i\right) - \sqrt{\frac{a_k}{a_i}\frac{b_i}{b_k}}\cos\left(\Phi(t) - \beta_k\right)}$$

An intimidating result...

$$\tan\Phi(t) = \frac{D_y(t)}{D_x(t)} = \frac{\left(\sqrt{\frac{\hat{a}_2}{\hat{b}_2}}\cos\beta_+ - \sqrt{\frac{\hat{a}_3}{\hat{b}_3}}\cos\beta_-\right)\tilde{D}_1 - \left(\frac{1-\sqrt{\frac{\hat{a}_3}{\hat{b}_3}}\cos\beta_-}{\hat{b}_2}\right)\tilde{D}_2 + \left(\frac{1-\sqrt{\frac{\hat{a}_2}{\hat{b}_2}}\cos\beta_+}{\hat{b}_3}\right)\tilde{D}_3}{\left(\sqrt{\frac{\hat{a}_2}{\hat{b}_2}}\sin\beta_+ + \sqrt{\frac{\hat{a}_3}{\hat{b}_3}}\sin\beta_-\right)\tilde{D}_1 - \left(\sqrt{\frac{\hat{a}_3}{\hat{b}_3}}\frac{\sin\beta_-}{\hat{b}_2}\right)\tilde{D}_2 - \left(\sqrt{\frac{\hat{a}_2}{\hat{b}_2}}\frac{\sin\beta_+}{\hat{b}_3}\right)\tilde{D}_3}$$

Quadrature signals D_x and Dy are weighted sums of the recorded signals (ref. offsets removed)

- **Seven parameters needed**
  - **Phase shifts and some combination of coupling ratios, beam block measurements, and ellipse parameters**
- **Reduces to a simple result in ideal conditions**
  - **Loss-less, symmetric coupler**
  - **Identical detectors**

$$\tan\Phi(t) = \sqrt{3}\,\frac{D_3(t) - D_2(t)}{2D_1(t) - D_2(t) - D_3(t)}$$

- **Why bother with the complicated solution?**

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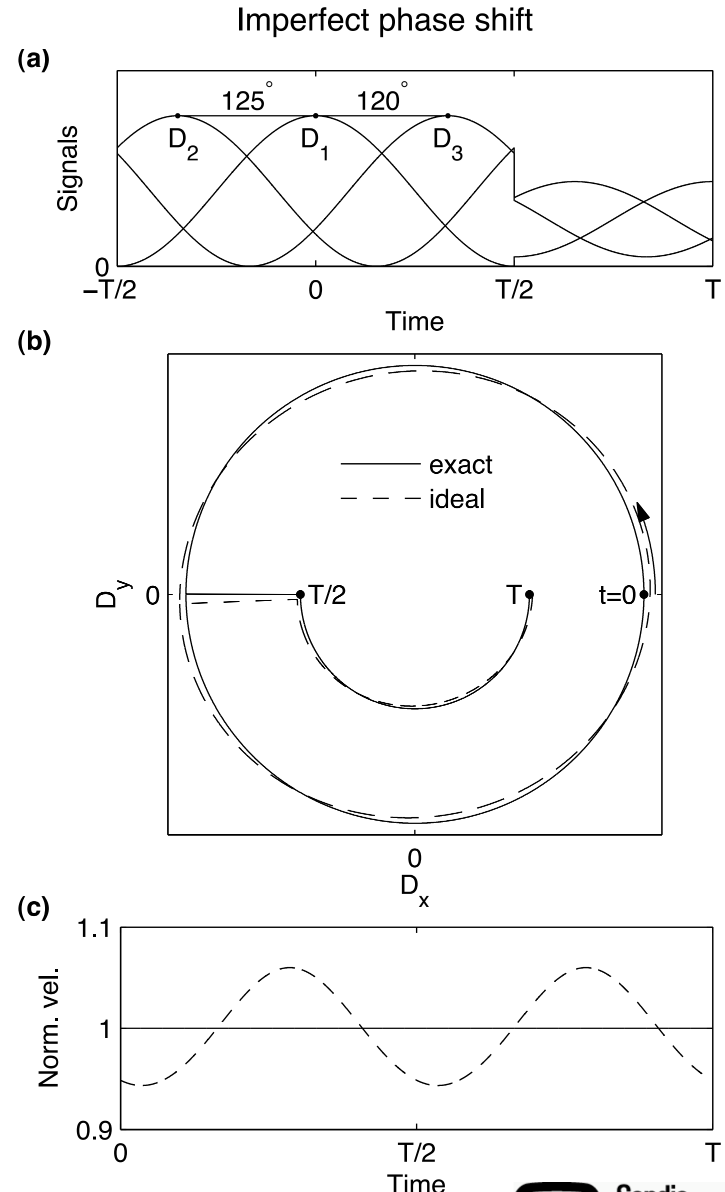
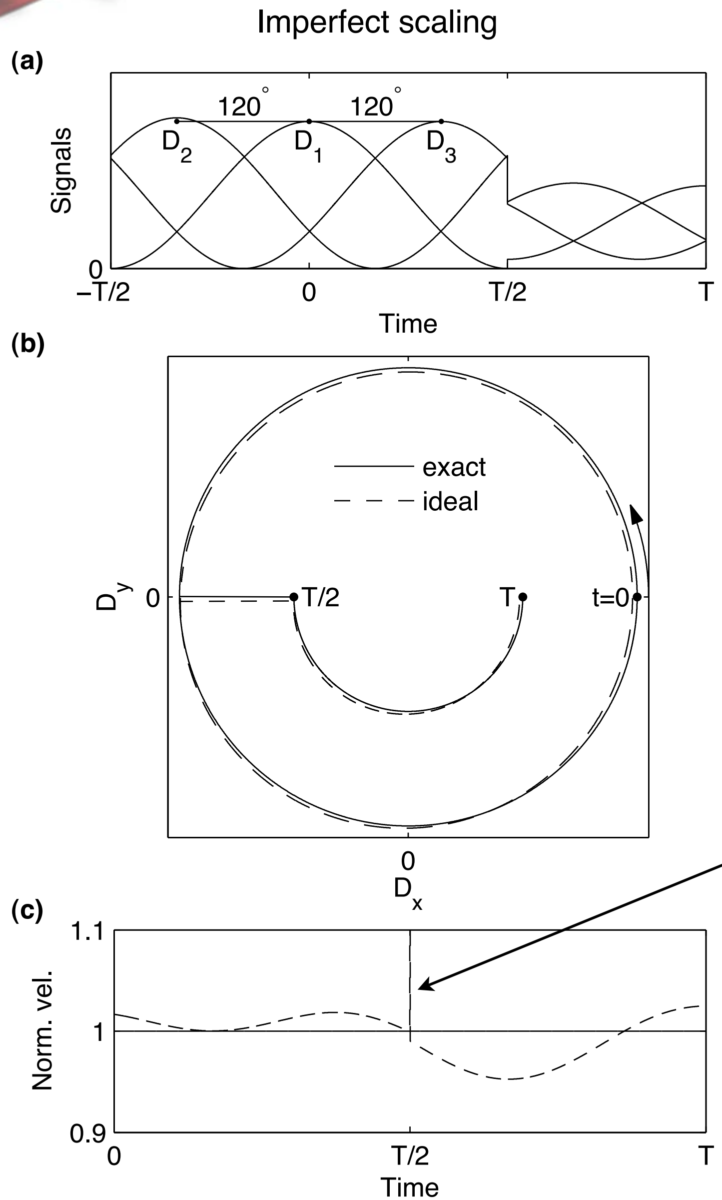
Simple example

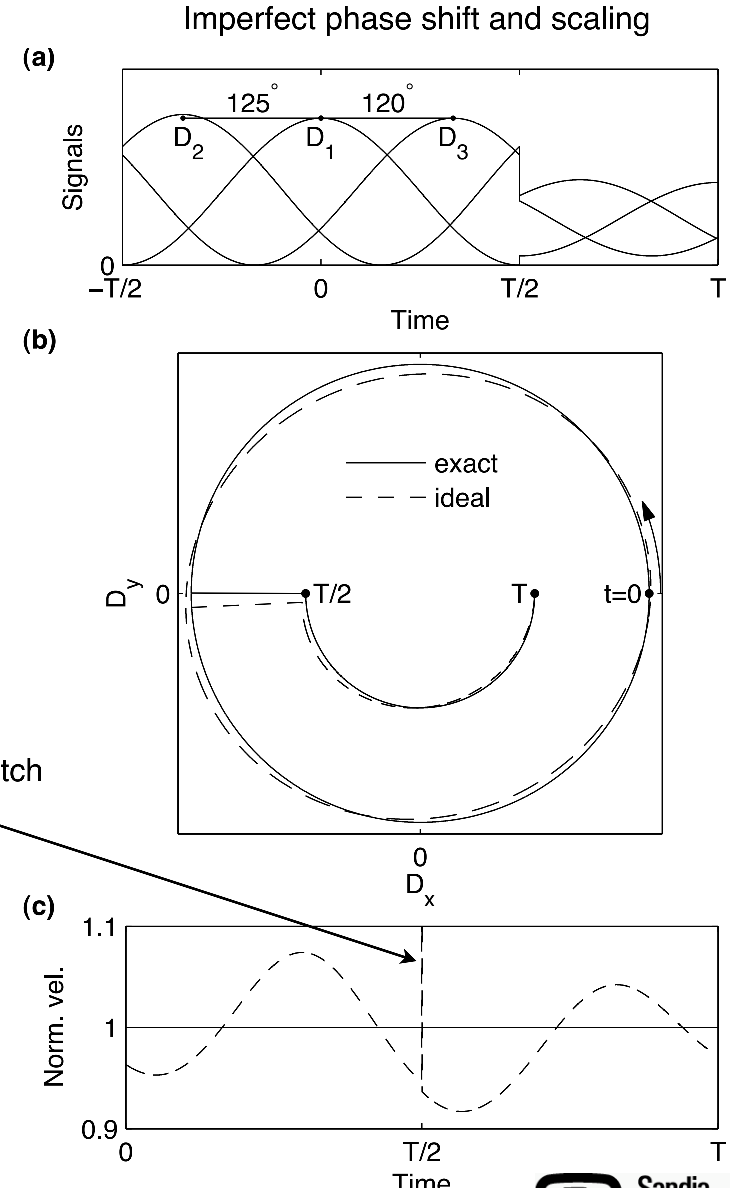
- **Constant velocity**
  - **Fringe period T ($v=\lambda_0/2T$)**
  - **Purely coherent input**
  - **Reference/target intensities match until t=T/2**
  - **Target light reduced to 25% of its initial value after T/2**

- **Consider imperfect phase shift**
  - **Ideal analysis yields a non-circular ellipse (sqrt(3)/2 scaling)**
  - **Calculated velocity oscillates about the true value**
    - **Equal area constructions (e.g., Kepler's second law)**



Unequal coupling effects (5% variation)





Derivative glitch

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What about that numerical derivative?

- **High frequency noise amplification is intrinsic to numerical derivatives**
  - **Data smoothing typically required**
  - **Time resolution sacrifice!**

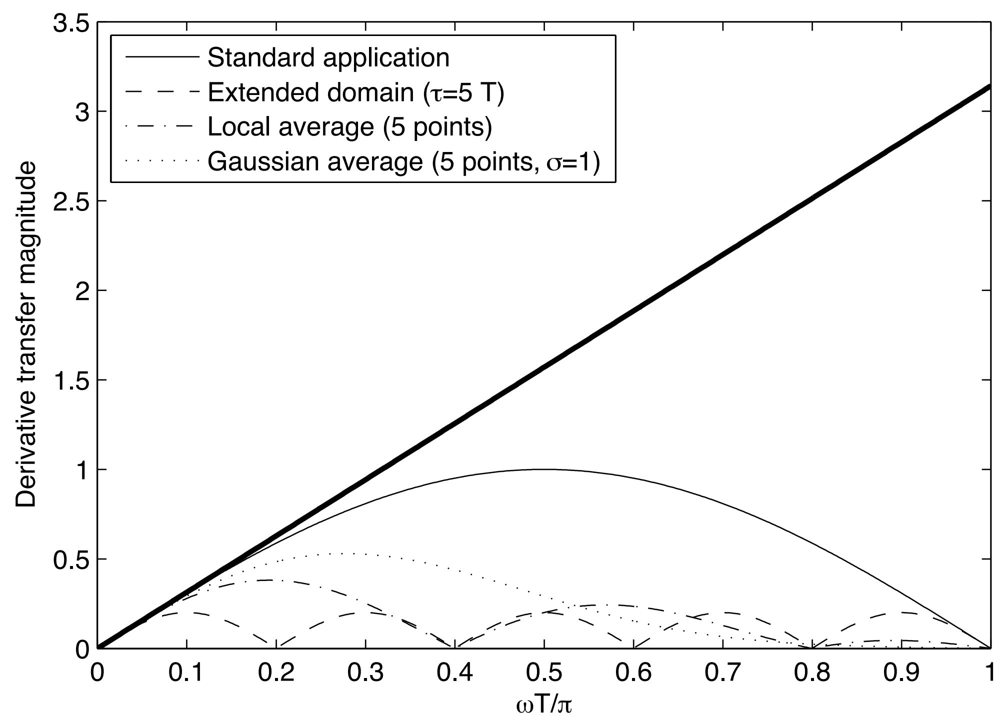
- **Considerations**
  - **Oversampling: how much faster is limiting velocity than the velocity of interest?**
  - **Signal-noise ratio**
  - **Dynamic range (8 bit limitation)**
    - **Similar issues in VISAR displacement mode**

See Hemsing, SPIE **1346**, p. 141 (1990).

Frequency transfer function

$$F'(\omega) = [-i\omega]F(\omega)$$

Centered finite difference derivative



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A question of time scales

- There is no information in a single point of a PDV measurement
  - Velocity calculation requires several data points
  - A time scale must be introduced into the problem
    - VISAR does this in hardware, we must do it in software
  - Uniqueness will always be an issue

- Sampling interval is never the limiting time resolution
  - Detection threshold: how long before motion can be distinguished from noise?
    - ~1 ps at 1 km/s (1/128 noise threshold)

$$\Delta t_{min} > \frac{\lambda_0}{4\pi\bar{v}}\frac{\delta D}{A}$$

  - Fringe threshold: how long to detect a complete fringe?
    - ~775 ps at 1 km/s

$$\Delta t_F = \frac{\lambda_0}{2\bar{v}}$$

  - For good SNR, push-pull analysis can be useful
    - Smoothing reduces time resolution to several sampling intervals

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Summary

- Push-pull analysis of multiple phase PDV measurements works on shorter time scales than time-frequency analysis
  - Only one source can be tracked
  - Intensity variations do not matter
- A lot more system characterization is needed
  - Beam-block measurements
  - Lissajous patterns/ellipse fitting
  - Improper characterization yields velocity oscillations
- Numerical differentiation needed to determine velocity
  - Signal noise is an issue
- PDV analysis introduces an arbitrary time scale to the problem
  - Limiting time resolution is not the sampling interval