Bounds for Kinetic Energy and Resistance for W wire array implosions on Z

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OUTLINE

- Schematics and equivalent circuit for Z (2006), measured and derived quantities
- Load current distribution implications
- Lower bound for pinch kinetic energy few ns before the main x-ray pulse and for pinch resistance at stagnation
- Application of the Energy Balance Procedure (EBP) to 12 tungsten bare axis 20x10mm single wire array Z shots
- Conclusions

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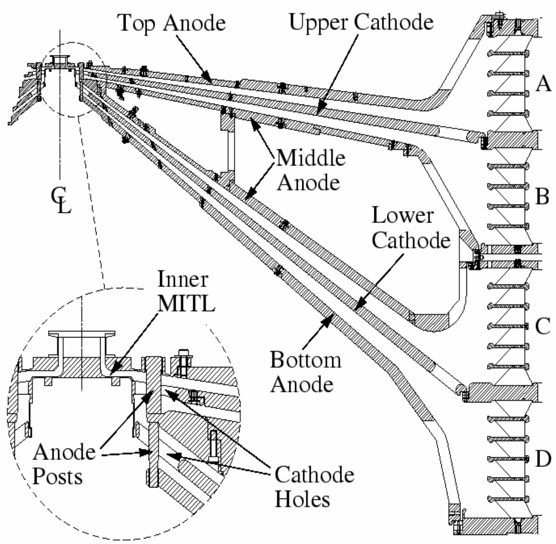
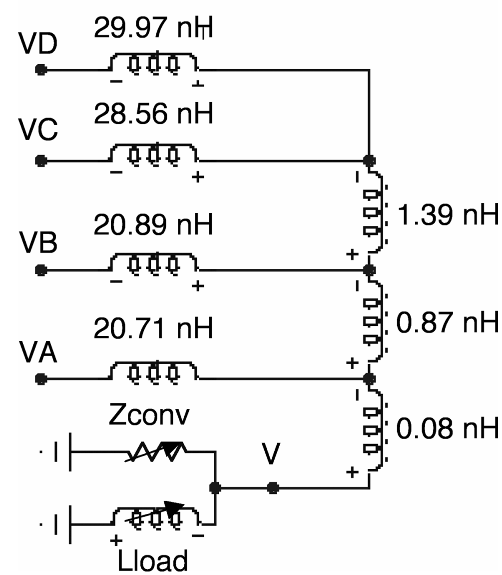
Z(2006) Vacuum Section Schematics and Equivalent Circuit





Experimentally known quantities

- **On the Z machine the quantities measured directly are:**

V_{stack}	Stack voltages	4 levels
I_g	MITL current	Sum of 4 levels
I	Load current	6 cm from axis
P_X	Radiated power	

- **From those quantities measured directly we can construct (and considered them known experimentally)**

V	Convolute voltage	Inductive translation (Ref. 1)
E_X	Radiated energy $\int_0^t P_x dt$	
P_P	Poynting power VI	
E_P	Poynting energy $\int_0^t VIdt$	

- **Nominal inductances are used (cold metal, no mechanical motion; from specifications) for MITL and load currents**

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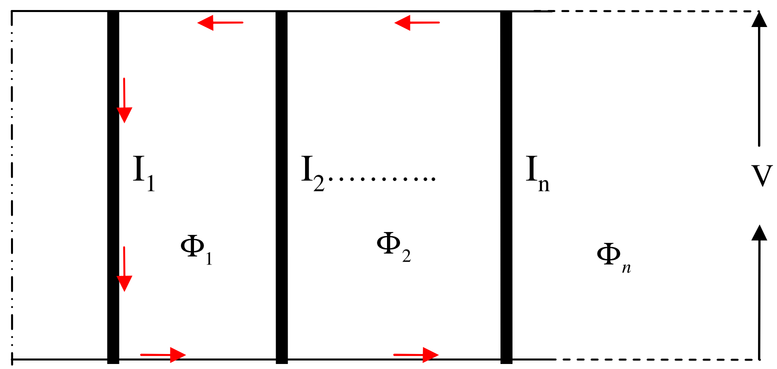
Convolute Voltage and Load Current

- **For any load current distribution the circuit equation can be written as:**

$$V = d(\tilde{L} I)/dt + R_{Wl} I + R_1 I_1 \qquad (1)$$



where R_{Wl} is the electrode resistance (cathode and anode) and,

$$\tilde{L} I \equiv \int drdz B_\theta / c \qquad (2)$$

Inductance Reconstruction Procedure (IRP)

- **Assume that for sufficiently high load current and up to the rapid rise of the x-ray power the pinch resistance is negligible, then:**

$$V_L \equiv V - R_{Wl} I = d(LI)/dt,\ t \geq t_0$$

$$L(t) \equiv \left(\int_{t_0}^{t} dt V_L + L_0 I(t_0)\right)/I, \quad I(t_0) = 2MA \qquad (3)$$

- **To determine end of ablation phase** t_a **look for minimum of**

$$R_{eff} \equiv (V - L_0 \dot{I})/I$$

- **Then;** $\tilde{L} \simeq L$ for $t_1 \geq t \geq t_0,\ t_1; P_x$ rapid rise start

and $L \approx \tilde{L}$ should be almost constant for $t_a \geq t \geq t_0$

- **Whether the pinch resistance is negligible or not from Eq(2):**

$$L \geq \tilde{L}$$

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Magnetic Energy and Flux Inductance

- **For one-sign current distributions (a sufficient condition is that** $\partial r B_\theta / \partial r$ be monotonic in r **) one has:**

$$W = \int drdzrB_\theta^2(r,z)/4 \le \tilde{L} I^2/2, \text{ where } \tilde{L} \text{ is the "true flux inductance"}$$

defined in Equation (2)

- **The equality** $W = \tilde{L} I^2/2$ **applies only for the thin single current path case.**

Energy Conservation related to experimental data on Z

- **Energy Conservation (from convolute to axis):**

$$Q(t) \equiv E_P - E_X = K + W + E + E_{Wl}$$

- **Where** K, W, E, E_{Wl} **are the kinetic, magnetic, plasma internal energies, and electrode Joule dissipation, respectively**

- **In the expression for energy conservation the left hand side is experimentally known**

- **Can we obtain estimates for** K, W **at relevant times of the implosion?**

Lower bound for pinch kinetic energy few ns before stagnation

- **Using the inequality (4) and energy conservation we obtain**

$$K = Q(t) - W - E \geq K_b \equiv Q(t) - LI^2/2 \ \text{ for } t \leq t_1$$

- K_b **is a useful lower bound for** K **at** $t \lesssim t_1$ **when** $K \gg E$
- **Example:** $E \approx 20kJ$ for tungsten at $100eV, \rho = 0.04g/cm^3, m = 6mg$
- **The internal energy is at about one order of magnitude smaller than** $K(t_1)$

Lower bound for pinch resistance at stagnation for thin single load current path

- At time t_2, when the power is half down of its peak value we have;

$$K(t_2), E(t_2) \ll W(t_2),$$

and thus; $W \lesssim Q$

- Assume the entire load current flows in a thin single path;

$$\tilde{L}(t)I(t) = 2W(t)/I(t) \lesssim 2Q(t)/I(t),\ t \gtrsim t_2$$

- From the circuit Equation (1);

$$\Delta\Phi \equiv \int_{t_1}^{t} R_1 I_1 dt = \int_{t_1}^{t_2} V_L dt - \tilde{L}(t)I(t) + L(t_1)I(t_1)$$

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Lower bound for pinch resistance at stagnation (cont.)

- **Thus, recalling the inequalities for** $Q \geq W = \tilde{L} I^2 / 2$ **,**
  the resistively dissipated flux satisfies:

$$\Delta\Phi = \int_{t_1}^{t} R_1 I_1 dt \leq F \equiv \int_{t_1}^{t} dt V_L - 2Q(t) / I(t) + L(t_1) I(t_1)$$

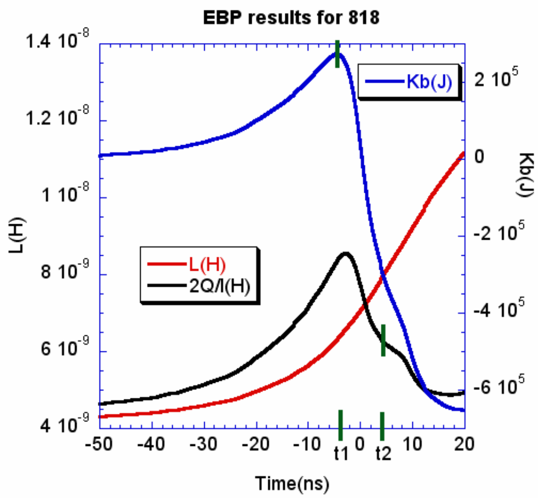
- **This bound for the dissipated flux depends on having assumed a single thin path for the current , and it is a good lower bound when**

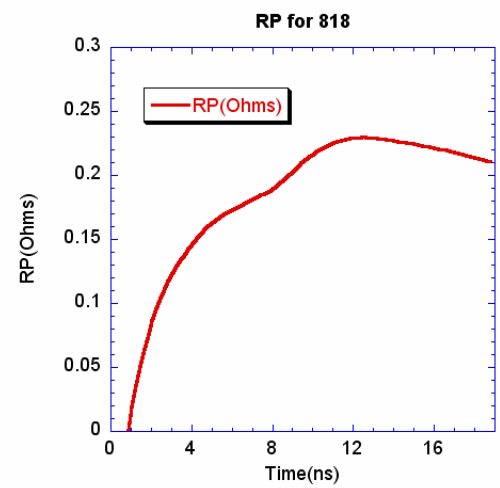
$$Q \geq\simeq W \gg E + K$$

- **We define a measure for pinch resistance from the dissipated flux lower bound;**

$$\overline{R}_P \equiv F / \int_{t_1}^{t} I dt$$

EPB results for 818: lower bounds for K and $\overline{R_P}$





- **Left frame: red curve is L from Eqn. (3), green curve is $2Q/I \geq\simeq \tilde{L}$ for $t \geq t_2$, and blue curve is K_b.Right frame shows $\overline{R_P}$.**
  The green vertical markers in left frame are t_1, t_2 as defined in a previous slide. Zero time is x-ray peak power.

Results for 12 bare axis single tungsten wire arrays on Z (20mmx10mm)

Shot	Mass(mg)	$I_{peak}(MA)$	$Peak\ P_x(TW)$	$Max\ K_b(kJ)$	$Max\ \overline{R_P}(\Omega)$
647	2.62	12.6	69	227	0.16
725	2.74	13.0	73	149	0.20
819	2.74	12.7	89	178	0.27
724	5.87	19.1	153	259	0.26
817	5.85	18.2	92	252	0.10
818	5.87	18.6	143	275	0.23
1099	1.15	11.6	114	310	0.29
1049	2.5	14.8	131	124	0.30
1097	6.0	17.9	141	133	0.40
1233	1.1	13.1	114	141	0.29
1735	2.42	15.2	149	142	0.38
1469	5.9	17.2	132	211	0.29

Conclusions

- **Obtained from shot data lower bounds for the pinch kinetic energy before peak x-ray power and its resistance at stagnation using circuit and energy balance equations.**

- **Within the data errors our analysis shows *(to our knowledge for the first time from actual data*) high pinch resistance for the stagnation phase in bare axis wire array implosions on Z**

- **To obtain the kinetic energy lower bound to about 10% we need about 2-3% accuracy for the electrical data. For the estimate of resistance we also need about 5-10% accuracy for the radiated power. Initial inductance values should be known with 5% accuracy**

- **This analysis will be employed for array implosions for the present configuration of Z. For the EBP to be conclusive efforts will be made to obtain the required data accuracy**