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Nonlinear Deflection Model for Corner-Supported, Thin Laminates Shape-Controlled with Moment Actuators

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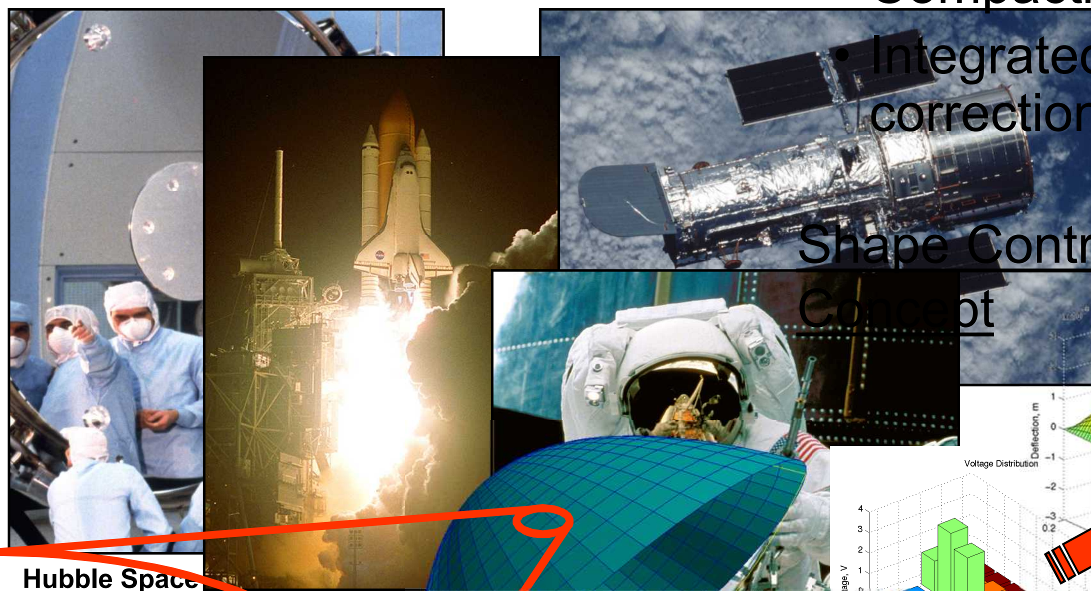
Shape-controlled Reflectors

Flexible Reflectors

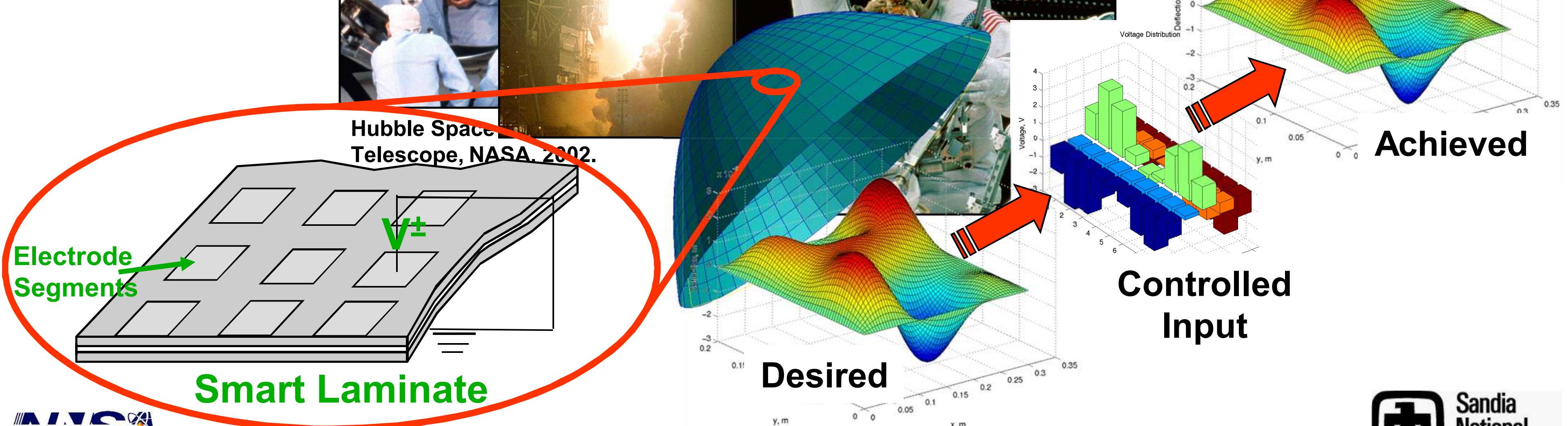
- Thin, light.
- Compactly deployed.
- Integrated surface error correction.

Traditional Rigid Reflectors



Hubble Space Telescope, NASA, 2002.

Shape Controlled Membrane Concept



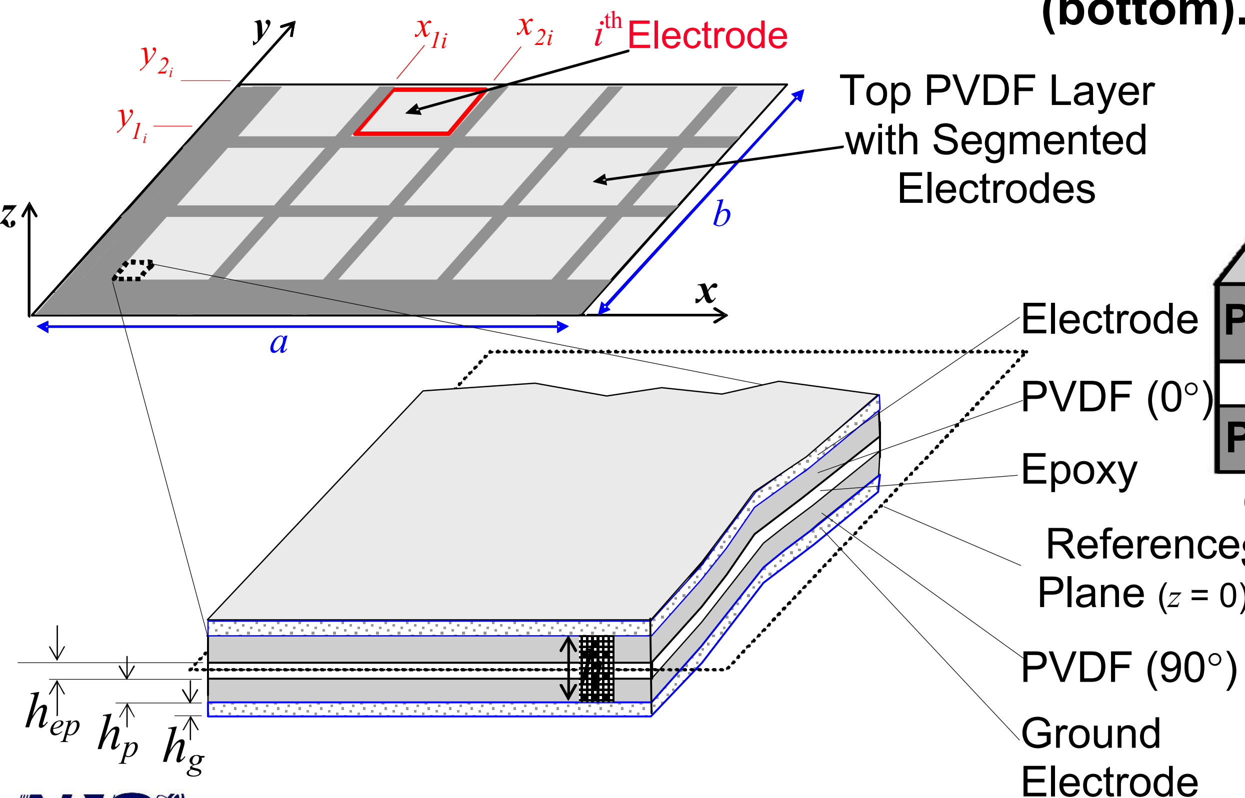
Smart Laminate with Moment Actuators

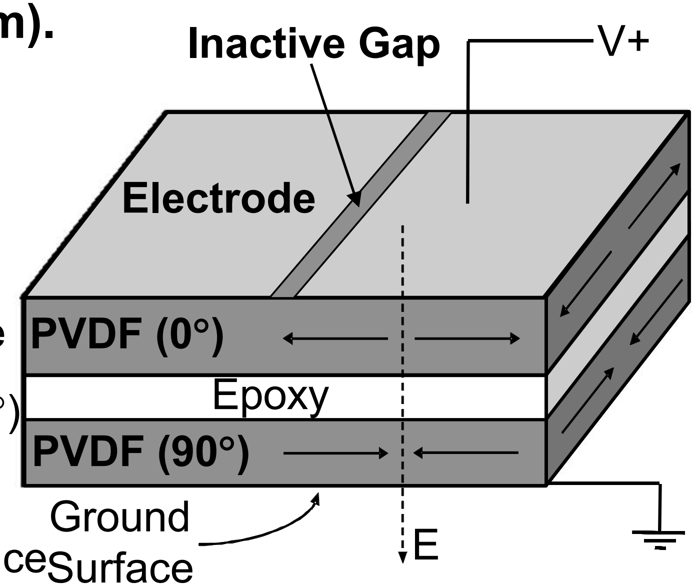
Thin, Square, Membrane with Corner Supports

- Natural actuation into paraboloid,
- Flexibility,
- Large deflections.

Bimorph Action

- PVDF layers have opposing poling directions.
- Positive field induces simultaneous expansion (top) and contraction (bottom).

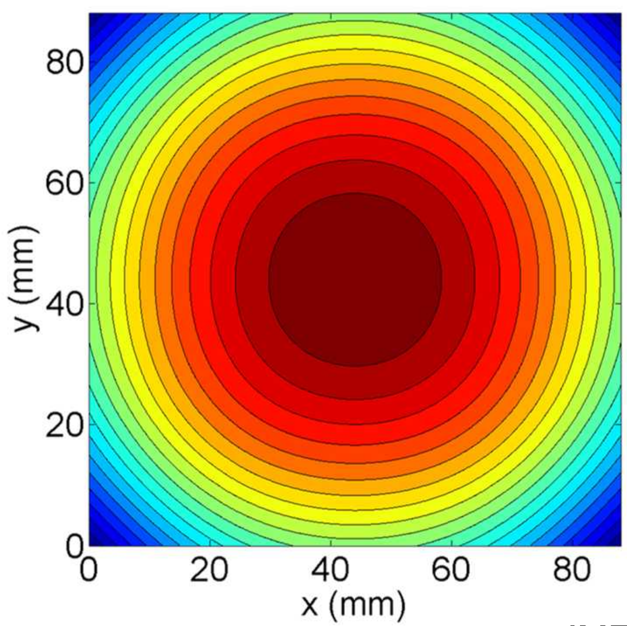




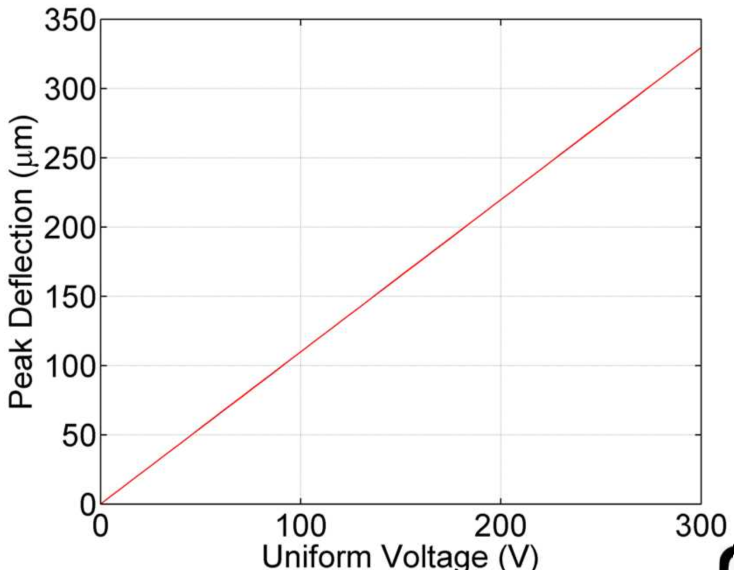
Previous Model

- Sumali, Massad, et alia, 2004-2005.
- Corner supports: sliding corners (out-of-plane constraint only).
- Based on Kirchhoff theory:
  - Describes bending action only;
  - Neglects in-plane membrane effects.
- Formulation facilitates inversion (shape control).
- Observations:
  - *uniformly circular contours*;
  - *linear* rise in peak deflection with increasing uniform actuation voltage.

Deflection Contours



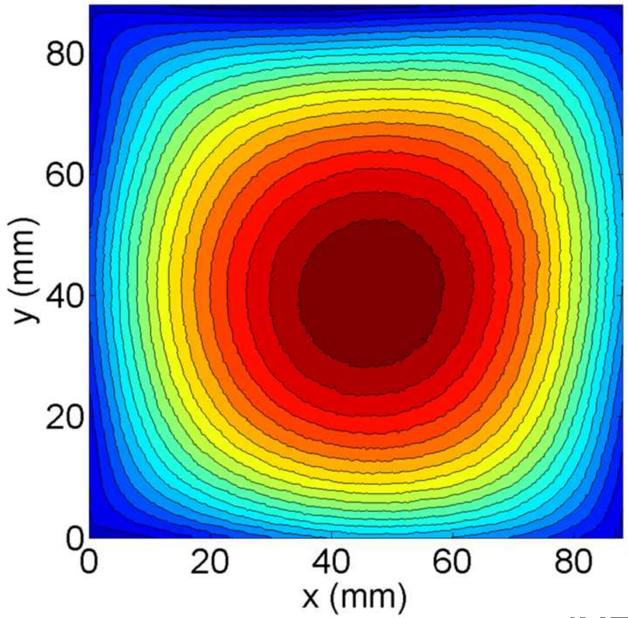
Peak Deflection vs. Uniform Voltage



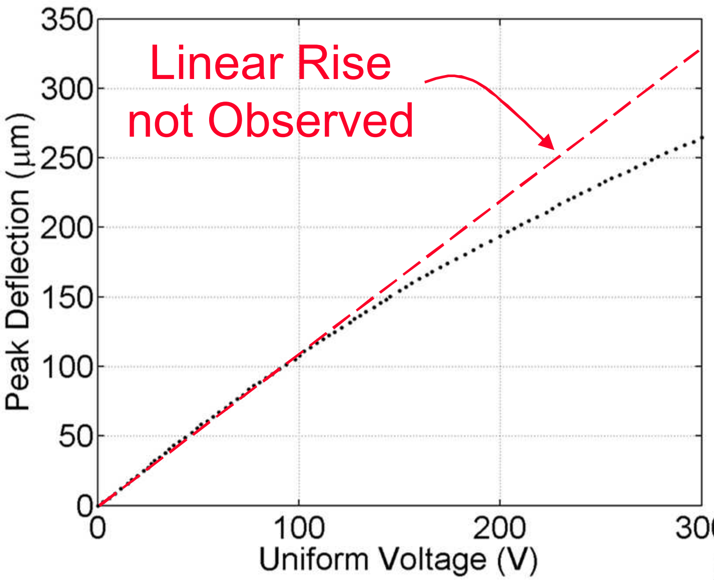
Preliminary Measurements

- **Fabricated corner-supported laminate with single electrode.**
- **Electronic speckle pattern interferometry (ESPI) measures *out-of-plane* deflection field.**
- **Corner supports: *fixed corners* to suppress vibrations and facilitate repeatable measurements.**
- **Observations:**
  - ***squared contours* become circular away from boundary;**
  - ***nonlinear* rise in peak deflection with increasing uniform actuation voltage.**

Deflection Contours



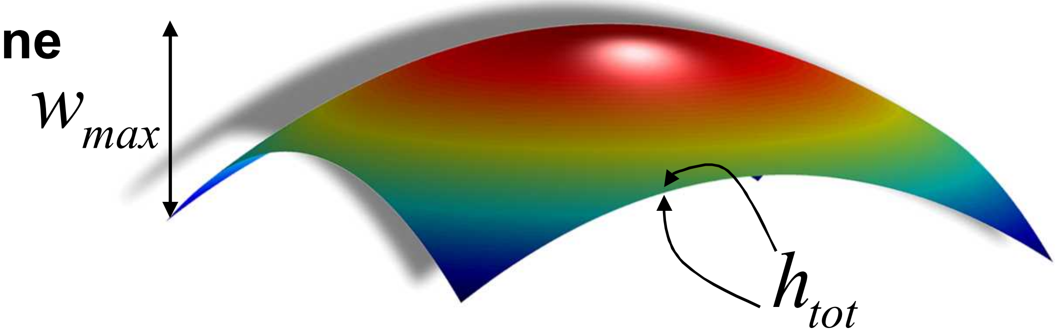
Peak Deflection vs. Uniform Voltage



Membrane Deflections

- **Structural severity of membrane deflections gauged by ratio**

$$\frac{\textbf{Peak Deflection } \ w_{max}}{\textbf{Total Membrane Thickness } \ h_{tot}}$$



Small Deflections

$$\frac{w_{max}}{h_{tot}} \leq 20\%$$

- **Negligible straining of middle surface.**
- **Bending is dominant.**
- **Kirchhoff linear theory adequate.**

Large Deflections

$$\frac{w_{max}}{h_{tot}} \geq 30\%$$

- **Measurable straining of middle surface.**
- **Membrane deformation ≥ bending.**
- **Nonlinear geometry changes and in-plane deformation.**

- **Desired and measured deflections ≥ 250 µm.**
- **Typical membrane thicknesses 100 - 250 µm.**

$$\frac{w_{max}}{h_{tot}} \geq 1$$

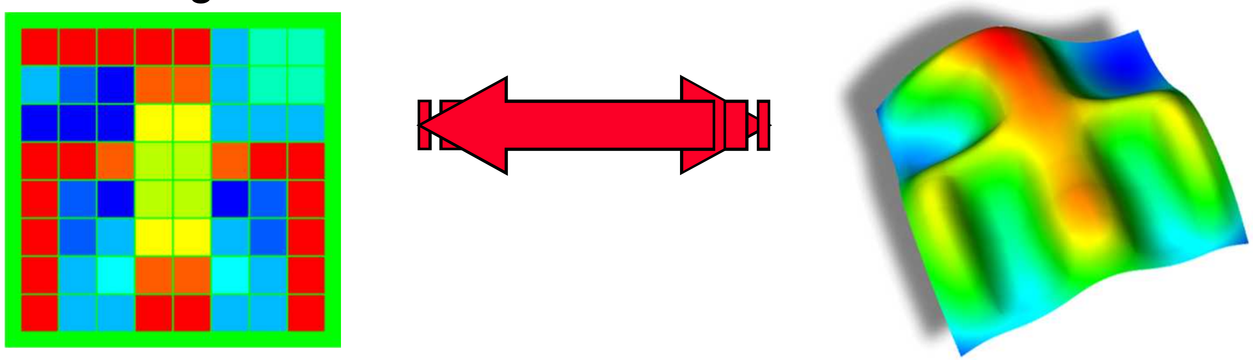
Large deflection theory of membranes must be used to accurately model laminate deflections.

Nonlinear (Large) Deflection Model

- Develop nonlinear model using framework of present linear, sliding-corner model.
- Predict large membrane deflections.
- Treat fixed corners.
- Preserve current model formulation as mapping:

Input Voltages

Output Deflection

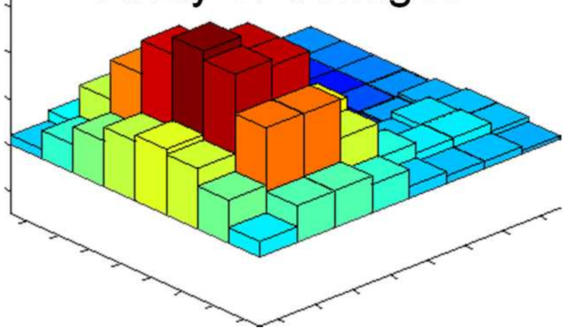


Critical: formulate model to be suitable for deflection control.

Energy-based Approach

Input

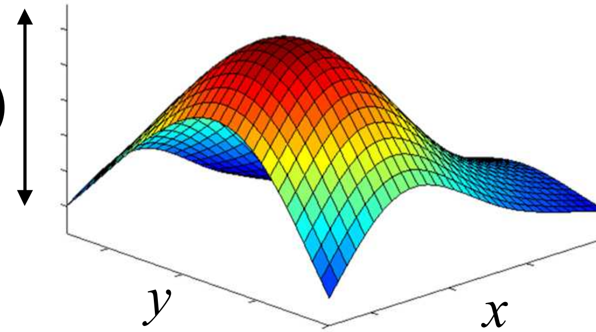
Array of Voltages



Output

Deflection

$w(x,y)$



Determine strain energy:

Total Strain Energy	=	Deflection Energy	+	Actuation Energy

In-plane Deformations
$u(x,y)$
$v(x,y)$

$$U \quad = \quad U_{\varepsilon}(u,v,w) \; + \; U_{act}(u,v,w;\mathbf{V})$$

Goal: find energy-minimizing deflection given voltage array $\mathbf{V}$.

Deflection Energy

$$U_\varepsilon = \frac{1}{2}\int_0^a\int_0^b\int_{-h_g-h/2}^{h_{el}+h/2} \boldsymbol{\varepsilon}(x,y,z)^{\mathsf{T}}\boldsymbol{T}(z)\,dz\,dy\,dx$$

Plane Stress

$$\boldsymbol{T}(z) = \boldsymbol{S}(z)\boldsymbol{\varepsilon}(x,y,z)$$

layer-dependent

von Karman Strain Relations

$$\boxed{\varepsilon = \varepsilon_b} + \varepsilon_m + \varepsilon_{nl}$$

Previous Model

Bending Strain

$$\boldsymbol{\varepsilon}_b(z) = -z\boldsymbol{\kappa}$$

Membrane Curvature

$$\boldsymbol{\kappa} = \begin{bmatrix} w_{xx} & w_{yy} & 2w_{xy}\end{bmatrix}^{\mathsf{T}}$$

Membrane Strain

$$\varepsilon_m = \begin{bmatrix} u_x & v_y & u_y + v_x\end{bmatrix}^{\mathsf{T}}$$

Nonlinear Strain

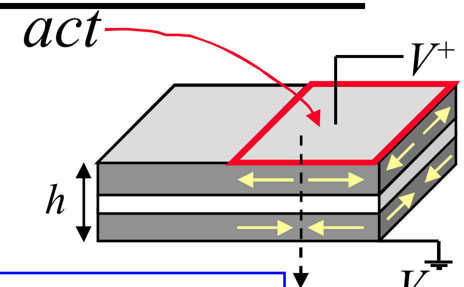
$$\varepsilon_{nl} = \frac{1}{2}\begin{bmatrix} w_x^2 & w_y^2 & 2w_x w_y\end{bmatrix}^{\mathsf{T}}$$

$$\boxed{U_\varepsilon = U_b + U_m + U_{lc} + U_{nlc} + U_{nl}}$$

- Energy includes Bending, Membrane, Linear-coupled, Nonlinear-coupled, and Nonlinear components.

Actuation Energy

$$U_{act} = \sum_{i=1}^{i_{max}} \iint_{act_i} \boldsymbol{\kappa}^{\mathsf{T}} \boldsymbol{M}_{act_i} \, dA$$



$$E_i = \frac{V_i}{h}$$

Membrane Curvature

$$\boldsymbol{\kappa} = -\begin{bmatrix} w_{xx} & w_{yy} & 2w_{xy} \end{bmatrix}^{\mathsf{T}}$$

Moment

$$\boldsymbol{M}_{act_i} = \int_{act_i} \boldsymbol{S}(z) \boldsymbol{\varepsilon}_{act_i}(z) z \, dz$$

Actuation Strain

$$\boldsymbol{\varepsilon}_{act_i}(z) = \begin{cases} \begin{bmatrix} d_{31} & d_{32} & 0 \end{bmatrix}^{\mathsf{T}} E_i & \frac{h_{ep}}{2} \le z \le \frac{h}{2} \\ 0 & -\frac{h_{ep}}{2} < z < \frac{h_{ep}}{2} \\ \begin{bmatrix} -d_{32} & -d_{31} & 0 \end{bmatrix}^{\mathsf{T}} E_i & -\frac{h}{2} \le z \le -\frac{h_{ep}}{2} \end{cases}$$

Integrate energy expression thru laminate thickness:

$$U_{act} = \frac{D_{act}}{h} \sum_{i=1}^{i_{max}} V_i \iint_{act_i} \left(w_{xx} + w_{yy} \right) dA$$

Voltages

V_i

Actuation Stiffness Constant

D_{act}

Energy Expansion

Assume expansions for tri-axial deformations:

$u(x,y)=\sum_{j=1}^{\infty}\mu_j(x,y)$	$\mu_j(x,y)=a_{u_j}\sin\left(n_j\pi\frac{x}{a}\right)\cos\left(m_j\pi\frac{y}{b}\right)$	Satisfy vanishing strain at edges.
$v(x,y)=\sum_{j=1}^{\infty}\psi_j(x,y)$	$\psi_j(x,y)=a_{v_j}\cos\left(m_j\pi\frac{x}{a}\right)\sin\left(n_j\pi\frac{y}{b}\right)$	
$w(x,y)=\sum_{j=1}^{\infty}\phi_j(x,y)$	$\phi_j(x,y)=a_{w_j}\cos\left(m_j\pi\frac{x}{a}\right)\sin\left(n_j\pi\frac{y}{b}\right)+ b_{w_j}\cos\left(m_j\pi\frac{y}{b}\right)\sin\left(n_j\pi\frac{x}{a}\right)$	Satisfies zero displacement at corners.

Truncate sums and simplify energy expression in terms of expansions:

$$\boxed{U(a_u,a_v,c_w,V)=U_\varepsilon(a_u,a_v,c_w)+(\mathbf{R}V)^T c_w}$$

Actuation Block Matrix
$\mathbf{R}$

Voltage Array
V

In-plane Expansion Coefficient Vectors
a_u, a_v

Out-of-plane Expansion Coefficient Vector
c_w

$$c_w=\begin{bmatrix}a_w\\ b_w\end{bmatrix}$$

Energy Minimization

Find energy-minimizing deformation. → Find energy-minimizing expansion coefficients.

Minimum conditions:

$$\nabla_{a_u} U = 0 \qquad \nabla_{a_v} U = 0 \qquad \nabla_{c_w} U = 0$$

Solve nonlinear system for expansion coefficients:

$$\boxed{G_\varepsilon\left(a_u, a_v, c_w\right) + \mathbf{R}V = 0}$$

Gradient Function

G_ε

couples expansion coefficients nonlinearly

Resulting Map:

Input: V → **Output:** ~~$u(x,y)$, $v(x,y)$~~, $w(x,y)$

- **Inverse map requires knowledge of *in-plane* deformation.**
- **Typically *out-of-plane* ($w(x,y)$) known/specified (e.g., ESPI, error surface), in-plane unknown.**

Must Decouple In-plane Deformations

In-plane Strain De-coupling

- **Minimum conditions yield linear dependence on a_u, a_v.**
- **In-plane coefficients explicitly cast in terms of out-of-plane coefficients.**

$$\nabla_{a_u} U = 0 \qquad a_u = F_u(c_w)$$

$$\nabla_{a_v} U = 0 \qquad a_v = F_v(c_w)$$

Recast nonlinear system:

$$\boxed{\mathbf{H}c_w + G_{nl}(c_w) + \mathbf{R}V = 0}$$

Decoupled Energy Hessian
$\mathbf{H}$

Resulting Map:

Input: V → **Output:** $w(x,y)$

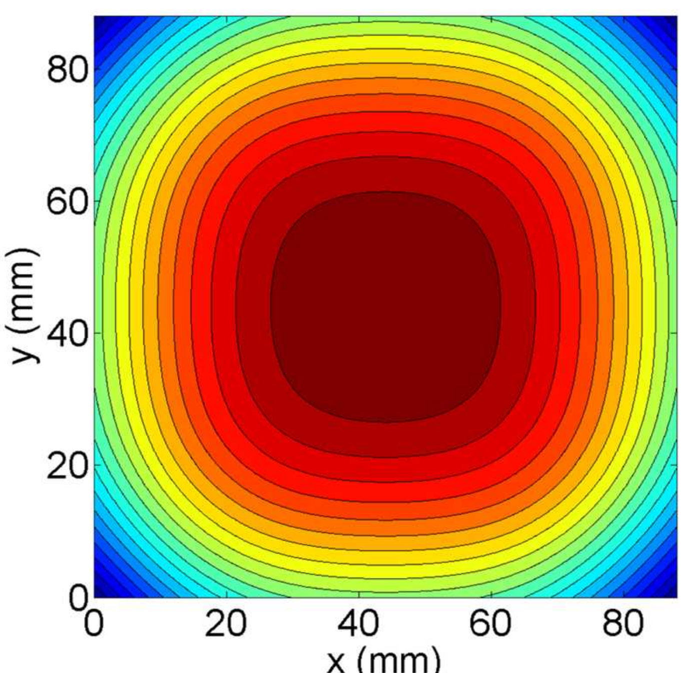
Nonlinear Gradient Function
G_{nl}
nonlinear component of decoupled gradient

- **Inverse map requires only *out-of-plane* deformation.**
- **Deflection control feasible.**

Model Results

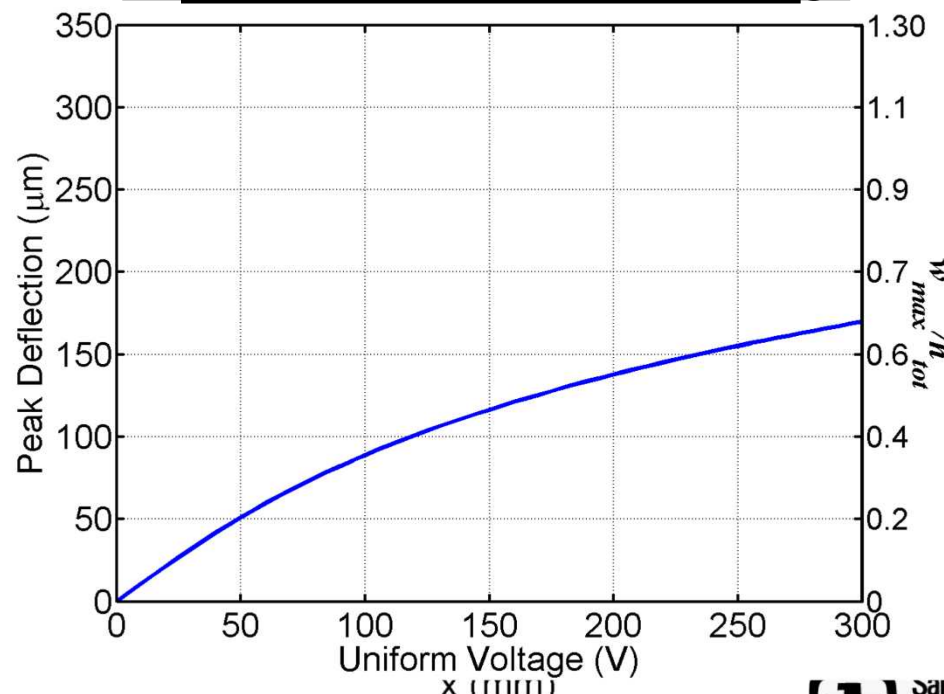
- Deflection contours show squaring effects.
- Nonlinear rise in peak deflection predicted.
- Source: nonlinear geometry changes and membrane forces due to large deflections and pinned corners.

Model Deflection Contours



Peak Deflection vs. Uniform Voltage

Measured Deflection Contours



Concluding Remarks

Summary

– Model computes large membrane deflections of active laminate given distribution of actuation voltages.

– Model accommodates clamped corner supports, membrane strains, and geometric nonlinearities.

– Formulation maps voltage input to deflection output, suitable for shape control.

– Model agrees qualitatively with measured deflections of an active laminate.

Future and Ongoing Considerations

– Validate model with coordinated experiments.

– Analyze convexity to guarantee admissible numerical solutions.

– Implement efficient inverse model for shape control.

– Improve computational efficiency.

– Integrate model with measurement system to control shape of fabricated laminate.