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Shock and Vibration Margin Testing for One-Shot Devices in the Energy-Based Capability Framework

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U.S. DEPARTMENT OF ENERGY

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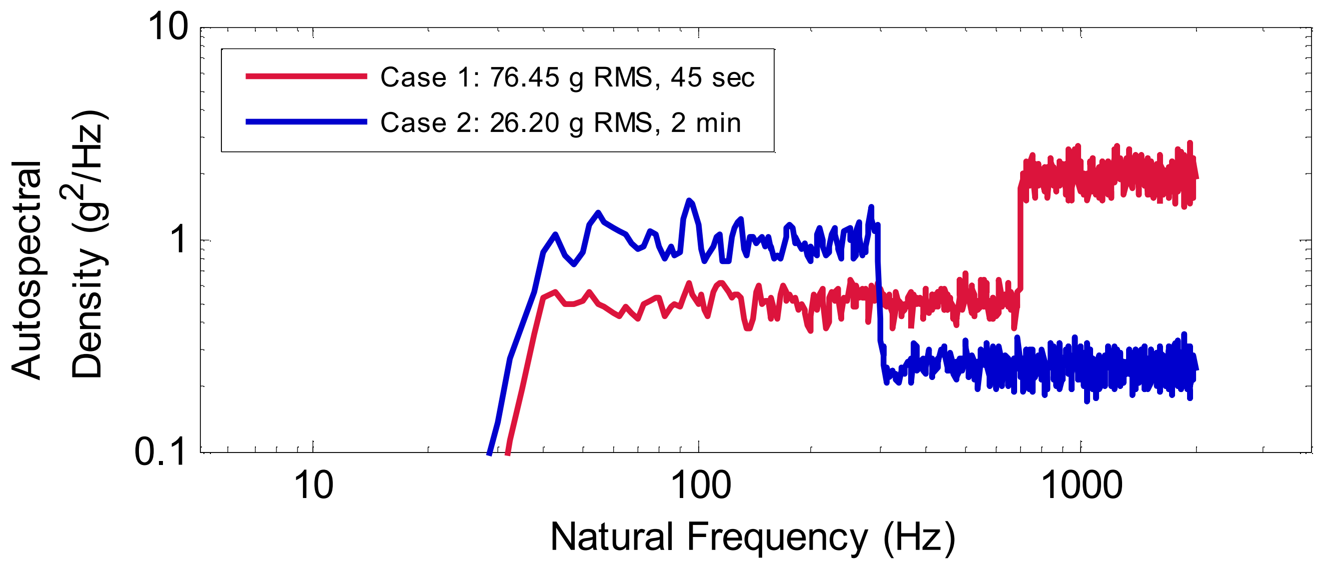
Introduction

- Motivation
  - To assess component-level margin to shock and vibration specs
- Framework
  - Energy-based failure modeling – modal energy as intensity metric
- Assumption
  - Failure threshold unique in single sample but distributed in population
- Implications due to limits on functional test capability
  - Multi-shot device failure threshold may be observed directly (continuous monitoring) or within an interval (periodic inspection)
  - One-shot device failure threshold cannot be observed directly
- Key issues in assessing margin for one-shot devices
  - Selecting shock & vibe test levels for maximizing information gained
  - Allocating test units for model uncertainty equal to multi-shot case

Energy Based Severity Analysis

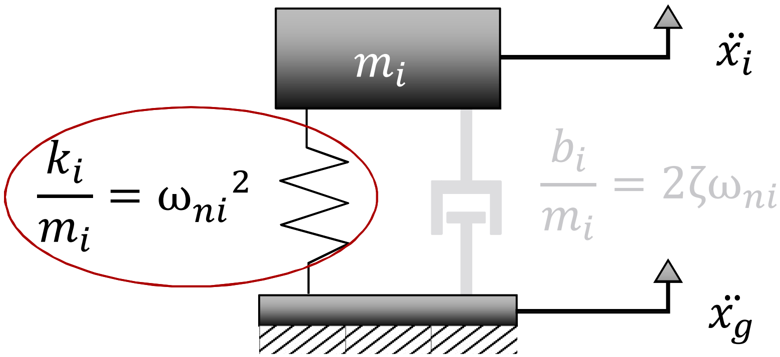
- Use modal energy as an intensity measure for predicting structural failure
- Requires approx. linear structure, fixed-base modal properties, base input
- Key Advantage: Once failure model is built, arbitrary input profiles can be assessed for relative severity



Which is more severe?

Understanding margins in modal energy enables risk-informed decision-making.

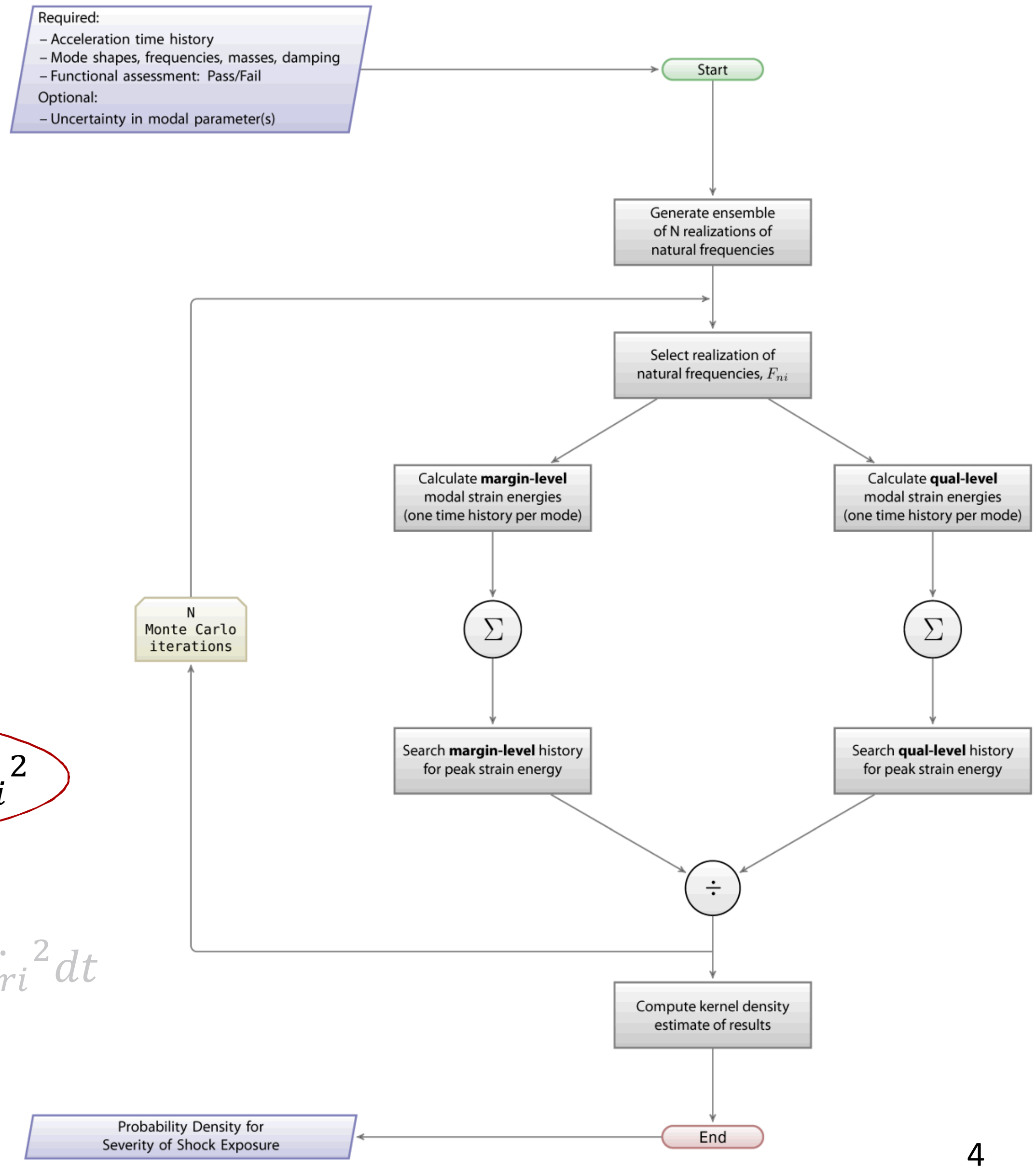
Shock: Peak Strain Energy



$$\ddot{x}_{ri} = \ddot{x}_i - \ddot{x}_g$$

Absorbed energy $$\frac{E_{ai}}{m_i} = \frac{1}{2}\omega_{ni}^2 x_{ri}^2$$

Dissipated energy $$\frac{E_{di}}{m_i} = \int 2\zeta\omega_{ni}\dot{x}_{ri}^2 dt$$



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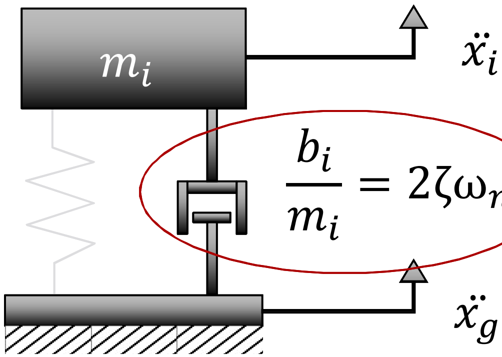
Vibration: Total Energy



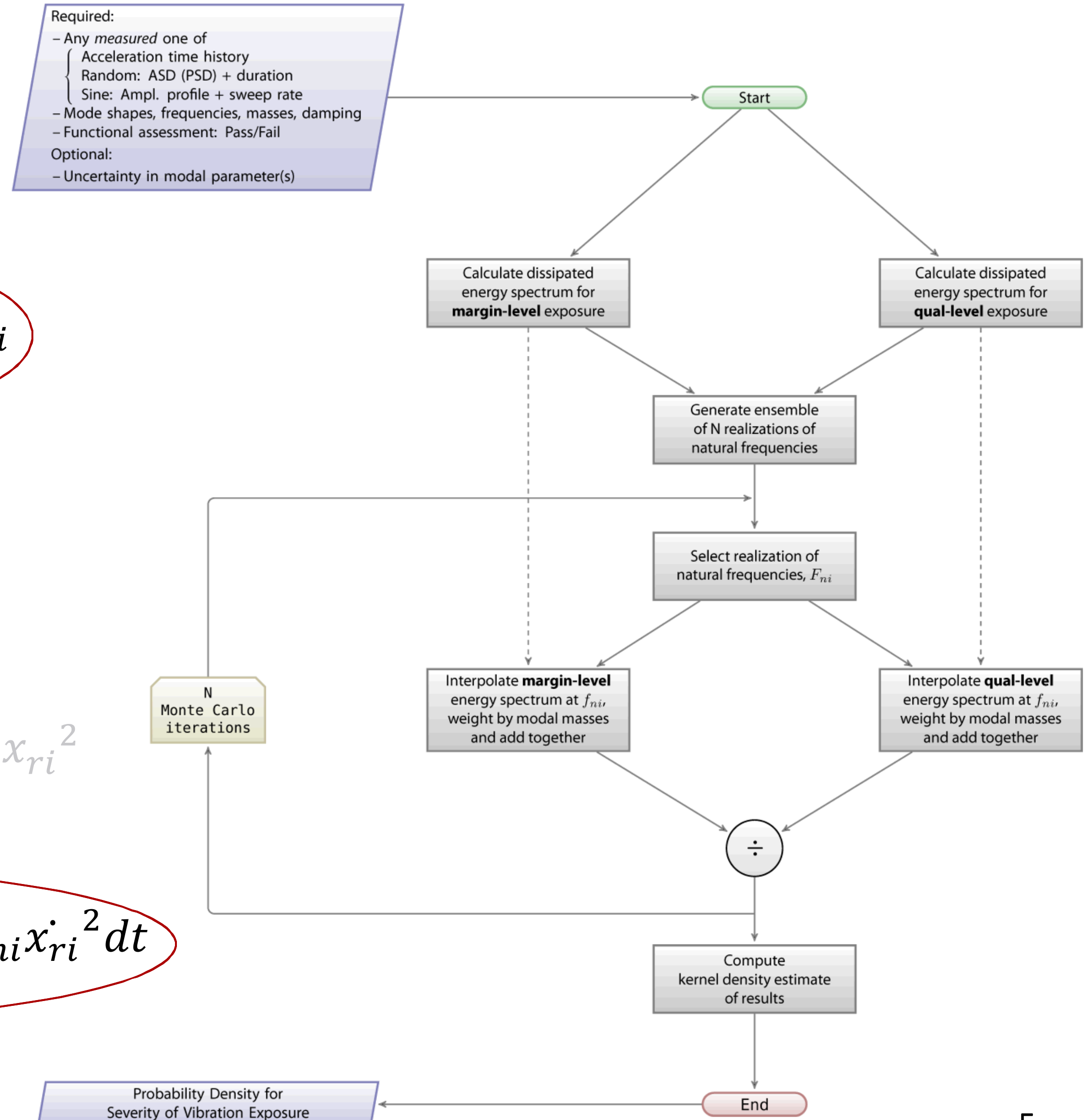
$$\frac{k_i}{m_i} = \omega_{ni}^2$$

$$\frac{b_i}{m_i} = 2\zeta\omega_{ni}$$

$$\ddot{x}_{ri} = \ddot{x}_i - \ddot{x}_g$$

Absorbed energy $\frac{E_{ai}}{m_i} = \frac{1}{2}\omega_{ni}^2 x_{ri}^2$

Dissipated energy $\frac{E_{di}}{m_i} = \int 2\zeta\omega_{ni}\dot{x}_{ri}^2 dt$



Literature Review

- One-shot device testing is still an active research area
  - Sensitivity testing or sequential sensitivity testing
- Early motivation was explosives drop testing
  - Find distribution for threshold on height to detonate
- One-shot test plans are sequential
  - Next level depends on all earlier levels and outcomes
- Many different plans proposed
  - Most frequently discussed in literature: Probit (Bliss, 1935), Bruceton (Dixon and Mood, 1948), Langlie (1962), Neyer's D-Optimal (1994)
- Three plans chosen for this study
  - Langlie
  - Neyer's D-optimal
  - LND sensitivity test (2013)

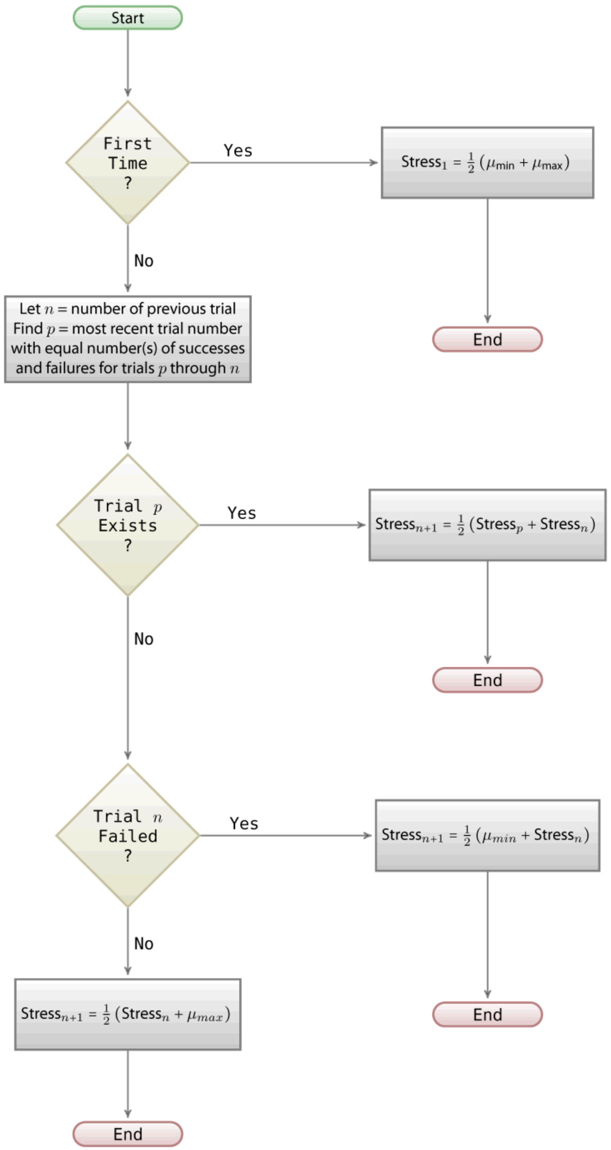
First Publication on Each Plan

H.J. Langlie, *A Reliability Test Method for "One-Shot" Items*, Proceedings of the Eighth Conference on the Design of Experiments in Army Research Development and Testing, Washington, D.C., October 24-26, 1962.

Barry T. Neyer, *A D-Optimality Based Sensitivity Test*, Technometrics, Vol. 36, No. 1 (Feb. 1994), pp. 61-70.

Lei Wang, Yukun Liu, Wei Wu, and Xiaolong Pu, *Sequential LND sensitivity test for binary response data*, Journal of Applied Statistics, Vol. 40, Iss. 11, pp. 2372-2384, 2013.

Langlie's Test



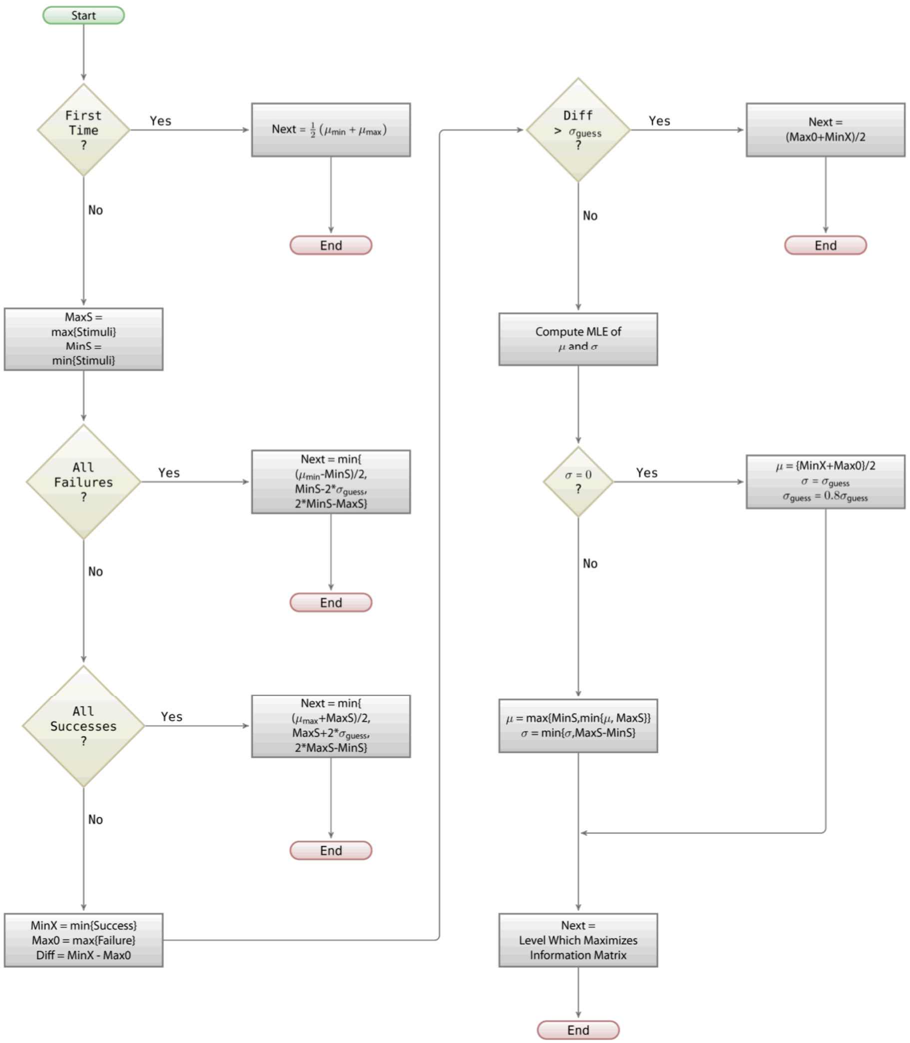
Advantages

- Requires only bounding estimates for the PDF mean
- Attempts to keep number of passes and failures equal
- Easy to calculate test levels

Disadvantages

- Suboptimal convergence
- Extrapolation outside mean search range not allowed
- Wider mean search range reduces efficiency

Neyer's D-Optimal Test



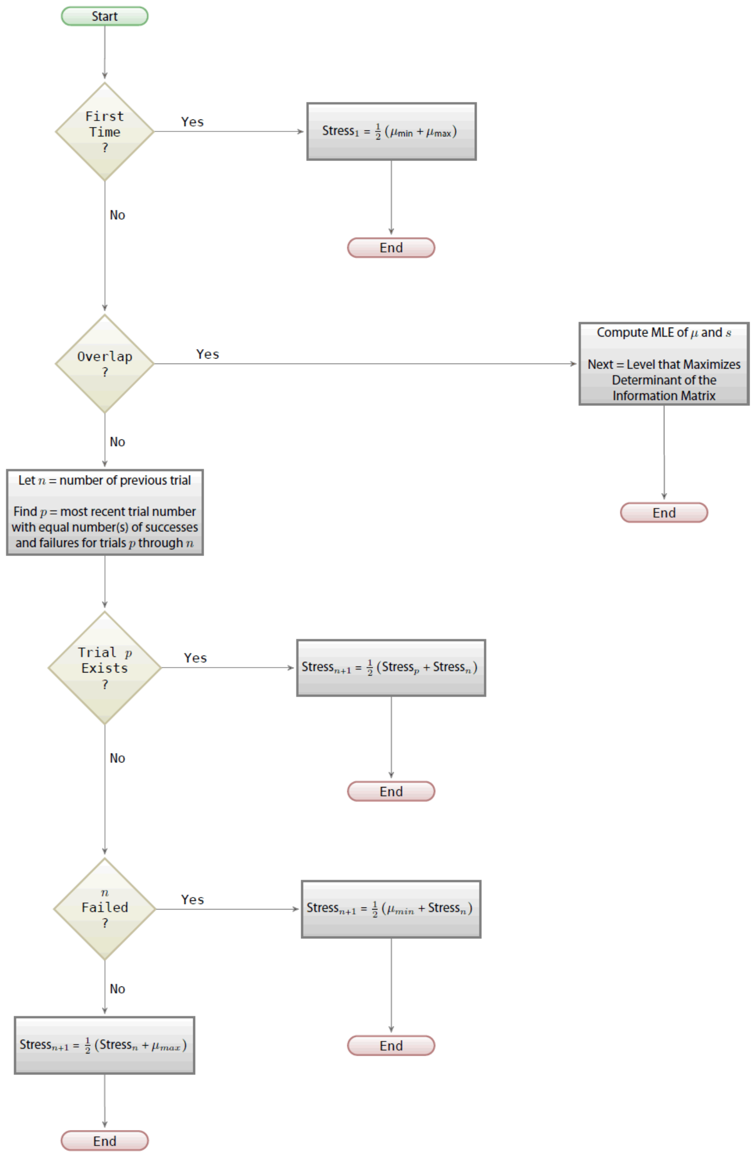
Advantages

- D-optimal convergence once "overlap" reached
- Extrapolation outside mean bounds if necessary

Disadvantages

- Requires estimate of scale parameter (inaccuracy reduces search efficiency)
- Complex: Software needed to perform MLE calculation to determine test levels

Langlie-Neyer D-Optimal (LND) Test

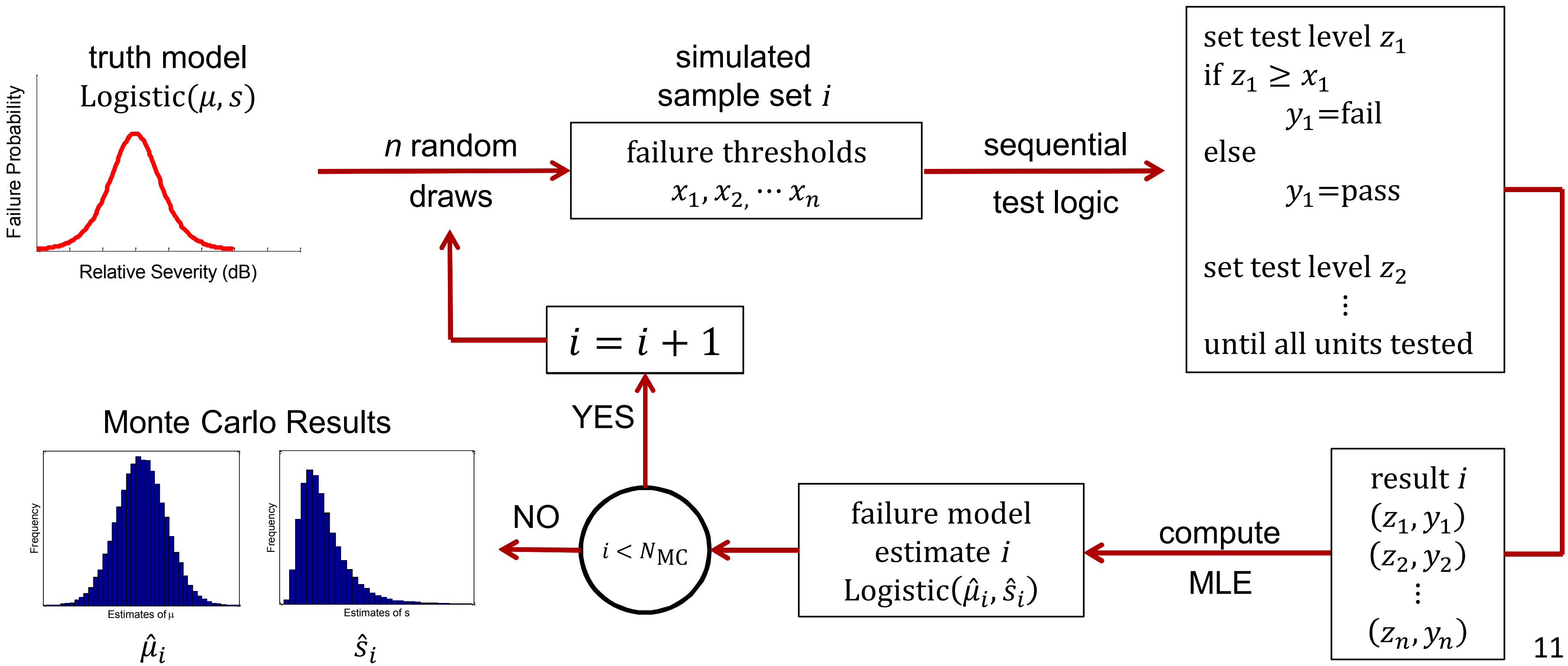


Leverages advantages of each approach

- Only estimated bounds on mean required to start
- Reaches overlap faster than Neyer if estimate of scale parameter is inaccurate
- D-optimal convergence once overlap reached
- Search allowed outside estimated bounds on mean

Evaluating Test Plans

- Sensitivity test comparisons in literature use method of Maximum Likelihood Estimation (MLE) to fit distribution parameters
- Use Monte Carlo to simulate N_{MC} realizations of each approach
- Compare Interquartile Ranges (IQR) of parameter estimates



Failure Model Comparison

Weibull PDF

$$W(t) = \begin{cases} 0, & t < 0 \\ \frac{\beta}{\eta} \cdot \left(\frac{t}{\eta}\right)^{\beta-1} \cdot e^{-\left(\frac{t}{\eta}\right)^{\beta}}, & t \geq 0 \end{cases}$$

- No negative "support"
- Cannot use any data from 0dB or lower tests

Logistic PDF

$$f(x_r|\mu, s) = \frac{e^{\frac{x_r-\mu}{s}}}{s\left(1 + e^{\frac{x_r-\mu}{s}}\right)^2}$$

$$f(x_r|\mu, s) = \frac{1}{4s}\operatorname{sech}^2\left(\frac{x_r - \mu}{2s}\right)$$

- Infinite support
- Physically meaningful for failure defined in dB

D-Optimal Test Plans

- Optimal designs set test levels in a way that allows efficient estimation of unknown distribution parameters.
- D-optimal designs achieve efficiency through maximizing the **D**eterminant of the information matrix.
- Publication (Neyer, 1994) gives plan for normal distribution
- "Optimization" step in Neyer's plan is a choice between two values that maximize information in parameter estimates
- Fisher Information Matrix (FIM)
  - Measure of information on distribution parameters from test data
  - Obtained by computing expected values of 2^{nd} derivatives of the log of the likelihood function with respect to distribution parameters θ.

$$\boldsymbol{I} = \mathrm{E}\left[-\frac{1}{(\log L)^2}\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)\right]$$

Fisher Information Matrix (FIM)

FIM for ***any*** location-scale distribution:

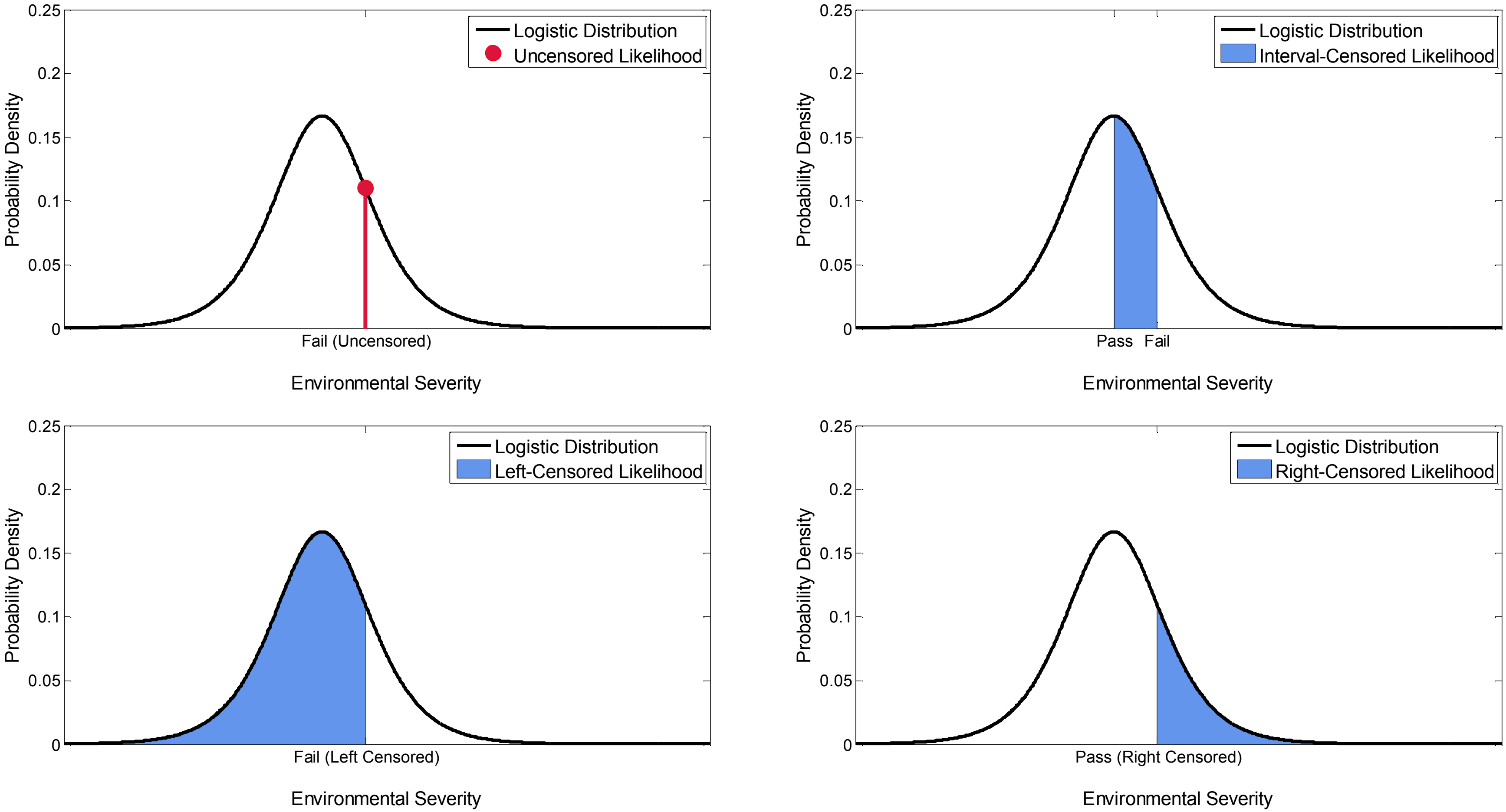
$$I_n(\mu, s) = \begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix} = \sum_{i=1}^{n} \frac{(f'(z_i))^2}{f(z_i)\big(1 - f(z_i)\big)s^2} \begin{pmatrix} 1 & z_i \\ z_i & z_i^2 \end{pmatrix}$$

where

n	number of observed data points
μ, s	location and scale parameters of the distribution
x_i	stimulus level for data point i in units of severity
$z_i = \frac{x_i - \mu}{s}$	normalized stimulus level for i^{th} data point
$f(z_i)$	cumulative density function (CDF) of the distribution
$f'(z_i)$	first derivative of $f(z_i)$ (i.e., the PDF)

Max. Likelihood Estimation

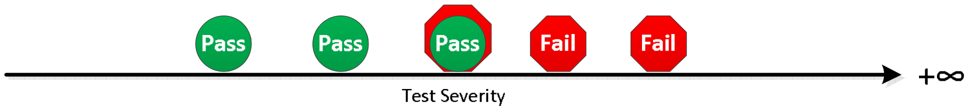
$$L(\mu, s) = \prod_{i=1}^{n} f(z_i)^{\delta_i}\left(1 - f(z_i)\right)^{1-\delta_i}, \qquad z_i = \frac{x_i - \mu}{s}$$



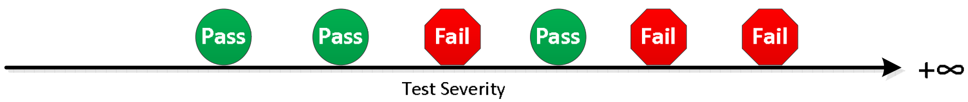
Separation and Overlap in MLE

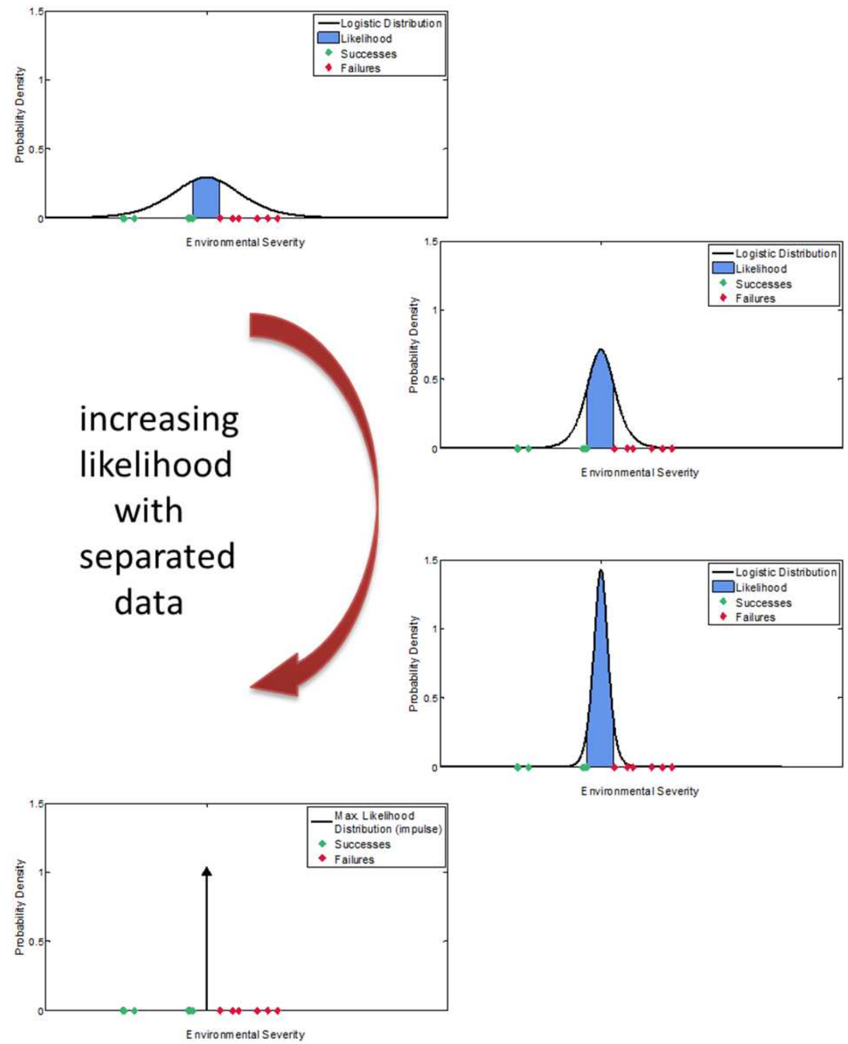
Completely Separated

Partially Separated

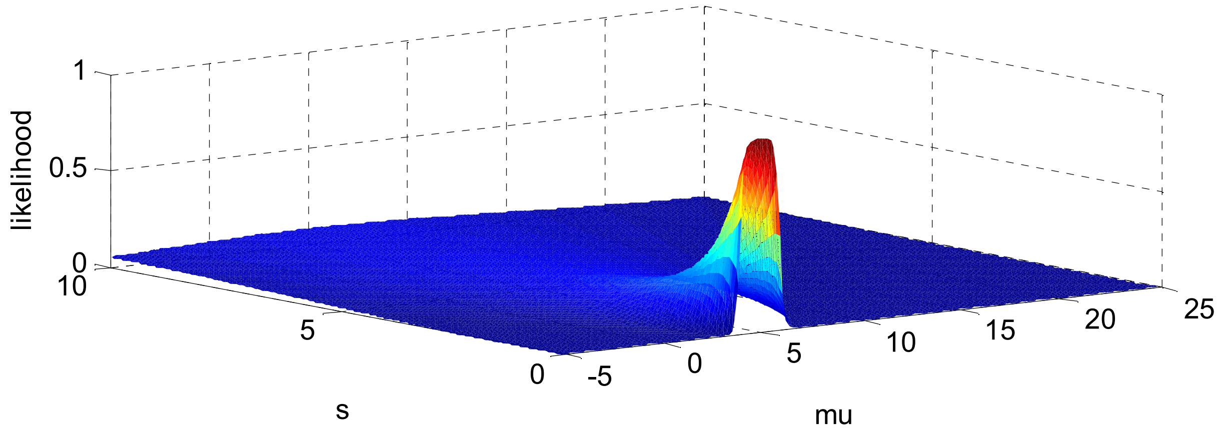


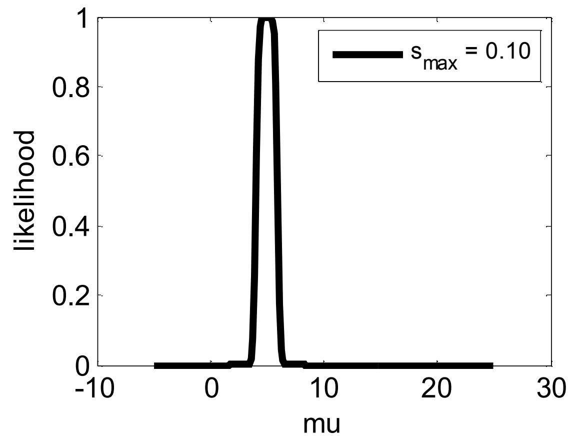
Overlapped

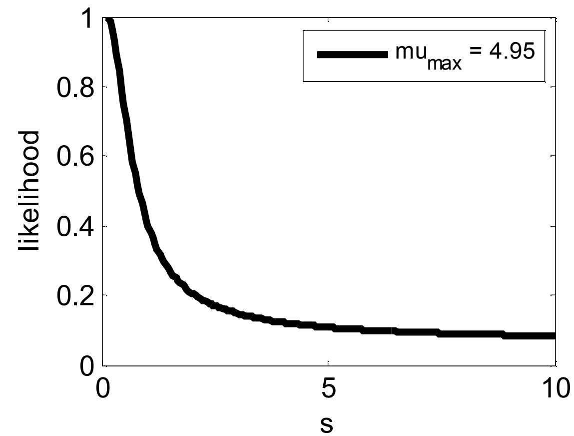




Separation in One-Shot Devices



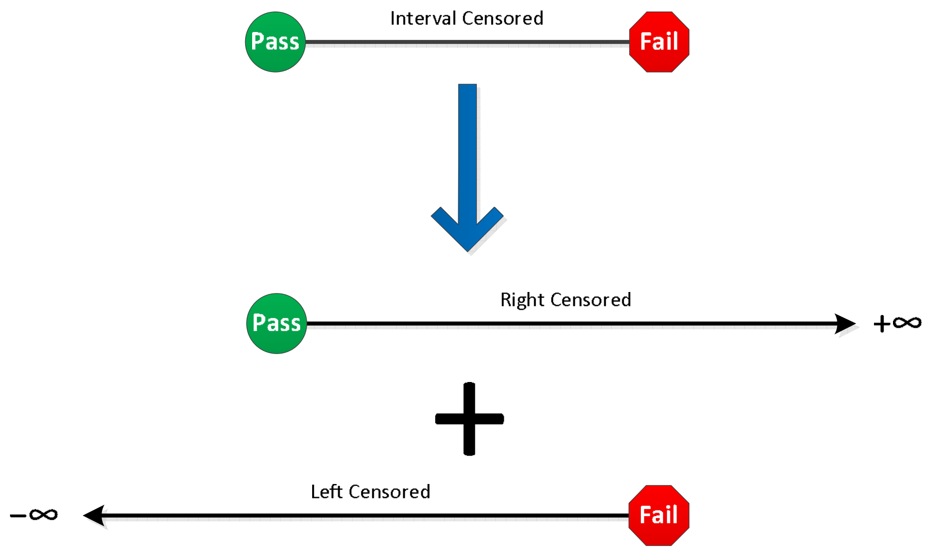




Likelihood surface for right-censored points at 3,4 and left-censored points at 6,7.

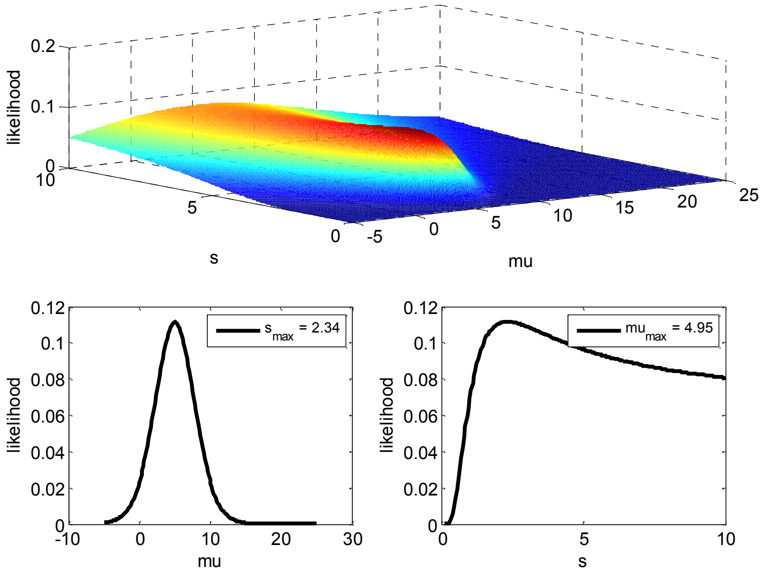
How Many Units for “Equivalence”?

- Contribution to likelihood for one interval censored result similar to contributions from two left-right censored results
- Assume min. 5 units to failure at 3dB intervals
- ~10 units to failure for one-shot devices, assuming P/F ratio = 1

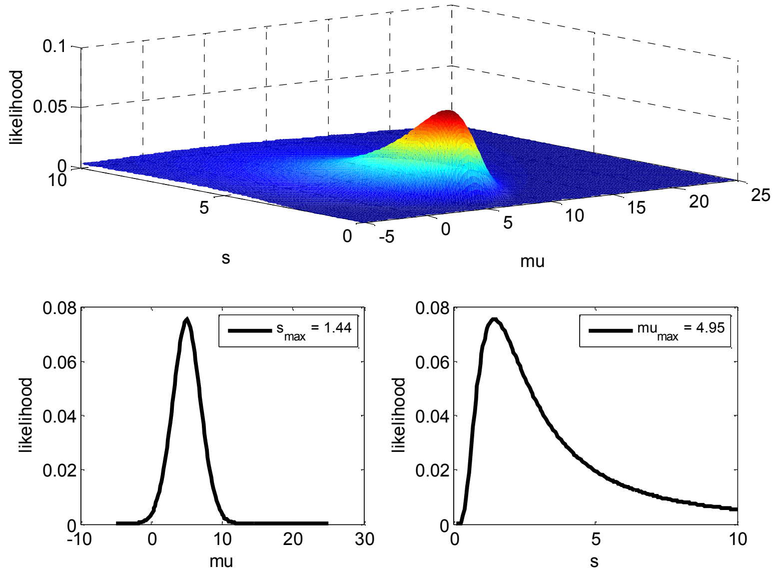


Likelihood Surface Differences

One-shot Overlap Case



"Equivalent" Multi-shot Case



Likelihood surface for right-censored points at 1,6 and left-censored points at 4,9.

Likelihood surface for failure intervals with 2dB gap ([1 4] and [6 9]).

MONTE CARLO SIMULATION STUDY

Monte Carlo Study Parameters

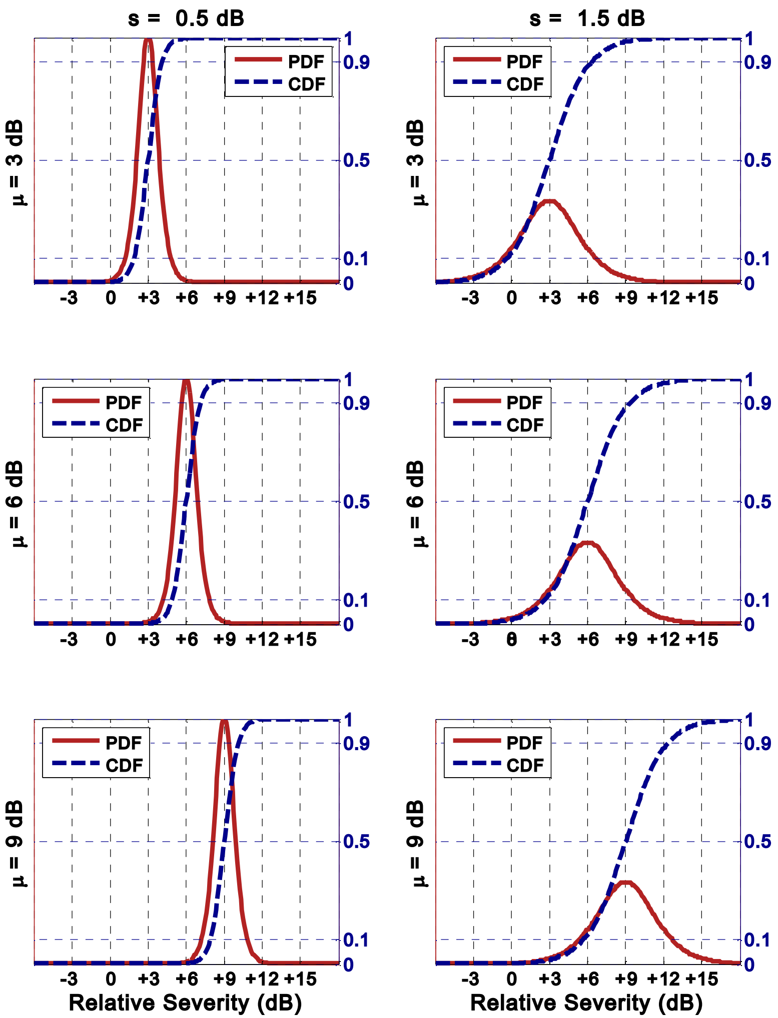
- Selected six representative truth models for component margin

- Simulated margin testing for methods (Langlie, Neyer, LND), sample sizes (5, 10, 20) and $N_{\mathrm{MC}} = 10{,}000$ realizations each

- Primary metric to compare test method performance is interquartile range (IQR): 25th to 75th percentile values from MC study

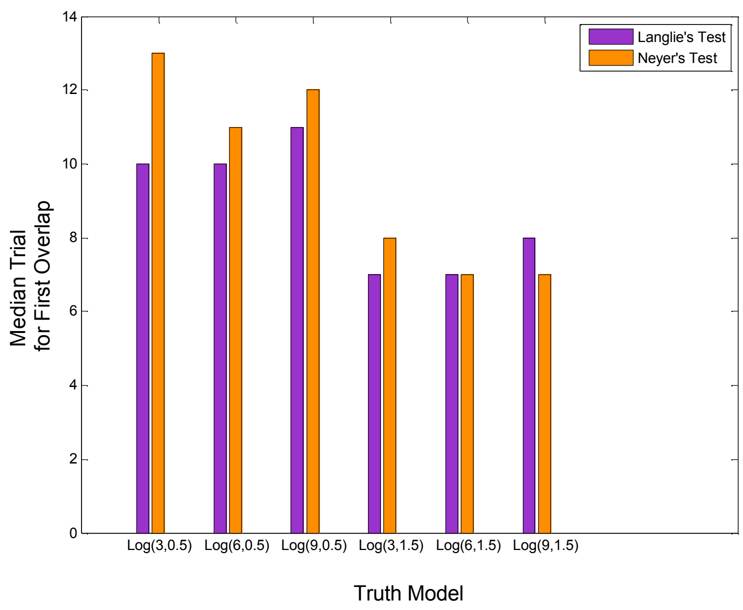
- Compared IQR values for LND method to IQR values for 5 units of multi-shot components with 3dB interval censoring

“Truth” Failure Models for Study



- Six combinations of logistic mean parameter μ and scale parameter s.
- Parameters cover range of failure models
  - Low end: Typical qualification tests multiple units at 0dB
  - High end: Resource limits on design robustness

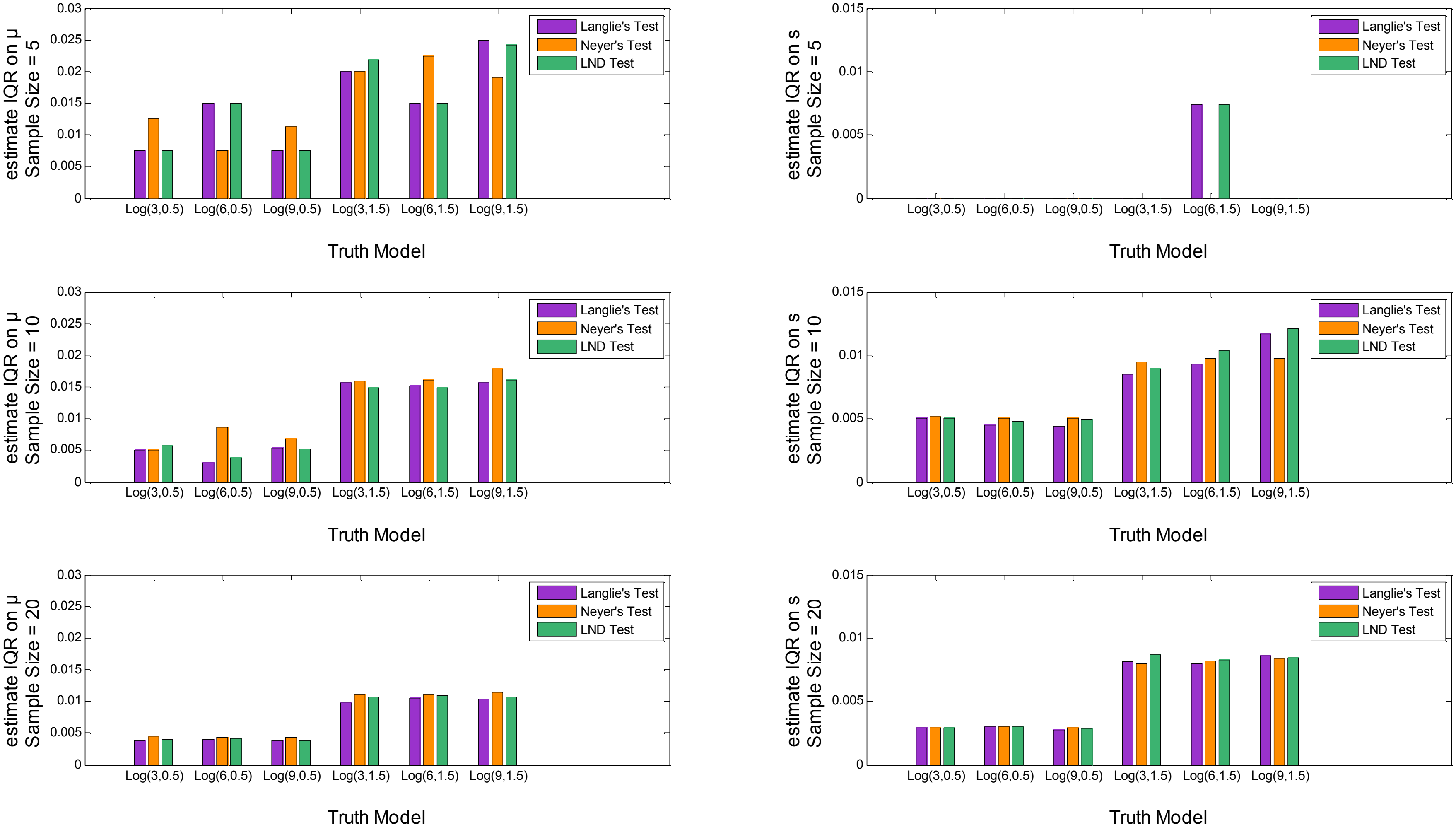
Number of Test Units for Overlap



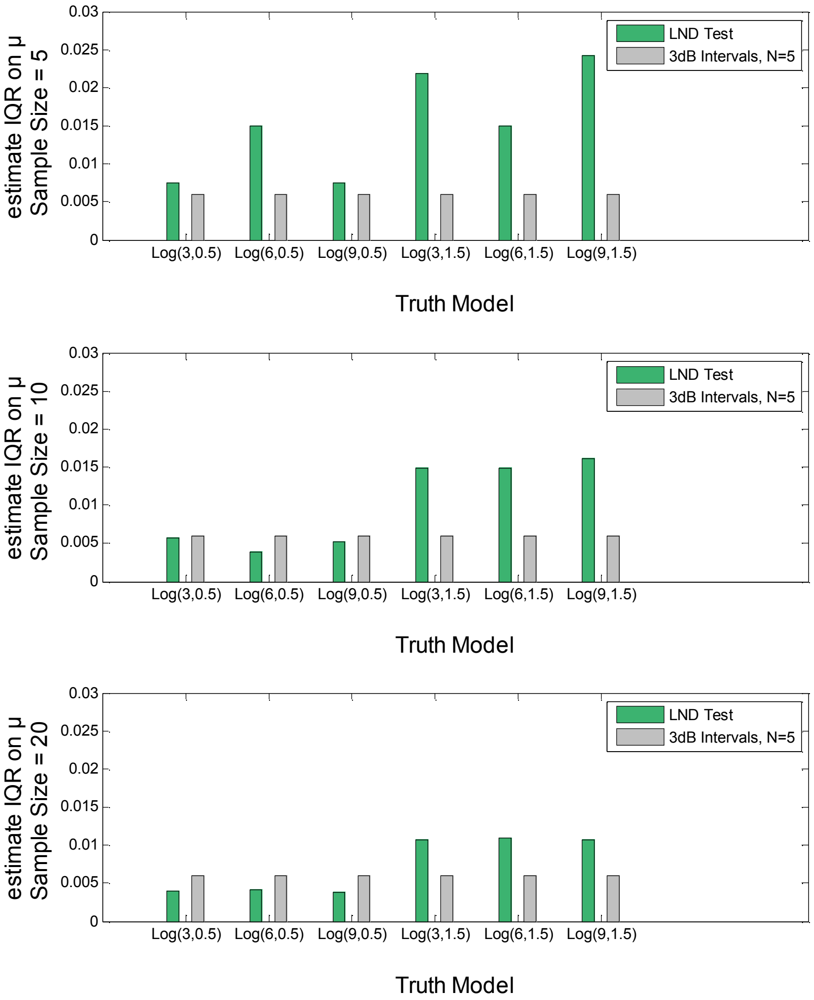
- Studied median number of trials to reach initial overlap condition for Langlie vs Neyer test.
- Practical constraints seem to improve rate of overlap attainment
  - Resolution 0.5dB
  - Maximum level 3dB above previous maximum level

Test Method: Langlie, Neyer or LND?

IQR differences become negligible for 20-unit sample size



Sample Size: One-shot vs Multi-shot

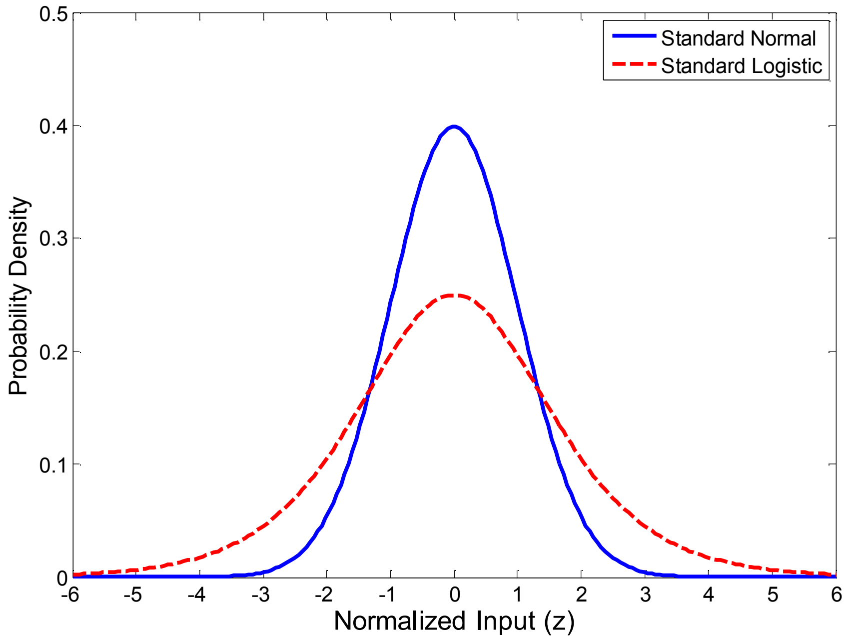


- Sample size 5 units in all multi-shot simulations
  - 3dB intervals with 5 units and these truth models means all units in 1 interval or 2 adjacent intervals
  - Estimate IQR values for s not compared
- Need at least 10 one-shot units, likely closer to 20 for approximately equal uncertainty

SUPPLEMENTARY SLIDES

Comparison of Logistic to Normal

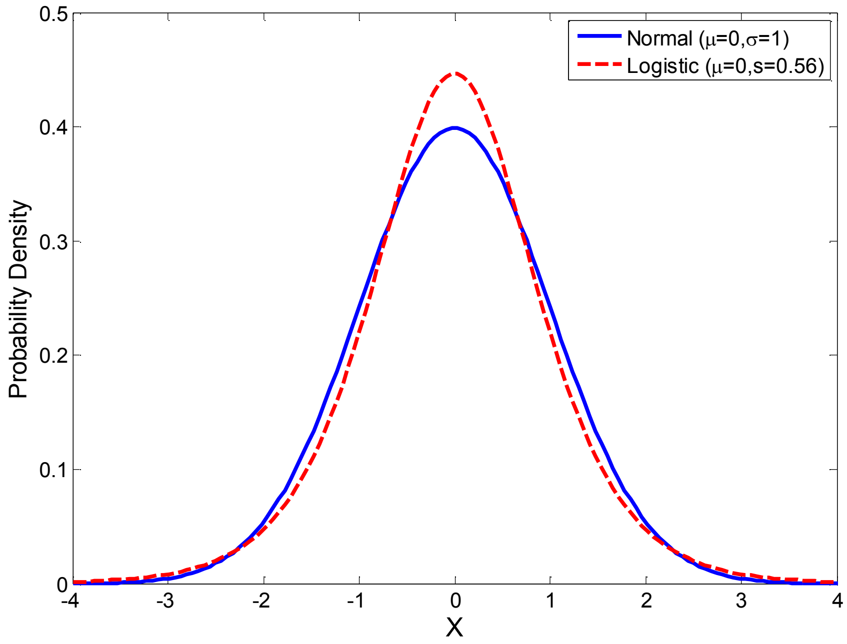
Interesting bits don't stop at ±3s !!



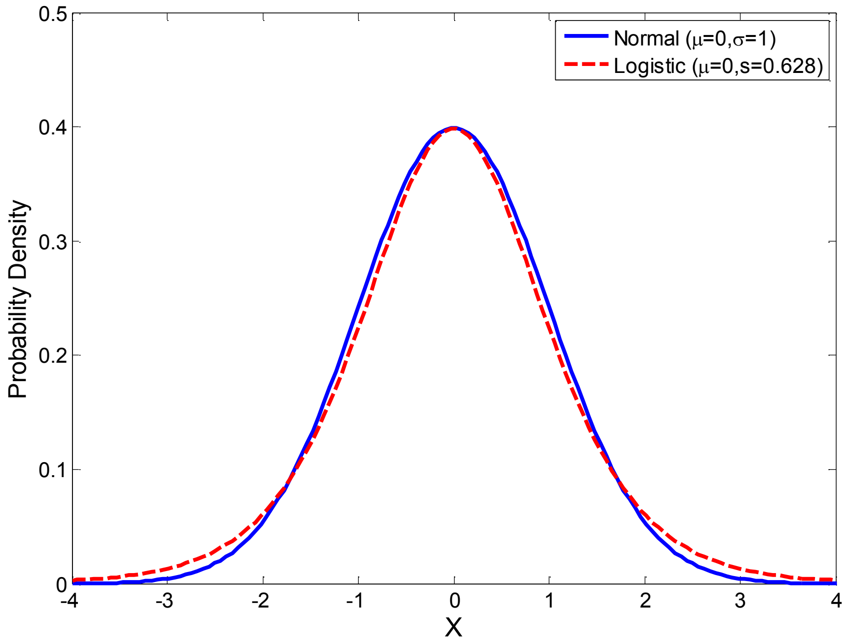
N	Normal PDF within (μ±N·σ)	Logistic PDF within (μ±N·s)
1	63.8%	46.2%
2	95.5%	76.2%
3	99.7%	90.5%
4	~100%	96.4%
5	~100%	98.7%
6	~100%	99.5%
7	~100%	99.8%

Leptokurtic versus Mesokurtic PDF

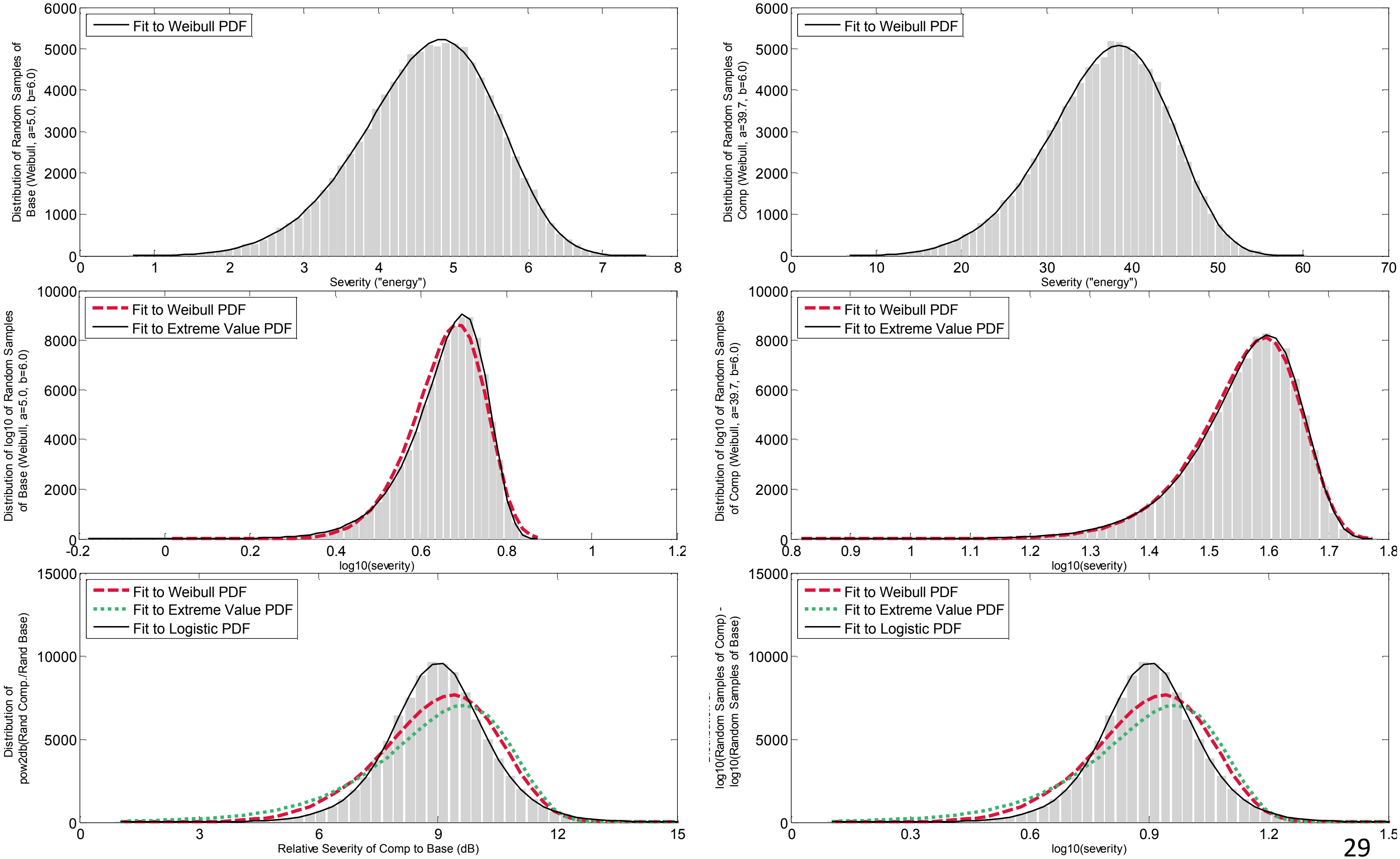
Match Tails → Peak Higher



Match Peak → Tails Fatter



Weibull vs. Logistic PDF



Strategy for Selecting Levels

- Let $I_{jk} = \sum_i n_i J_{j+k}(z_i)$

  where $J_j(z_i) = \frac{P'^2(z_i) {z_i}^j}{P(z_i)Q(z_i)s^2}$

- Asymptotic variances

$$\text{var}\,\hat{\mu} = {I_{11}}/{\left(I_{00}I_{11} - {I_{01}}^2\right)}$$

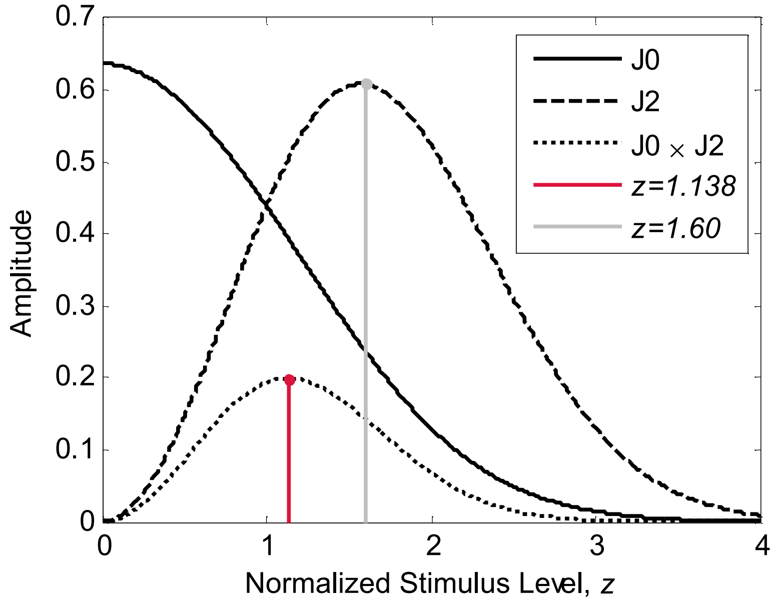
$$\text{var}\,\hat{s} = {I_{00}}/{\left(I_{00}I_{11} - {I_{01}}^2\right)}$$

- For large sample sizes and symmetric selection

$$I_{01} \ll I_{00}, I_{11}$$

  - To minimize var $\hat{\mu}$, maximize I_{00}
  - To minimize var $\hat{s}$, maximize I_{11}
  - To minimize both equally, maximize $I_{00}I_{11}$

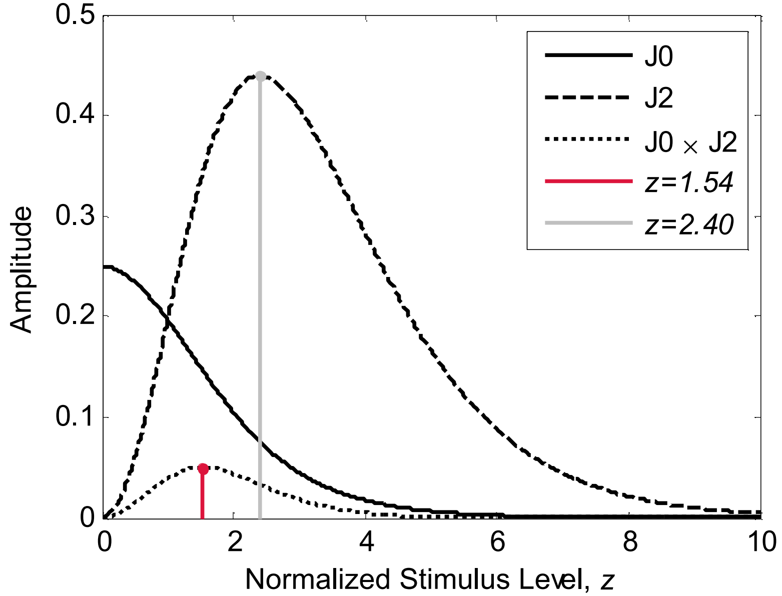
FIM Components for Normal PDF



$$z = \frac{x - \mu}{s}$$

- If μ most important
  - Test at $z = 0$
- If s most important
  - Test at $z = \pm 1.60$
- If μ, s equally important
  - Test at $z = \pm 1.138$

FIM Components for Logistic PDF

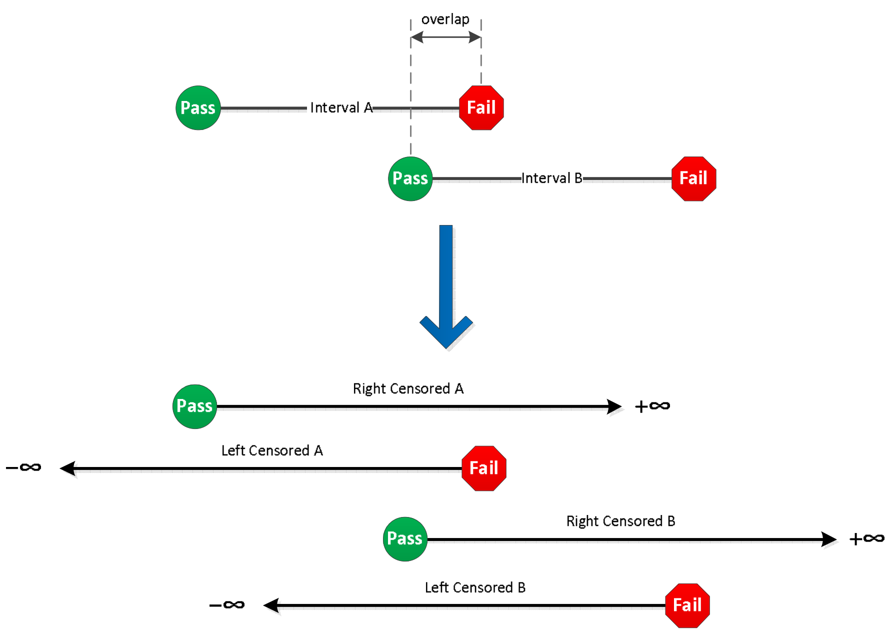


$$z = \frac{x - \mu}{s}$$

- If μ most important
  - Test at $z = 0$
- If s most important
  - Test at $z = \pm 2.40$
- If μ, s equally important
  - Test at $z = \pm 1.54$

Overlap in Multi-Shot Devices

Interval-Censored w/ Overlap



Interval-Censored w/ Gap