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A Partition of Unity FEM for Cohesive Zone Modeling of Fracture

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Applied Mechanics Development (1526)

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Preview

- Introduction
- PUFEM Displacement Field Enrichment
- Results for Model Problems
- Ongoing and Future Work

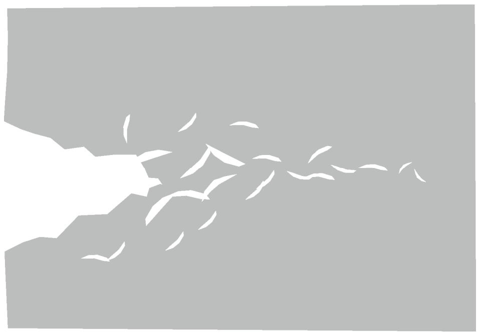
Preview

- Introduction
  - Cohesive crack model
  - Objective, goals, and approach
  - Background
  - Motivating problems

Fracture Models

- *Cohesive crack model* -- assumes the process zone can be idealized as a surface (*i.e.*, a curve in a 2D representation).

Example: quasibrittle material
with bridging between microcracks



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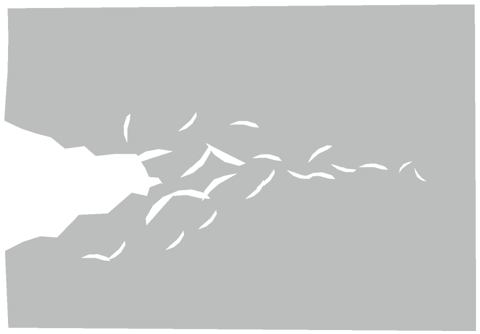
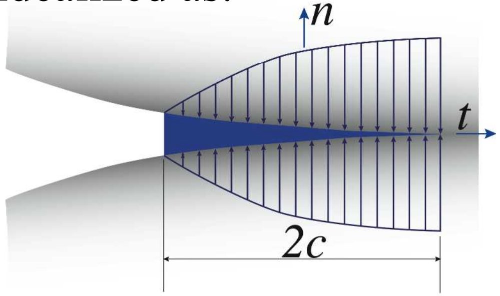


Idealized as:

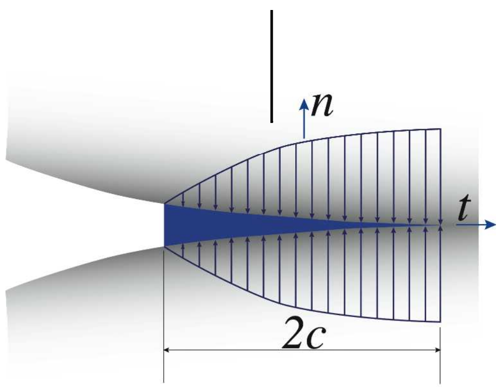


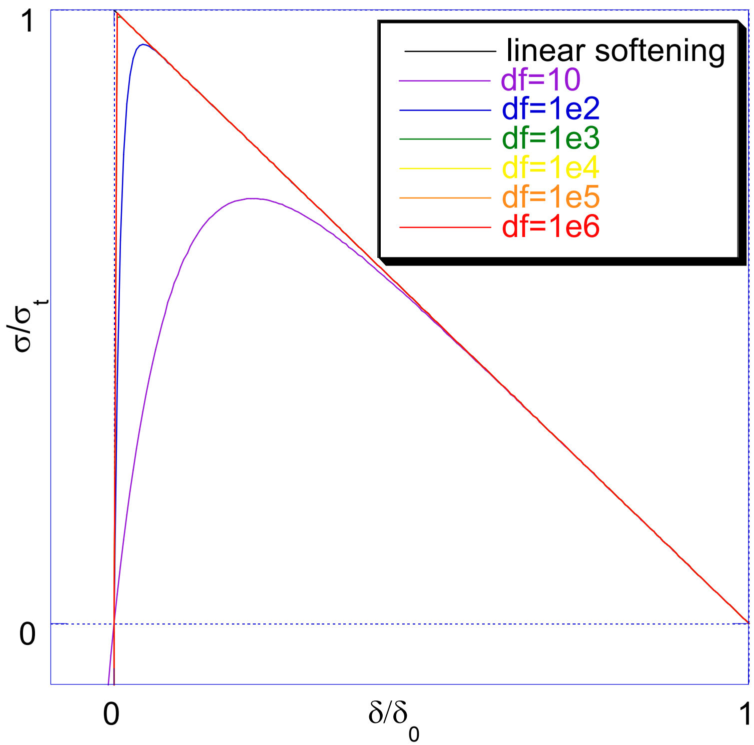
- Actual tractions are homogenized (σ)
- Kinematic effects of micro-cracks are lumped to cohesive zone surface
  δ ~ a fictitious crack opening

Fracture Models

- *Cohesive crack model* -- assumes the process zone can be idealized as a surface (*i.e.*, a curve in a 2D representation).

where the relationship between σ and δ is given by the cohesive zone model





Study Introduction

Objective: A "valid" means of modeling material localization in finite element analyses.

Goals:

- applicable to cohesive zone modeling
- "continuous discontinuity"
- arbitrary orientation of discontinuity relative to mesh

Approach: Develop a partition of unity FEM (PUFEM) that allows the displacement field to be enriched in the neighborhood of a strong discontinuity.

- can represent a discontinuity without mesh refinement
- can potentially represent the gradients near a surface of localization without mesh refinement

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Background

Initial related studies

- Melenk and Babuska (1996)
  Theory for PUFEM
- Belytschko and Black (1999)
  - developed PUFEM for LEFM -- XFEM
  - used asymptotic displacement fields near a crack tip for enrichment

Origins of this study

ARL (2001)

motivating problem: armor penetration

SNL

Initial problem: HDBT

Fracture modeling (LDRD fatigue)

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Recent Related Studies

PUFEM-Cohesive Zone Studies

- Wells and Sluys (2001)
- Moës and Belytschko (2002)
- Zi and Belytschko (2003) -- tip function addresses tip position but not the field
- de Borst *et al.* (2004) & Mariani *et al.* (2003) tip at element edges

Current work is closer to:

- Strouboulis, Copps, Zhang, and Babuska (2000, 2001, 2003) numerical enrichment functions -- handbook functions

Origins of this study

ARL (2001)

motivating problem: armor penetration

SNL

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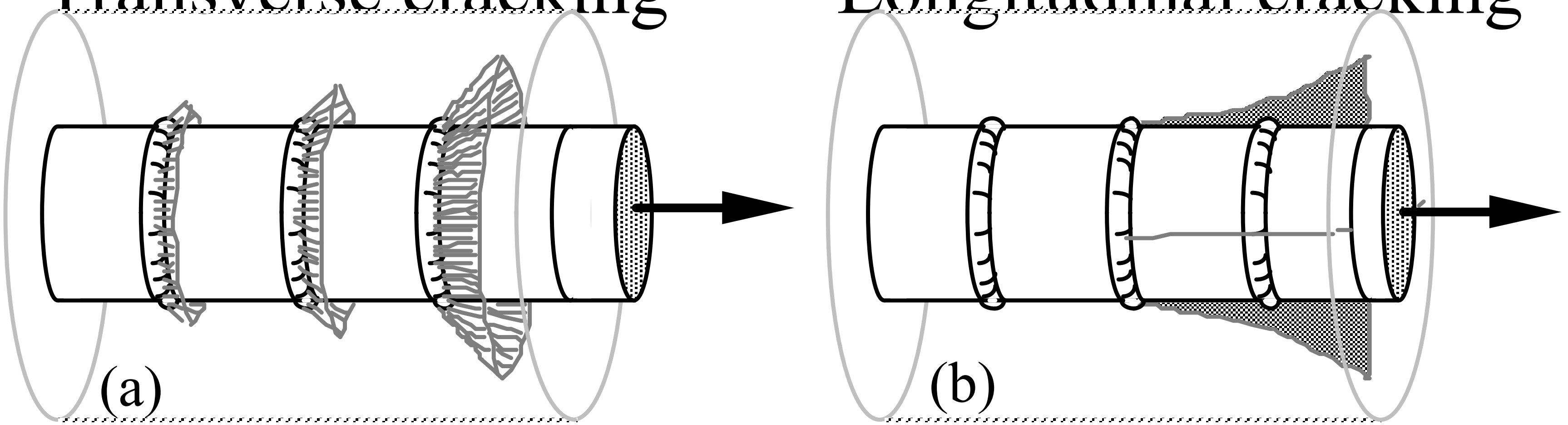
Mechanical Interlocking Effects in Bond

- Concrete Cracking and Crushing

Transverse cracking | Longitudinal cracking

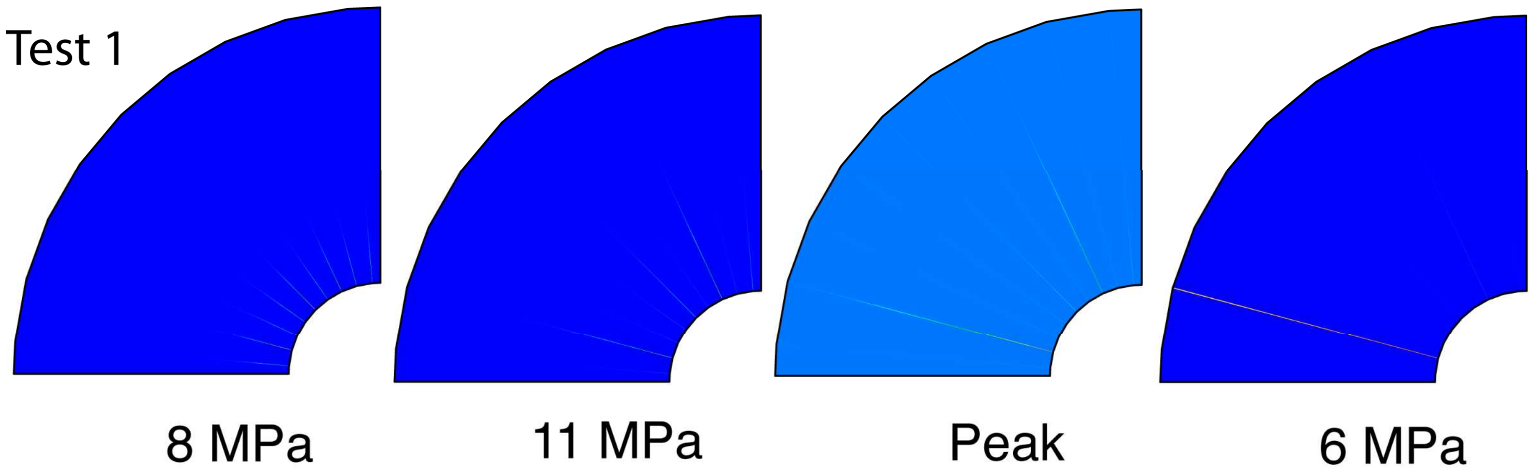


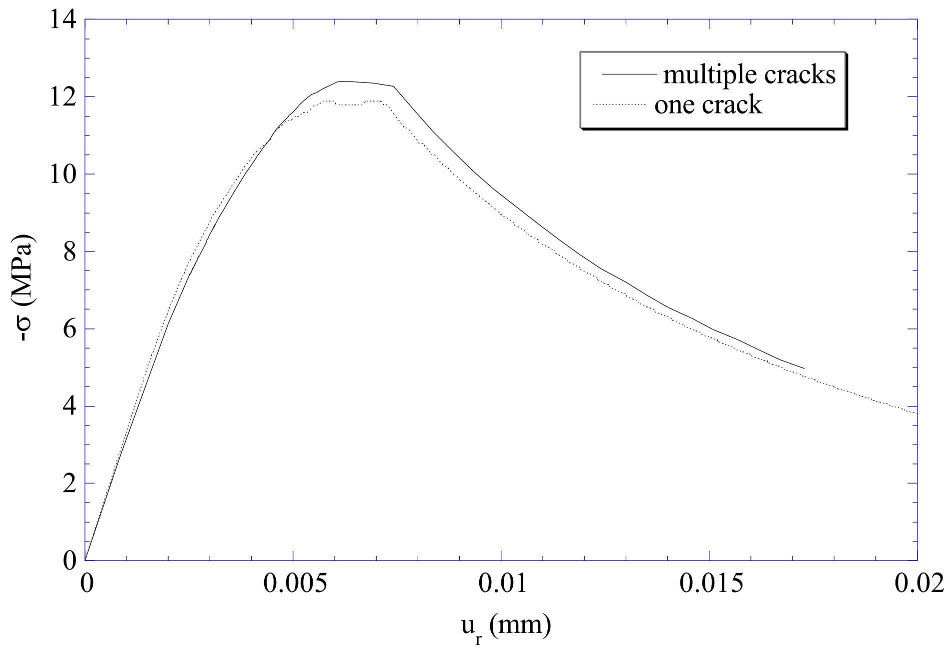
AKA bond cracks | AKA splitting cracks

- Surface structure failure of the FRP bar

QuickTime™ and a BMP decompressor are needed to see this picture.

Cohesive Zone Modeling of Splitting Cracks





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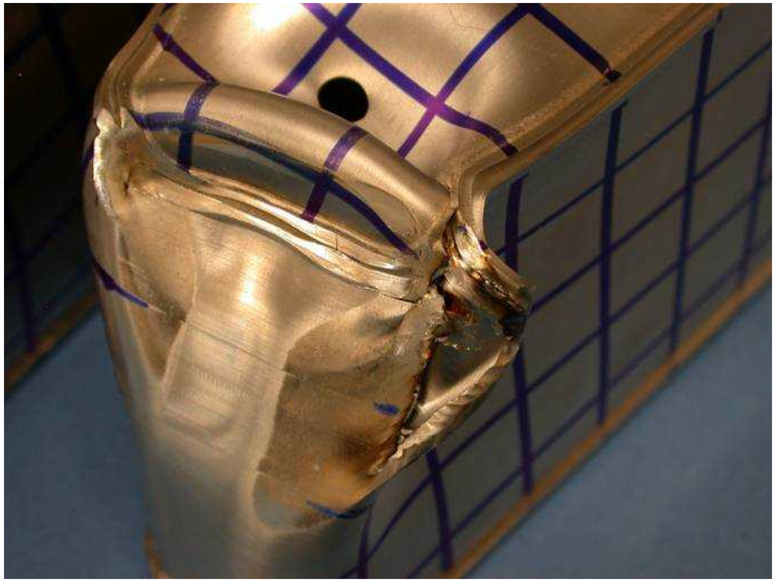

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Preview

- PUFEM Displacement Field Enrichment
  - General formulation
  - My path to enrichment
  - Analytical enrichment functions

PUFEM Displacement Field Enrichment

- Standard FEM
- PUFEM

Global displacement approximations

$$u(x) = \sum_{i=1}^{N_\Phi} \Phi_i(x) u_i$$

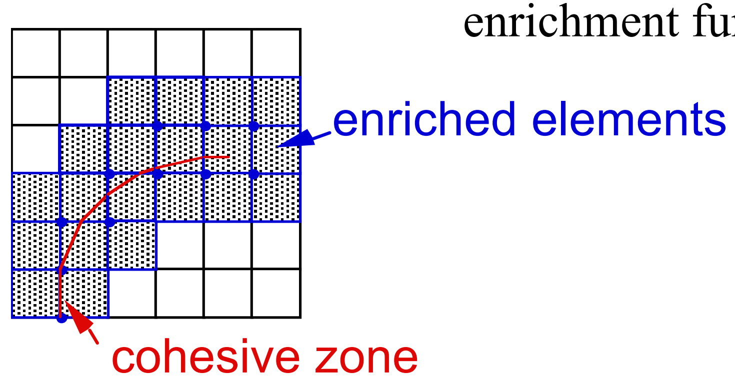
$$u(x) = \sum_{i=1}^{N_\Phi} \Phi_i(x) u_i + \sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_\Phi} \Lambda_j(x) \Phi_i(x) \alpha_{ij}$$

Element displacement approximations

$$u(x) = \sum_{i=1}^{N_N} N_i(x) u_i$$

$$u(x) = \sum_{i=1}^{N_N} N_i(x) u_i + \sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_N} \Lambda_j(x) N_i(x) \alpha_{ij}$$

enrichment functions



Path to Enrichment

Enrichment Functions: An Analytical Source

Muskhelishvili formalism

Hong & Kim (2003) obtained a series solution to the inverse problem

Zhang & Deng (2006) obtained asymptotic solutions

– both assumed linear elastic isotropic material (except for cohesive zone)

Additional analysis has been used to:

verify the proposed solutions

extend them for field variables required by the PUFEM

- Displacements

$$u_1 + iu_2 = \frac{1}{2\mu}\left\{\kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}\right\}$$

where φ and ψ are analytic functions, and $z = x+iy$.

- Another set of analytic functions simplify $u_{i,j}$ and σ_{ij} expressions

$$\Phi(z) = \varphi'(z) \qquad \Omega(z) = \left[z\varphi'(z) + \psi(z)\right]'$$

Enrichment Functions: An Analytical Source

- Displacement gradients

$$u_{1,1} + iu_{2,1} = \frac{1}{2\mu}\left[(\bar{z} - z)\overline{\Phi'(z)} + \kappa\Phi(z) - \overline{\Omega(z)}\right]$$

$$u_{2,2} - iu_{1,2} = \frac{1}{2\mu}\left[(z - \bar{z})\overline{\Phi'(z)} + \kappa\Phi(z) + \overline{\Omega(z)} - 2\overline{\Phi(z)}\right]$$

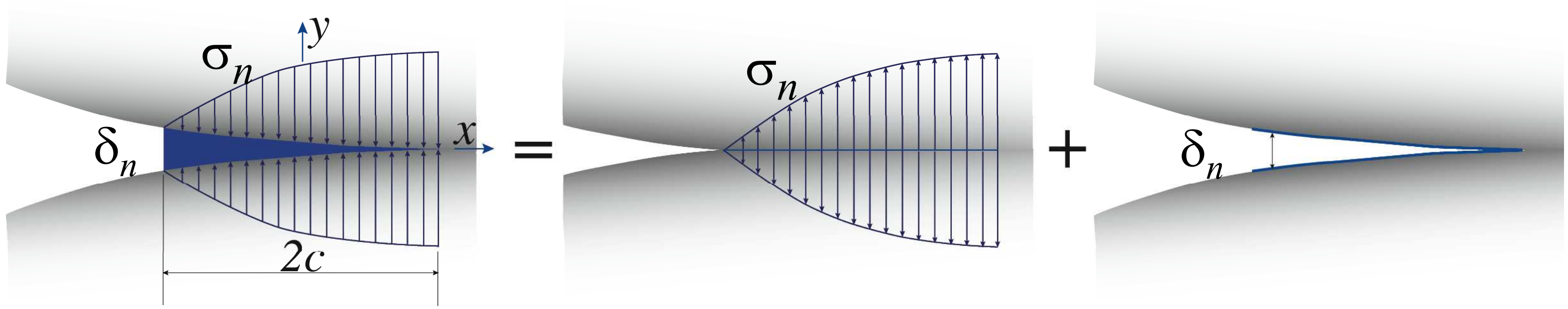
- Stress components

$$\sigma_{11} + i\sigma_{12} = (\bar{z} - z)\overline{\Phi'(z)} + \Phi(z) - \overline{\Omega(z)} + 2\overline{\Phi(z)}$$

$$\sigma_{22} - i\sigma_{12} = (z - \bar{z})\overline{\Phi'(z)} + \Phi(z) + \overline{\Omega(z)}$$

Enrichment Functions: An Analytical Source

- Super-position of two solutions yields a convenient solution form



- Analytic functions

$$\Phi(z)=\frac{1}{2}\left[\sqrt{z+c}F(z)-\sqrt{z-c}G(z)+H(z)\right]$$

$$\overline{\Omega}(z)=\frac{1}{2}\left[\sqrt{z+c}F(z)-\sqrt{z-c}G(z)-H(z)\right]$$

where F, G, and H are *entire* (analytic over the whole domain)

Enrichment Functions: An Analytical Source

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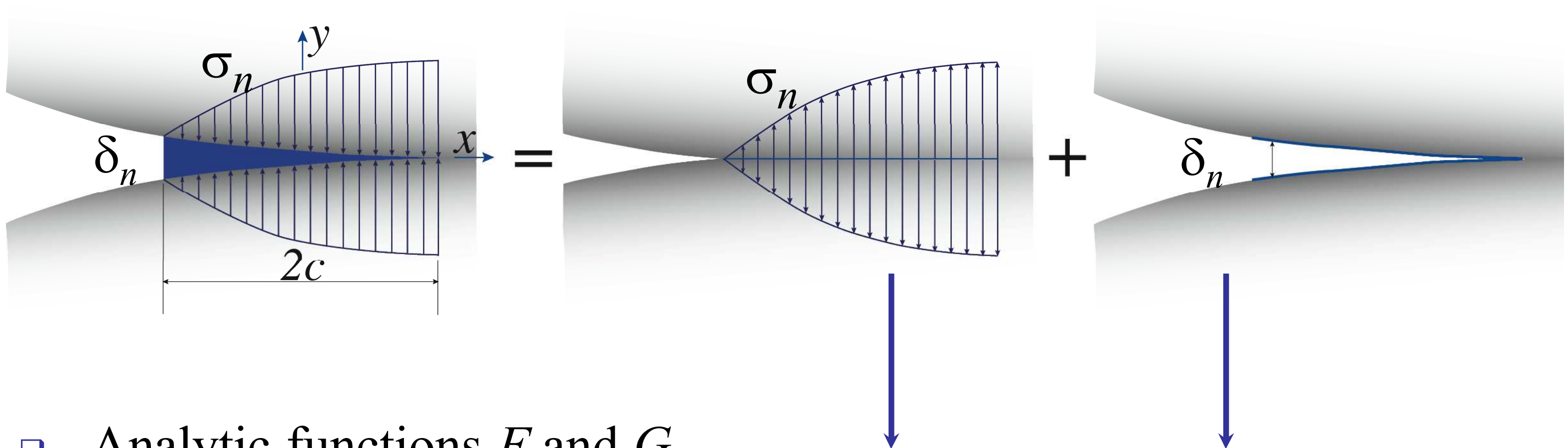


- Analytic functions F and G

$$F(z/c) = \sum_{n=0}^{N} A_n U_n(z/c) \qquad G(z/c) = \sum_{n=0}^{N} B_n U_n(z/c)$$

where A_n and B_n are complex coefficients and
U_n are Chebyshev polynomials of the second kind

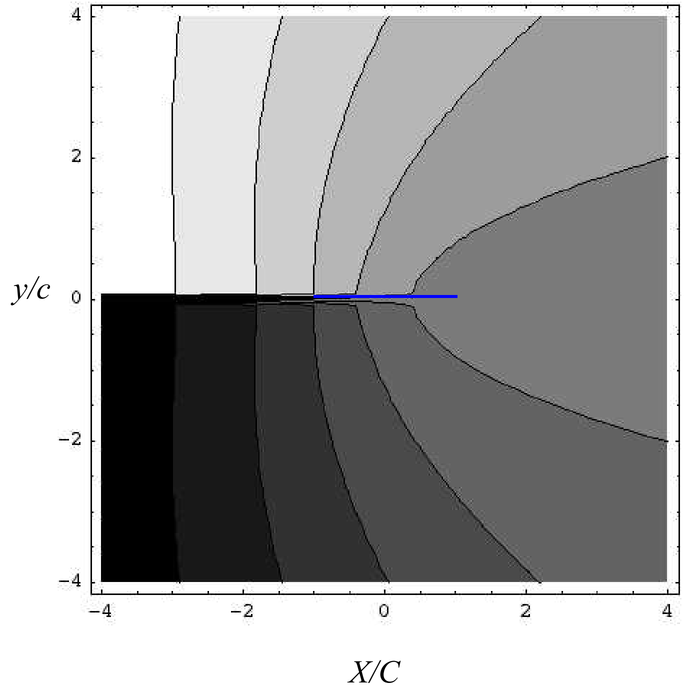
Enrichment Functions: An Analytical Source

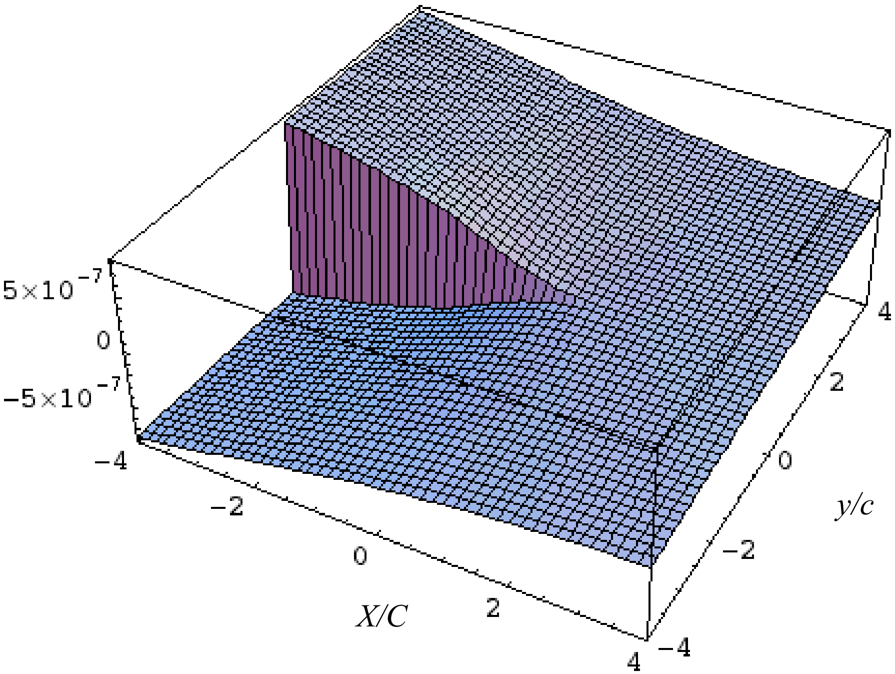
- First term considered
  - $F(z)=G(z)=1$
  - $H(z)=0$

Problem for plots

$E=10^7$ psi, $\nu=0.3$

- u_2





Enrichment Functions: An Analytical Source

- First term considered
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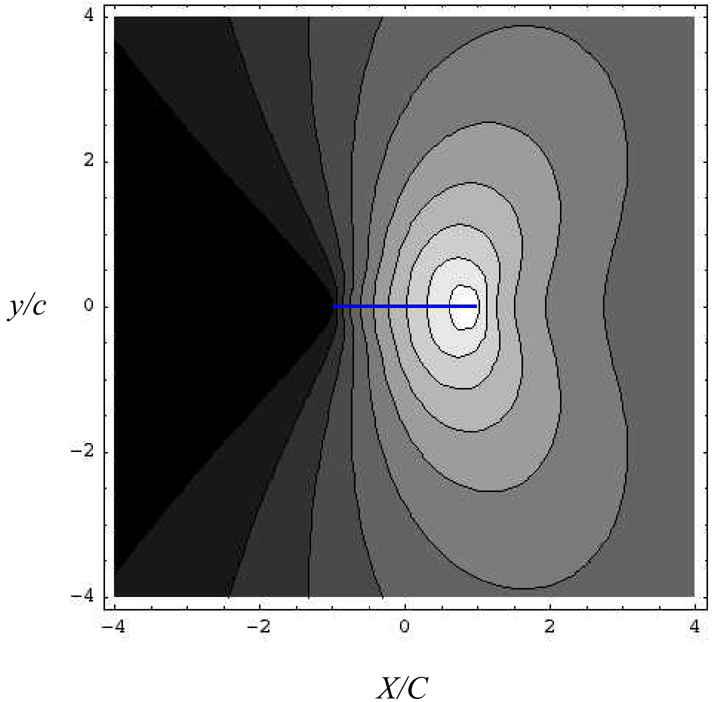
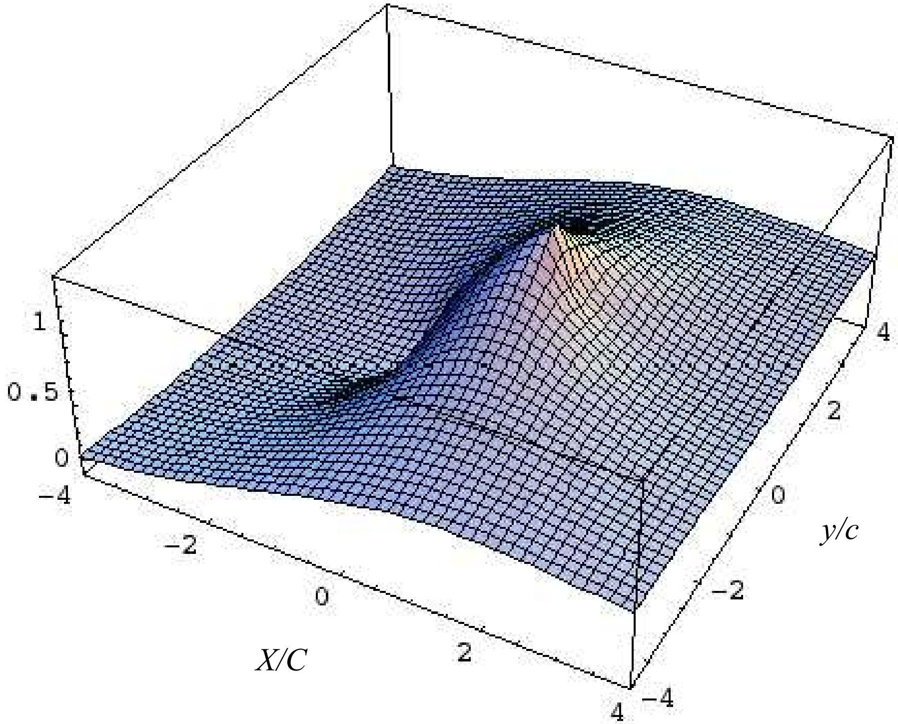
Problem for plots

$E=10^7$ psi, $\nu=0.3$

- σ_{22}





Enrichment Functions: An Analytical Source

- First term considered
  - $F(z)=G(z)=1$
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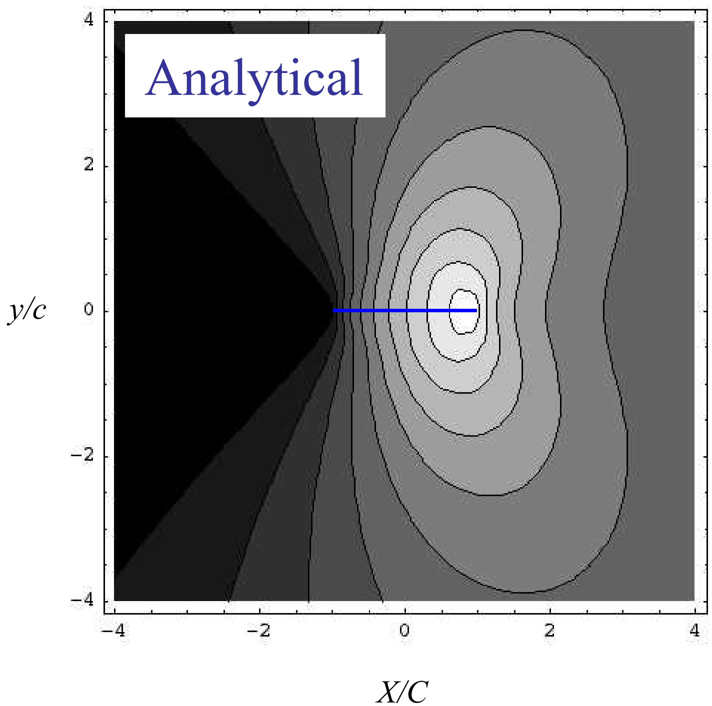
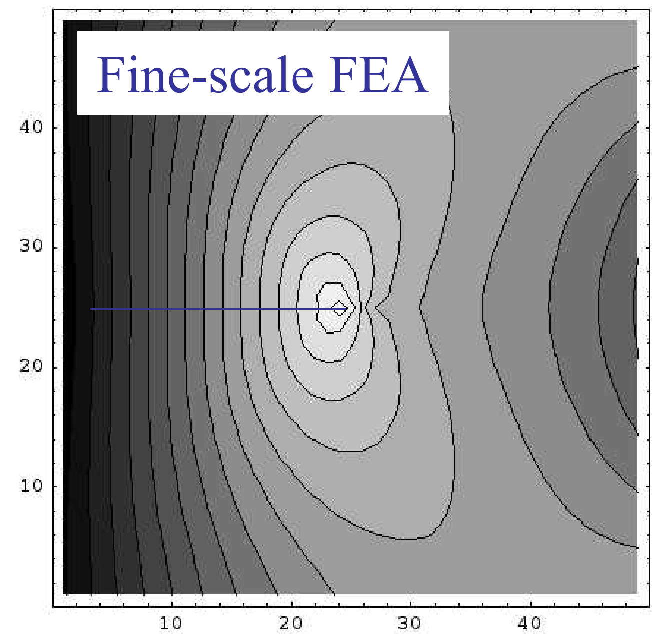
Note: problems differ and CZ sizes are not to the same scale.

- Qualitative comparison of σ_{22} with fine-scale FEA





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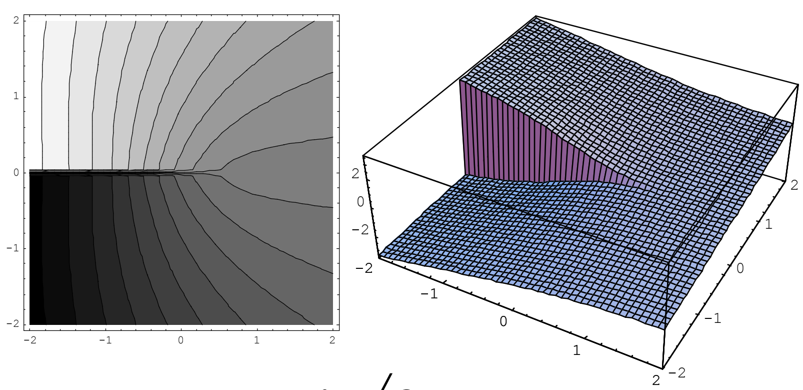
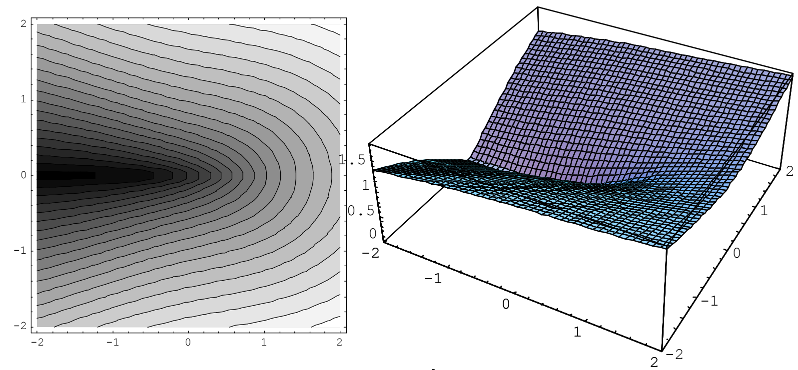
Enrichment Functions

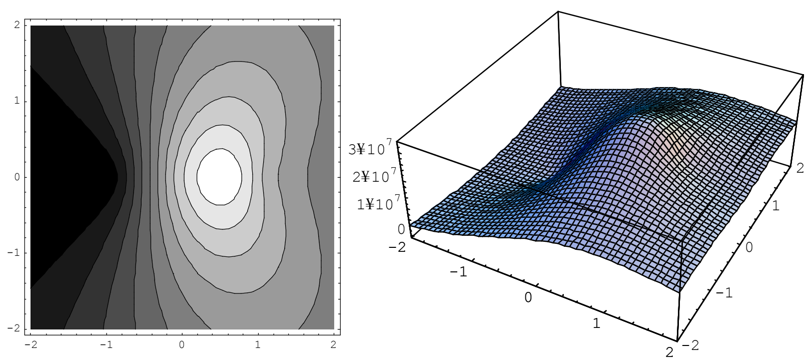
- Based upon the asymptotic solutions of Zhang & Deng



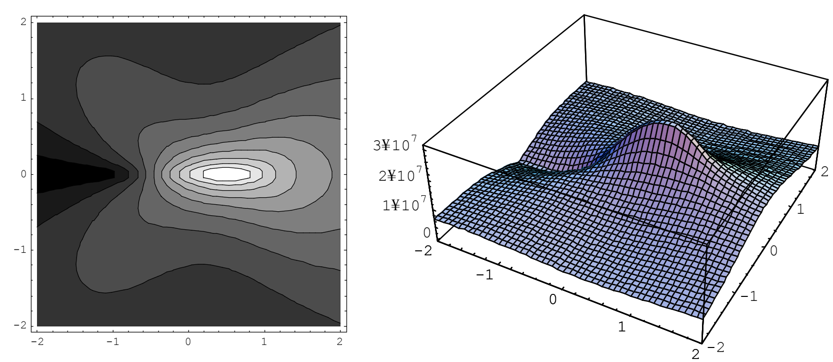
u_2/c



u_1/c



σ_{22}/c



σ_{11}/c

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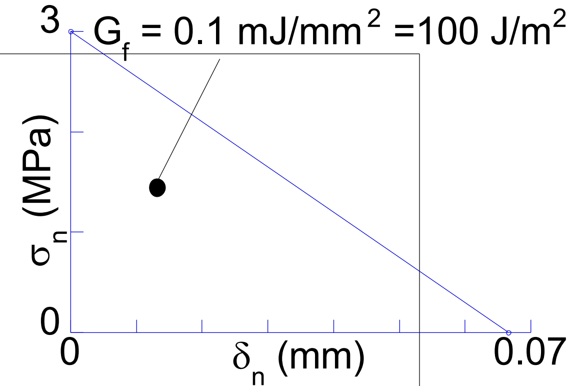
Preview

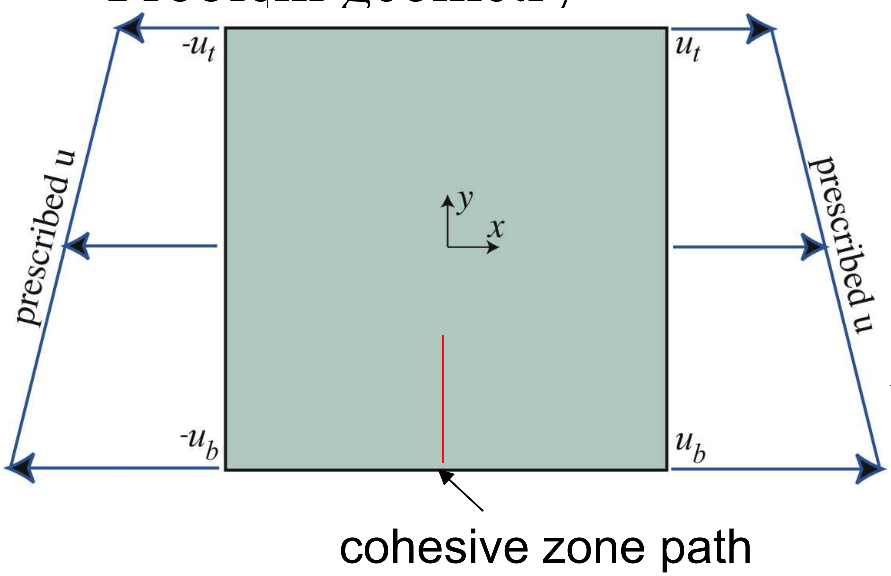
- Results for Model Problems
  - Simple model problems & Meshes
  - Example showing how enrichment->crack
  - Results for aligned meshes
  - Results for skewed meshes

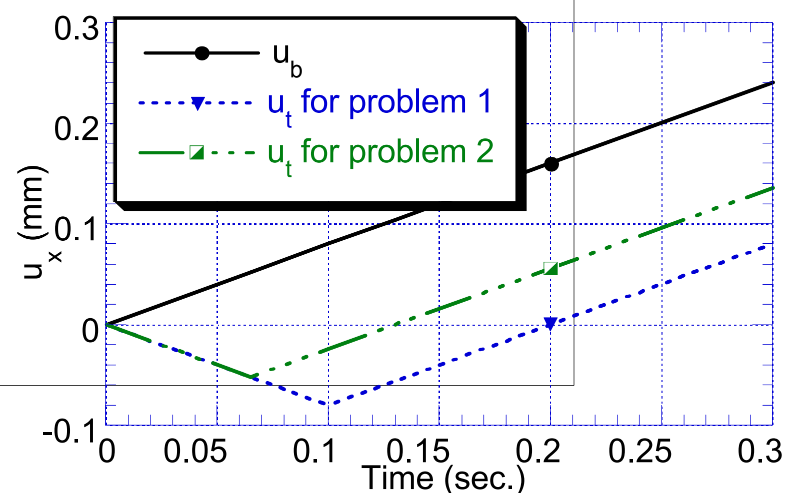
Initial Simple Test Problems

- Concrete test problems
  - relevant to HDBT
  - domain 1 m x 1 m
  - process-zone size ~ O(250 mm)
  - representative concrete tensile properties (except for simplified linear softening)
  - mode I quasistatic crack propagation



Problem geometry





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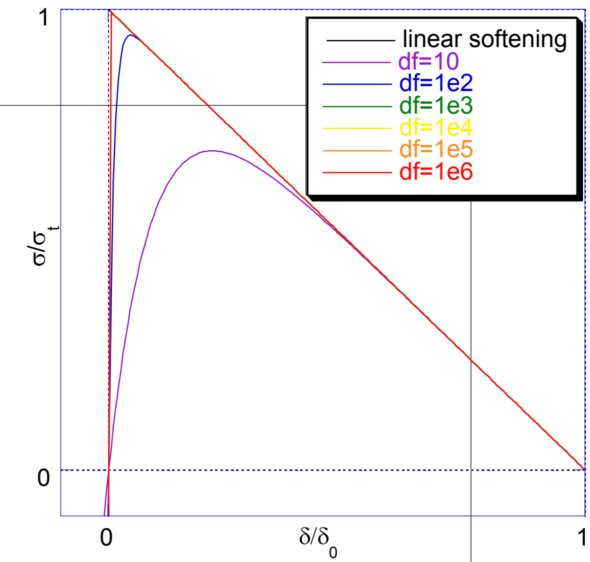

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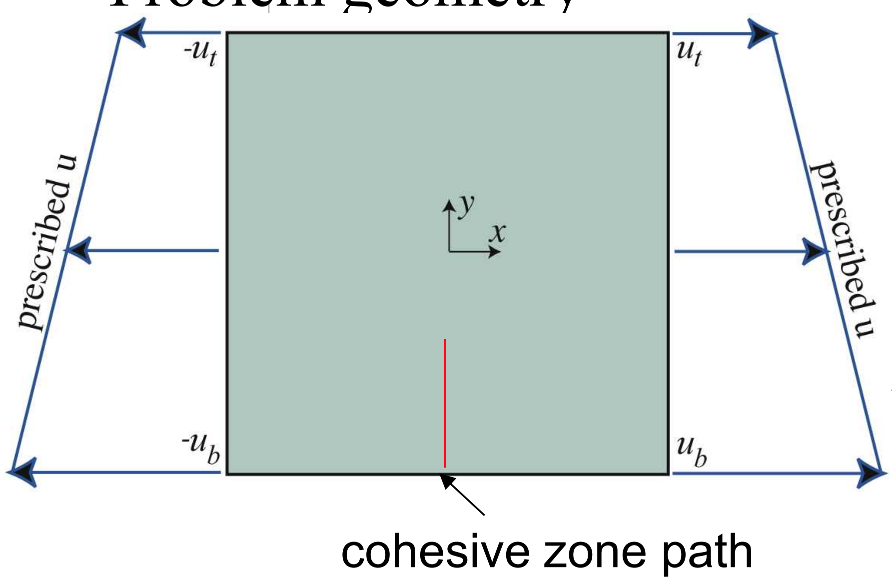

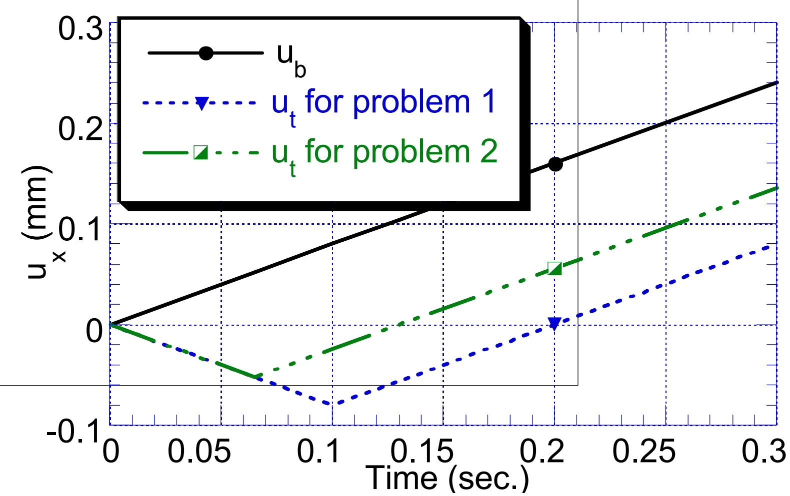

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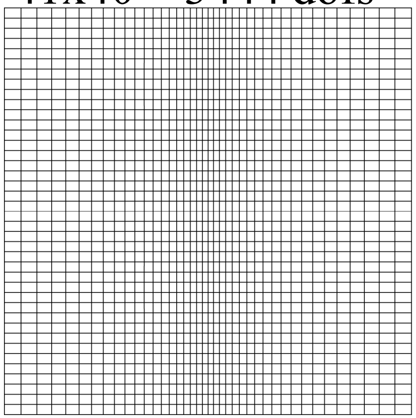
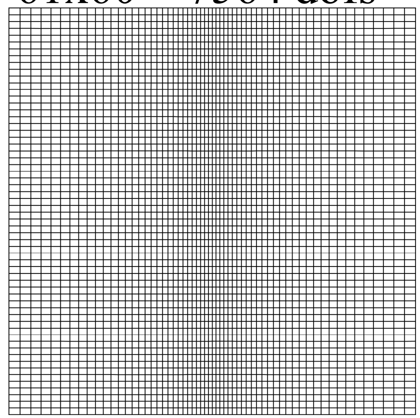
Spacial Discretizations

- Fine FEM meshes – accurate reference solution

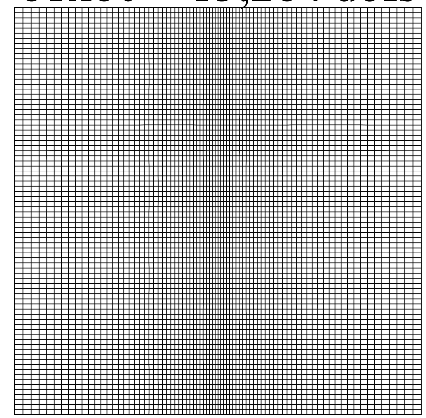
41x40 ~ 3444 dofs



61x60 ~ 7564 dofs

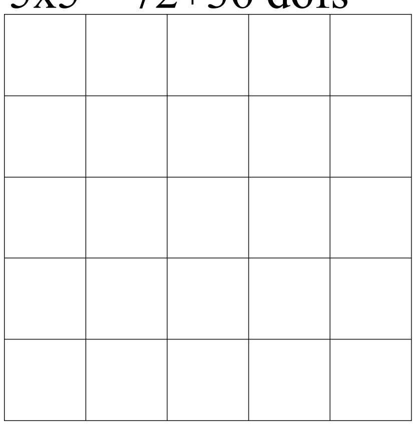


81x80 ~ 13,284 dofs

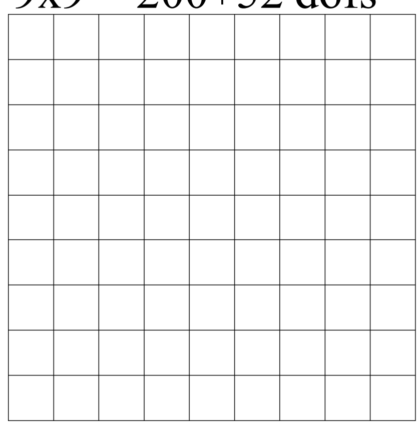


- PUFEM – Aligned Meshes

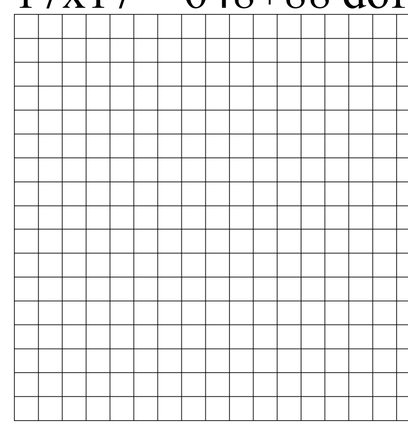
5x5 ~ 72+36 dofs



9x9 ~ 200+52 dofs

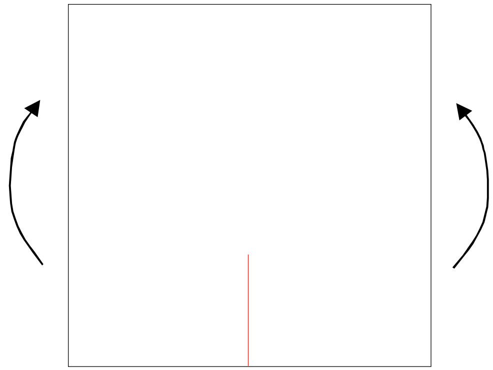


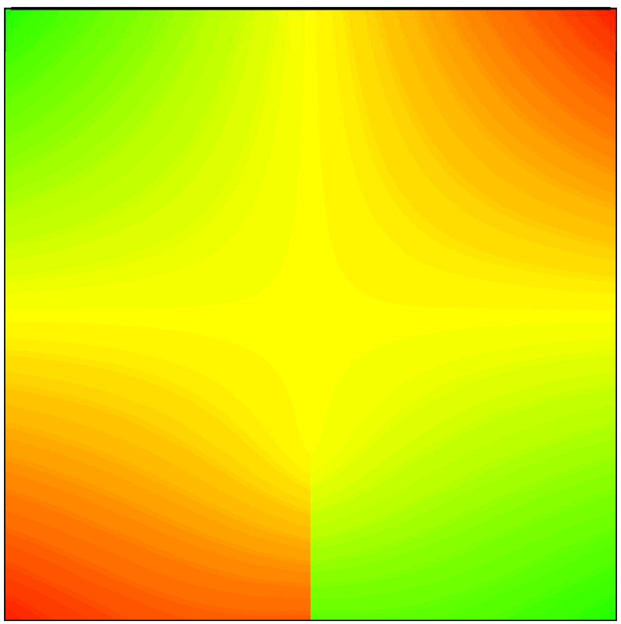
17x17 ~ 648+88 dofs



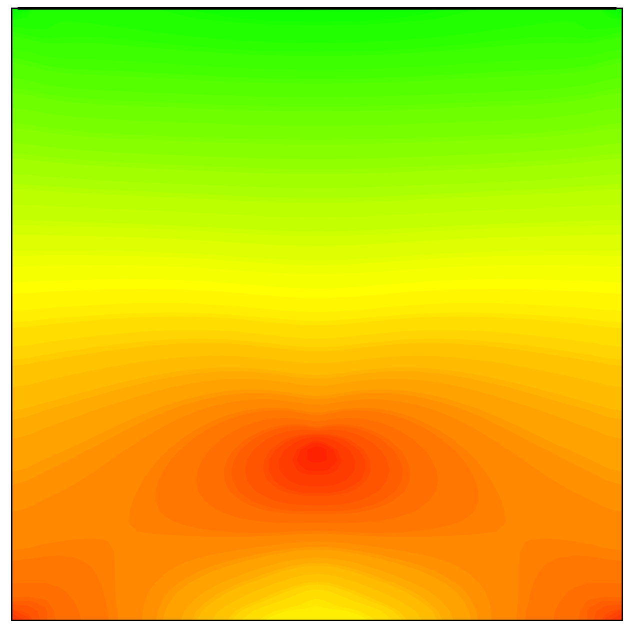
PUFEM Displacement Field Enrichment

Example Problem:
concrete 1 m x 1 m
in bending





fine-scale FEM solution: u_x



fine-scale FEM solution: σ_{xx}

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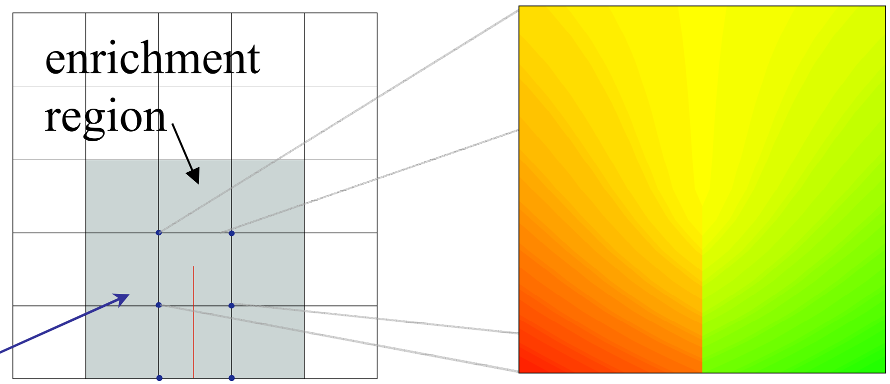
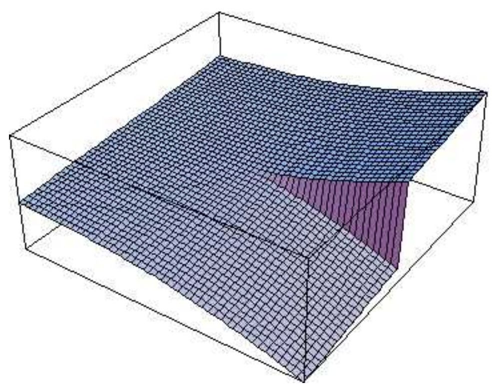
Example response in the "tip-element"

Fine-scale



Surface plot view

u_x for tip-element

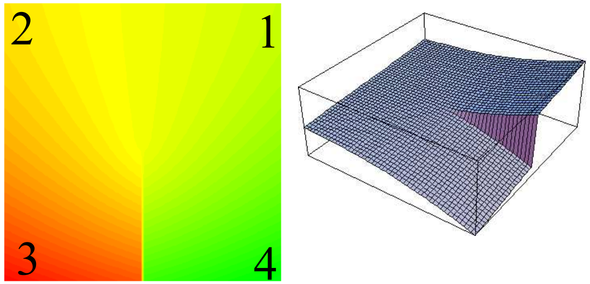


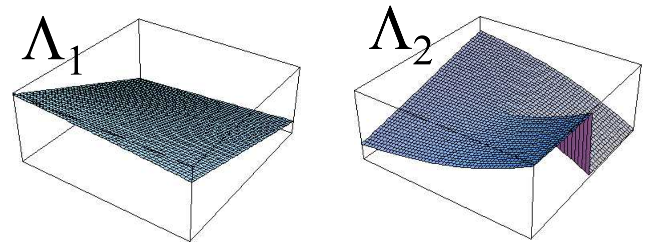
PUFEM

u_x for tip-element

Example enrichment in the “tip element”

$$u(x) =$$

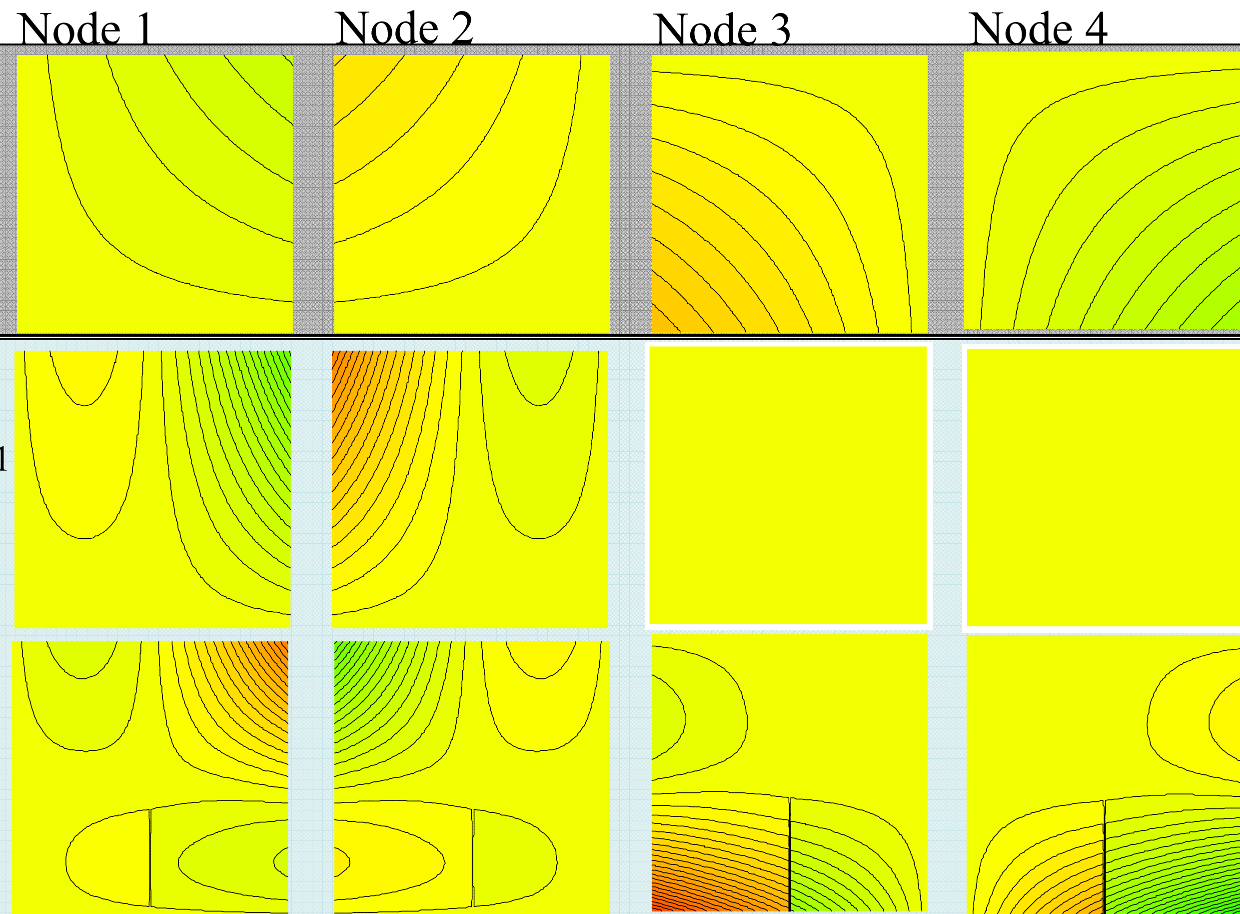




$$\sum_{i=1}^{N_N} N_i(x) u_i$$

$$\sum_{i=1}^{N_N} \Lambda_1(x) N_i(x) \alpha_{i1}$$

$$\sum_{i=1}^{N_N} \Lambda_2(x) N_i(x) \alpha_{i2}$$

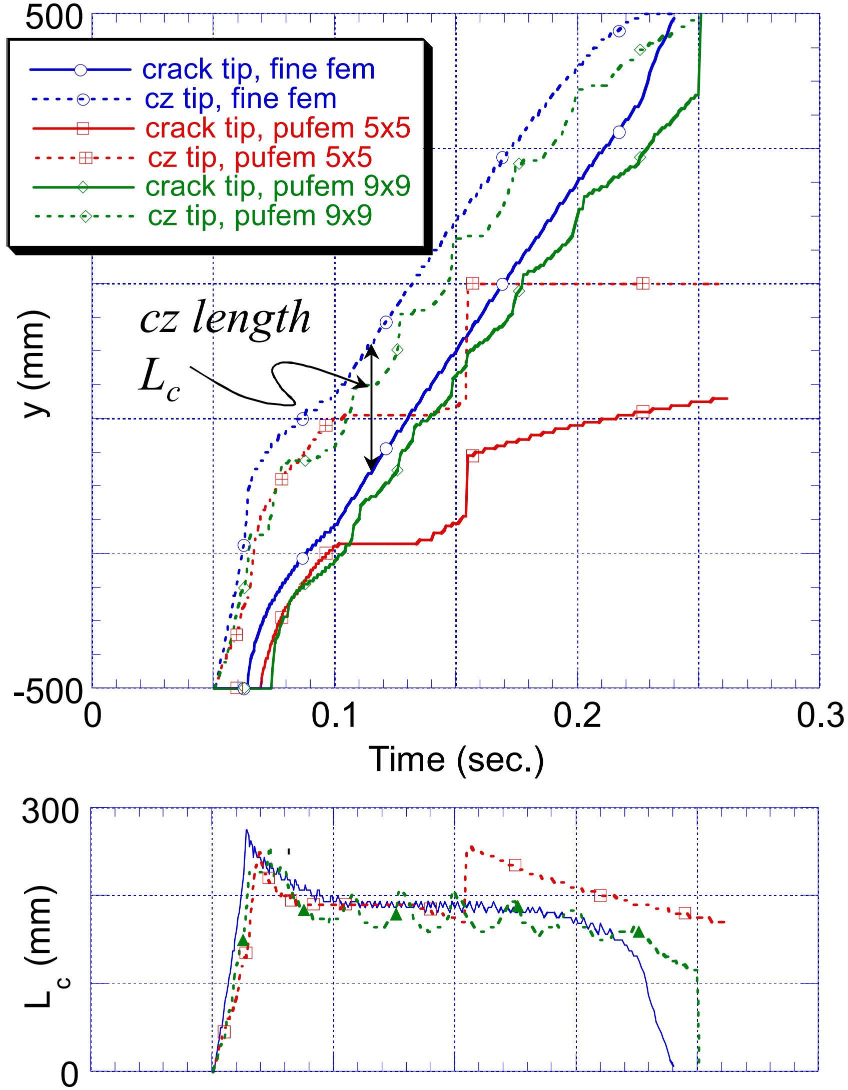


Extremes & Length History

F&G enrichment functions

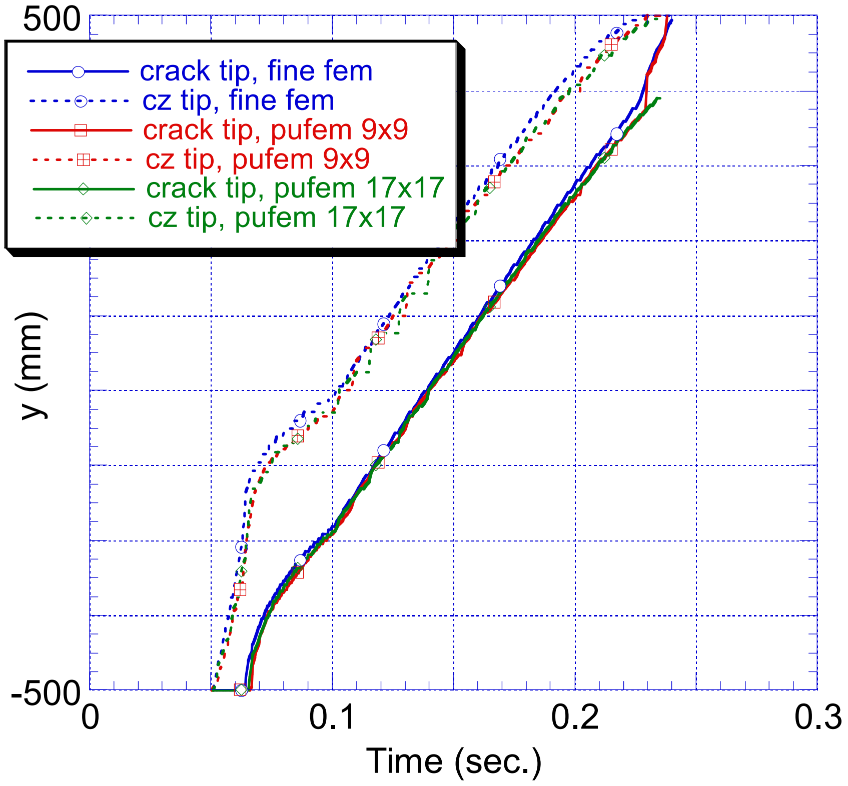
2 PUFEM meshes
c = 125 mm

Grid lines represent element spacing in the coarsest mesh.



Extremes Histories

aligned meshes with the λ enrichment functions



Grid lines represent element spacing in the coarsest mesh.

c = 50 mm

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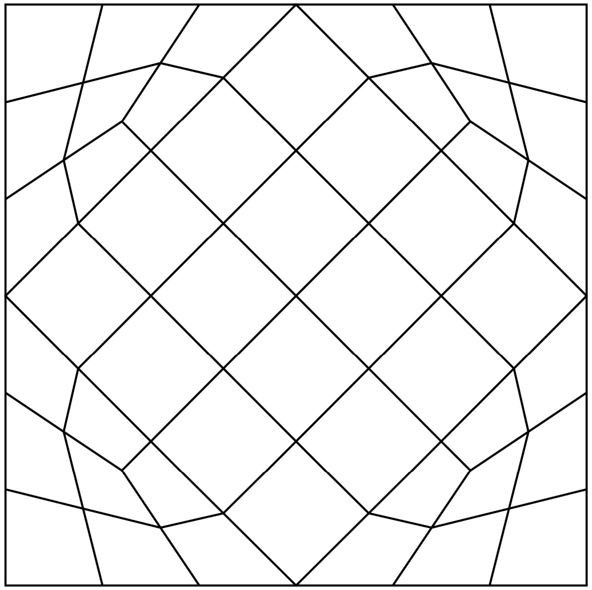
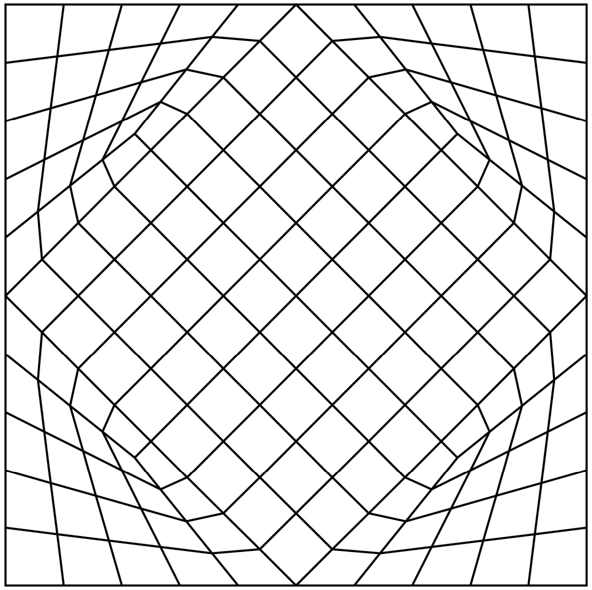
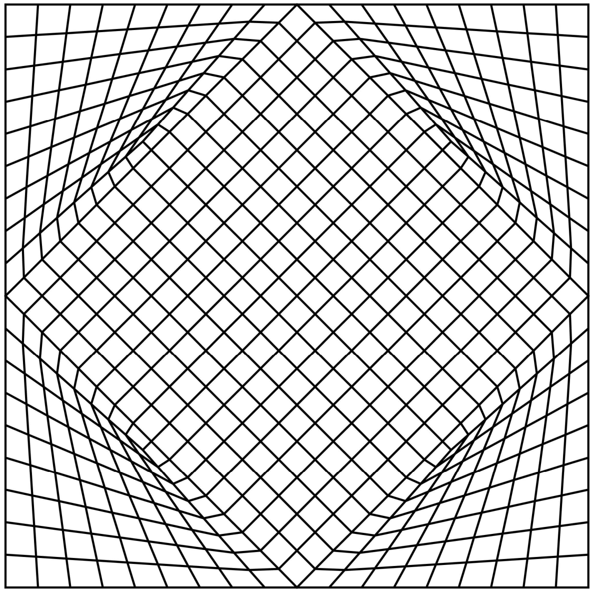
PUFEM Skewed Mesh Tests



4x4 @ 45°

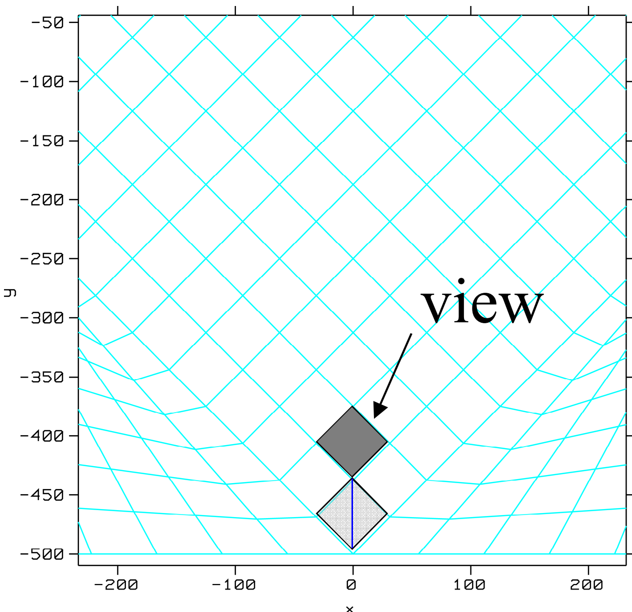


8x8 @ 45°

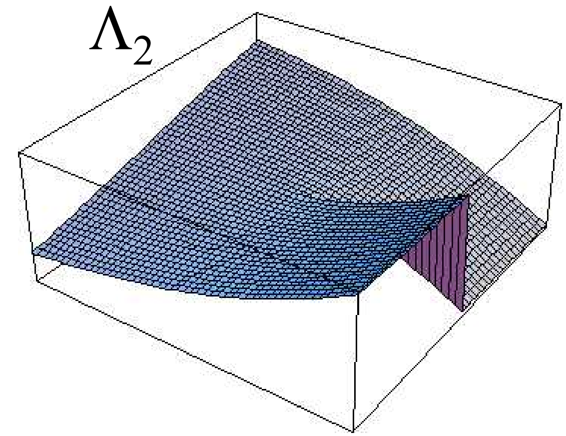


16x16 @ 45°

Skewed-mesh: Results and Enrichment

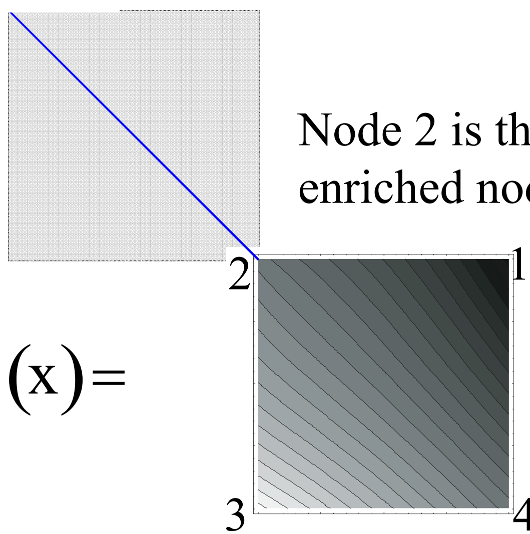


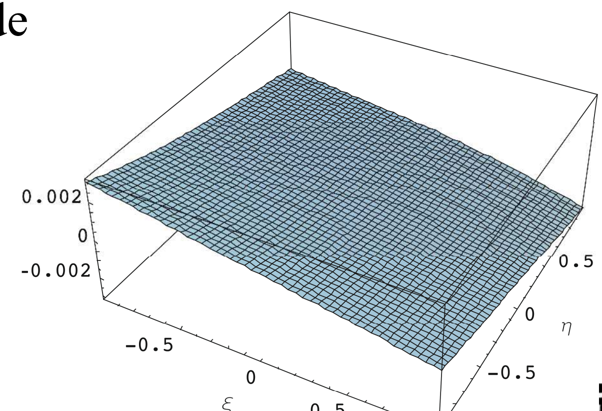
Using a single enrichment Function (G=1)



Node 2 is the only enriched node

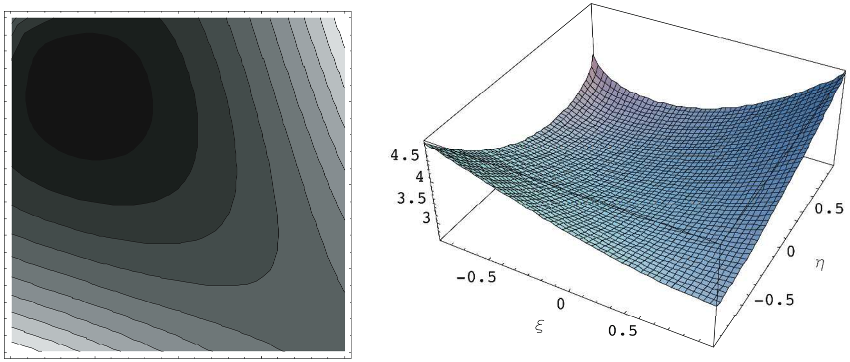
$u_1(x) =$



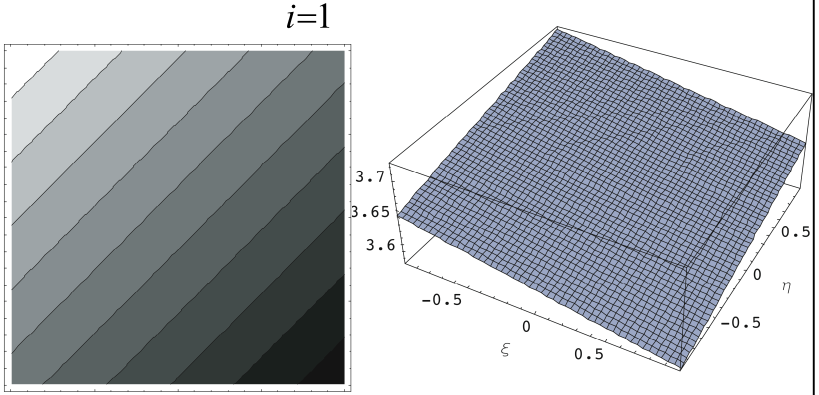


Skewed-mesh: Enrichment

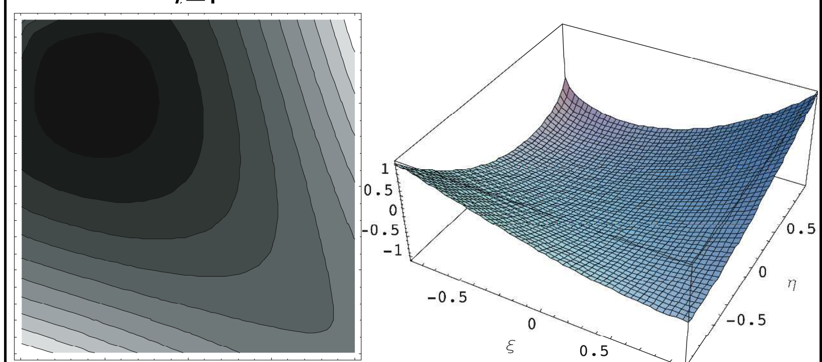
$$\sigma_{11}(\mathrm{x})=$$



$$\sigma_{11}(\mathrm{x})=\sum_{i=1}^{N_N}\mathrm{N}_i(\mathrm{x})\mathrm{u}_i+\sum_{i=1}^{N_N}\Lambda_2(\mathrm{x})\mathrm{N}_i(\mathrm{x})\alpha_{i2}$$



"within 2% of being flat"



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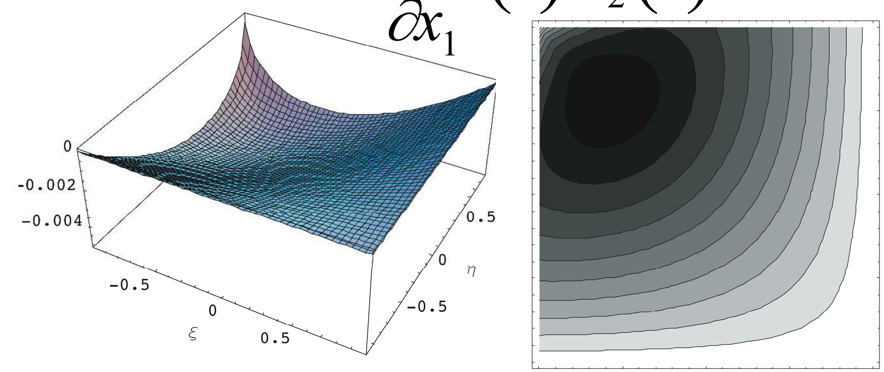
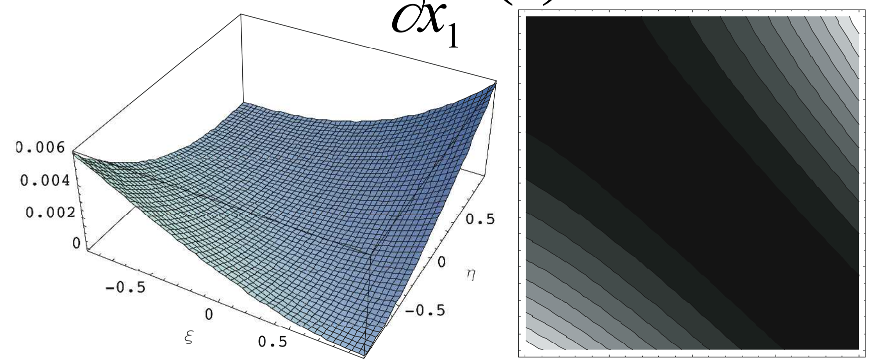
Skewed-mesh: Enrichment

Enrichment contribution to $\varepsilon_{11}(\mathrm{x}) = \frac{\partial}{\partial x}\left[\sum_{i=1}^{N_N} \Lambda_2(\mathrm{x})\mathbf{N}_i(\mathrm{x})\alpha_{i2}\right]$

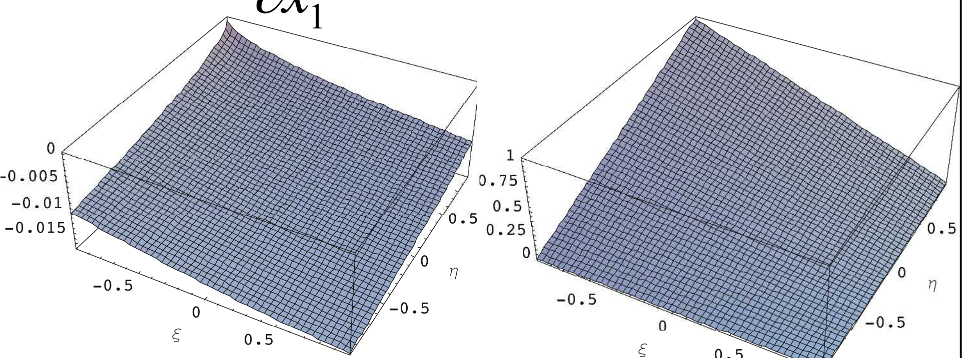
$\alpha_{22} < 0$ for crack opening

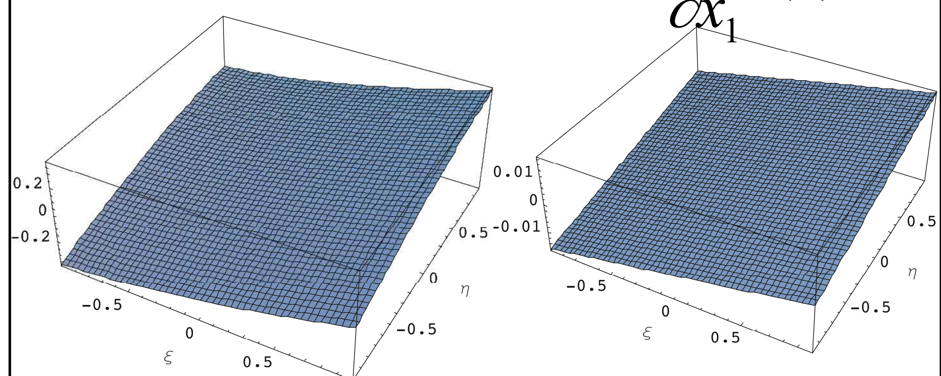
$-\frac{\partial \Lambda_2}{\partial x_1}(\mathrm{x})N_2(\mathrm{x})$ + $-\Lambda_2 \frac{\partial N_2}{\partial x_1}(\mathrm{x})$





$-\frac{\partial \Lambda_2}{\partial x_1}(\mathrm{x}) \times N_2(\mathrm{x})$ + $-\Lambda_2 \times \frac{\partial N_2}{\partial x_1}(\mathrm{x})$

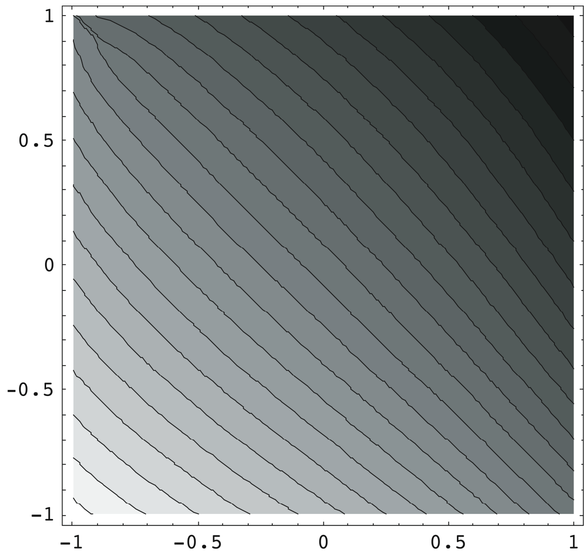


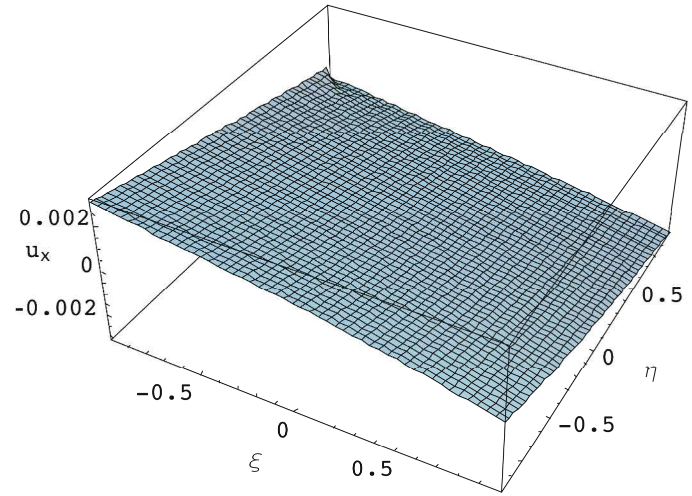


Skewed mesh: Results

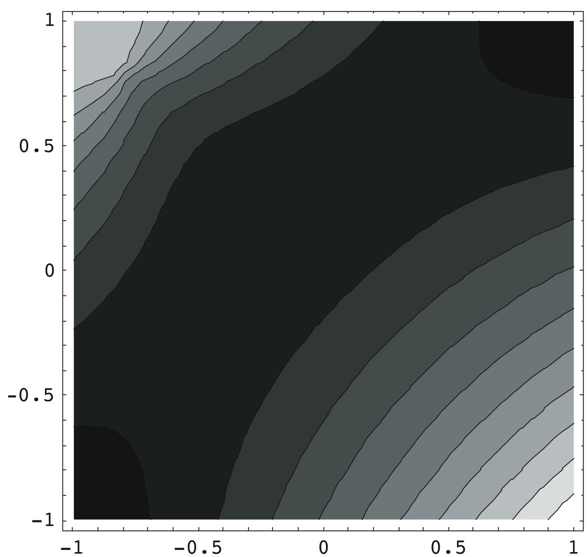
Results for the tip 1/10 of the way through the second element

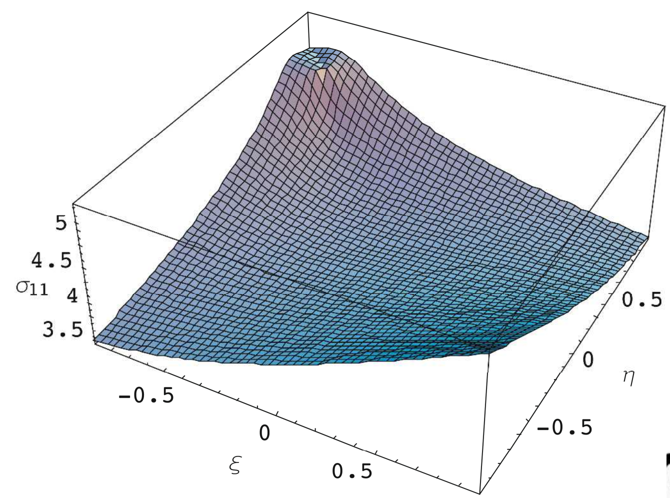
u_1





σ_{11}





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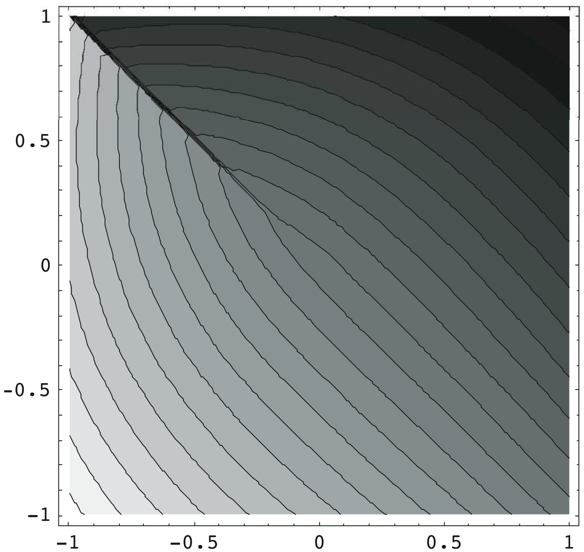
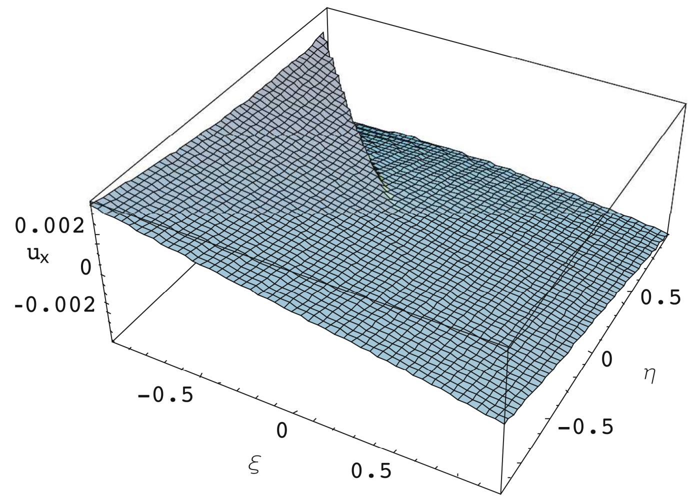
Skewed mesh: Additional Results

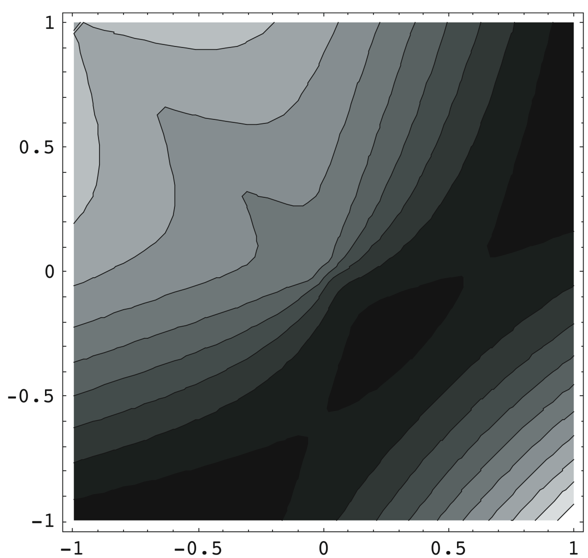
Results for the tip midway through the second element

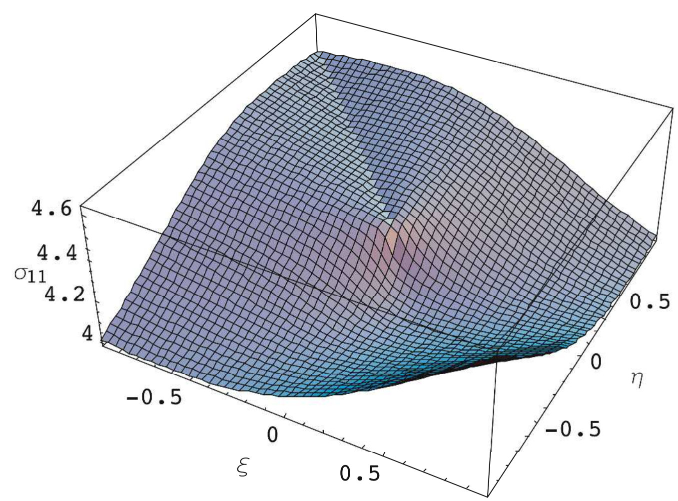
u_1





σ_{11}





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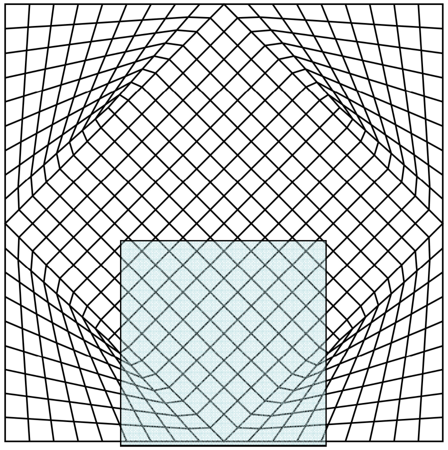
Neighborhood Enrichment

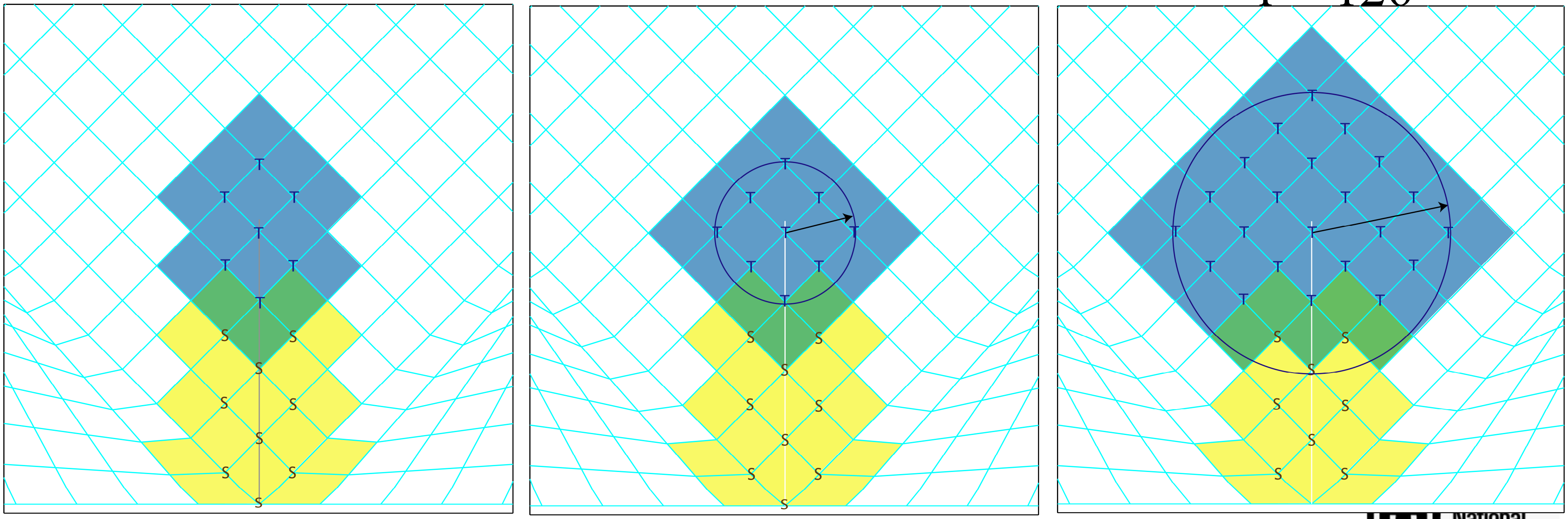
Aka the Mr. Roger's modification



Enriches additional nodes within a user-defined neighborhood of the tip.

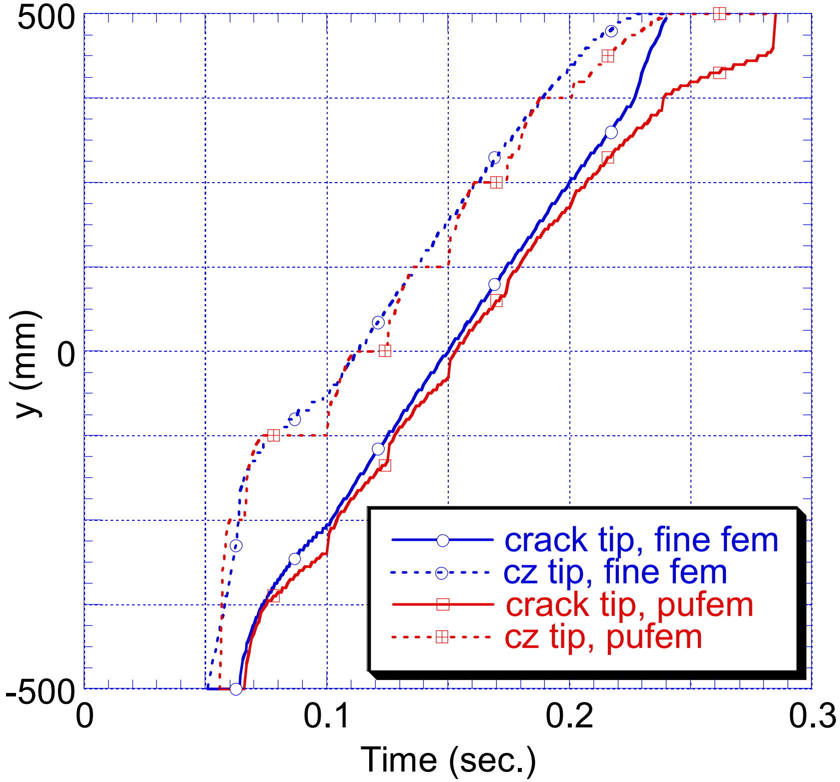
Done each time the tip enters a new element.

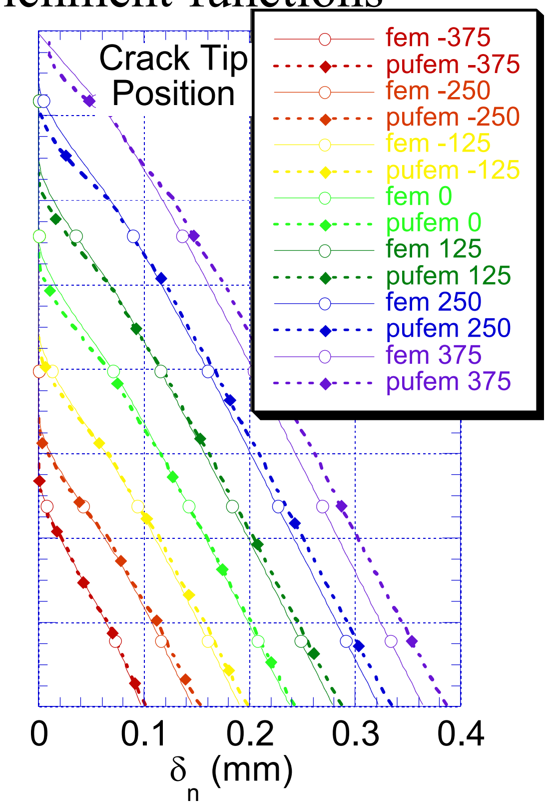
r = 0 r = 63 r = 126

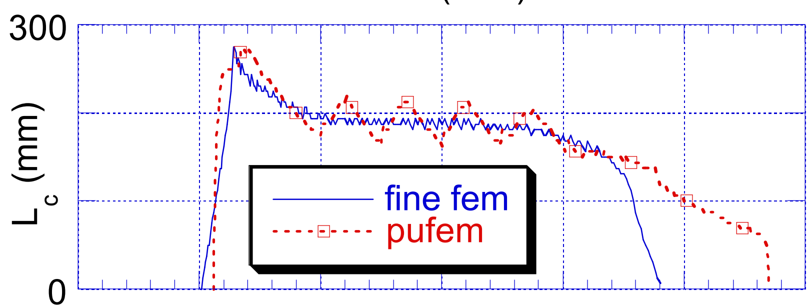


Extremes & Length Histories

skewed meshes with the λ enrichment functions







8x8 mesh, c = 75 mm

Intersect enrichment

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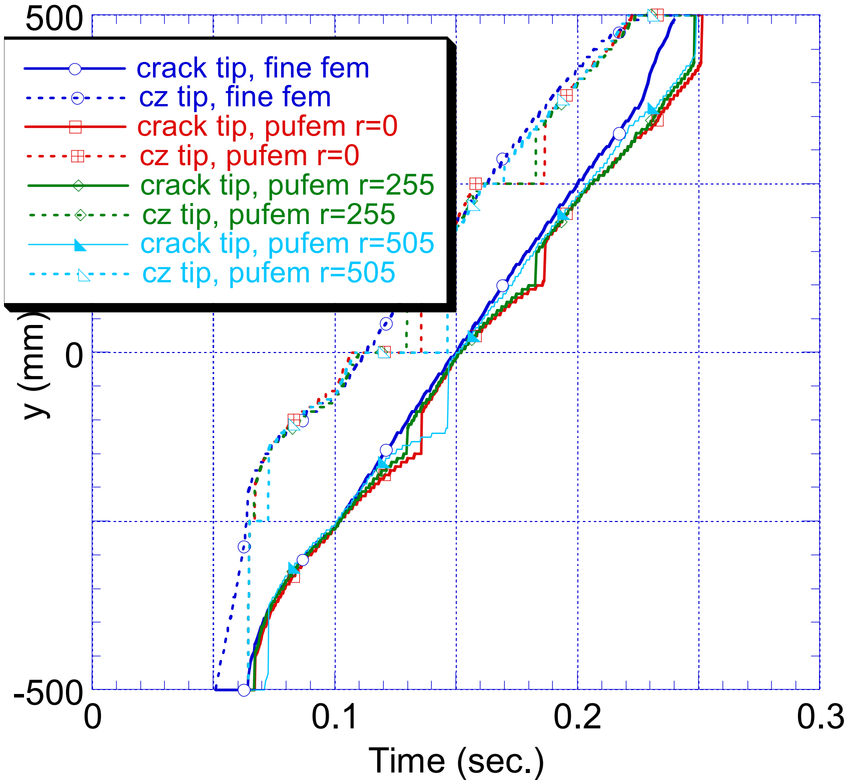
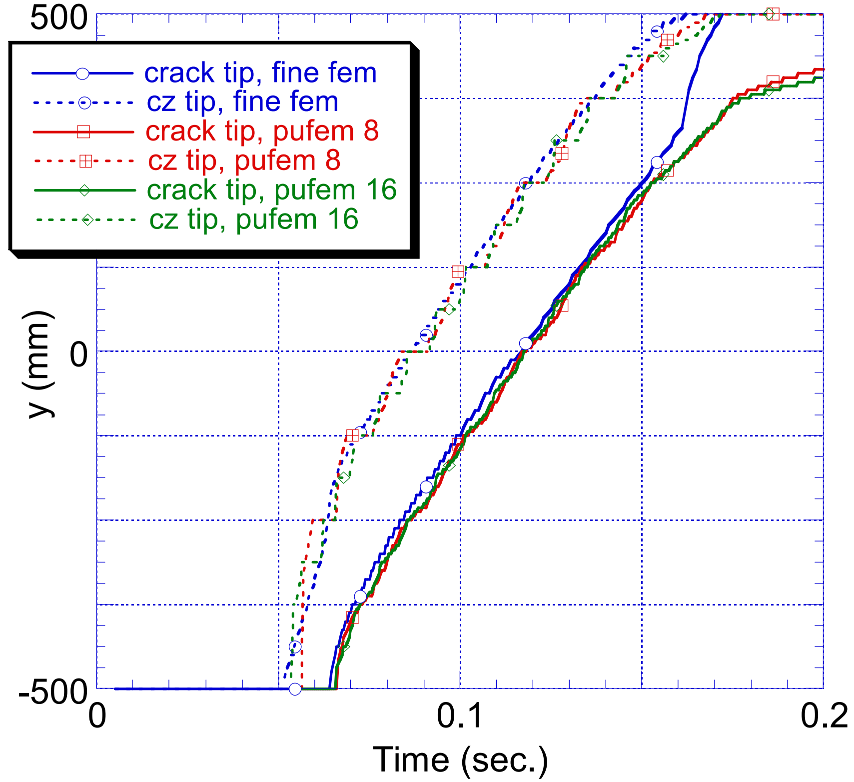
Extremes Histories

skewed meshes with the λ enrichment functions





4x4 mesh, c = 75 mm
Neighborhood enrichment
Average deviations: 29 mm for r=0
20 mm for r=255 mm
19 mm for r=505 mm

Problem 2
c = 75 mm
Intersect enrichment

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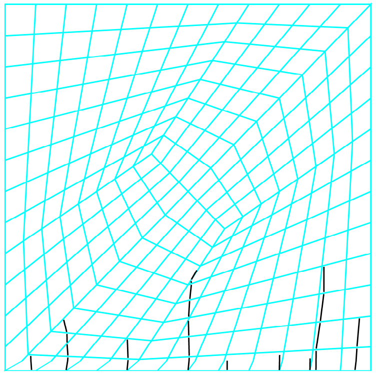
Ongoing and Future Work

- Multiple cracks -- stress relief in quasistatic propagation – cracks “compete”



- Mixed-mode cracking
- Enrichment function applicability
  - Inelastic materials
  - Inhomogeneous materials
  - Anisotropic materials
- 3D

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Observations & Conclusions

- Both forms of enrichment give good results for the model problem with aligned meshes.
- Other enrichment strategies can improve results but the added complexity may not be merited.
- Product form of enrichment has negative effects with a “coarse” skewed-mesh for F&G enrichment.
- λ-enrichment yields much better results for skewed meshes.
- Initial results are not very sensitive to *c*, but adjustment of *c* for the tip-functions may be necessary for some classes of problems.
- PUFEM is exhibiting convergence (with mesh refinement)
- PUFEM for cohesive zone modeling of localization has potential and merits further investigation.
- No free-lunch -- algorithm complexity is high.

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Acknowledgements

- Initial funding was provided by the Materials Directorate, Army Research Laboratory.
- Current funding is from the Engineering Science Research Foundation, Sandia National Laboratories.

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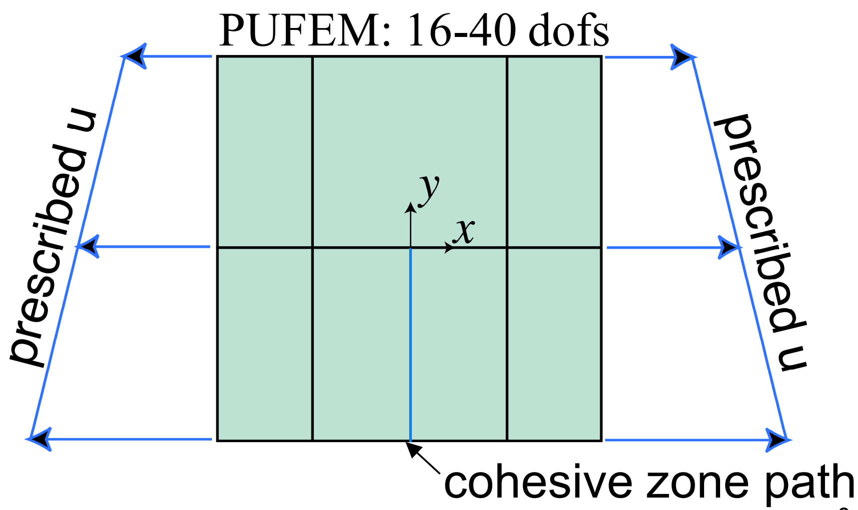
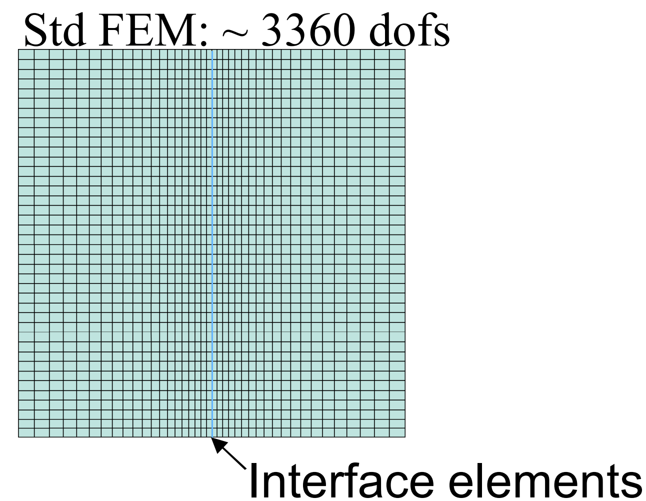
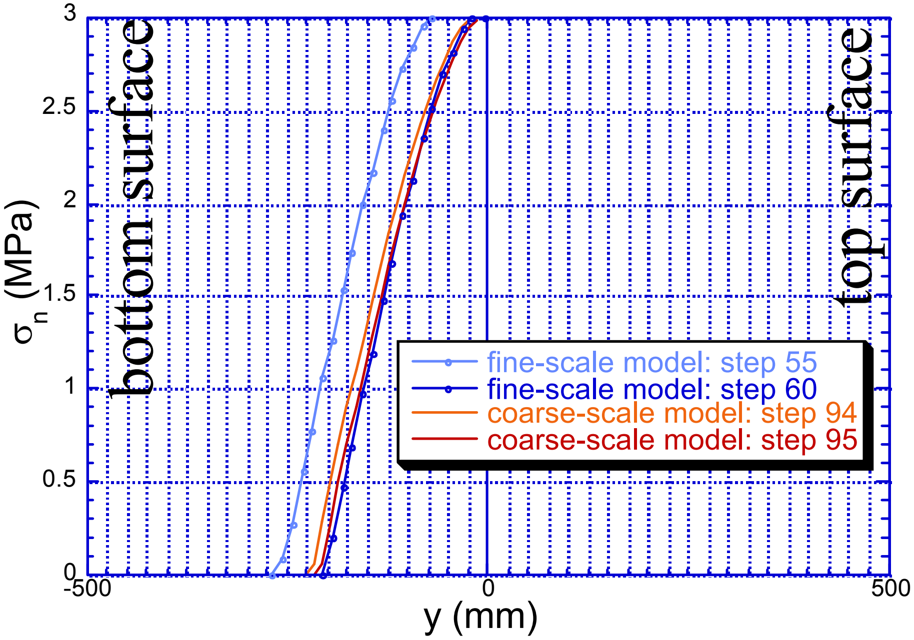
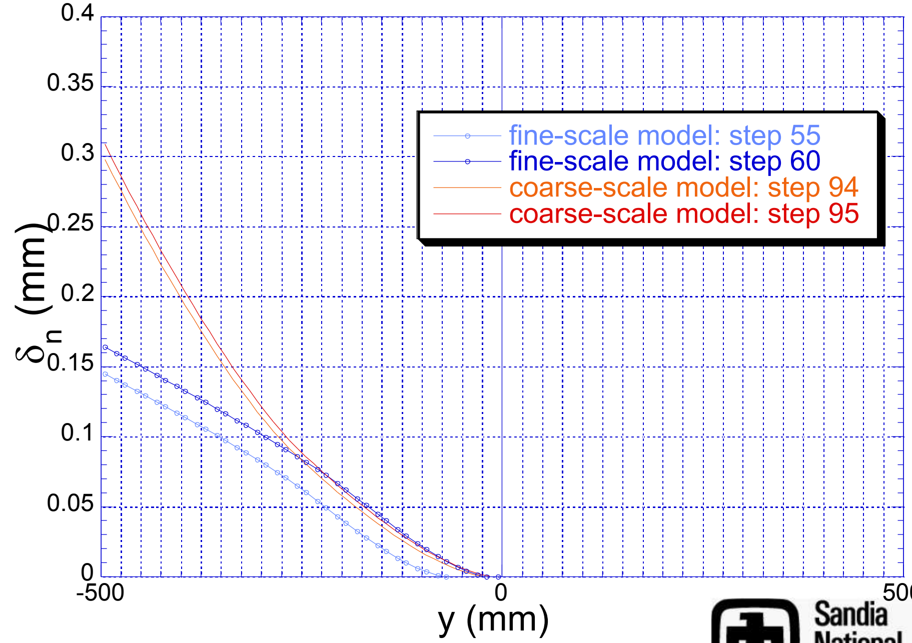
Backup material~~~~~~~~~~~~~~~

Coarse-scale *vs*. Fine-scale: Qualitative Evaluation









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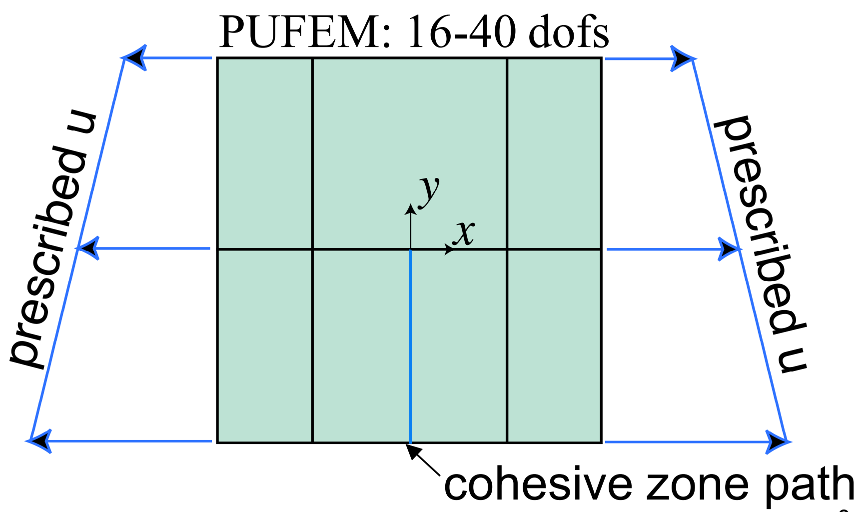
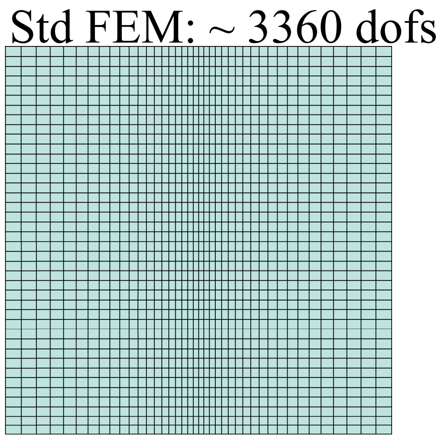
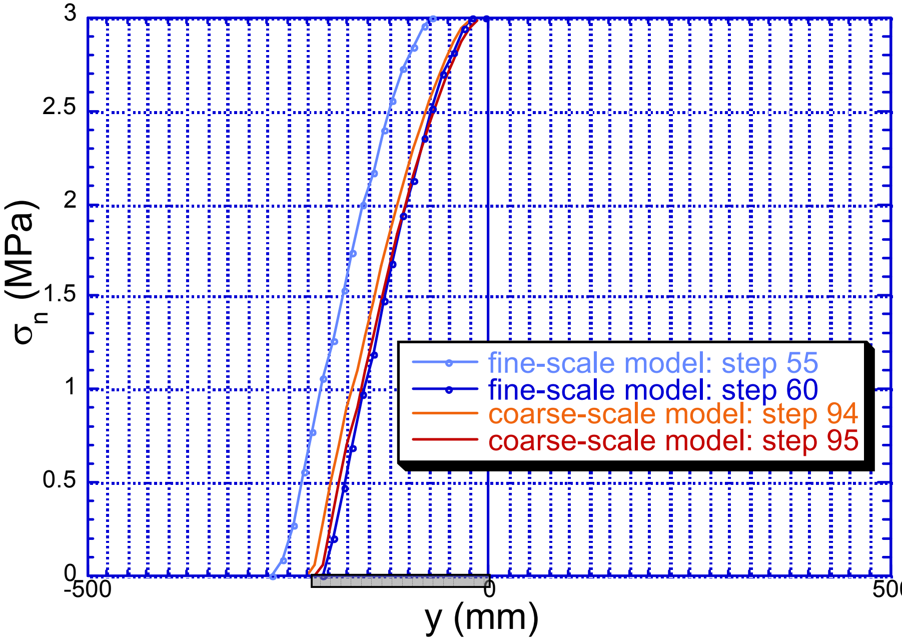
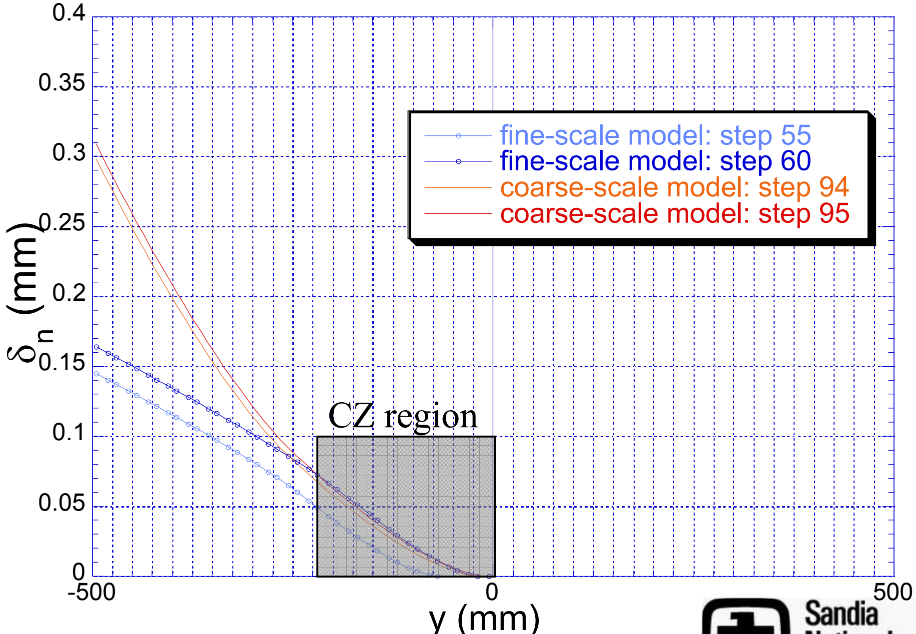
Coarse-scale *vs*. Fine-scale: Qualitative Evaluation







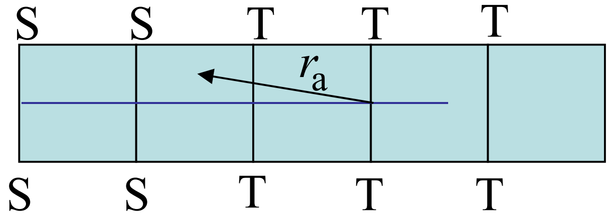


Initial PUFEM Issues

- Crack profiles differed significantly with fine-scale results in the traction free region.
- If several terms are needed to obtain better crack profiles the efficiency will be reduced.
- A length scale exists in the enrichment functions.

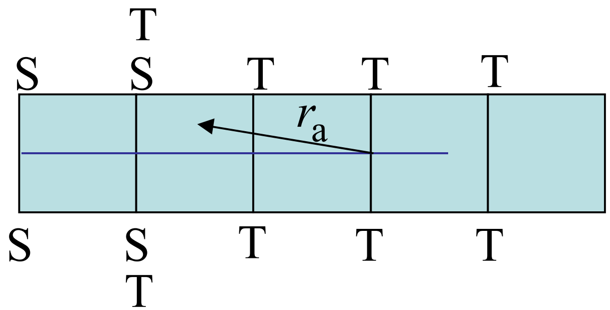
Enrichment Modification

- Change to step enrichment
- Analytical radius -- could be applied to the whole plane



T ~ tip enrichment

S ~ step enrichment



Fine *vs*. Coarse -- Cohesive Zone Response