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Exact results and field-theoretic bounds for randomly advected propagating fronts, and implications for turbulent combustion

Jackson Mayo and Alan Kerstein

Combustion Research Facility

Sandia National Laboratories, Livermore, CA

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There is no established theory of the turbulent burning velocity in fuel-air mixtures

- Measurements show wide scatter, indicating sensitivity to apparatus details
- Usual modeling strategy: Start from a generic idealization (e.g., homogeneous isotropic flow, constant density), then add complicating details empirically
- Key obstacle: Even for idealized problems, neither a consensus on the governing physics nor a sound mathematical framework for analysis has been established
- Our study of idealized front propagation in random flows has yielded:
  - A novel systematic approach to analysis of the weak turbulence limit
  - Numerical verification of predictions of the turbulent burning velocity
  - Implications for parameter dependencies in the strong turbulence limit
  - A strategy for further analysis of that limit

Studies to date yielded diverse predictions, but there has been no reliable way to evaluate them

u': rms velocity fluctuation
S_L: laminar flame speed
u_T: turbulent burning velocity

Weak turbulence ($u' << S_L$):

- $u_T/S_L - 1 \sim (u'/S_L)^2$
  - Clavin and Williams (1979)
- $u_T/S_L - 1 \sim (u'/S_L)^{4/3}$ predicted for random flow; quadratic dependence attributed to periodic flow
  - Kerstein and Ashurst (1992)
- Quadratic dependence demonstrated for a random flow
  - Akkerman and Bychkov (2003)

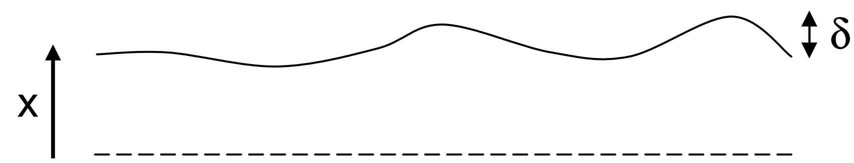
Strong turbulence ($u' >> S_L$):

- $u_T \sim u'/[\log (u'/S_L)]^{1/2} << u'$
  - Yakhot (1988); others propose $u_T/S_L \sim (u'/S_L)^p$ for $0<p<1$
- $u_T \sim u'$
  - Pocheau (1994) and others
- $u_T \sim u'Re^{1/4} >> u'$
  - Upper bound implied by Fedotov (1997)

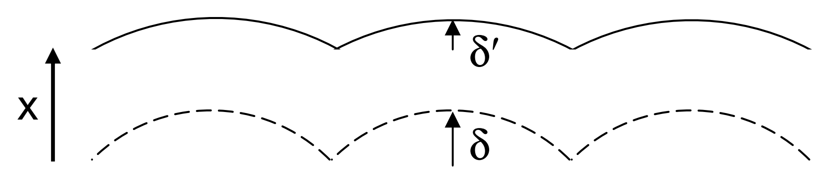
Weak-turbulence 4/3 scaling: Derived heuristically, supported by simulations of idealized flows

- For $u' << S_L$, $u_T/S_L - 1 \sim (\delta/\xi)^2$ (δ is fluctuation amplitude, ξ is correlation length)
- Balance of growth (by advection) and decay (by propagation) processes determines δ
- Growth: $\delta \sim \delta_\xi (x/\xi)^{1/2}$ (random walk scaling, where $\delta_\xi \sim \xi u' / S_L$)
- Decay: $d\delta/dx \sim -\delta^2$
- Balance occurs at downstream distance $x \sim (u'/S_L)^{-2/3}\xi$, giving $u_T/S_L - 1 \sim (u'/S_L)^{4/3}$

Growth by advection



Decay by propagation



(Kerstein and Ashurst 1992)

For weak turbulence, the problem reduces to a formulation amenable to quantum field theory

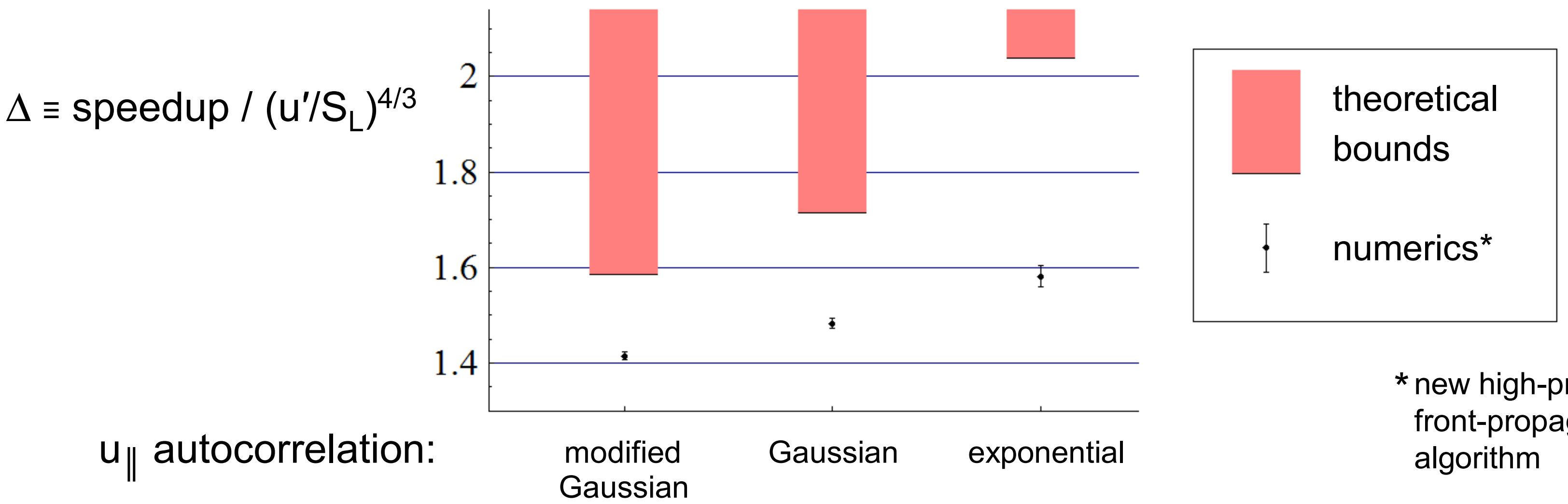
Steps in the analysis

- For $u' << S_L$, propagation + advection reduces to propagation for **x**-dependent S_L (which idealizes heterogeneous propellant combustion; Kerstein, 1987)
- A Lagrangian formulation allows exact problem reduction, giving
  - Immediate extraction of 4/3 scaling
  - Prefactor = energy density of Burgers flow driven by white-in-time noise
- Previous field-theory analysis of Burgers flow enables the derivation of a bound on the prefactor, Δ, as a function of the spatial autocorrelation of the 'noise' $S_L(\mathbf{x})$, which is the motionless-medium analog of $u_{\|}(\mathbf{x})$

New contributions

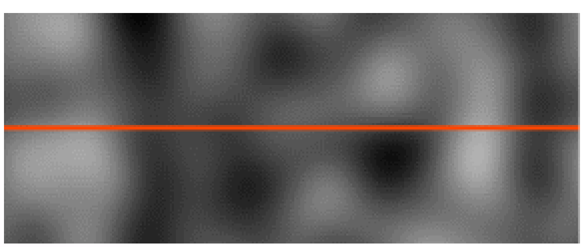
- Formally established this intuitive result
- Generalized a cusp analysis by White (1984)
- Applied a theorem of Iturriaga and Khanin (2003)
- Obtained a new explicit solution within Blum's (1994) formal framework
- Found new relationships among advection/propagation, polymer conformation statistics, and a quantum multi-particle system

2D simulations demonstrate bound accuracy, raising confidence in untested 3D predictions

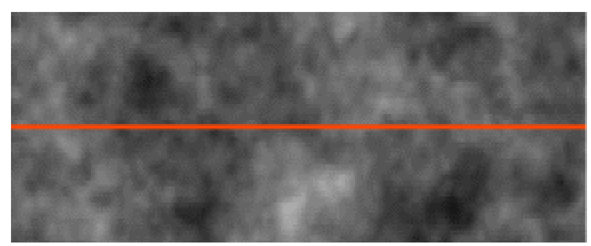


* new high-precision front-propagation algorithm

Gaussian



exponential



The results resolve an apparent discrepancy and have important implications for $u' >> S_L$

- Analysis assumes isotropic random flow
  - Certain unphysical forms of anisotropy can change the scaling
  - This anomaly explains the quadratic scaling found by Akkerman and Bychkov

- Significant dependence of u_T on flow structure is found
  - For $u' << S_L$, both the power spectrum of velocity fluctuations and u'/S_L affect u_T
  - Analysis suggests even greater sensitivity to details for $u' >> S_L$
  - Implication: no expression for u_T that depends only on u'/S_L can capture all physics

- Because u_T must decrease as S_L decreases,
  - The derived bound on the normalized speedup Δ constrains u_T scaling for $u' >> S_L$
  - In particular, it rules out $u_T >> u'$ for physically relevant flows

- *Renormalization* analysis of u_T dependencies must be modified

Renormalization, a key tool for $u' \gg S_L$, must be reformulated to accommodate 4/3 scaling

- Renormalization:
  - Partition turbulent flow into wavenumber (k) bands
  - Represent the effect of the highest-k band as a slight change of S_L
  - Iterate from large to small k to find the aggregate effect of all bands, giving u_T
- This motivates Pocheau's (1994) 'scale-invariant' law $u_T^{\alpha} = S_L^{\alpha} + \beta u'^{\alpha}$
- Yakhot's (1988) renormalization approach predicts
  - Quadratic scaling for $u' \ll S_L$ vs. the demonstrated 4/3 scaling
  - u_T/S_L dependence only on u'/S_L vs. the demonstrated dependence on spectrum
  - $u_T \ll u'$ for $u' \gg S_L$ vs. intuition that flame can't lag flow
- Both analyses are problematic in light of 4/3 scaling
  - They assume infinitesimal k-bands
  - This is a well behaved limit only for quadratic scaling ($\alpha = 2$ in Pocheau's law)
- ***Can renormalization be reformulated using finite k-bands?***

Finite-band renormalization is evident in an algebraic form of the theoretical bound on Δ

- The algebraic form of the weak-turbulence bound on Δ depends on details of the normalized power spectrum, D(k), of $u_{\|}(\mathbf{x})$
- In 2D, for Gaussian (and some other) spectra, the following is obtained:

$$\Delta \leq \frac{3}{4^{5/3}\pi^{2/3}} \int_0^\infty \frac{dz}{z} [B(z)]^{2/3}$$

where

$$B(z) = z^{3/2} \int_0^\infty dk\, k^4 e^{-zk^2/2} D(k)$$

- Written this way, the dk integral defines overlapping finite-width bands parameterized by z, and the dz/z integral is a scale-invariant aggregation of band contributions
- ***This result is suggestive, but its utility for developing a general theory of turbulent combustion is yet to be demonstrated***