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Analysis of Cross-hole Tests in Fractured Systems

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Abstract

The estimates of transmissivity (*T)* and storativity (*S)* obtained from analysis of observation-well responses to constant-rate pumping tests are affected by the correlation between the specified (measured) flow rate and the subsequent pressure response, with both the magnitude and rate of change of the pressure response being a function of the diffusivity (*D*) – the ratio of *T* to *S.* Traditional well-test analysis methods assume that the fluid withdrawal during a constant-rate test is evenly distributed within the domain, but in a heterogeneous/fractured system, this assumption will be violated to some extent.

For example, drawdown in higher *T* zones within a heterogeneous system will be increased relative to drawdown in a homogeneous system with the same "high" *T* value, simply because the water is preferentially withdrawn from the higher *T* zones in the heterogeneous system. This relative increase in drawdown in a heterogeneous system, coupled with the implicit assumption of evenly distributed pumping, will result in underestimating the *T* value in the "high-*T*" areas of the heterogeneous system. It is because of the uncertainty in the effective pumping rate at any observation point within the tested domain that *D,* rather than *T* and *S* individually, is actually the parameter that can be determined. In addition, errors in assumed flow-path length between pumping and observation wells may render even the estimate of *D* meaningless.

Analysis of synthetic data sets obtained from numerical simulations of flow in binary heterogeneous fields (analogous to fractured systems) shows that individually estimated *T* and *S* values can be in error by orders of magnitude. These systems, like fractured systems we have tested, often exhibit a subradial flow signature, and the correlation between the assumed system geometry and the estimates of *T* and *S* makes those values virtually meaningless. Traditional well-test analysis of complex flow-system observation-well responses should be largely abandoned and the observed responses should instead be used to calibrate areal (2D or 3D) models.

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Introduction

In a homogeneous system, one could expect that the values of the fitting parameters derived from the analysis of different cross-hole responses (observation wells) would be relatively similar, given the absence of varying boundary condition effects. *Oliver* (1993) indicates that the average hydraulic properties estimated from analysis of individual observation-well responses in a heterogeneous system will depend on the distribution of those heterogeneities within the system. The ability to detect a zone of T with a particular value in a heterogeneous system with spatially variable T, for example, will depend on the shape, extent, and distribution of that zone with respect to both the testing and observation well locations. While numerous studies spanning multiple decades can be found that have looked at the effects of heterogeneity at various levels of complexity on hydraulic-test responses and the subsequent analysis (*Warren and Price*, 1961; *Vandenberg*, 1977; *Butler*, 1988; *Barker*, 1988; *Butler and McElwee*, 1990; *Oliver*, 1993; and *Indelman et al*., 1996), the approach to analyzing well-test response data to assign unique hydraulic properties to heterogeneous systems, such as those in fractured rock, continues to be a challenge.

A variety of factors affect the estimates of hydraulic parameters obtained from well-test analysis of cross-hole tests performed in heterogeneous systems. The estimates of transmissivity (T) and storativity (S) obtained from constant-rate tests are affected by the correlation between the specified (measured) flow rate and the subsequent pressure response, with both the magnitude and rate-of-change of the pressure response being a function of the diffusivity (D) – the ratio of T to S. In basic well-test analysis, it is assumed that the fluid withdrawn during a constant-rate test is withdrawn equally from all directions within the domain, but in a heterogeneous/fractured system, this assumption will be violated to some extent. Variations in T and S will result in an unequal drawdown distribution within a heterogeneous aquifer, and consequently, application of the expected relationship between the pumping rate and drawdown derived from homogeneous systems will result in parameter estimation errors. As variations in T and S increase, flow will become more channelized – the extreme case being a discretely fractured system. In such a system, if T and S are constant and the matrix is effectively impermeable, then it is only the geometry of the pathways, i.e., the way(s) in which the fractures are connected, that determines the shape of an observation-well pressure response during a constant-rate test. This pathway geometry will also control the amount of observed drawdown for a given T, S, and pumping rate, and it will be shown that knowledge of the complete fracture-pathway geometry between the pumping and observation well is necessary for accurate estimates of T and S.

The two- and three-dimensional characteristics of actual heterogeneous systems can only be approximated in any well-test analysis code in a very simplified way, generally making it necessary to analyze individual observation-well responses, leaving the analyst to piece together these individual analyses in an attempt to describe the complete system. Unfortunately, correlations among the various fitting parameters and vital information to which an individual observation-well match is completely insensitive render this piecemeal approach effectively useless.

The relatively simple representation of a heterogeneous system in which *T* and *S* are constant and pathway geometry is variable will be investigated and discussed in this paper. Data from one- and two-dimensional models will be used to demonstrate potential sources of error when inferring *T*, *S*, and *D* from cross-hole data using well-test analysis techniques.

The numerical well-test analysis code nSIGHTS solves the generalized diffusivity equation to simulate pressure responses within a hydraulic system with variable flow geometry. The generalized diffusivity equation (*Barker*, 1988) is given by:

$$\frac{S_s}{K}\frac{\partial h}{\partial t} = \frac{1}{r^{n-1}}\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial h}{\partial r}\right) \qquad (1)$$

where:	S_s	=	specific storage, *1/L*
	h	=	hydraulic head, *L*
	t	=	elapsed time, *T*
	K	=	hydraulic conductivity, *L/T*
	r	=	radial distance from borehole, *L*
	n	=	flow dimension

The empirically derived relationship between fluid flux (flow rate per unit area) and gradient (pressure difference) is Darcy's law (*Freeze and Cherry,* 1979), which can be written as follows:

$$Q = -KiA \qquad (2)$$

where:	Q	=	flow rate, L^3/T
	K	=	hydraulic conductivity, *L/T*
	i	=	hydraulic gradient
	A	=	cross-sectional flow area, L^2

It is assumed in virtually all well-test analysis that Darcy's law applies. In the nSIGHTS simulator, the cross-sectional flow area *A* at any distance *r* from the borehole is a function of the flow dimension *n* and is calculated as:

$$A(r) = \frac{A(r_w)}{r_w^{n-1}}\, r^{n-1} \qquad (3)$$

where:	r_w	=	wellbore radius, *L*

Given the relationship between flow area and distance in (3), the initial flow area will be equal to the surface area of the borehole within the test interval. The variable describing the system geometry, the flow dimension (n), determines the rate at which this initial flow area increases or decreases with distance from the borehole – n and $A(r)$ are directly proportional. Flow area will increase with distance for all values of n greater than 1, it will decrease for all values less than 1, and it will be constant if n is equal to 1. Familiar values of n are 1, 2, and 3, representing linear, radial, and spherical flow, respectively, but there is no reason to assume that the flow dimensions observed in actual systems would be restricted to integer values.

From (3) and Darcy's law (2) it follows that K (and therefore T) and n (the parameter controlling $A(r)$) are inversely proportional for a given Q and i. No relatively unique estimation of T is possible from well-test analysis of observation-well responses without knowledge (or an assumption) of the flow-path geometry described by n. In addition, knowledge of the flow-path length between the pumping and observation well is necessary to estimate S, given that the diffusivity (D), the ratio of T to S, controls the rate at which a pressure signal propagates through an aquifer.

The following examples demonstrate possible errors in the estimates of T, S, and D derived from observation-well analysis when n and the true flow-path length between the pumping well and the observation well are not well known.

One-Dimensional Model: Case 1

In this first example, nSIGHTS was used to simulate an observation-well response to a constant-rate pumping test where the flow geometry between the pumping well and the observation well changed with distance – a variable-n model. The T and S values used to generate the simulation data were constant and equal to 1E-05 m^2/s and 1E-04, respectively, and the observation well was located 100 m from the pumping well. Figure 1 shows the geometry (flow dimension) variations used in the simulation. Note that the radial flow geometry (n = 2) that begins at about 50 m from the pumping well continued to the model boundary – effectively an infinite distance from the pumping well.

As noted above, the flow dimension in nSIGHTS simply controls the rate at which the initial specified flow area changes with distance in the simulator. Figure 2 shows both the flow area changing with distance ($A(r)$) corresponding to the flow geometry variations shown in Figure 1 and also the $A(r)$ for a radial system with the same initial flow area.

Relative to the radial-geometry model, Figure 2 shows that the variable-n model has significantly less flow area. This reduction in flow area will result in increased drawdown in the variable-geometry system relative to the radial system for given values of T and S.

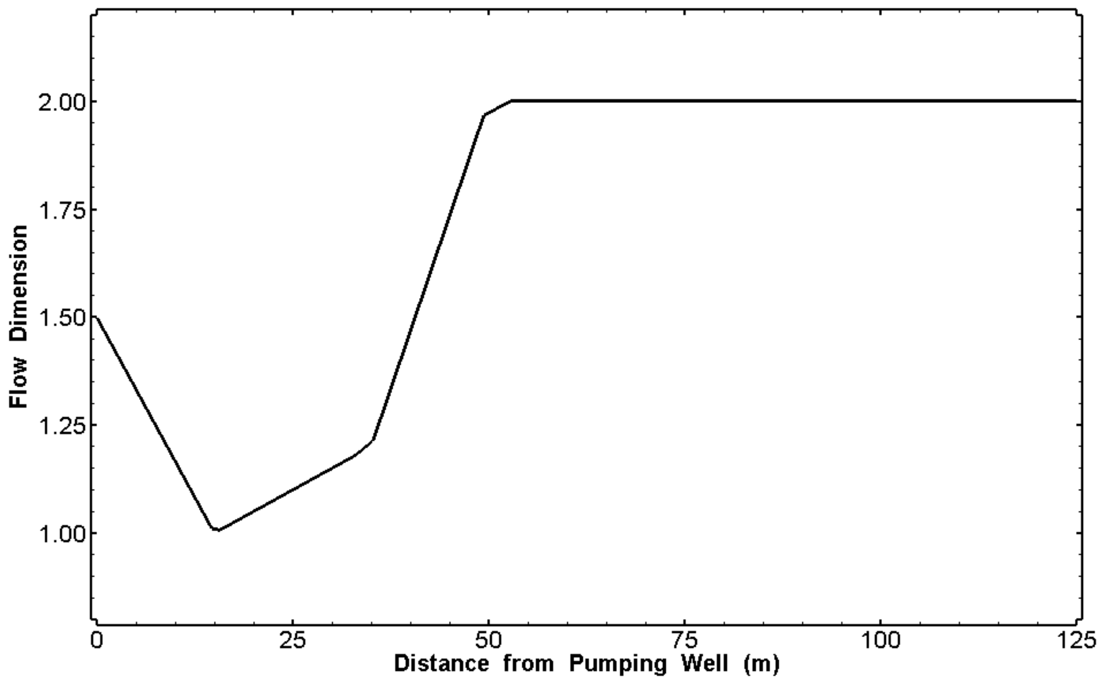

Figure 1. Flow dimension changing with distance from the pumping well, transitioning from subradial to radial.

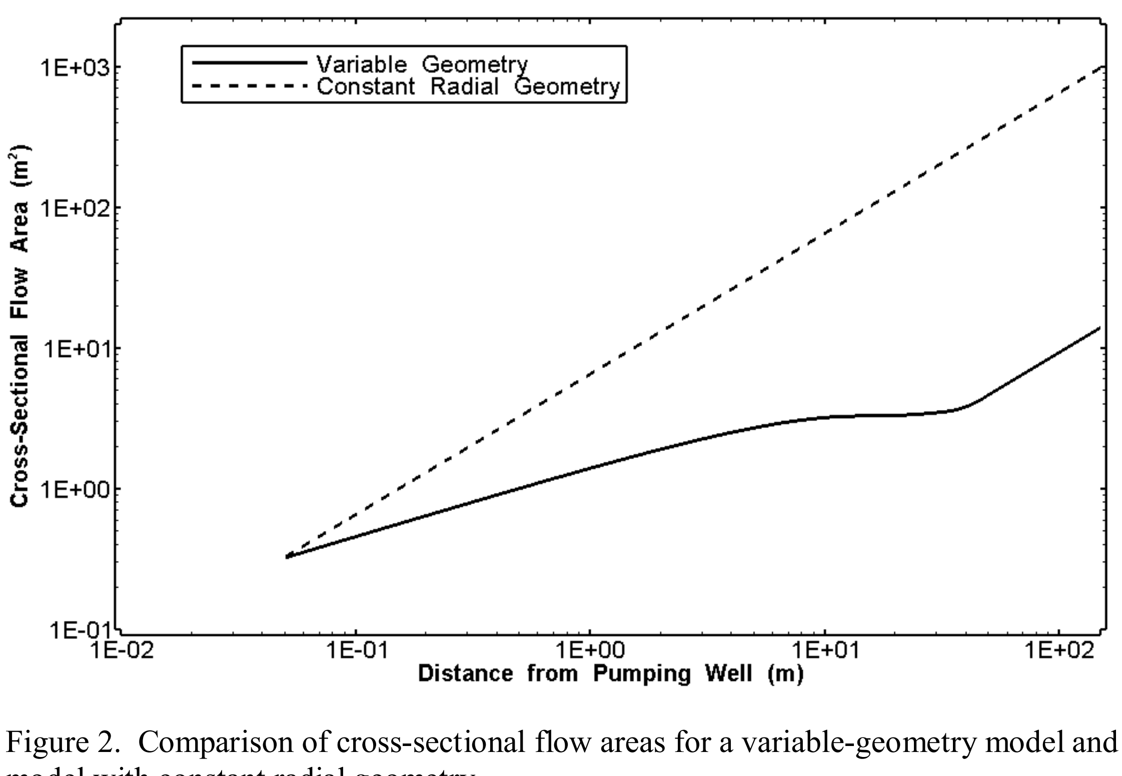

Figure 2. Comparison of cross-sectional flow areas for a variable-geometry model and a model with constant radial geometry.

As noted above, the observation well in this simulation was placed at a distance of 100 m from the pumping well – a distance at which the flow geometry was constant and radial (Figure 1). Given that *T* and *S* are constant in the model, the shape of the observation-well pressure response is affected by the system flow geometry (*n*) only, and the flow geometry evident in the observation well response will be only the geometry at distances equal to or greater than the observation-well distance. This means that only the radial geometry will affect the shape of the observation-well pressure response in this simulation, and consequently, the response will be well fit with a simple radial model. The subradial geometry between the borehole and the observation well will not affect the shape of the observation-well response, but it will affect the observed drawdown at the observation well.

A radial model will reproduce the shape of the simulated observation-well response for the reasons noted above – a radial model would be the obvious choice for analysis of the observation-well response. However, the use of a radial model implies an *A(r)* equal to the constant-radial geometry shown in Figure 2 – the radial model will overestimate the true flow area. This error in assumed flow area means that the *T* and *S* values estimated from a match to the observation-well response using a radial model will have to be reduced relative to their true values – drawdown differences resulting from a conceptual model error (use of the radial model) will be unknowingly compensated by errors in the estimates of *T* and *S*.

Figure 3 shows a radial-model match to the observation-well data generated with the variable-*n* model. The estimated *T* and *S* values were 2.8E-07 m^2/s and 7.1E-07, respectively – approximately two orders-of-magnitude less than the true values of 1E-05 m^2/s and 1E-04. The resulting diffusivity estimate of 4E-01 m^2/s, however, was only in error by a factor of four.

When analyzing observation-well responses, the assumed distance between pumping and observation well is typically the straight-line distance. The actual flow path along the fracture(s) connecting these wells, however, may be greater than the simple straight line distance between them. Therefore, fitting-parameter values that depend on knowing the true flow-path distance between the test and observation intervals in fractured media have additional uncertainty resulting from uncertainty in the flow-path distance. The parameter most affected by uncertainty in the flow-path length is *S*. To assess the parameter estimation errors caused by underestimating the fracture flow-path lengths, data generated by simulating the observation-well response at a distance of 100 m from the pumping well in the variable-*n* model were subsequently matched using a radial model and assuming the observation well was at distances of 25, 50, 75, and 100 m. The resulting errors come from both geometry and flow-path length uncertainty. Figure 4 shows the estimates of *T* and *S* for this scenario.

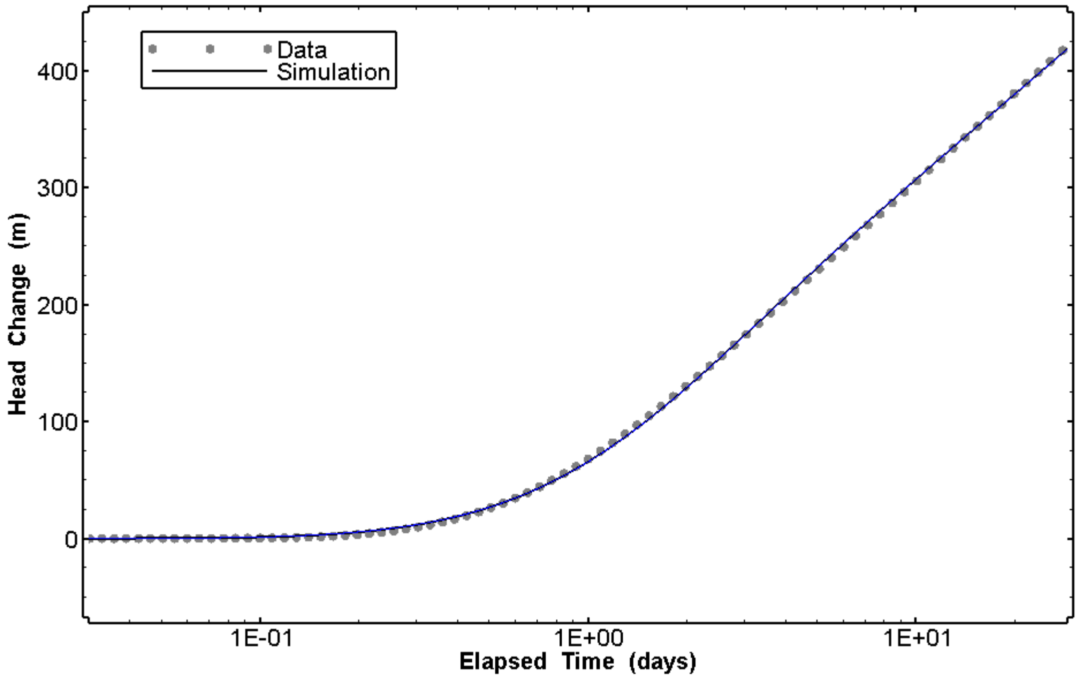

Figure 3. Semi-log plot of simulation and data showing the match obtained to the observation-well data using a radial model.

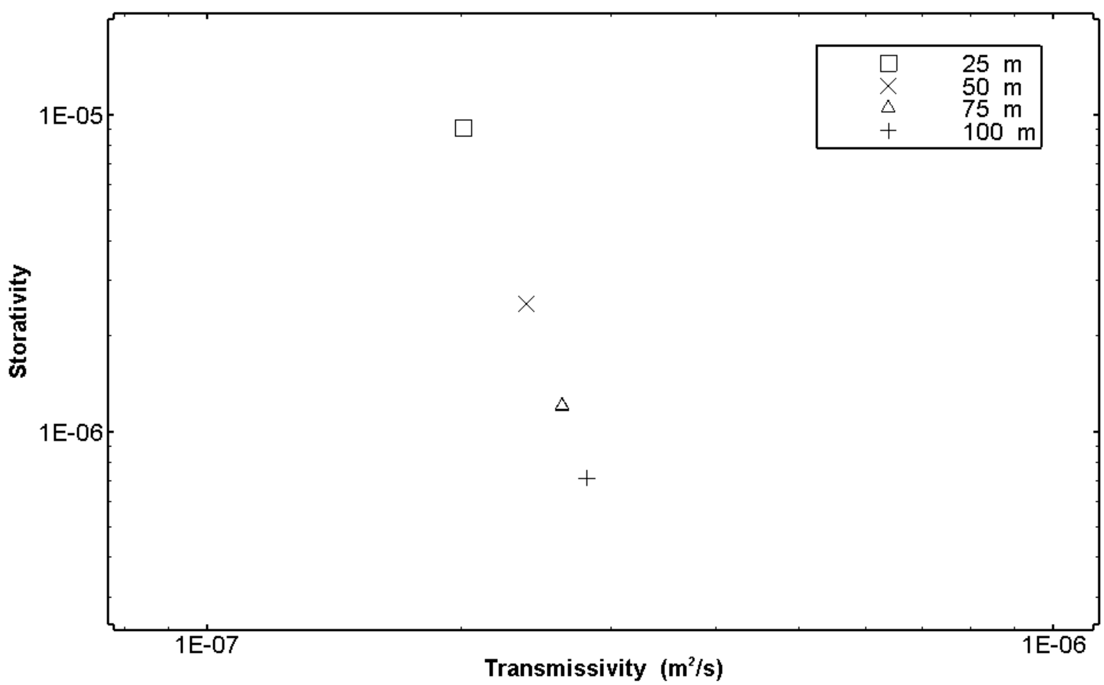

Figure 4. Estimates of transmissivity and storativity showing errors resulting from overestimating cross-sectional flow-path area and underestimating the flow-path length.

In addition to the previously described underestimation of *T* and *S* due to errors in the flow geometry (corresponding to the 100-m solution shown in Figure 4), errors are seen to result when the flow-path length is underestimated by various amounts. The flow-path distance errors affect the estimate of *S* more than *T* (Figure 4) – the *S* estimates vary by greater than a factor of ten while the *T* estimates vary by less than a factor of two. Figure 5 shows the corresponding estimates of *D* and *S*, where the relatively large variability in *D* results from the differing *T* and *S* sensitivities to errors in flow-path length.

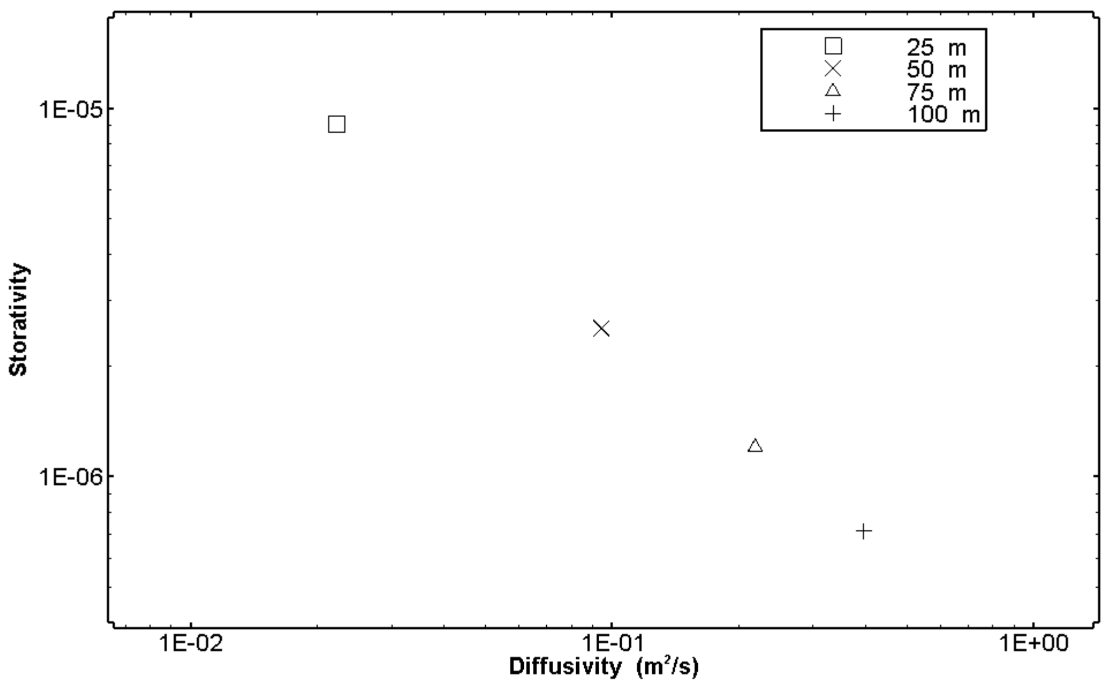

Figure 5. Estimates of diffusivity and storativity showing errors resulting from overestimating cross-sectional flow-path area and underestimating the flow-path length

One-Dimensional Model: Case 2

In the example above, the values of *T* and *S* were underestimated when the model chosen to match the observation-well response overestimated the true flow area. The following example shows the effect on the estimates of *T* and *S* when the conceptual model used for parameter estimation underestimates the flow area. Figure 6 shows *n(r)* where the near pumping-well geometry begins as radial and then transitions to subradial at some distance away. Figure 7 shows *A(r)* corresponding to *n(r)* in Figure 6, along with the constant $n = 1.5$ *A(r)*. Again, the response of an observation well located 100 m from the pumping well was simulated using the variable-*n* model and was subsequently matched using the constant $n = 1.5$ model to estimate the values of *T* and *S*.

As in the previous example, the only geometry evident in the observation-well response will be the geometry at the distance of the observation well and beyond, i.e., n = 1.5 in this example, making this the obvious conceptual-model choice for analysis (for those bold enough to believe that all flow is not radial). Choosing a model with a constant n = 1.5 geometry will underestimate the true flow area. The T and S estimates in this example were 1.5E-04 m^2/s and 1.2E-03, respectively – overestimates of the true T and S values of 1E-05 m^2/s and 1E-04, because these parameters must compensate for the under-predicted flow area resulting from the constant n = 1.5 model geometry. The T and S estimates differ by approximately a factor of ten from the true values while the resulting D estimate is approximately 1.3E-01 m^2/s – very close to the actual value of 1E-01 m^2/s.

Underestimating the distance between the observation and pumping well again resulted in larger errors in the estimates of S than in T (Figure 8) and the wide range of estimated D values shown in Figure 9 is due to the differing T and S sensitivities to errors in flow-path length. Note that an unfortunate combination of underestimating both the flow area and the distance between the pumping and observation well could result in anomalously high estimates of S (Figure 8).

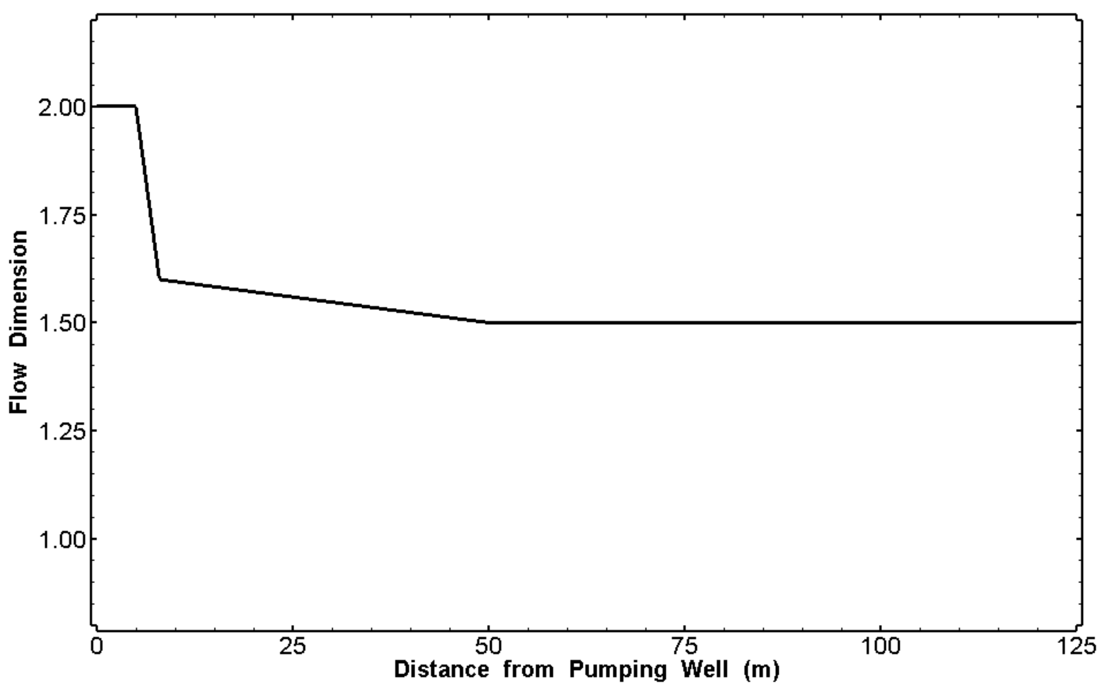

Figure 6. Flow dimension changing with distance from the pumping well, transitioning from radial to subradial.

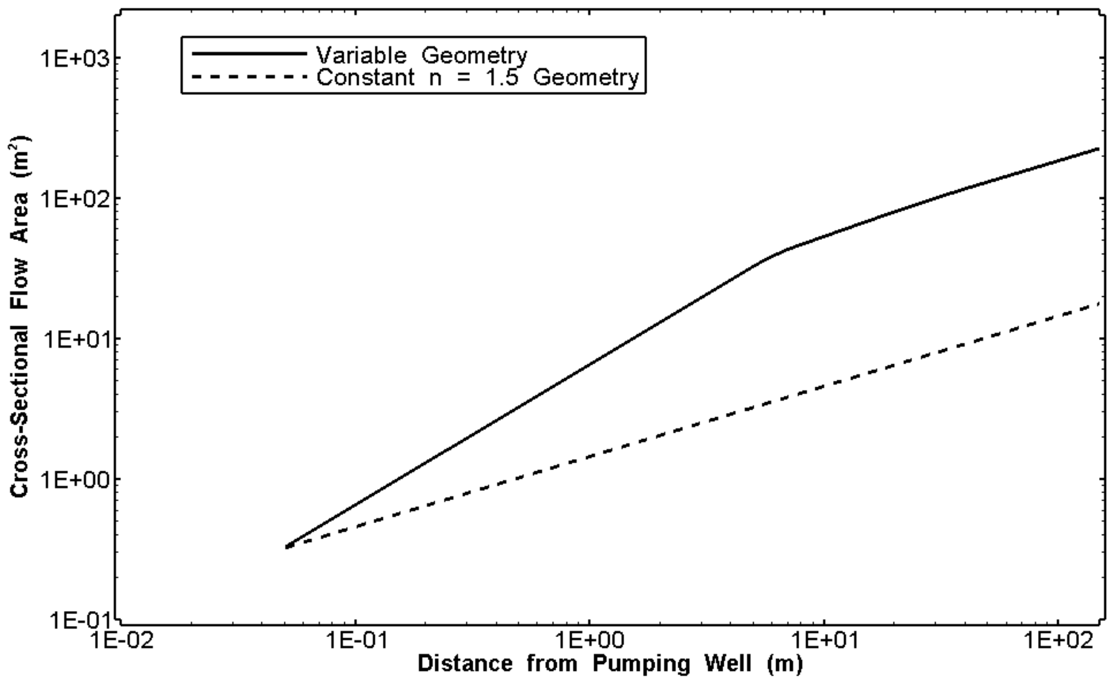

Figure 7. Comparison of cross-sectional flow areas for a variable-geometry model and a model with constant subradial geometry.

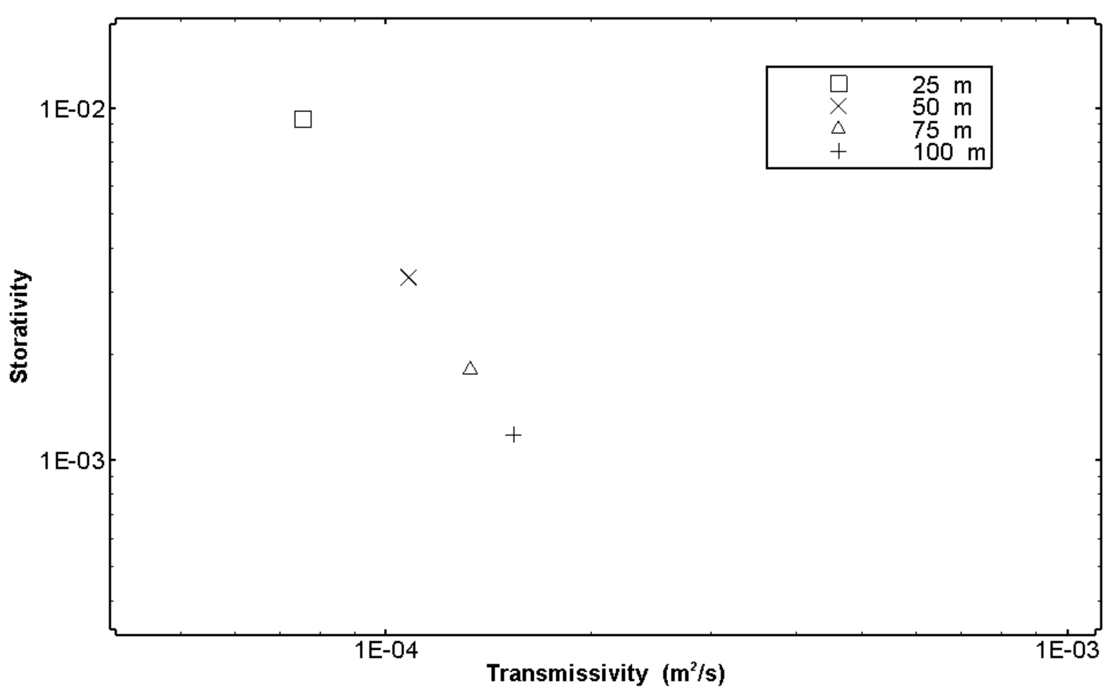

Figure 8. Estimates of transmissivity and storativity showing errors resulting from underestimating cross-sectional flow-path area and underestimating the flow-path length

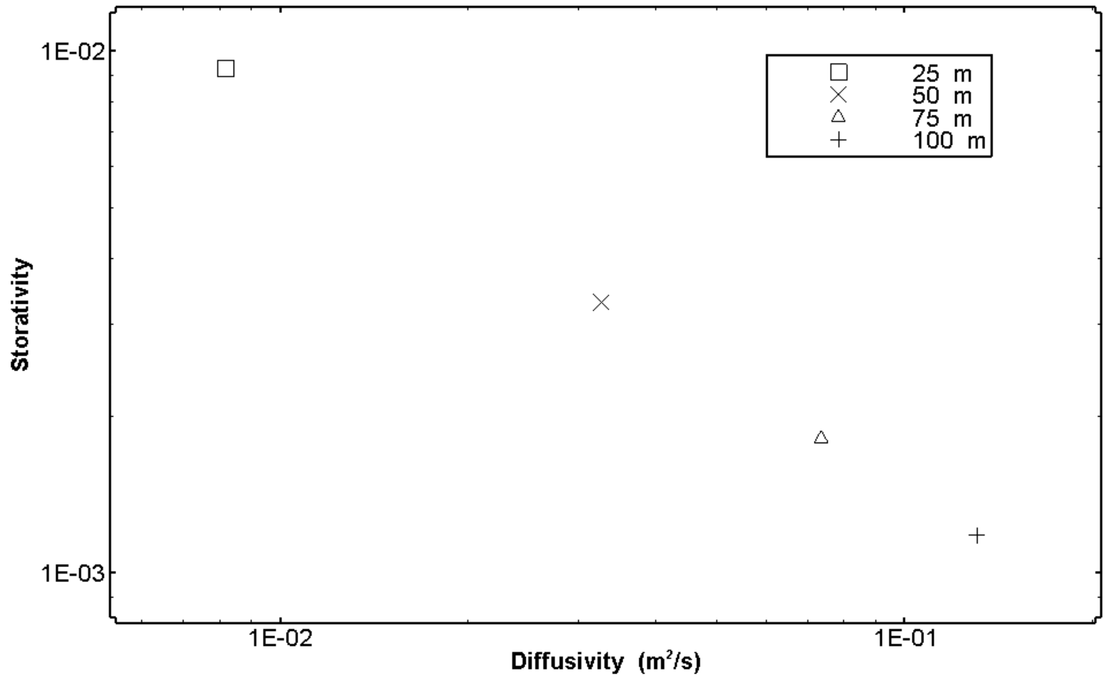

Figure 9. Estimates of diffusivity and storativity showing errors resulting from overestimating cross-sectional flow-path area and underestimating the flow-path length

Two-Dimensional Model

A series of two-dimensional, binary random fields were generated to model a system with constant *T* and *S* and variable geometry to further investigate the results developed with the one-dimensional models above. Gaussian random fields were created with a specified mean, variance, and a 1:10 anisotropy ratio applied to the x-to-y directional correlation lengths. A divide value in the field values was then selected and all field values greater than the divide value were designated T = 1.0E-5 m^2/s. All field values less than the divide value were designated impermeable. A constant value of S = 1E-04 was used in the field. The anisotropy in the model resulted in simulated pressure responses whose geometry characteristics ranged from subradial to radial. Incorporating this field into MODFLOW (Harbaugh et al., 2000), observation-well responses to a constant-rate pumping test were generated and these data were then analyzed using nSIGHTS. Figure 10 shows one of these fields and the locations of the observation wells, which are numbered 1 through 6, and the pumping well. Note that Figure 10 shows only a detailed part of the 7700 x 7700 m total field. The transmissive part of the field is shown in white and impermeable blocks are shown in black.

Figure 11 shows the estimates of *T* and *S* and Figure 12 shows the corresponding estimates of *D* and *n* obtained from the nSIGHTS analysis of the MODFLOW output. The flow dimension in these analyses was a fitting variable along with *T* and *S*. The specified distance between the pumping well and the observation wells was the straight-line distance between the grid points. In all six cases, a model with a single flow dimension provided a good fit to the observation-well response.

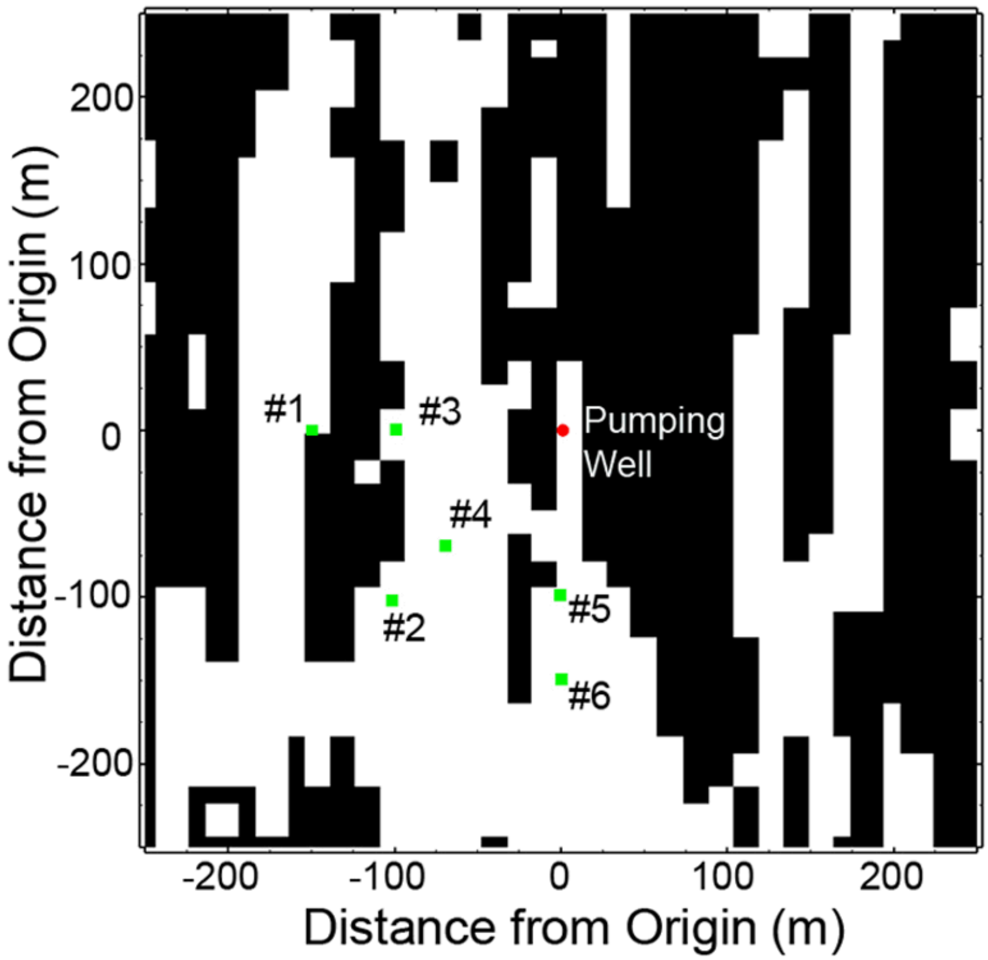

Figure 10. A binary flow field showing the locations of the pumping and observation wells.

Two sets of observation-well parameter estimates obtained from the binary-field analyses seemed to correspond clearly to the sources of parameter estimation error predicted by the one-dimensional models: observation wells #1 and #6. The flow-path length in the binary field (Figure 10) between the pumping well and observation well #1 is clearly greater than the straight-line distance between the two wells. As would be expected from the one-dimensional modeling results, the corresponding estimate of S is high (Figure 11) and the estimated D is much lower than the true value (Figure 12). The previous one-dimensional simulations indicated that errors in flow-path length caused greater errors in the estimated D than did errors in path geometry. While some of the model-input flow-path lengths for the other five observation wells were likely somewhat shorter than the true path length, only the large discrepancy in the observation well #1 path length resulted in an estimated D much lower than the true 1E-01 m^2/s value (Figure 12).

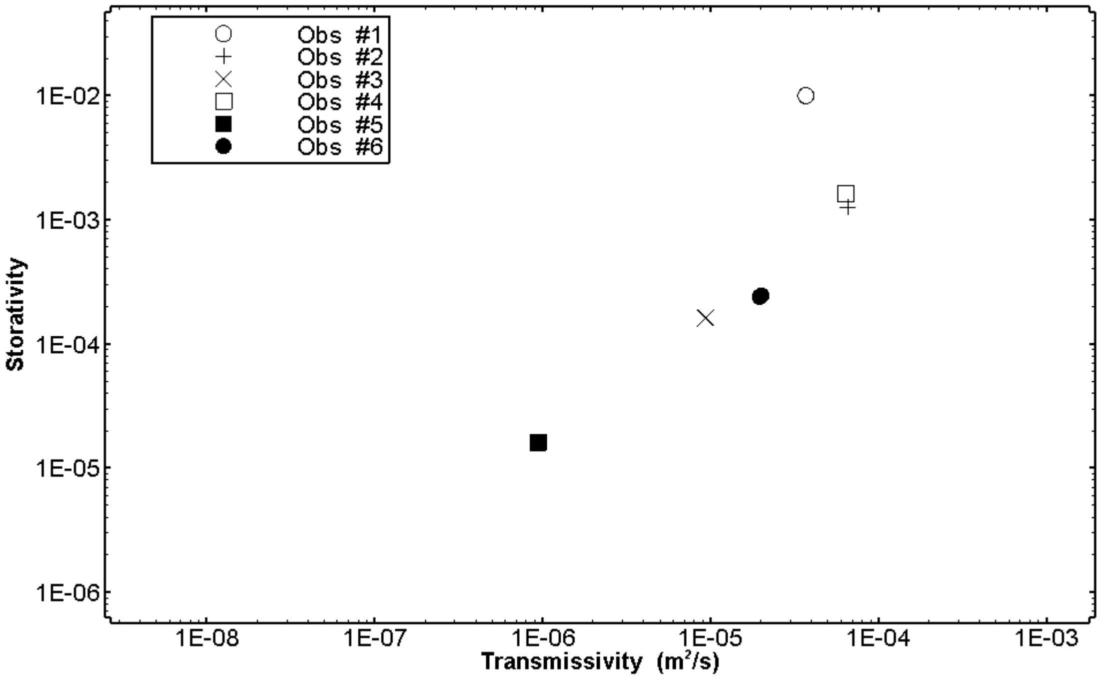

Figure 11. Estimates of transmissivity and storativity obtained from analysis of simulated pumping tests in a two-dimensional binary field.

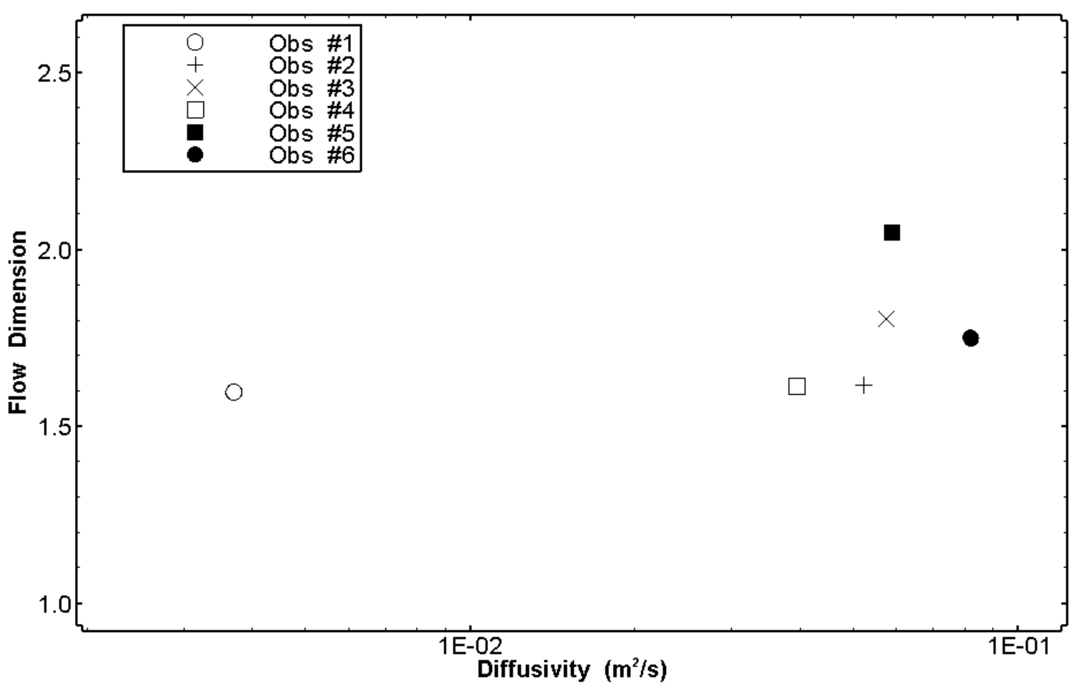

Figure 12. Estimates of diffusivity and flow dimension obtained from analysis of simulated pumping tests in a two-dimensional binary field.

Figure 12 shows that the geometry estimated from observation well #5 was approximately radial (note that a flow pathway in the shape of a "pie-slice" wedge will have a radial flow signature), while the flow path between the pumping well and observation well #5 is distinctly subradial. Use of a radial geometry to match the observation well #5 response resulted in *T* and *S* estimates that were lower than the true values (Figure 11), as predicted by the one-dimensional modeling.

Summary and Conclusions

Attempting to accurately estimate *T* and *S* from observation-well data collected in heterogeneous/fractured systems using standard well-test analysis methods is a questionable (if not impossible) task. An analyst may well obtain excellent matches to field pressure responses, but that can have little or nothing to do with the accuracy of the estimated parameters.

The characteristics of the system that do not affect the shape of observation-well pressure response, and consequently, the obvious choice of conceptual model – the flow-path geometry changes between the pumping well and the observation well – have been shown to greatly affect the accuracy of estimated *T* and *S*. The resulting estimate of *D* appears to be less sensitive to errors in flow-path geometry than the individual *T* and *S* estimates.

Errors in the assumed flow-path length between the pumping and observation wells produce greater errors in the estimate of *S* than *T*. This difference in sensitivity will produce larger errors in estimated *D* than result from flow-path geometry errors.

Unfortunate combinations of these two error sources will likely produce *T* and *S* estimates that are effectively meaningless. Analysis of the pumping-well data would provide more unique estimates of *T* and *S* at some scale than could be obtained from observation-well analysis, but the one-dimensional nature of well-test analysis codes means that even pairs of well responses measured in heterogeneous/fractured systems typically cannot be matched simultaneously in an attempt to better constrain the problem. This inability by well-test analysis to combine all possible constraints when attempting to characterize complex systems means that parameter estimation in heterogeneous/fractured systems would be better addressed by calibration of two- or three-dimensional models that consider all of the well responses simultaneously.

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Biographical Information

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