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Sandia National Laboratories

Predicting Fatigue Failure of a Circuit Board in Random Vibration

(Adapted from presentation given at the 84th Shock & Vibration Symposium)

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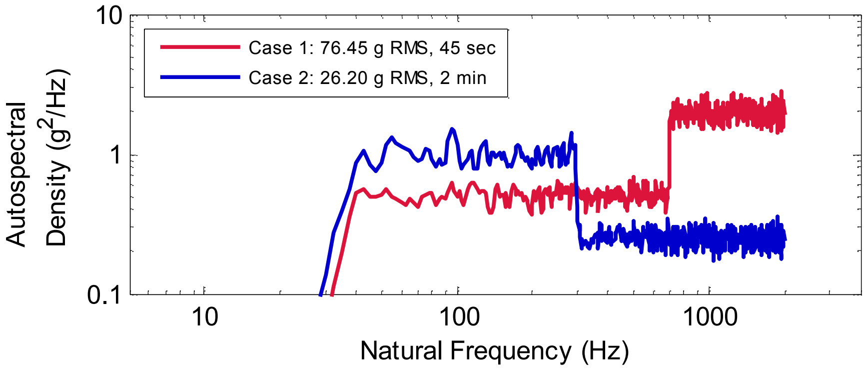
U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

Introduction

- Energy Based Failure Prediction Methods
  - Use modal energy as an intensity measure for predicting structural failure
  - Requires approx. linear structure, fixed-base modal properties, base input
  - Key Advantage: Once failure model is built, arbitrary input profiles can be assessed for relative severity – hedge against environmental uncertainty.

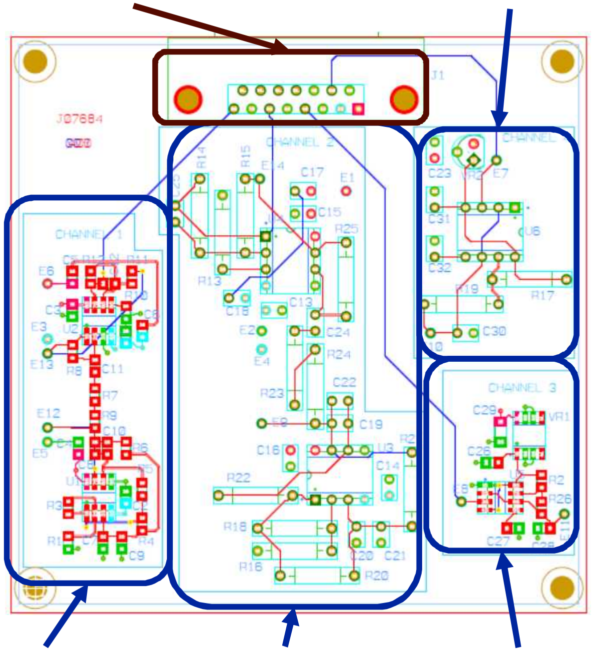


Which is more severe?

- Project Objective
  - To compare the prediction efficiency of energy-based fatigue failure models to a traditional fatigue failure model.

This talk is about the reference fatigue model

Printed Circuit Board



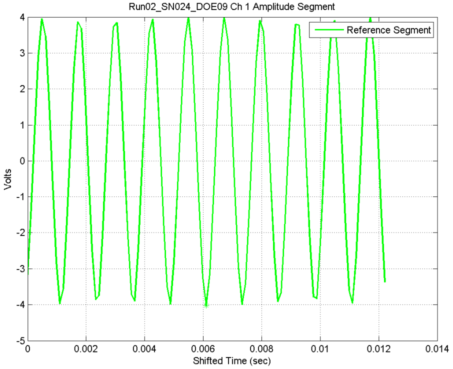
Connector

Circuit 4

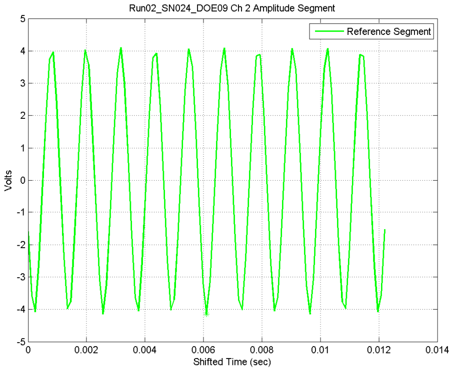
Circuit 1

Circuit 2

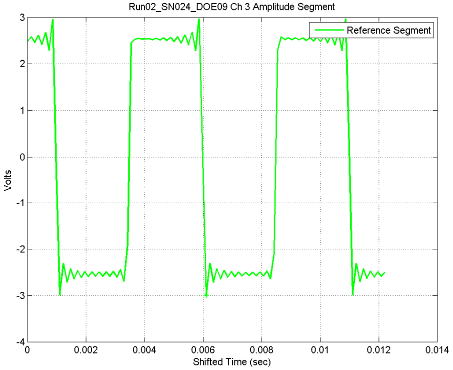
Circuit 3



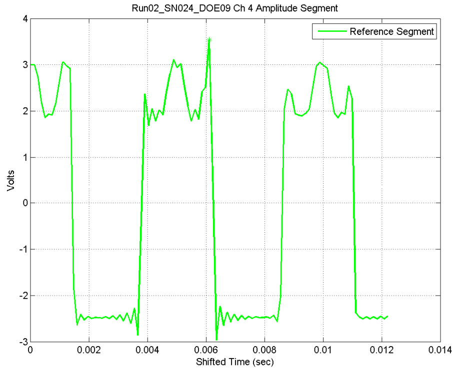
Circuit 1



Circuit 2



Circuit 3

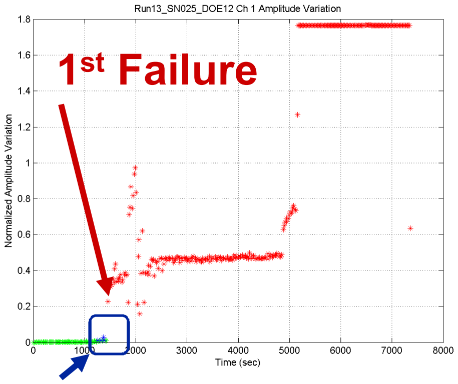


Circuit 4

Quasi-isotropic Material Properties
E = 3.795x10^{6} psi
ν = 0.266

Circuit Failure Criterion

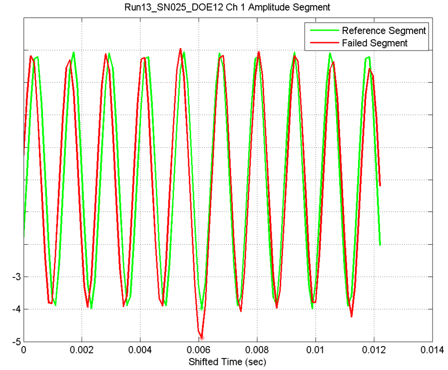
- Failure was based on the outputs of the PCB circuits
- The signals were subdivided into 20 sec segments
  - The 2nd segment is the reference segment
- Circuit failure criterion:
  - Peak response exceeds the reference value by 1% in at least 3 consecutive segments

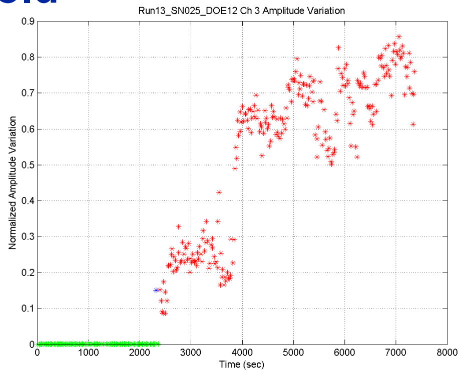


1st Failure

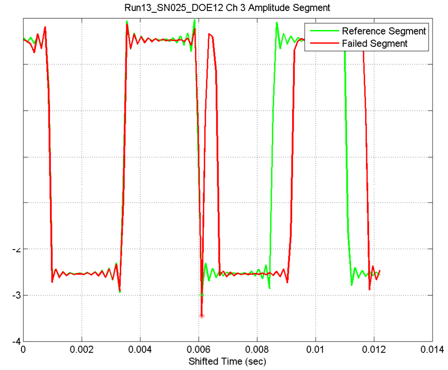
Amplitude exceeds threshold

Circuit 1





Circuit 3



The Reference Model

- A standard technique for approximating the fatigue life of a structure subjected to vibratory loads is Miner's method.
  - It is based on the assumption that every structural component has a specific fatigue life and every stress cycle uses up a portion of that fatigue life.

$$D = \sum_{j=1}^{M} \frac{n_j}{N_j}$$

$D =$ fraction of consumed fatigue life;
$n_j =$ number of cycles experienced by the structure at stress S_j;
$N_j =$ number of cycles to failure at stress S_j determined from S-N curves.

  - $\boldsymbol{D} = 0.3$ for critical life-cycle electronics
  - $\boldsymbol{D} = 0.7$ for typical structures

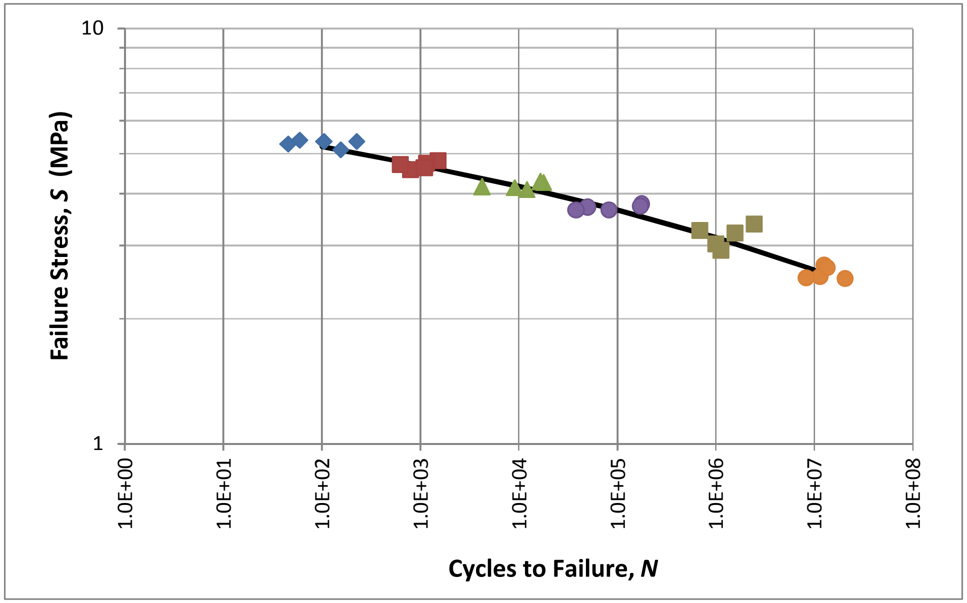
The key to using Miner's method is the S-N curve

S-N Curves

- S-N curves are failure definition specific
  - Failure is not necessarily physical failure / breakage
  - It can be some change in a structural property like loss of stiffness
- S-N curves are based on sinusoidal loading
  - Experimental determination can be time consuming and expensive
- Basic assumption: Fatigue strength is a linear function in log-log space

$$NS^b = A$$

  - $\boldsymbol{N}, \boldsymbol{S}$ are measured
  - $\boldsymbol{A}, \boldsymbol{b}$ are empirically derived constants



Notional S-N curve

The Challenge

- No S-N curves exist for the PCB and the conditions of interest
- The first part of the study generated failure data with which to characterize the fatigue endurance of the PCB
- Preliminary experiments indicated that long duration exposures were necessary
- Sinusoidal testing was not feasible (experiment was designed to populate energy-based failure models for PCB in wideband random vibration)

The Challenge: Develop fatigue damage properties from wideband random vibration data measured during test to failure.

RMS Stress Approach

- Replace the S-N curve with a $\sigma - N_T$ curve
  - σ - stress standard deviation
  - N_T - total number of cycles to failure

- Advantages
  - Straightforward computation
- Disadvantages
  - Cannot account for combined loading

Cumulative Damage Index Approach

- Assume M units have been tested to failure with random excitation and the stress amplitude distribution has been determined for each unit
- Discretize each distribution into at most M bins, centered on stress levels $S_1, S_2, \ldots S_M$ to obtain the rainflow cycles at each stress, $n_1, n_2, \ldots n_M$.
- The number of cycles to failure at each stress level, $N_1, N_2, \ldots N_M$ can be estimated with Miner's Equation

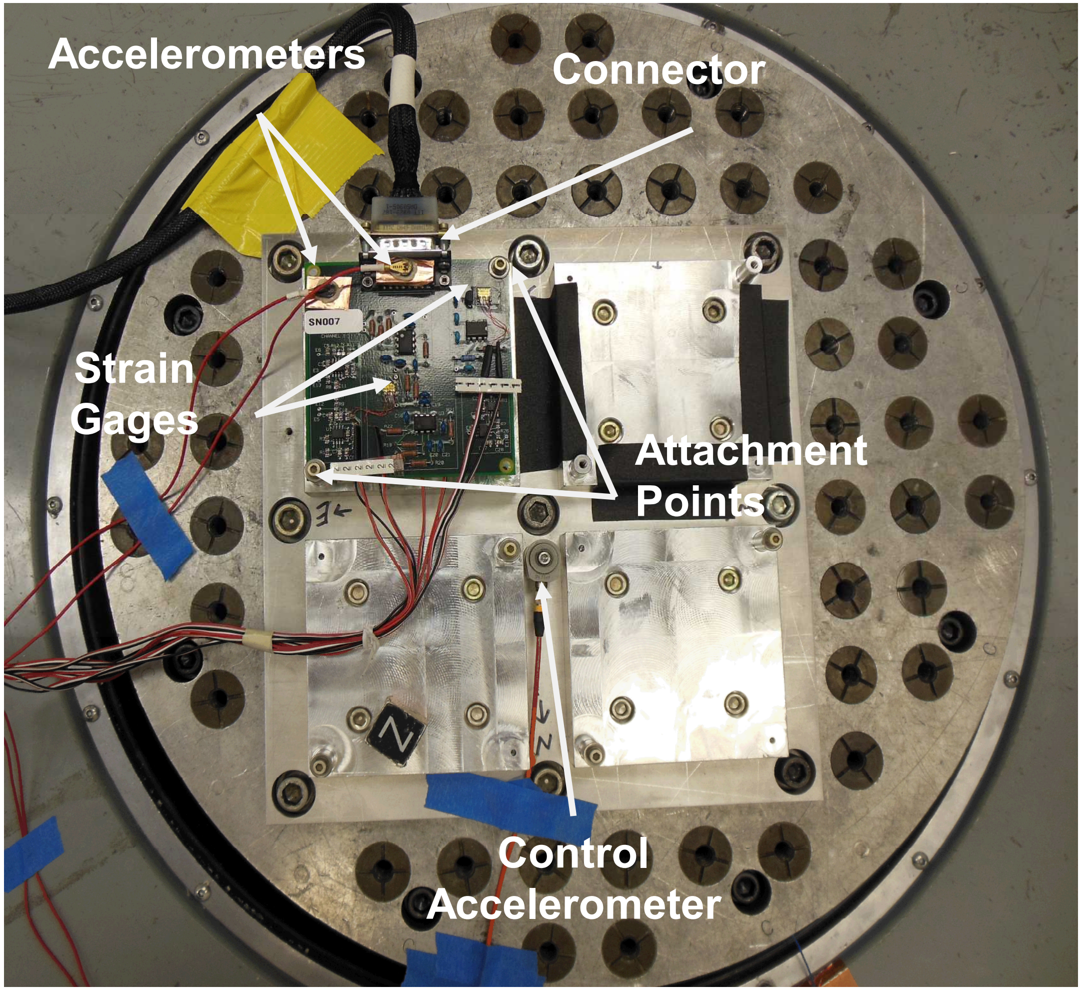
$$\underbrace{\begin{bmatrix} n_1^1 & \cdots & n_M^1 \\ \vdots & \ddots & \vdots \\ n_1^M & \cdots & n_M^M \end{bmatrix}}_{[n]} \underbrace{\begin{Bmatrix} \frac{1}{N_1} \\ \vdots \\ \frac{1}{N_M} \end{Bmatrix}}_{\left\{\frac{1}{N}\right\}} = \underbrace{\begin{Bmatrix} D \\ \vdots \\ D \end{Bmatrix}}_{\{D\}}$$

- This yields a constrained least squares optimization problem

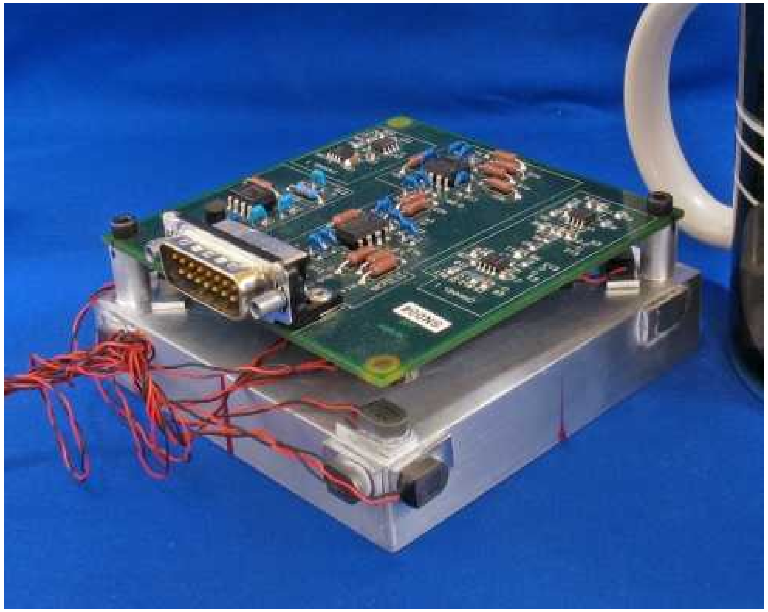
$$\min_{N_i} \left\| \{D\} - [n] \left\{\frac{1}{N}\right\} \right\|_2^2 \text{ subject to } N_i > \max_{k=1\ldots M} n_i^k > 0$$

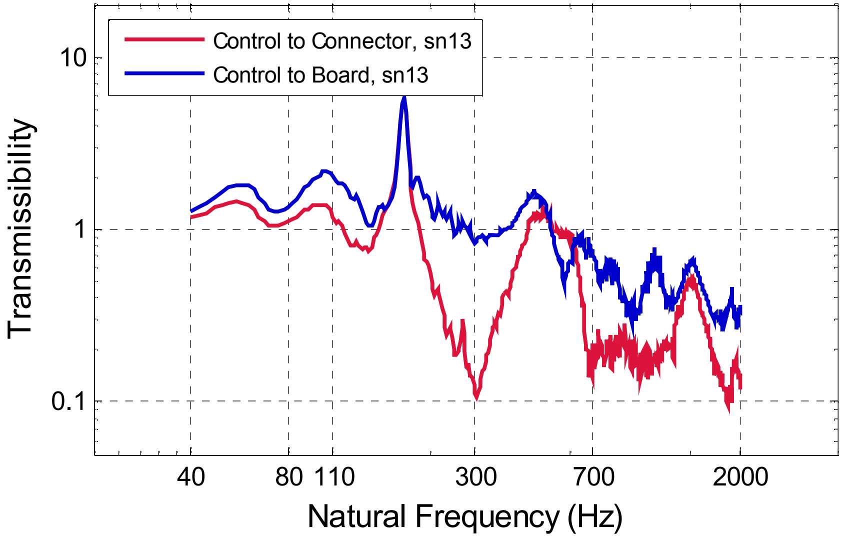
Vibration Test

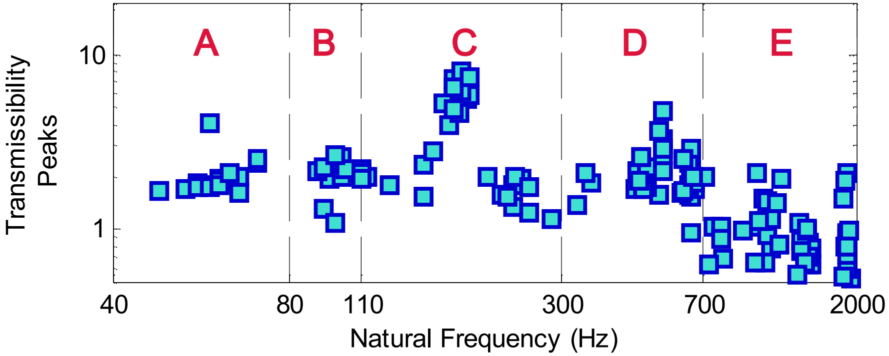
- Each PCB was mounted to a fixture on a shaker via two standoffs diagonally
- Vertical excitation
- Bandwidth: 40 Hz – 2000 Hz
- 16 PCBs were tested, each with a unique excitation profile



Partitioning Input Bands



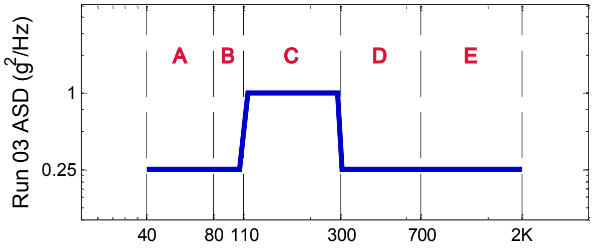


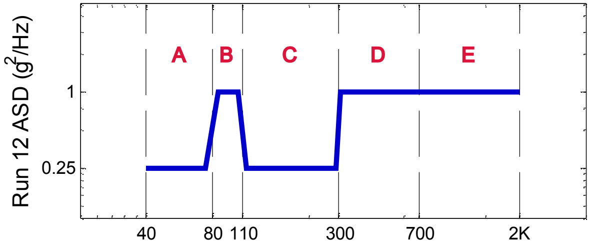


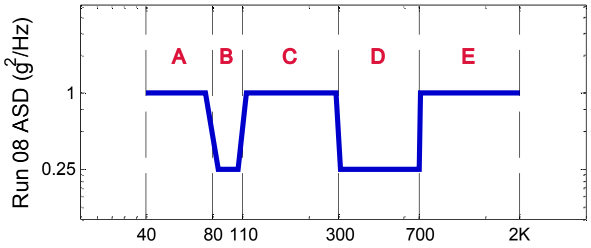
2_V^{5-1} Design Matrix

Trial	A	B	C	D	E
1	−	−	−	−	+
2	−	−	−	+	−
3	−	−	+	−	−
4	−	−	+	+	+
5	−	+	−	−	−
6	−	+	−	+	+
7	−	+	+	−	+
8	−	+	+	+	−
9	+	−	−	−	−
10	+	−	−	+	+
11	+	−	+	−	+
12	+	−	+	+	−
13	+	+	−	−	+
14	+	+	−	+	−
15	+	+	+	−	−
16	+	+	+	+	+

16 Input Profiles



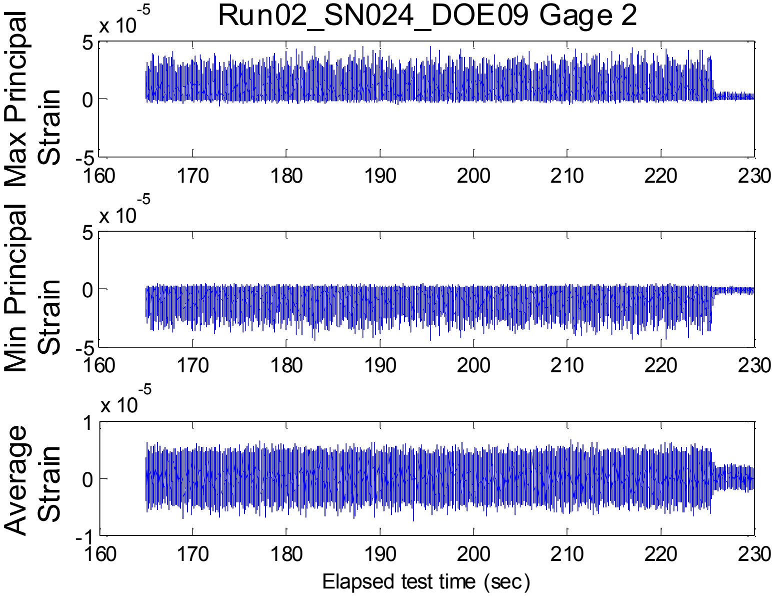


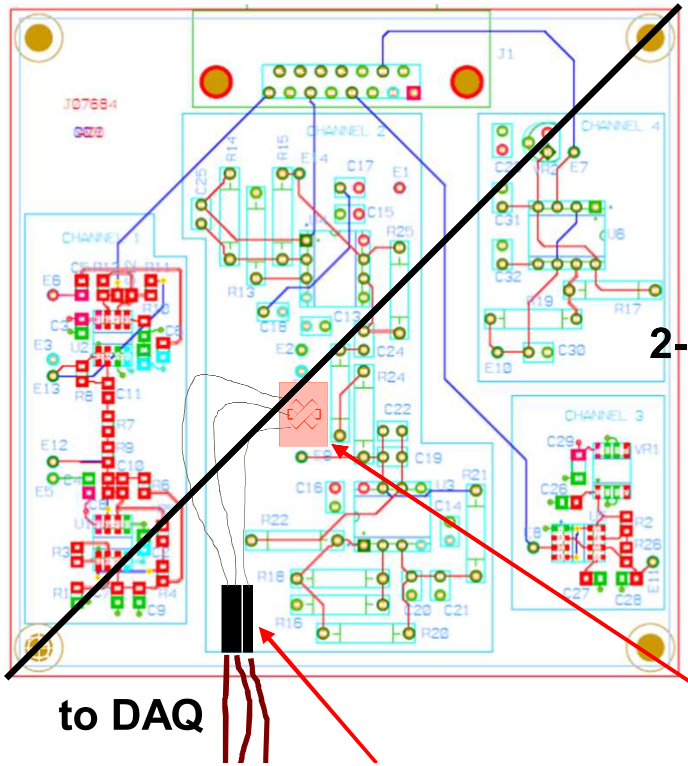


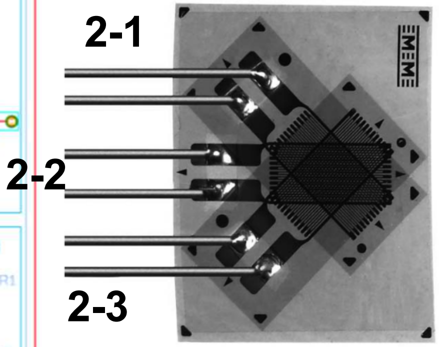
Strain Gage Measurements

- The center strain gage (Gage 2) was used for making the reference model
- Average stress was the quantity of interest

$$\sigma_{P,Q} = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_3}{1-\nu} \pm \frac{\sqrt{2}}{1+\nu}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2}\right]$$







to DAQ

cable “collars” epoxied to board with strain relief for gage lead wires

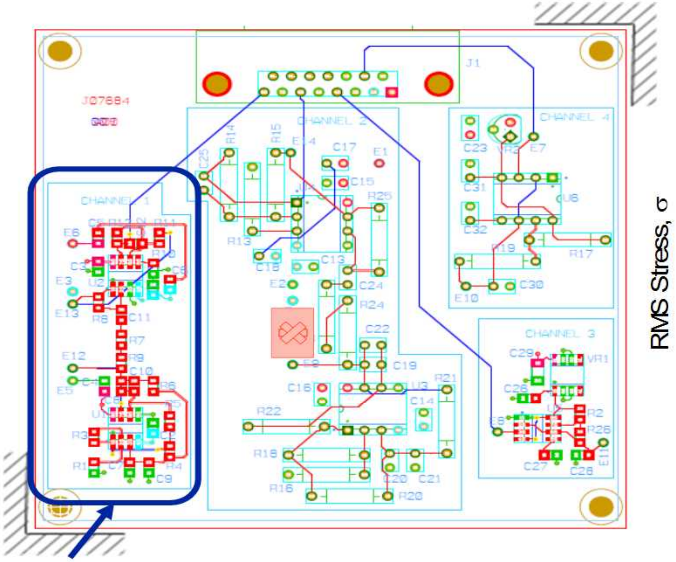
2-1 : ⊥ to diagonal
2-2 : center field
2-3 : ‖ to diagonal

Results

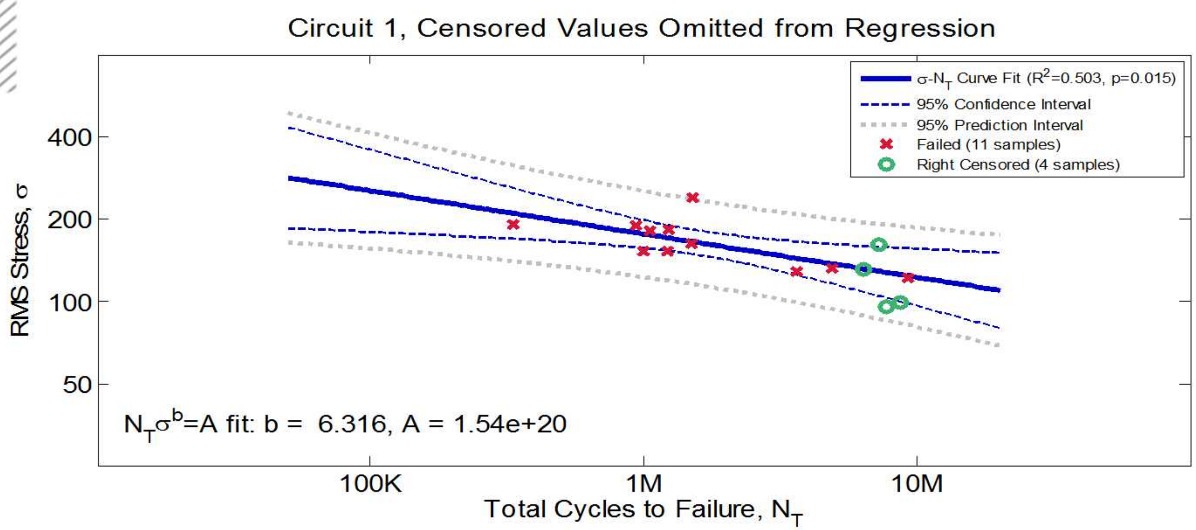
Test	Input RMS (g)	Gage 2 Avg Stress RMS (psi)	Circuit 1		Circuit 2		Circuit 3		Circuit 4	
			Time to Failure (min)	# of Cycles to Failure	Time to Failure (min)	# of Cycles to Failure	Time to Failure (min)	# of Cycles to Failure	Time to Failure (min)	# of Cycles to Failure
Run01 SN021 DOE01	38.26	101.14	1.18	-91554	No Failure	9584543	16.96	1165361	2.16	-14120
Run02 SN024 DOE09	22.78	95.22	No Failure	7730505	No Failure	7730505	No Failure	7730505	No Failure	7730505
Run03 SN029 DOE03	25.12	152.49	21.86	989869	32.10	1508675	No Failure	6089038	118.65	5896048
Run04 SN027 DOE04	43.67	239.93	25.19	1501047	66.32	4242794	79.75	5137359	No Failure	8007150
Run05 SN014 DOE14	28.98	132.06	90.64	4887099	90.64	4887099	90.64	4887099	57.89	3068032
Run06 SN015 DOE02	28.08	130.9	No Failure	6364557	No Failure	6364557	No Failure	6364557	No Failure	6364557
Run07 SN028 DOE07	25.56	152.43	23.97	1220546	35.70	1892196	No Failure	7042003	35.18	1862498
Run08 SN026 DOE11	40.42	180.49	16.90	1052013	25.94	1705100	No Failure	8703575	16.67	1035335
Run09 SN030 DOE10	42.35	161.64	No Failure	7230877	No Failure	7230877	No Failure	7230877	No Failure	7230877
Run10 SN017 DOE16	44.27	190.04	16.32	936524	84.67	5510966	53.30	3411678	36.90	2313535
Run11 SN022 DOE15	26.16	162.78	22.99	1497379	53.65	3756436	74.27	5275618	15.99	981344
Run12 SN013 DOE06	42.25	128.33	50.98	3627986	No Failure	8916185	No Failure	8916185	49.33	3506343
Run13 SN025 DOE12	30.98	182.33	24.31	1226594	56.59	3085377	39.94	2126822	21.03	1038277
Run14 SN018 DOE13	38.93	121.92	120.55	9279470	No Failure	9428775	No Failure	9428775	84.52	6458859
Run15 SN016 DOE05	22.6	98.90	No Failure	8700199	No Failure	8700199	No Failure	8700199	No Failure	8700199
Run16 SN020 DOE08	30.88	190.87	10.66	331473	106.21	4859450	0.85	-133293	9.72	286692

🟩	No failure
🟥	Circuit failed before full level

Results: Circuits 1 and 4

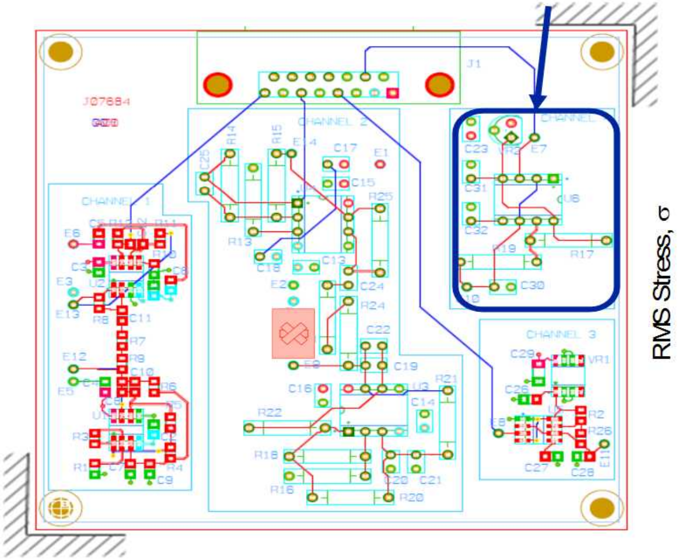


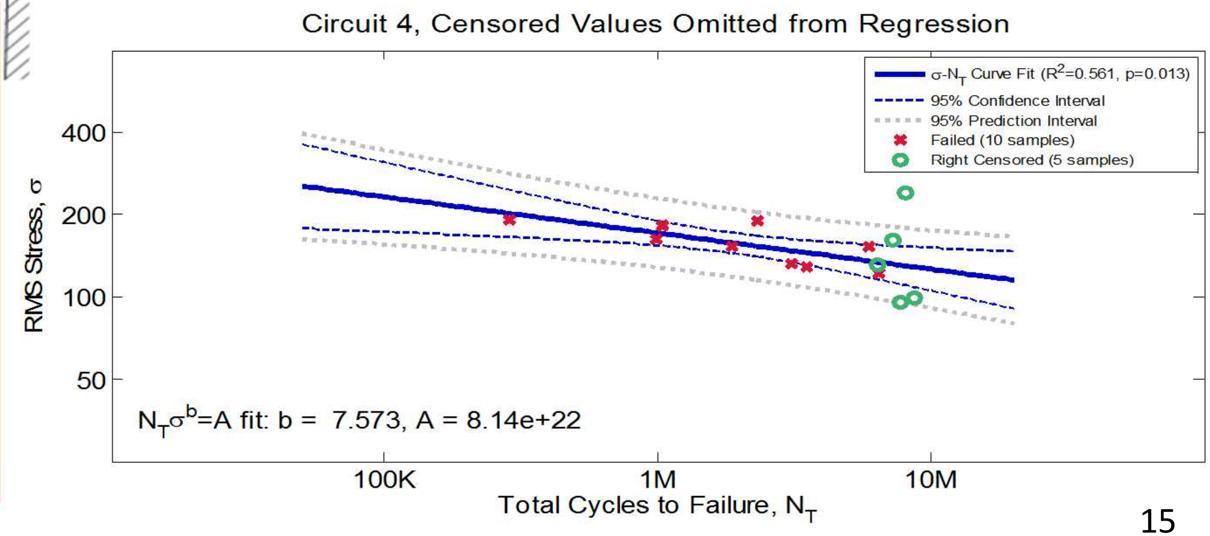
Circuit 1



Circuit 1, Censored Values Omitted from Regression

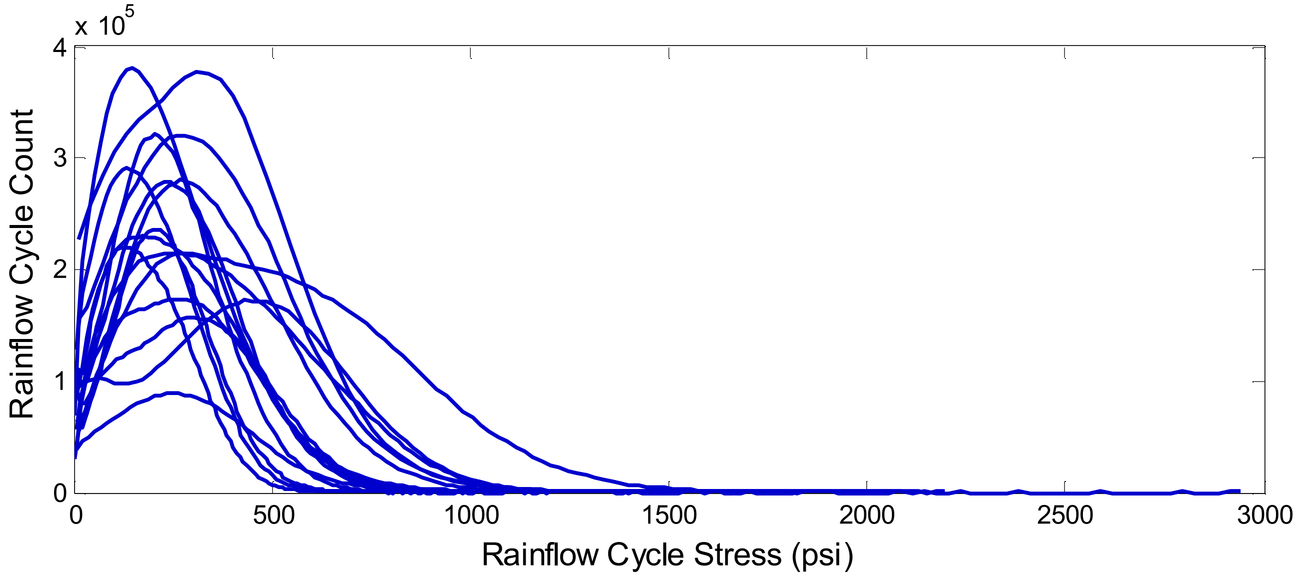
Circuit 4





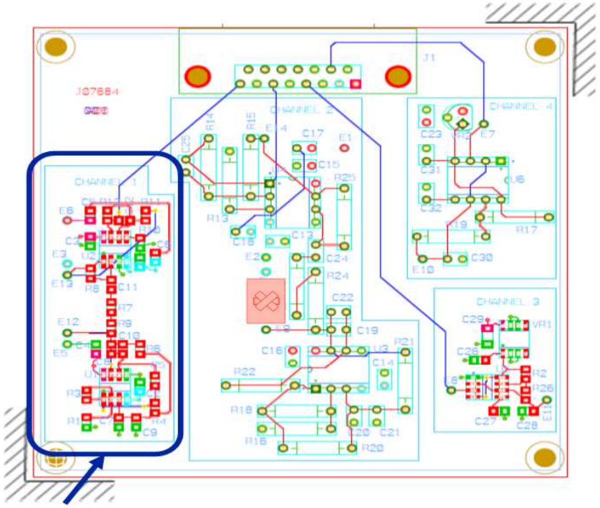
Circuit 4, Censored Values Omitted from Regression

Results: Cumulative Damage Index



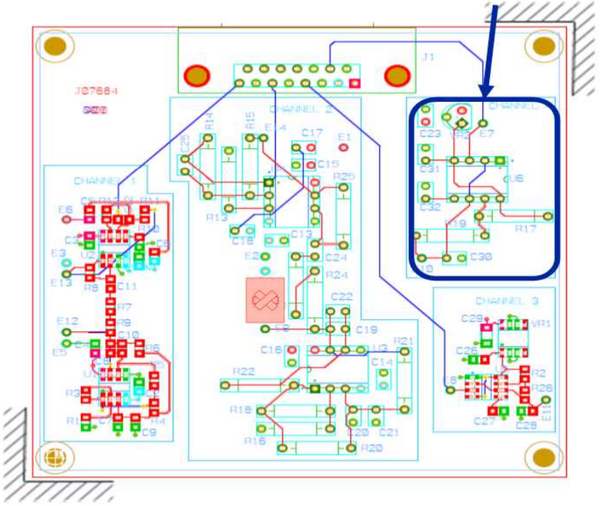
- Distribution differences led to matrix inversion problems in constrained least squares approach
- Solution found by assuming S-N curve standard form and minimizing residuals, but no way to assess goodness of fit

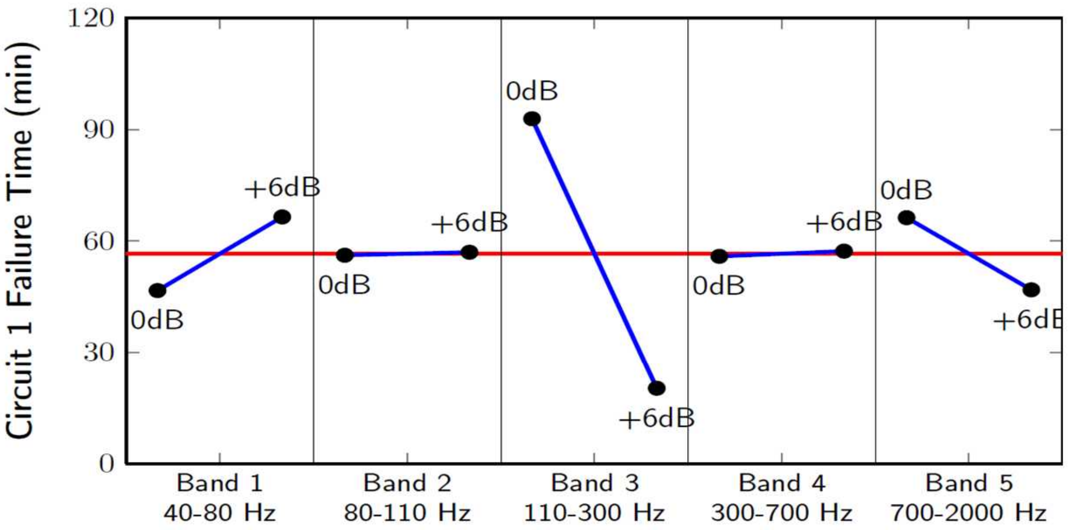
Results: ANOVA with Censored Data

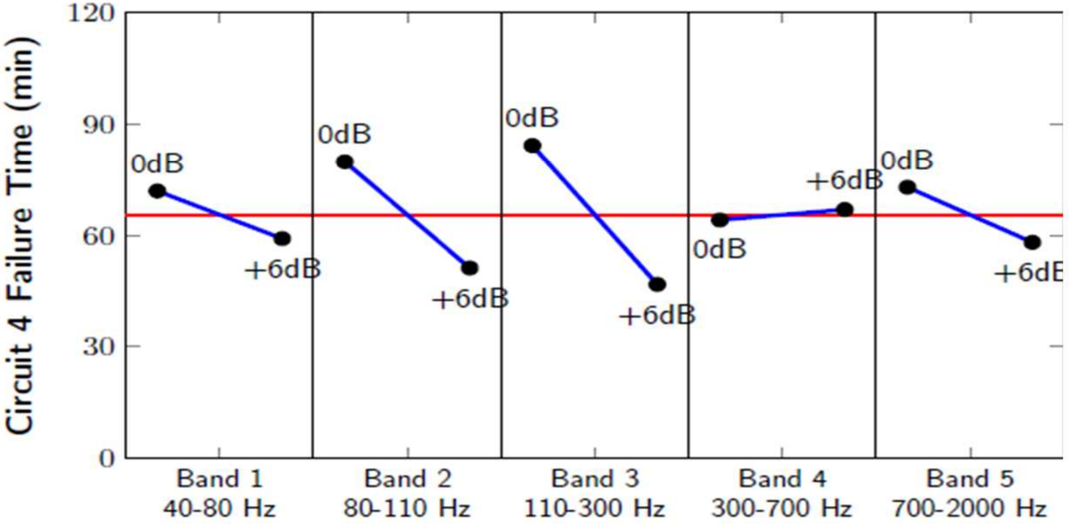


Circuit 1

Circuit 4







Scaling Modal Energy in Intensity Metrics

- Early approach: IMs treat all modal energy equally
- What about resonant modes that don't contribute to failure?
  - Design margin test plan to find modal weighting factors
- Guthrie's proposed intensity metric for random vibration:

$$\mu_j = \sum_{i=1}^{n} \sqrt{\frac{Q_i}{\varepsilon_{\text{fail},i}\big|_j}}$$

$$\left[\text{Compare to Miner's rule: } D = \sum_{\text{k}=1}^{M} \frac{n_k}{N_k}\right]$$

where Q_i is input energy for mode i and $\varepsilon_{\text{fail},i}$ is the strain energy at failure of element j

- Other energy-based variants?

$$D = \sum_{i=1}^{n} \frac{e_i}{E_i}$$

Conclusions

- Reference model feasibility
  - RMS stress approach showed reasonable correlation with failure in Circuits 1 and 4 but not Circuits 2 and 3 (too many suspensions)
  - Cumulative damage index approach failed due to problems in constrained least squares solution with sparse data

- Take-away messages
  - Failure correlation demonstrated with somewhat arbitrary location for stress (similar to superposition of modal energy)
  - Isolating spectral energy contributions through designed experiments should lead to more predictive failure models
  - Next steps in project fill in missing data and determine precise failure locations to study differences between overall and local intensity metrics

- Lessons learned (next slide)

Lessons Learned

- Many issues due to specific test article choice
  - Massive connector/cable had a strong influence on low-frequency dynamics and made setup-to-setup repeatability difficult
  - Connector block failed before the circuits in some cases, confounding the failure modes under study
  - Handmade test units expensive, difficult to obtain in quantity and likely exhibit large unit-to-unit variability in robustness
  - Somewhat contrived boundary condition (diagonal post attachment) required to obtain failures in reasonable amount of shaker time

- Proposed solution for next phase (shock) is to obtain mass-produced hobbyist function generator boards that power and monitor through USB port rather than 15-pin D-Sub connector