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Resiliency of the Power Grid

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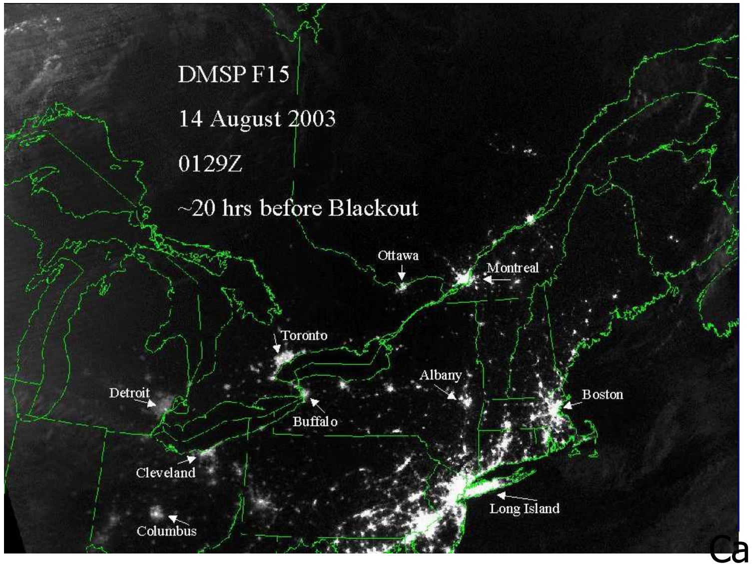
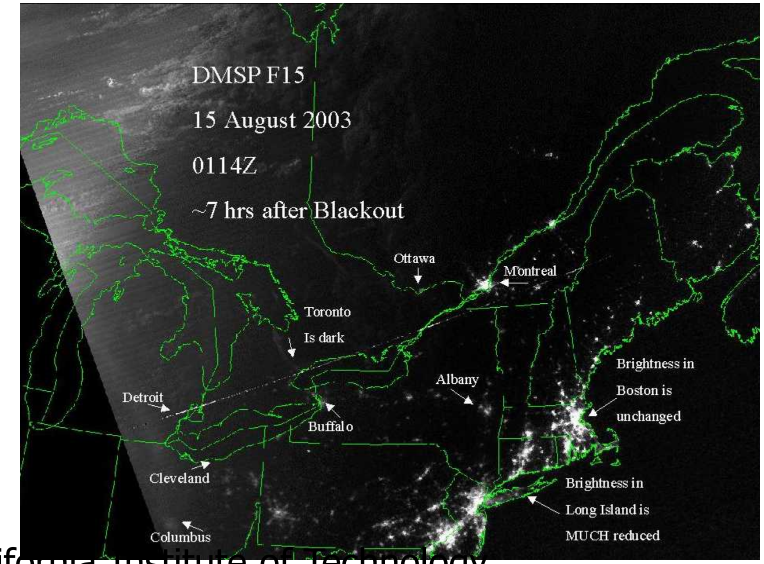
Power blackouts are a global problem

- August 2003 blackout affected 50 million people in New York, Pennsylvania, Ohio, Michigan, Vermont, Massachusetts, Connecticut, New Jersey, Ontario.
- The time to recover from the blackout was as long as 4 days at an estimated cost of $4-10 B
- Similar occurrences elsewhere: Brazil (1999), France-Switzerland-Italy (2003)





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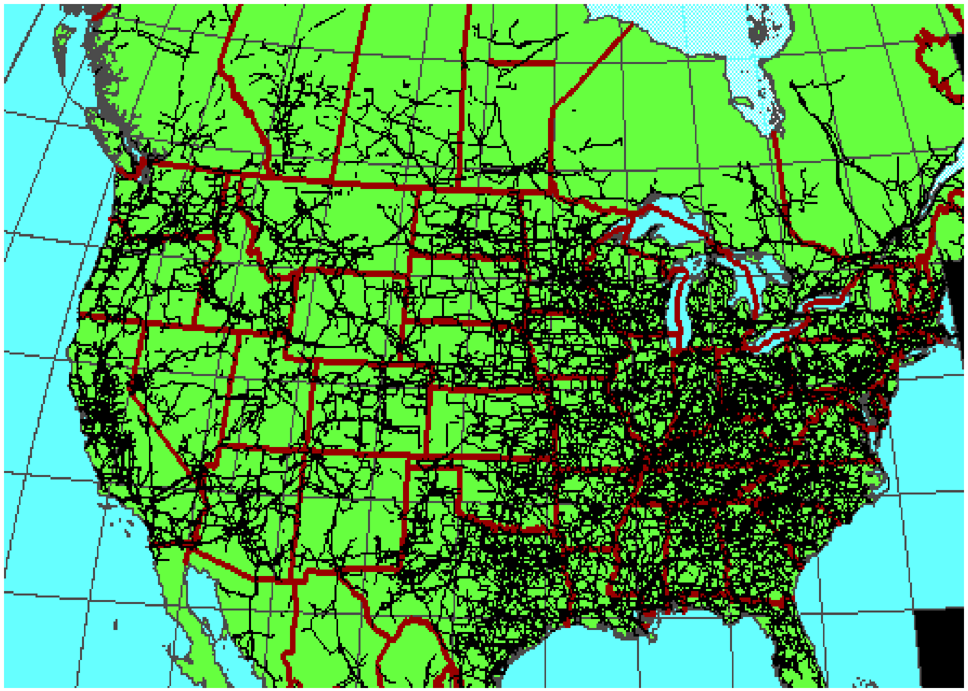
The grid's vulnerability increases with its growing complexity



Northeast blackout started with three broken lines.

- **Problem:** the current standard requires the system to be resilient to only one failure, because higher standards are not enforceable.
  - Uncertainty inherent in many renewable resources and the increasing load on the system force us to operate closer to the feasibility boundary.
- **Goal:**
  - detect vulnerabilities of the power network
  - Include contingency analysis as a constraint in systems planning
- **Challenge:** large-scale tri-level combinatorial optimization

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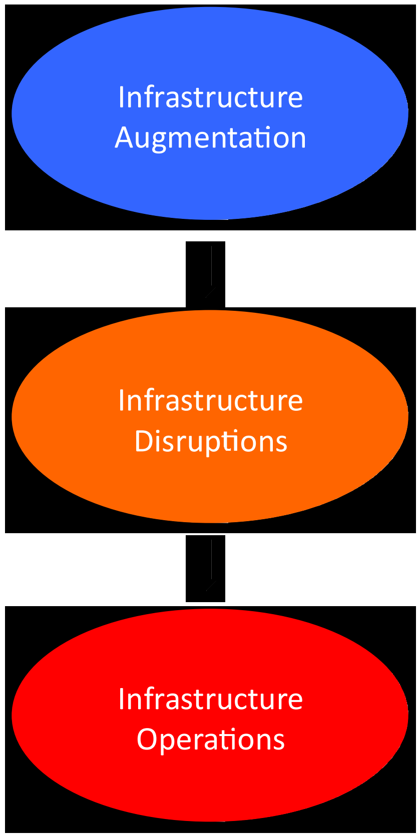
Tri-level optimization

1st Level

2nd Level

3rd Level



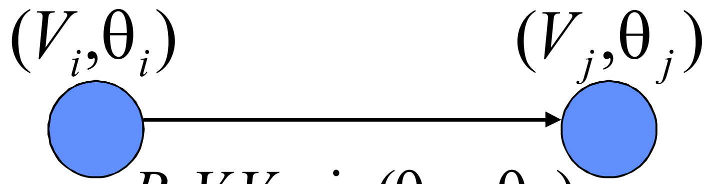
Medium and long term planning (e.g. capacity expansion, new transmission corridors, unit-commitment)

Loss of components (e.g. maintenance, equipment failure, attacks)

Respond to loss of components (e.g. load shedding)

Hierarchy of optimization problems with a modular structure

Power flow equations



$$B_{ij} V_i V_j \sin(\theta_i - \theta_j)$$ Active power

$$B_{ij} V_i V_j \cos(\theta_i - \theta_j)$$ Reactive power

$$\frac{-\pi}{2} \leq \theta_i - \theta_j \leq \frac{\pi}{2}$$

$$V_l \leq V \leq V_u$$

V: voltage

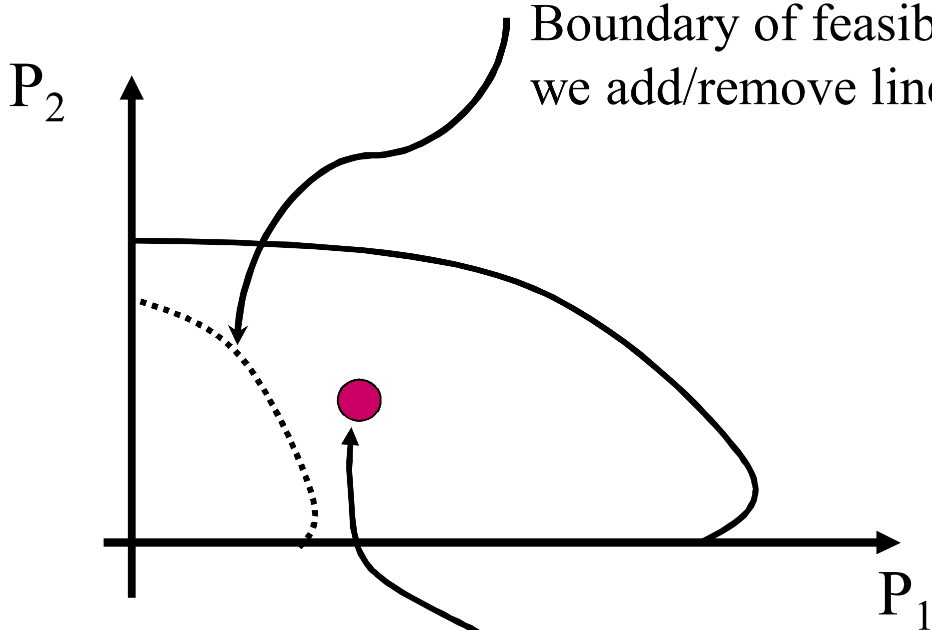
θ: phase angle

B: susceptance

- **Simplified model**
  - **Fix voltages at 1.**
  - **Work only on active power equations.**

$$F(A, \theta, p) = A^T B \sin(A\theta) - p = 0$$

Graphical representation of a blackout

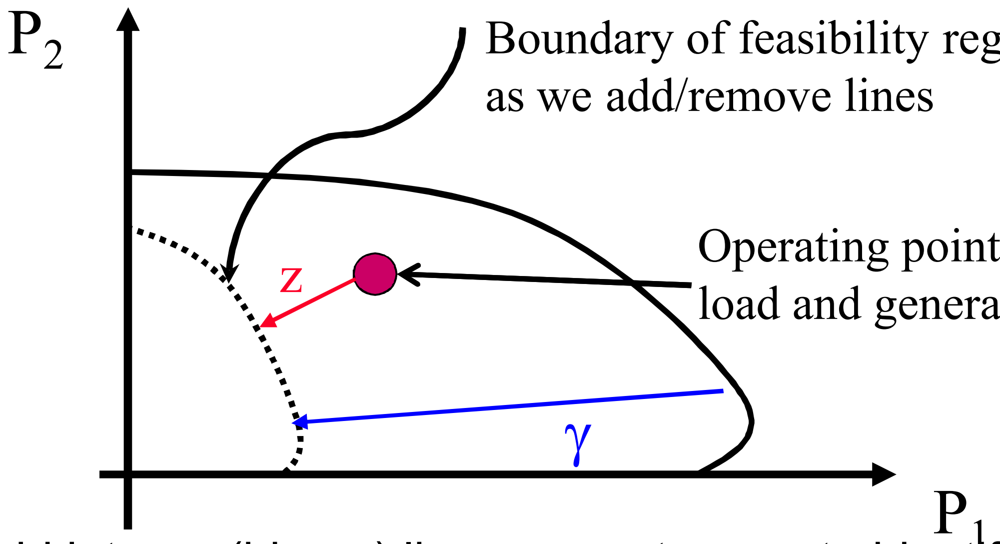


Boundary of feasibility region shifts as we add/remove lines

Operating point corresponding to current load and generation

- Blackout corresponds to infeasibility of power flow equations.
- Cascading is initiated by a significant disturbance to the system.
- Our focus is detecting initiating events and analyzing the network for vulnerabilities.

Contingency analysis as a bi-level optimization problem



- Add integer (binary) line parameters, γ, to identify broken lines
- Measure the blackout severity as the distance to feasibility boundary
- Bilevel-MINLP problem
  - cut minimum number of lines so that
  - the shortest distance to feasibility (i.e. severity) is at least as large as a specified target
- Mangasarian Fromowitz constraint qualification conditions are satisfied for a slightly modified system.

This approach leads to a Mixed Integer Nonlinear Program (MINLP)

$$\min_{\lambda,z,\theta,\mu_1,\mu_2,\mu_3,\mu_4,\mu_5,\mu_6} \quad |\gamma|$$ → minimize number of lines cut

s.t.

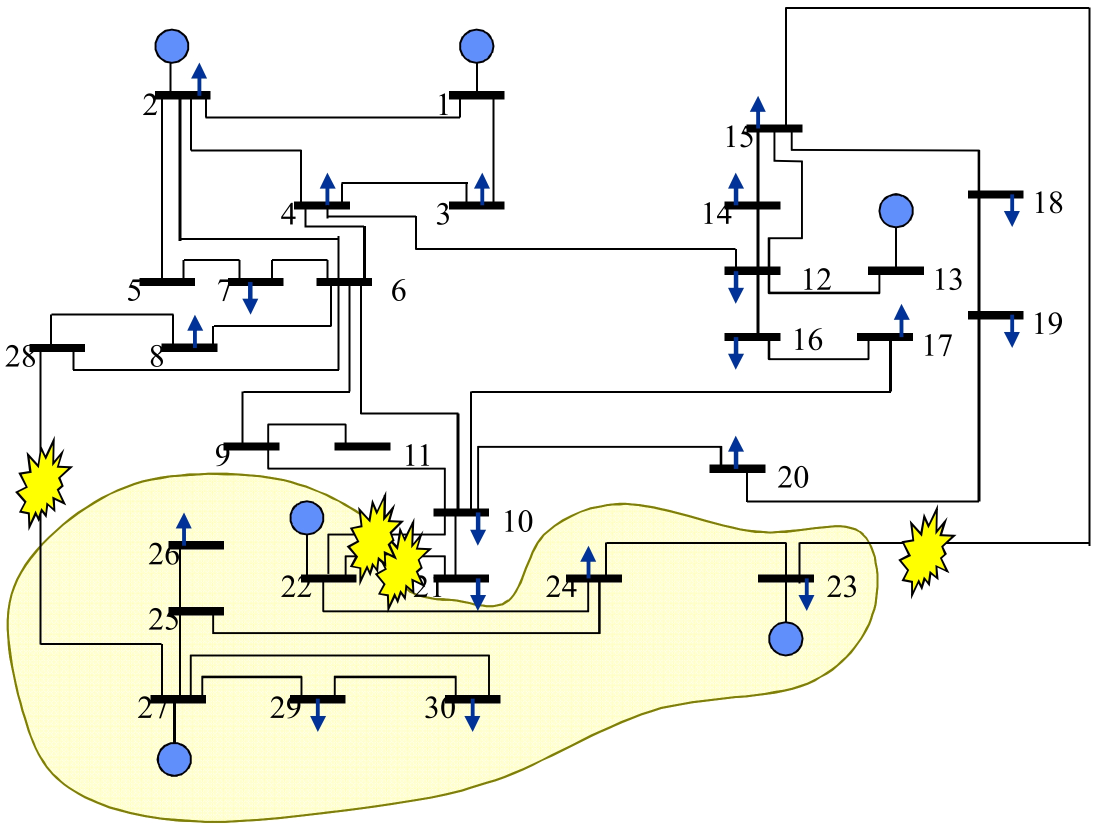
$$F(AD(1-\gamma),\theta,p+z)=0$$
$$-\pi/2 \le AD(1-\gamma)\theta \le \pi/2$$
→ feasible power flow

$$-e^T z_g \ge S$$ → severity above threshold

$$0 \le p_g + z_g \le p_g$$
$$p_l \le p_l + z_l \le 0$$
→ feasible load shedding

$$\begin{pmatrix}-e\\0\end{pmatrix} - \begin{pmatrix}\lambda_g\\\lambda_l\end{pmatrix} + \begin{pmatrix}\mu_4-\mu_3\\\mu_2-\mu_1\end{pmatrix} = 0$$
$$\lambda^T \frac{\partial F}{\partial \theta} + A^T D(1-\gamma)(\mu_6-\mu_5) = 0$$
$$\mu_1 z_l = 0; \quad \mu_2(p_l + z_l) = 0$$
$$\mu_4 z_g = 0; \quad \mu_3(p_g + z_g) = 0;$$
$$\mu_5(\pi/2 + AD(1-\gamma)\theta) = 0;$$
$$\mu_6(\pi/2 - AD(1-\gamma)\theta) = 0;$$
$$\mu_1,\ldots,\mu_6 \ge 0$$
$$\gamma \in \{0,1\}$$
→ *s*atisfy the KKT optimality conditions

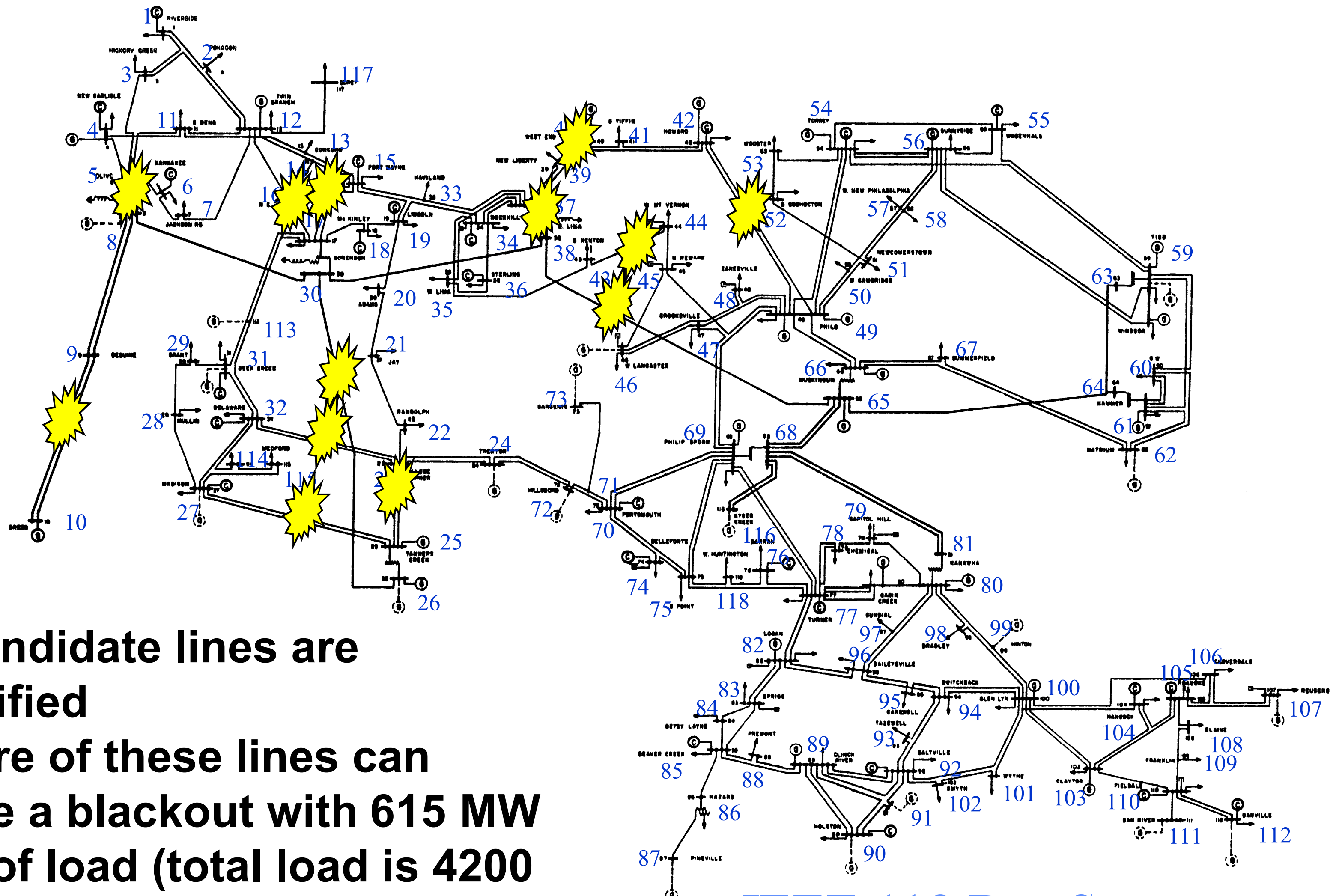
Relaxation works on small problems



IEEE 30-Bus System

- Four candidate lines identified.
- Two are sufficient to cause a blackout.
- Failure of these lines can cause a blackout with 843 MW loss out of a total load of 1655 MW).
- Solutions found using SNOPT.

…. but not on larger problems



- **13 candidate lines are identified**
- **Failure of these lines can cause a blackout with 615 MW loss of load (total load is 4200 MW)**
- **Better solutions exist**

IEEE 118 Bus System

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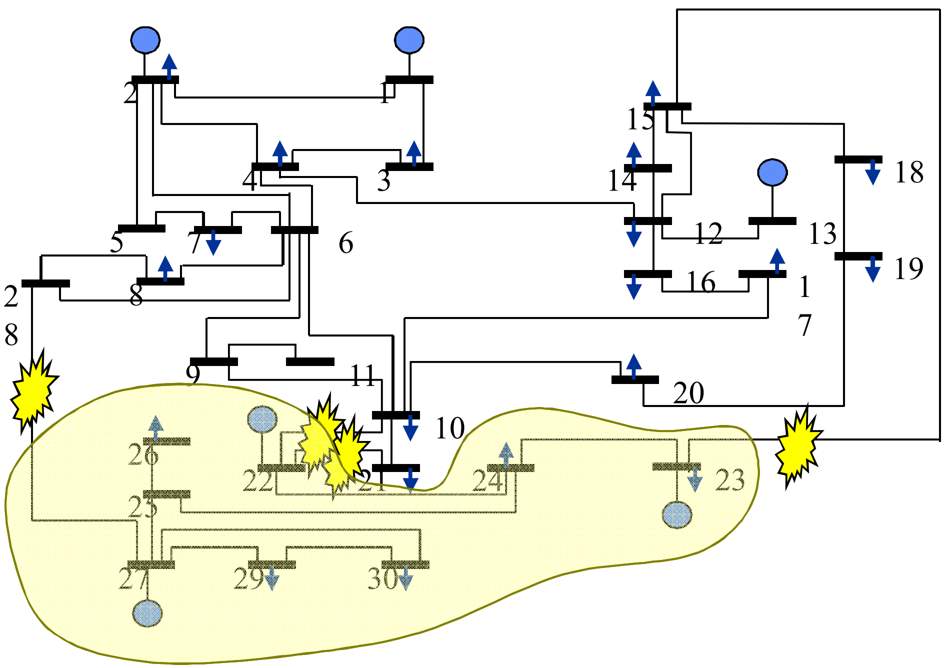
Exploiting the combinatorial structure



Theoretical analysis of the bilevel MINLP formulation shows:

- System is split into load-rich and generation-rich regions.
- There is at least one saturated line from the generation rich region to the load rich region.
- Blackout size can be approximated by the generation/load mismatch and capacity of edges in between.

Practical application: Exploit the combinatorial structure to find a loosely coupled decomposition with a high generation/load mismatch

Power-flow Jacobian corresponds to the graph Laplacian

- Key new observation: *The Jacobian matrix, which characterizes the feasibility boundary, has the same structure as the Laplacian matrix in spectral graph theory.*

$$\frac{\partial F}{\partial \theta} = J = A^T \underbrace{BD((1-\gamma)\cos(A\theta)}A$$

Node-arc incidence matrix

Diagonal matrices with non-negative weights

Node-arc incidence matrix

Can we work on a nonlinear model without solving nonlinear equations?

"'There's no such thing as a free lunch' — that'll be ten bucks."

- Practical application: Exploit the combinatorial structure
  - Find a loosely coupled decomposition with a high generation/load mismatch
- It is not free lunch, it is a very good deal.
- Why does it work?
  - We are not proposing power flow model, we only find why it is not flowing.
  - This is a flow problem.
  - The goal of the load shedding problem is to make this model work.

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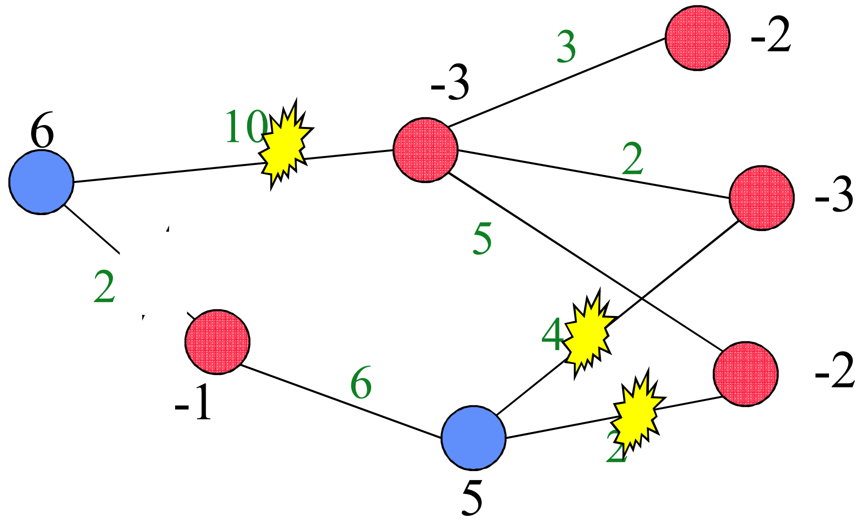
Network inhibition problem



k =0, max-flow= 11

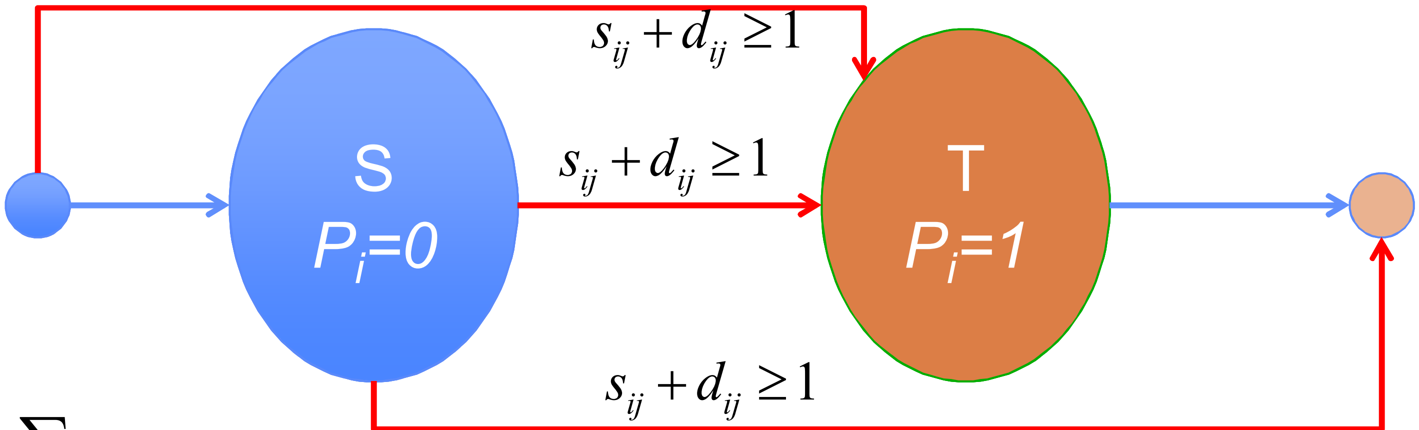
k =1, max-flow= 7

k =2, max-flow= 5

k =3, max-flow=1

- Cut min. number of lines so that max flow is below a specified bound.
- Shown to be NP-complete (Phillips 1991).
- The classical min-cut problem is a special version of network inhibition, where max-flow is set to zero.
- Can be formulated as MILP with |V|+|E| binary variables.

MILP formulation for network inhibition



$$\min \quad \sum d_{ij}$$

$$s.t. \quad \forall (v_i, v_j) \in E \quad p_i - p_j - s_{ij} - d_{ij} \leq 0$$

$$p_i - p_j + s_{ij} + d_{ij} \geq 0$$

$$\sum_{(v_i, v_j) \in E} c_{ij} s_{ij} \leq B$$

$$p_s = 0; \quad p_t = 1$$

$$p_i, d_{ij} \in \{0,1\}; \quad s_{ij} \in [0,1]$$

$$p_i = \begin{cases} 0 & v_i \in S \\ 1 & v_i \in T \end{cases} \qquad d_{ij} = \begin{cases} 1 & if \quad e_{ij} \quad is \quad cut. \\ 0 & otherwise \end{cases}$$

$$s_{ij} = \begin{cases} 1 & d_{ij} = 0 \wedge p_i \neq p_j \\ 0 & otherwise \end{cases}$$

The integrality gap is small, leading to fast solutions.

C. Burch, R. Carr, S. Krumke, M. Marathe, C. Phillips and E. Sundberg, A decomposition-based approximation for network inhibition, Network Interdiction and Stochastic Integer Programming, D.L. Woodruff, eds., (2003), pp. 51–66.

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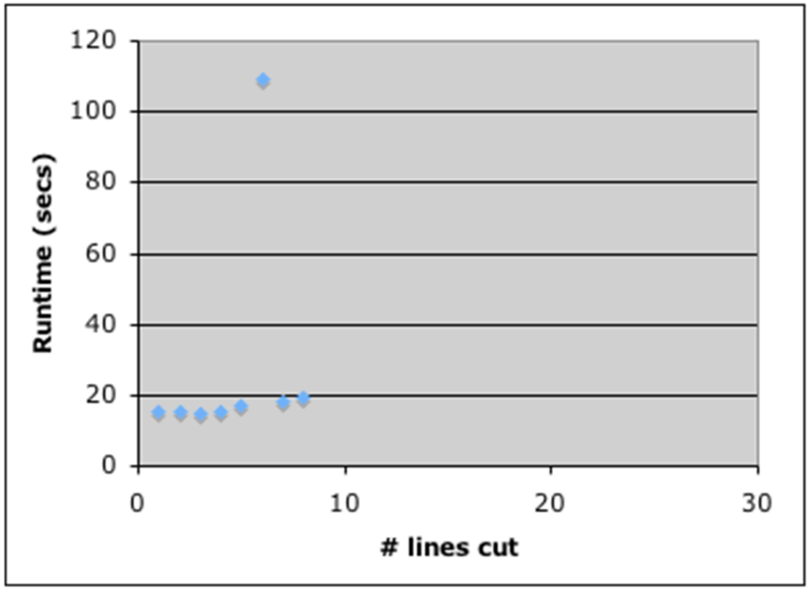
This is a tight formulation



- The integrality gap is provably small.
- Only one fractional variable after each solution.
- Experimented on a simplified model for Western states with 13,374 nodes and 16,520 lines, used PICO for solving the MILPs.
- Even the largest instances can be solved in small time, motivating us for more higher objectives.

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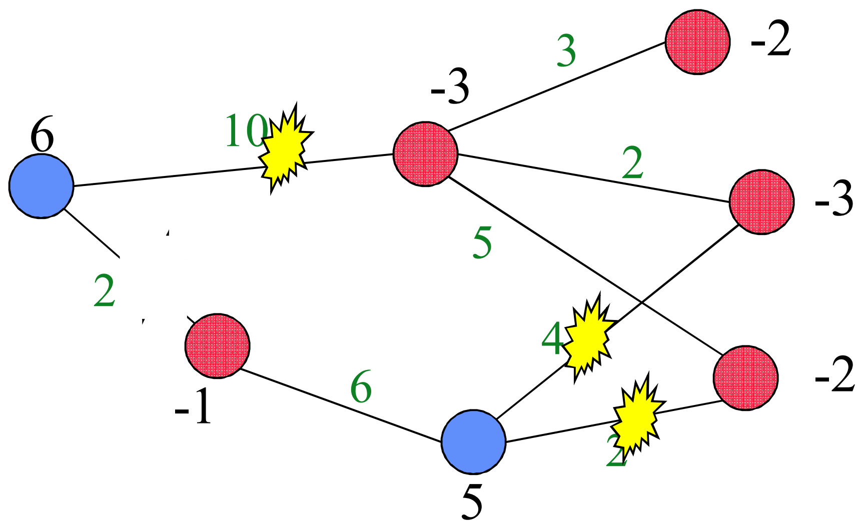
Take 2: Network inhibition problem



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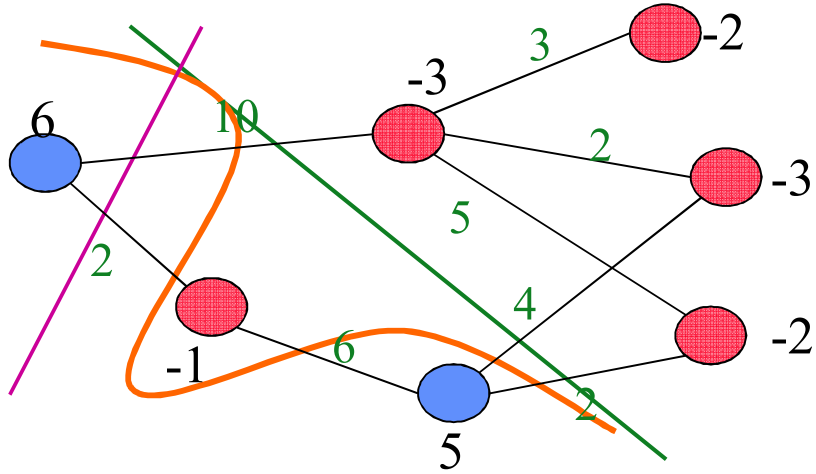
Take 3: Inhibiting bisection problem



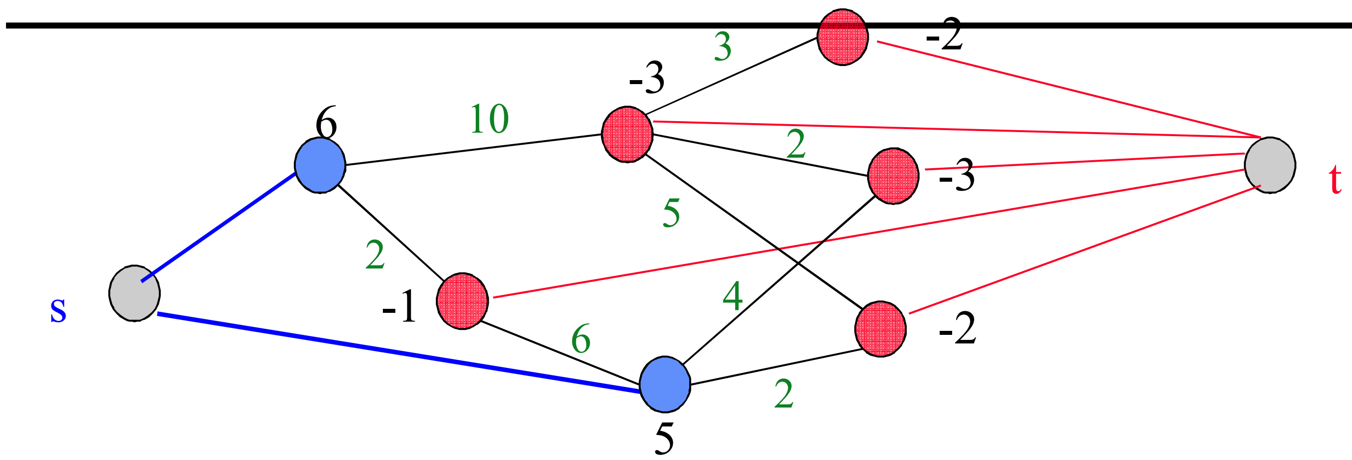
- Divide graph into two parts (bisection) so that
  - load/generation mismatch is maximum.
  - cutsize is minimum.

imbalance= 6; cutsize=2

imbalance=10; cutsize=3

imbalance=11; cutsize=5

Solving the inhibiting cut problem



- Constrained problem is NP-complete.
- Goal: minimize α (cutsize) - (1- α) imbalance
  - α is the relative importance of cutsize compared to imbalance.
- Solution: use a standard min-cut algorithm.
- Min-cut gives an *optimal* solution to the linearized inhibiting bisection problem.
- Other versions are solvable
  - Minimize cutsize/imbalance
  - Minimize capacity*(cutsize-1)/cutsize

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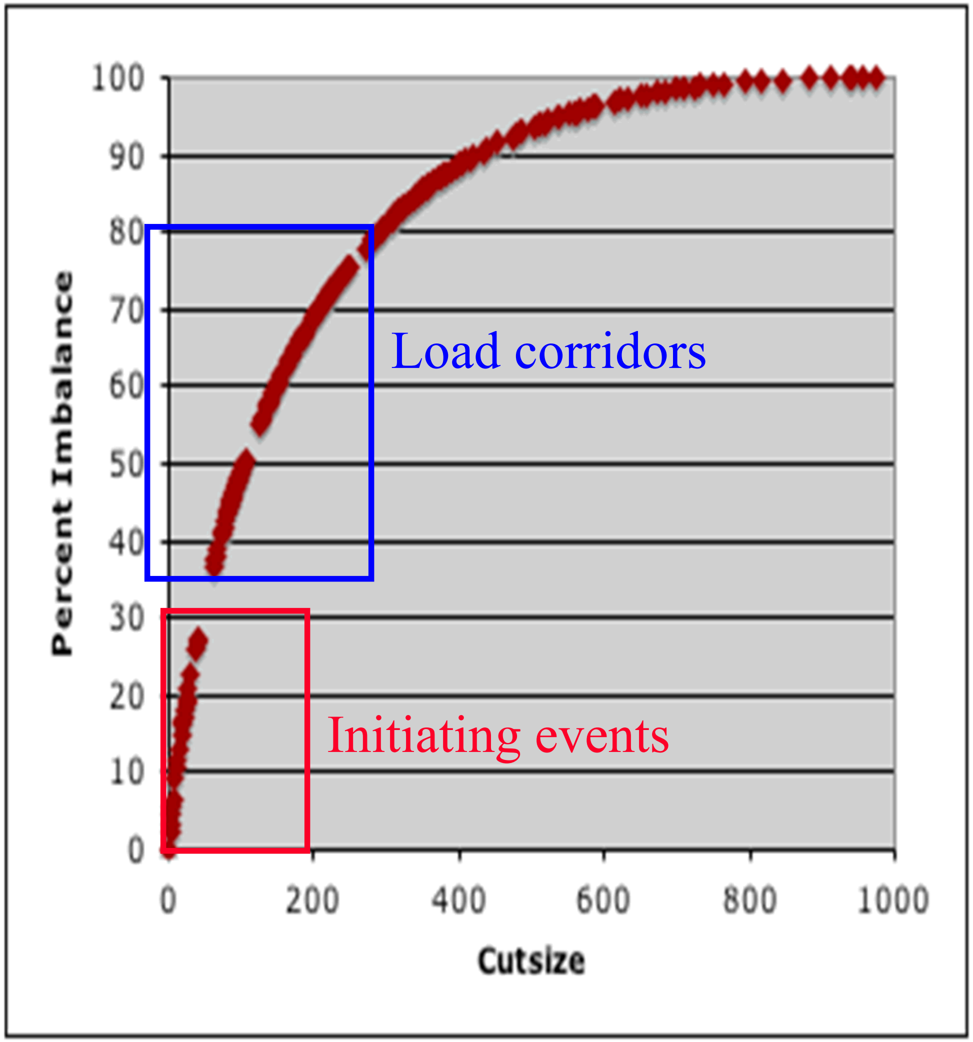
Inhibiting bisection enables fast analysis of large systems



- Experimented on a Simplified model for Western states with 13,374 nodes and 16,520 lines.
- Complete analysis using Goldberg's min-cut solver takes minutes
- Solutions with small cutsize can be used to detect initiating events and groups of vulnerabilities
- Solutions with medium cutsize reveal load corridors that can be used to contain cascading

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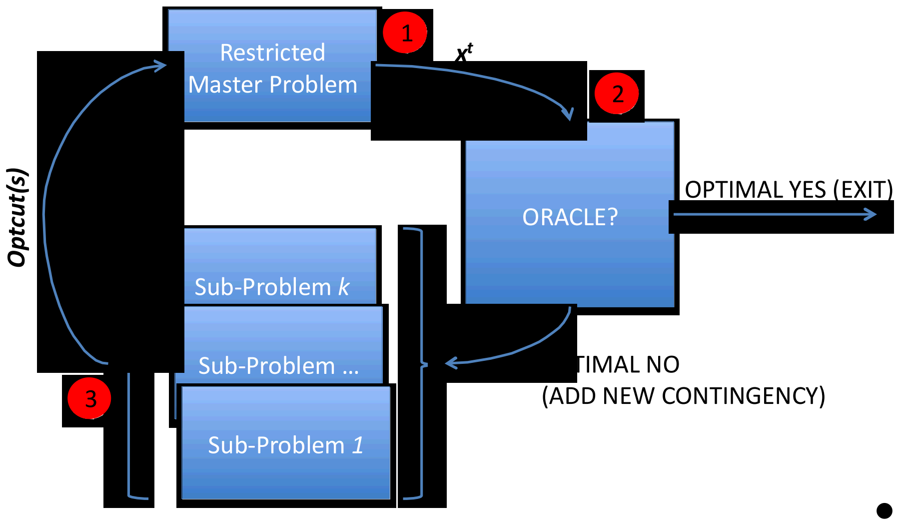
N-k survivable network design problem

- Improve a network efficiently to make it resilient to contingencies
  - Minimize the improvement cost such that the minimum number of lines for the maximum flow to be below a threshold B is above a threshold C.
- Solution approaches:
  - A single problem with a separate set of constraints for each contingency
    - forms a giant problem
  - Bender's decomposition
    - limits the memory requirements
    - the number of subproblems is still very large, prohibitively expensive for large N and k.
  - Proposed Method: Delayed Contingency Generation

Delayed contingency generation



- **Outline of the algorithm**
  - **Solve a restricted master problem to identify candidate lines to add.**
  - **Solve the network inhibition problem**
  - **If we cannot break the network, current solution is optimal**
  - **If not, add a constraint to the master problem for the identified vulnerability.**
- **Efficient solution of the interdiction problem is the key enabler.**

Initial results show scalability

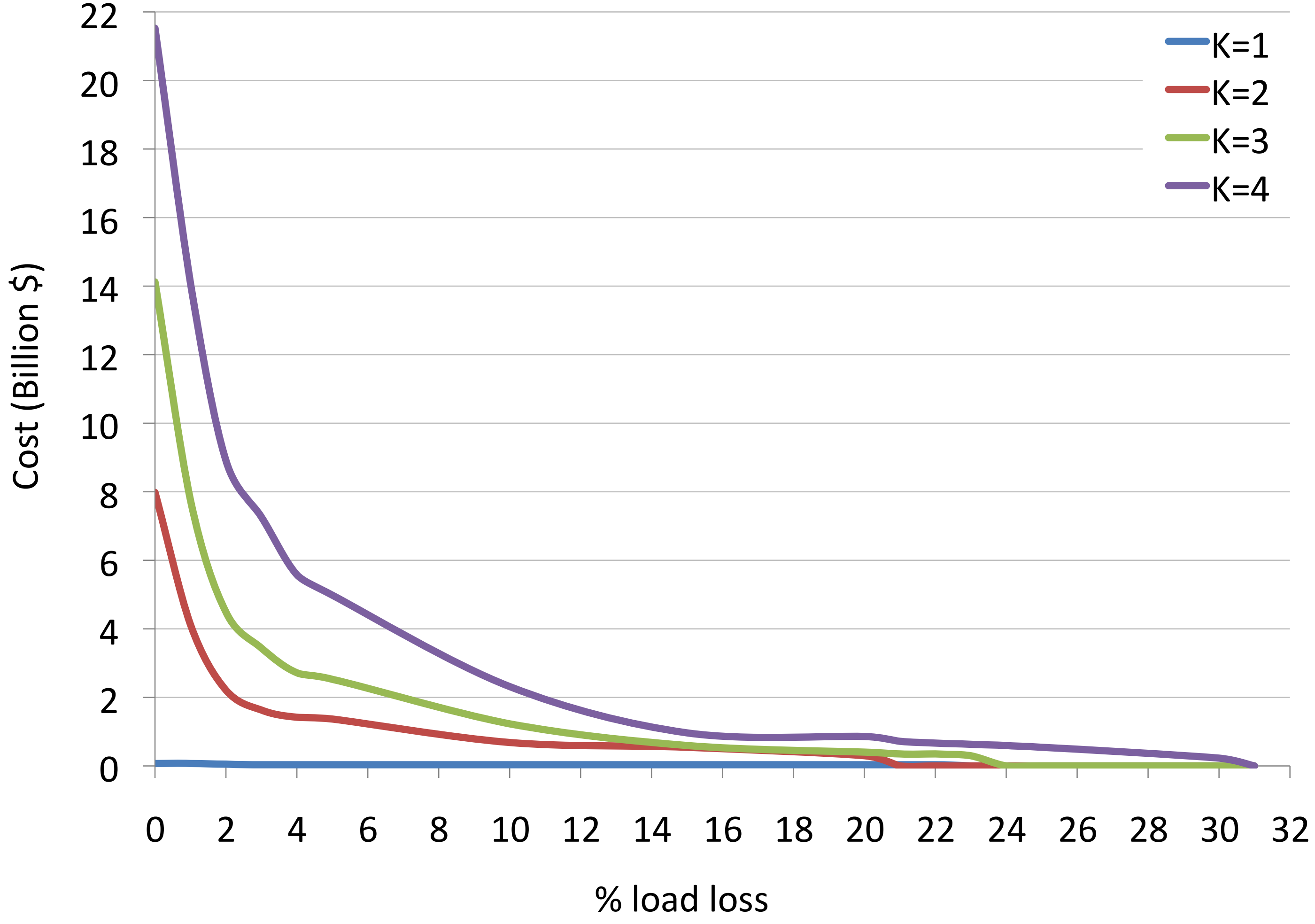
IEEE Test Systems	K	N	# of possible contingencies	EF (sec.)	CPA (sec.)	DCG (sec.)
30	1	82	82	0	0	0
118	1	358	358	20	4	4
179	1	444	444	33	11	19
30	2	123	>7K	81,722	34	1
118	2	537	>140K	x	2,142	41
179	2	666	>200K	x	5,924	174
30	3	164	>700K	x	3,045	9
118	3	716	>60M	x	x	398
179	3	888	>116M	x	x	653
30	4	205	>72M	x	x	67
118	4	895	>26B	x	x	2,708
179	4	1110	>63B	x	x	11,999

Using Cplex to solve MILPs

Only a small subset of potential contingencies are considered.

IEEE Test Systems	N	K	No. of contingencies	Total time	MP time	NIP time	SP time	No. of contingencies evaluated
30	82	1	82	0	0	0	0	3
118	358	1	358	4	0	2	1	17
179	444	1	444	19	1	7	10	51
30	123	2	>7K	1	0	1	0	15
118	537	2	>140K	41	3	26	12	58
179	666	2	>200K	174	6	50	118	158
30	164	3	>700K	9	2	5	2	43
118	716	3	>60M	398	25	303	70	128
179	888	3	>116M	653	21	193	439	284
30	205	4	>72M	67	7	23	37	156
118	895	4	>26B	2,708	399	1,698	612	359
179	1110	4	>63B	11,999	4,939	1,822	5,237	899

Cost analysis



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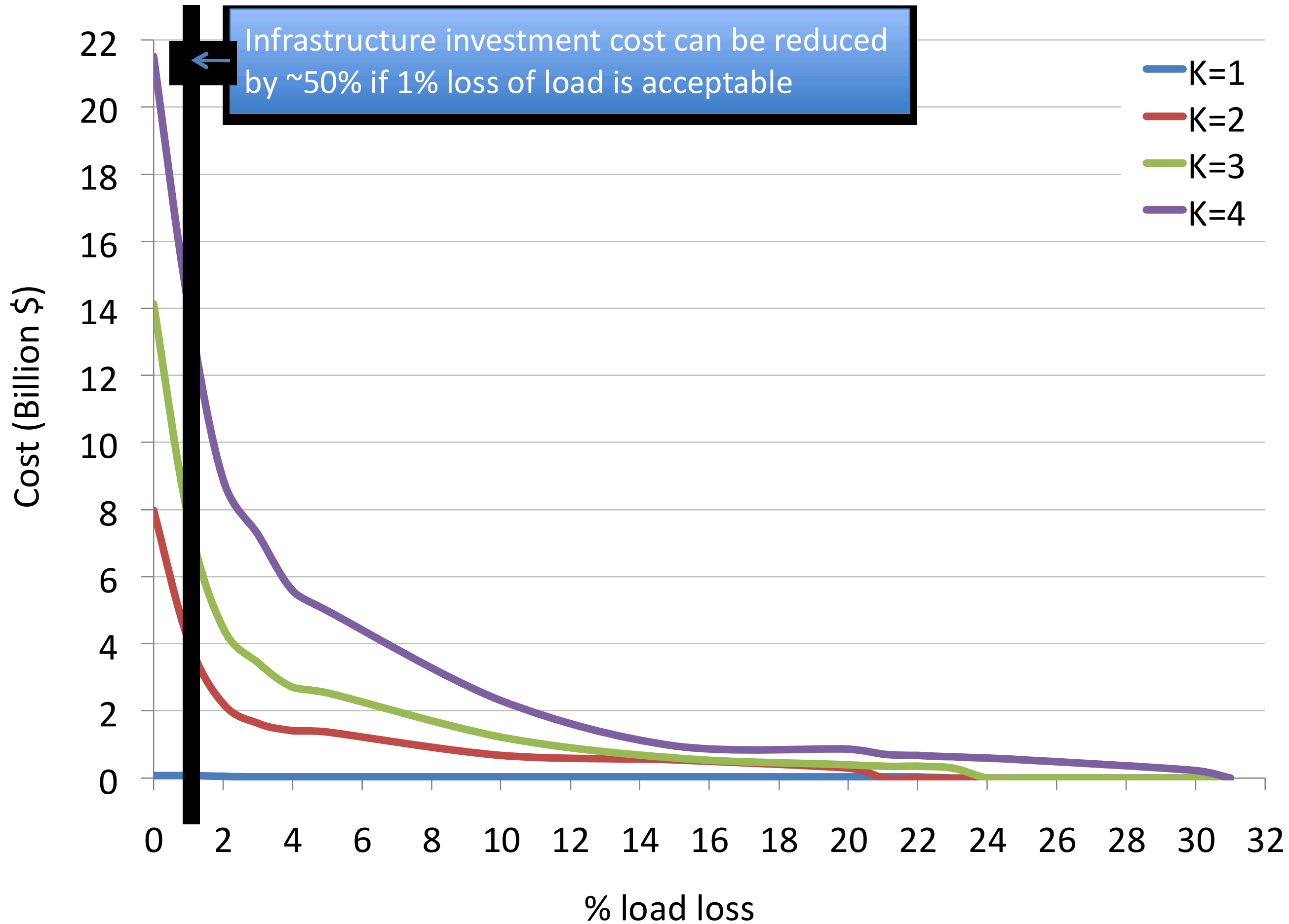
Cost of perfectness



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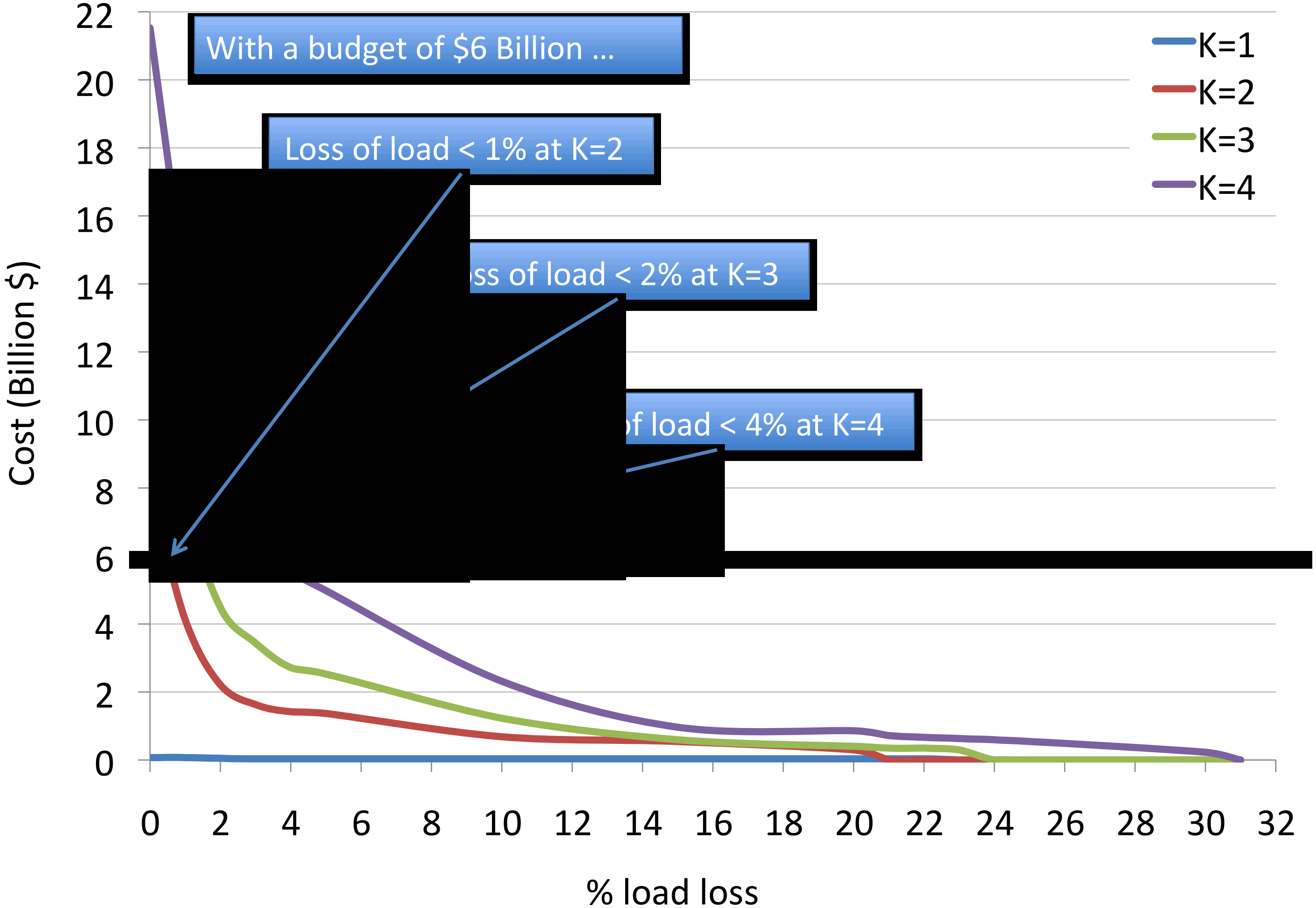
Benefits of humbleness



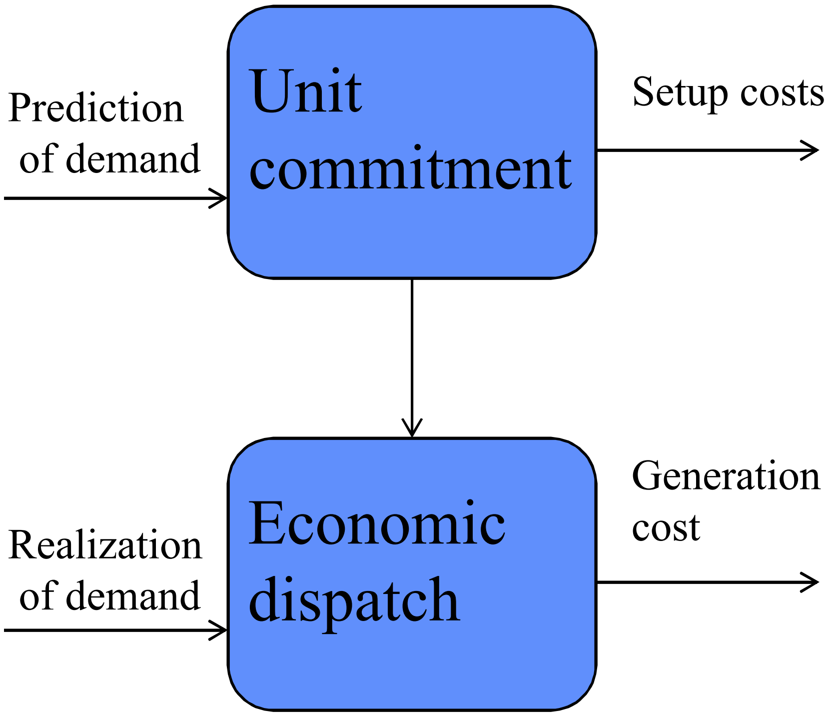
Uncertainties of renewables pose a crucial challenge for grid operations

Source: http://saferenvironment.wordpress.com Full link

Source: http://www.thesierraleonetelegraph.com/?p=5393

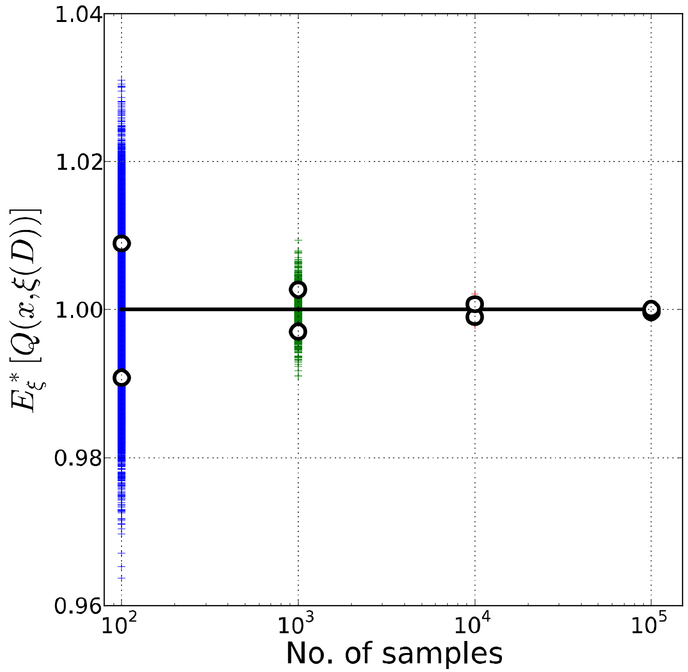
- Most renewable resources cannot be controlled and involve significant uncertainties.
- High penetration of renewables lead to a significant change in operations due to uncertainty.
- Storage technologies are not adequate enough, yet.
- Operations require decision making under uncertainty.
  - Stochastic optimization is essential.
  - Better models for handling uncertainty are needed.

Operational problems require stochastic optimization



- Sample Problem: Unit Commitment
- Fundamental problem in operations
- Two stage problem
  - Decide on the state of big and slow generators under a prediction of demand/ renewables
  - Operate on a realization of uncertainties to minimize generation costs
- Standard approach Monte Carlo Sampling

Efficient Model for Uncertainty: Polynomial Chaos Expansion



Thiam and DeMarco: "Simply put, when uncertainty is credibly accounted for such methods yield solutions for economic benefit of a transmission expansion in which the "error bars" are often larger than the nominal predicted benefit."

- Error for Monte Carlo Var(f)/sqrt(S)
- Accurate estimations render optimization problems impractical.
- Proposed Solution: Polynomial chaos expansion
  - Commonly used for uncertainty quantification in CSE applications
  - Core idea: preprocess the random variables to build a surrogate that represents random variables compactly
- Promising Initial results:
  - Currently working on adding this to the optimization loop

Concluding remarks

- **Optimization problems with contingency constraints lie at the heart of many problems in power systems operations and planning.**
- **Recent progress in contingency analysis has paved the way for higher objectives.**
  - Vulnerability analysis of a power system can be studied as bi-level MINLP problem.
  - Special structure of a feasible solution to our MINLP formulation can be exploited for a simpler approach for vulnerability detection.
  - Our combinatorial techniques can analyze vulnerabilities of large systems in a short amount of time.
- **Delayed contingency generation approach shows promising results for N-k contingency constrained network improvement problem.**
- **Current work**
  - **Improve current results**
  - **Use DC power flow**
  - **Apply the same approach to unit-commitment**

Acknowledgements

- Richard Chen, **Amy Cohn, Jean-apul Watson,** Bernard Lesieutre, Juan Meza, Vaibhav Donde, Vanessa Lopez, Chao Yang, Adam Reichert, Yonatan Fogel, Sandip Roy, and Aydin Buluc contributed to this work.

Related Publications

- P., Meza, Donde, and Lesieutre, “Optimization Strategies for the Vulnerability Analysis of the Power Grid,” *SIAM Optimization*, 20 (4), pp. 1786-1810, 2010.
- Donde, Lopez, Lesieutre, P., Yang, and Meza, “Severe Multiple Contingency Screening in Electric Power Systems,” *IEEE T. Power Systems*, 23(2), pp. 406-417, 2008.
- Lesieutre, P., and Roy, “Power System Extreme Event Detection: The Vulnerability Frontier," in *Proc. 41st Hawaii Int. Conf. on System Sciences*, p. 184, HI, 2008.
- P., Reichert, and Lesieutre, “Computing Criticality of Lines in a Power System,” *Proc. 2007 IEEE Int. Symp. Circuits and Systems*, pp. 65—68, New Orleans, LA, May 2007.
- Lesieutre, Roy, Donde, and P., “Power system extreme event analysis using graph partitioning,” *Proc. the 39th North American Power Symp.*, Carbondale, IL, 2006.
- Donde, Lopez, Lesieutre, P., Yang, and Meza, “Identification of severe multiple contingencies in electric power networks,” *Proc. the 38th North American Power Symp.*, Ames, IA, October 2005.
- P., Fogel, and Lesieutre, ``The Inhibiting Bisection Problem,” Technical Report: LBNL-62142, Lawrence Berkeley National Laboratory, Berkeley, CA.

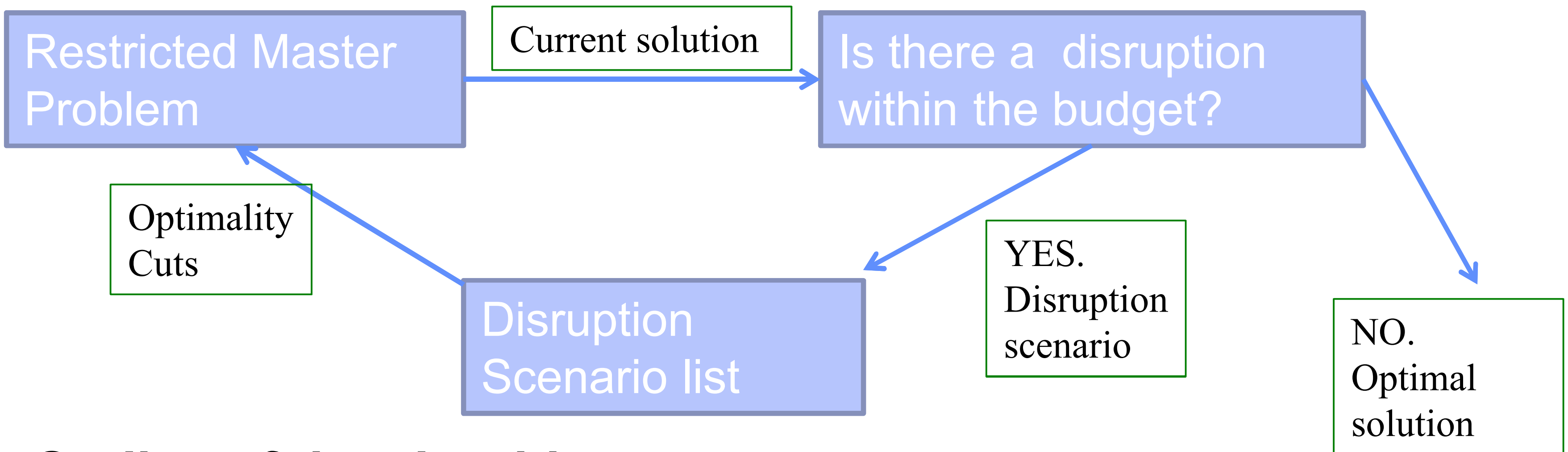
- **Questions?**

Breakdown of runtimes

Test Systems	No. poss. scen.	No. eval. scen.	Total time	RMP time	NDP time	SP time
1	82	3	0	0	0	0
2	358	17	4	0	2	1
3	444	51	19	1	7	10
4	$> 7K$	15	1	0	1	0
5	$> 140K$	58	4	3	26	12
6	$> 200K$	158	174	6	50	118
7	$> 700K$	43	9	2	5	2
8	$> 60M$	128	398	25	303	70
9	$> 116M$	284	653	21	193	439
10	$> 72M$	156	67	7	23	37
11	$> 26B$	359	2708	399	1698	612
12	$> 63B$	899	11999	4939	1822	5237

Experiments on IEEE-30, K=1

Alternative Formulation



- **Outline of the algorithm**
  - **Solve a restricted master problem to identify candidate lines to add.**
  - **Solve the network inhibition problem**
  - **If we cannot break the network, current solution is optimal**
  - **If not, add a constraint to the master problem for the identified vulnerability.**
- **Efficient solution of the interdiction problem is the key enabler.**

N-k survivable network design

- How do we improve a network cost effectively to make it resilient to contingencies?
- It is a tri-level discrete optimization problem
  - **Minimize** the improvement cost such that the **minimum** number of lines for the **maximum** flow to be below a threshold B is above a threshold C.
- We can
  - build on our work on network interdiction
  - rely on the reasonable resilience of the existing network.
- The same paradigm can be applied to various other optimization problems in power systems.

Take home lessons

All models are wrong; some are useful.
George E.P. Box

Graph models are useful, because

- *they have the flexibility to model a variety of problems*
- *we have the ability to solve associated problems.*

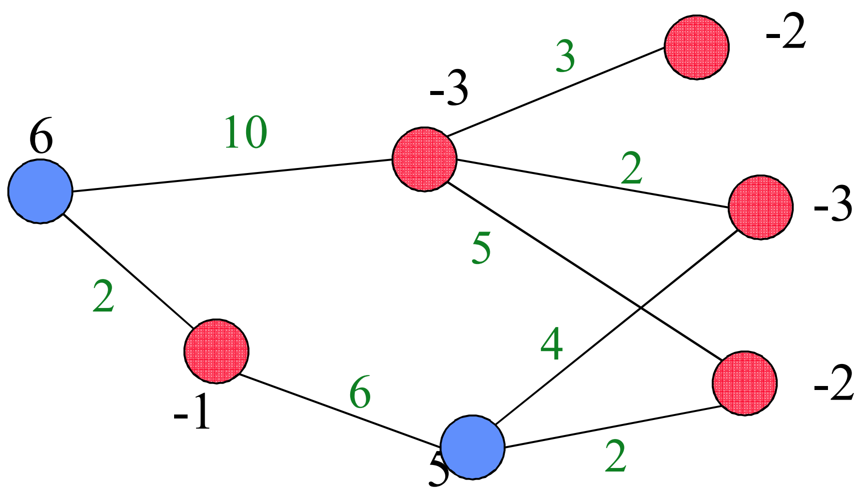
- *That your problem involves a graph does not imply graph algorithms will provide a solution.*
- *That you cannot see a graph problem immediately does not mean it does not exist.*

Vulnerability analysis as a combinatorial problem

Given a graph *G=(V,E)* with weights on its vertices

- positive for generation,
- negative for loads,

find a partition of *V* into two loosely connected regions with a significant load / generation mismatch.



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Concluding remarks

- Graph models are useful!
- That your problem involves a graph does not imply graph algorithms will provide a solution.
- That you cannot see a graph theoretical problem immediately does not mean it does not exist
- Vulnerability analysis of a power system can be studied as bi-level MINLP problem.
- Special structure of a feasible solution to our MINLP formulation can be exploited for a simpler approach for vulnerability detection.
- Our combinatorial techniques can analyze vulnerabilities of large systems in a short amount of time.

Future work

- Study the gap between the combinatorial model and the nonlinear flow model
  - quantify the gap
  - find better approximations
  - understand its effect on dynamics
- Include vulnerability analysis as a constraint in decision making
  - daily operations unit commitment
  - system upgrade, maintenance scheduling
- Surrogates challenge:
  - We do not need a power flow model, we need a certificate that a solution exists.
- Generalizations of the inhibiting bisection problem
- New project starting in FY11 will look at long term planning for the power grid.