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Estimation of Transmissivity using Non-radial Flow Dimensions

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Outline

- Background
- Flow dimension
  - Conceptualization
  - Mathematics
- Conduits
  - Formation
  - Simulations
    - Impermeable host material
    - Variable transmissivity host material
- Binary random fields
- Conclusions

Background

- Flow dimension (*n*)
  - Description of how cross-sectional area of flow changes with respect to distance from a source
  - Developed by Barker (1988) for use in studies of flow in fractured rock

- Frame of reference
  - $n = 1$ – Linear flow
  - $n = 2$ – Radial flow
  - $n = 3$ – Spherical flow

Background – Flow Dimension

$$S_s \frac{\partial h}{\partial t} = \frac{K}{r^{n-1}} \frac{\partial}{\partial r}\left(r^{n-1} \frac{\partial}{\partial r} \right)$$

S_s = specific storage [1/L]
h = hydraulic head [L]
t = elapsed time [T]
K = hydraulic conductivity [L/T]
r = radial distance from the borehole [L]
n = flow dimension

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Research Questions

- Can we devise a way to produce non-radial flow dimensions in a two-dimensional context?
- Can we use diagnostic analysis techniques to extract aquifer property estimates (*Transmissivity (T)*, *Storage (S)*, *n*) in the two-dimensional context?
- Can we mimic the diagnostic response characteristics seen in field data analysis with simulated pumping tests with non-radial flow dimension?

Mathematics of Conduit Simulation Formulation

Barker's formulation can be simplified if we specify the value of $A(r_w)$ for a constant flow area at the well or source that does not change with n. Solving for b and creating a simplifying term $a=2\pi^{n/2}/\Gamma(n/2)$,

$$A(r) = b^{3-n} \frac{2\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)} r^{n-1} \longrightarrow b = \left(\frac{A(r_w)}{a r_w^{\,n-1}} \right)^{\frac{1}{3-n}}$$

For any distance r in a constant n system we can substitute for b:

$$A(r) = \left[\left(\frac{A(r_w)}{a r_w^{\,n-1}} \right)^{\frac{1}{3-n}} \right]^{3-n} a r^{n-1} \longrightarrow A(r) = \frac{A(r_w)}{r_w^{\,n-1}} r^{n-1}$$

This relationship defines the cross-sectional area of flow at some distance r from the source.

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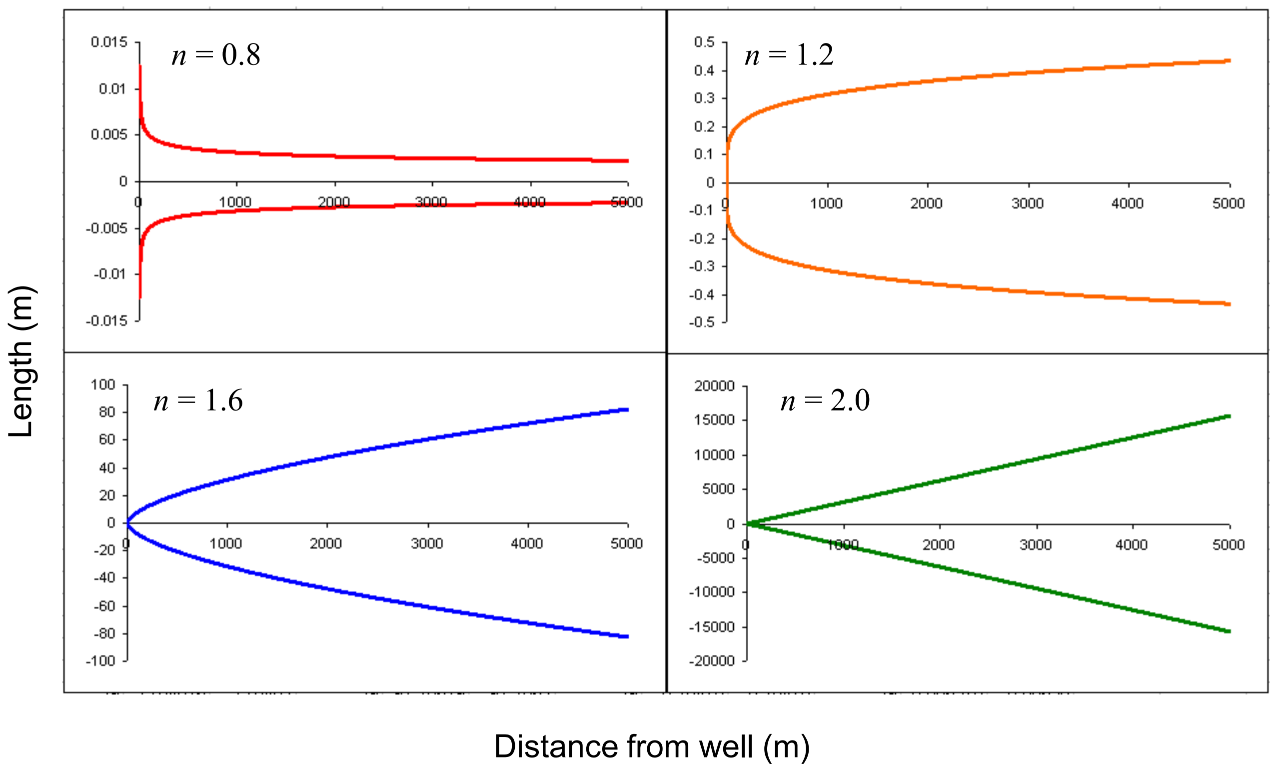
n-dimensional Conduits

Using our numerical relationship, we can generate representations of increasing cross-sectional area of flow for any input flow dimension.



$$A(r) = \frac{A(r_w)}{r_w^{n-1}} r^{n-1}$$

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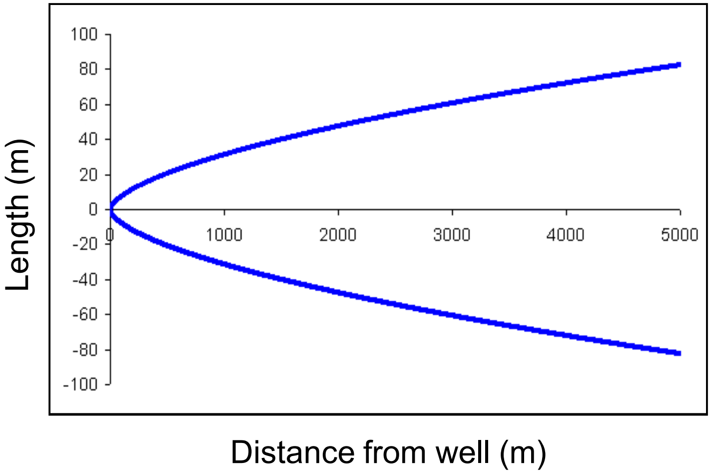
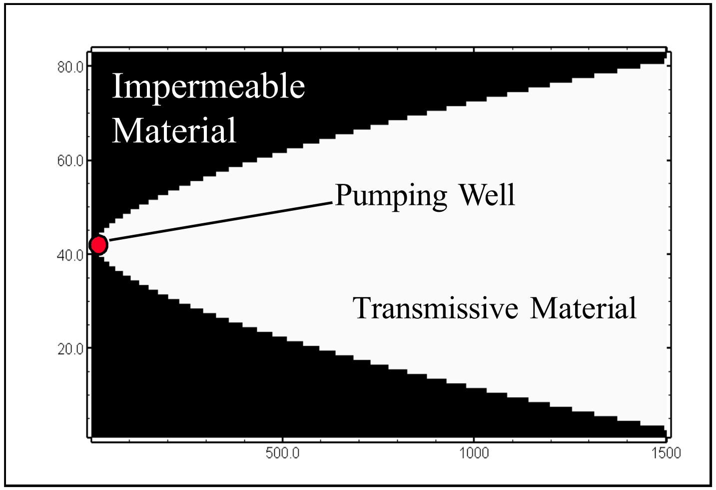
Finite Difference Conduit Representations

Using our cross-sectional area of flow to radius relationship, 2-D linear representations were calculated and transformed into a finite difference grid for simulated pumping tests.





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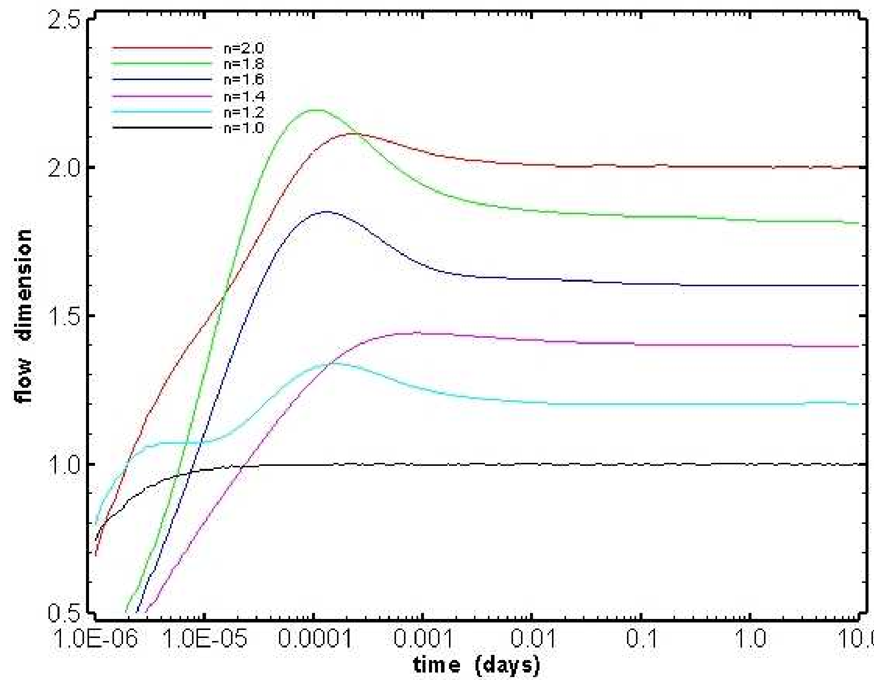
Conduit Simulation Results

- Conduits of n = 1.0, 1.2, 1.4, 1.6, 1.8, and 2.0 were tested
- Diagnostic analysis of each simulation produced the same flow dimension used to create each conduit



Flow dimension can be connected to geometry using our assumption

We provide a method to visually describe non-radial flow dimensions

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Perturbation Analysis

A method used to adequately sample the parameter space about a user-supplied estimate

- User choices:
  - Baseline value
  - Plus/minus range for parameter space
- Fitting parameters are randomly perturbed
- Re-optimization of the perturbed fitting parameters

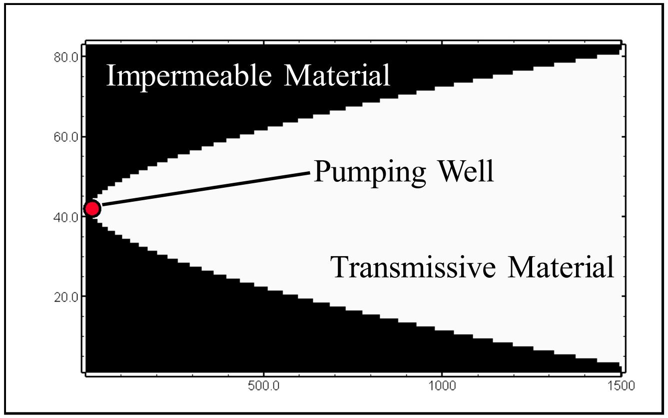
Perturbation Analysis Results
Conduits in an Impermeable Host Material

Input T = 1E-5 m^2/s, S = 1E-4

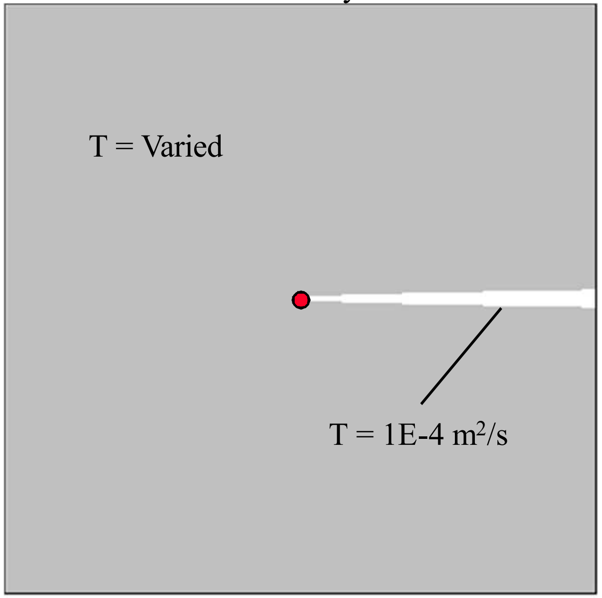
	Estimated Mean Value				Estimated Variance		
Input n	S	T (m^2/s)	n	Input n	S	T (m^2/s)	n
1.0	2.02E-04	1.28E-05	1.00	1.0	3.20E-09	8.10E-12	9.33E-05
1.2	8.80E-05	1.02E-05	1.20	1.2	4.84E-10	5.07E-12	1.32E-07
1.4	2.47E-04	1.08E-05	1.40	1.4	2.94E-07	7.19E-11	1.04E-04
1.6	3.29E-04	1.21E-05	1.61	1.6	1.45E-07	1.50E-10	2.83E-04
1.8	1.27E-04	6.44E-06	1.83	1.8	1.06E-09	2.46E-12	4.81E-04

Conduit Simulations – Variable Host Material Transmissivity

Impermeable Host Material



Variable Transmissivity Host Material



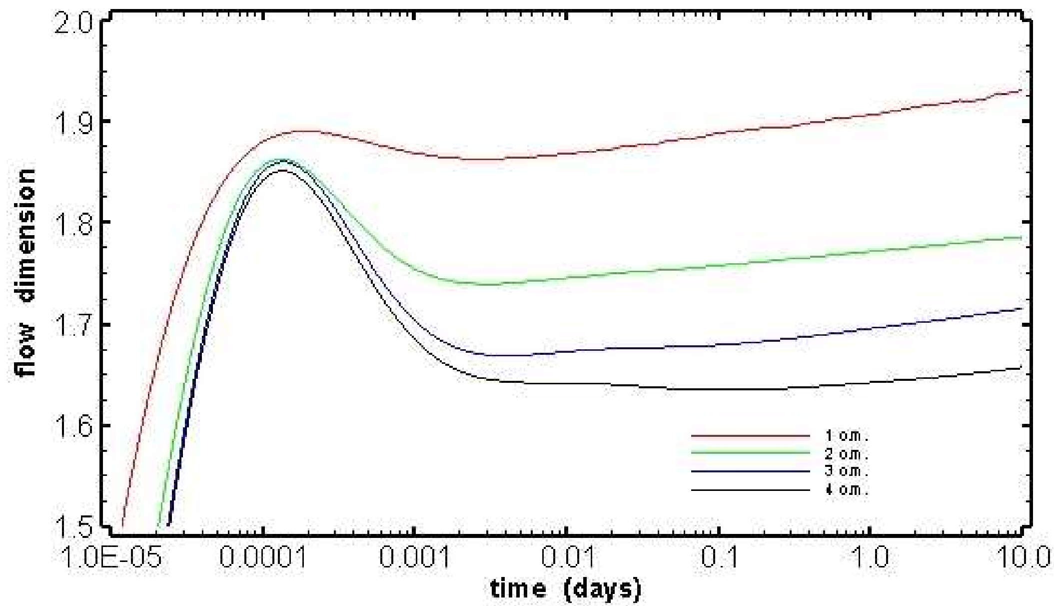
- Conduit transmissivity held constant
- Host material transmissivity varied per simulation over four orders of magnitude (1E-5 m^2/s -- 1E-8 m^2/s)

Results - Conduit Simulations in Variable Host Material Transmissivity

- Steady positive slope

- As we approach homogeneity, *n* tends toward radial

- *T* contrasts greater than four orders of magnitude between the conduit and host material responds similar to a conduit imbedded in an impermeable host material



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Perturbation Results – Conduits in a Variable Transmissivity Host Material

Input conduit *T* = 1E-5 m^2/s, *S* = 1E-4, *n* = 1.6

	Estimated Mean Value				
Host *T* (m^2/s)	*S*	*T* (m^2/s)	*n1*	*n2*	*n3*
1E-6	1.10E-04	5.44E-06	1.82	1.97	-
1E-7	1.10E-04	7.76E-06	1.67	1.72	1.82
1E-8	1.32E-04	7.88E-06	1.63	1.75	-
1E-9	1.35E-04	8.27E-06	1.61	1.67	-

	Estimated Variance				
Host *T* (m^2/s)	*S*	*T* (m^2/s)	*n1*	*n2*	*n3*
1E-6	1.07E-09	1.57E-12	3.39E-3	1.65E-2	-
1E-7	7.00E-05	3.41E-05	6.01E-6	3.43E-6	3.84E-5
1E-8	1.42E-10	4.18E-13	5.41E-4	1.31E-4	-
1E-9	3.09E-11	5.75E-14	4.42E-5	3.03E-5	-

Binary Random Fields

We wanted a way to generate fields that would result in purely geometric effects in their diagnostic analysis.

- Binary Random Fields (BRFs)
  - Originally Gaussian fields
  - Input values of mean, variance, x- and y-directional correlation length
  - A division of values about the mean of the original field
    - At or above the mean, T = 1E-4 m^2/s
    - Below the mean, T = 0 m^2/s
- Fields tested
  - Isotropic
  - Anisotropic
    - 1:10 correlation length ratio

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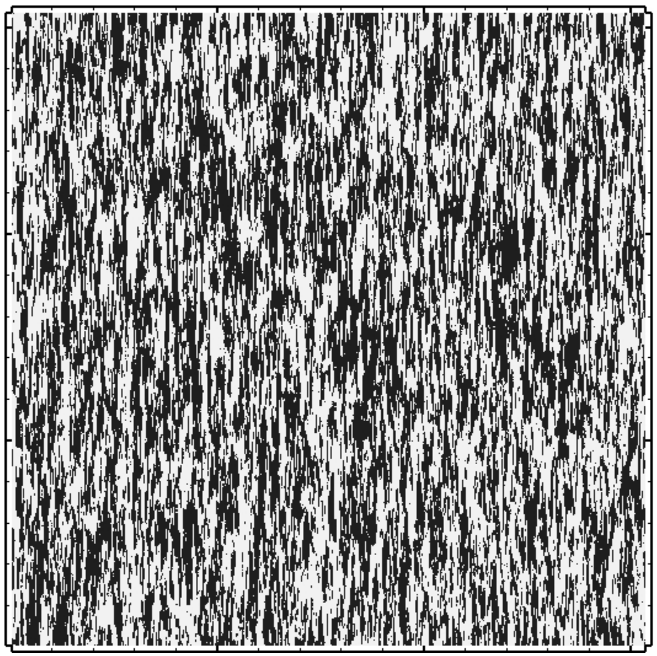
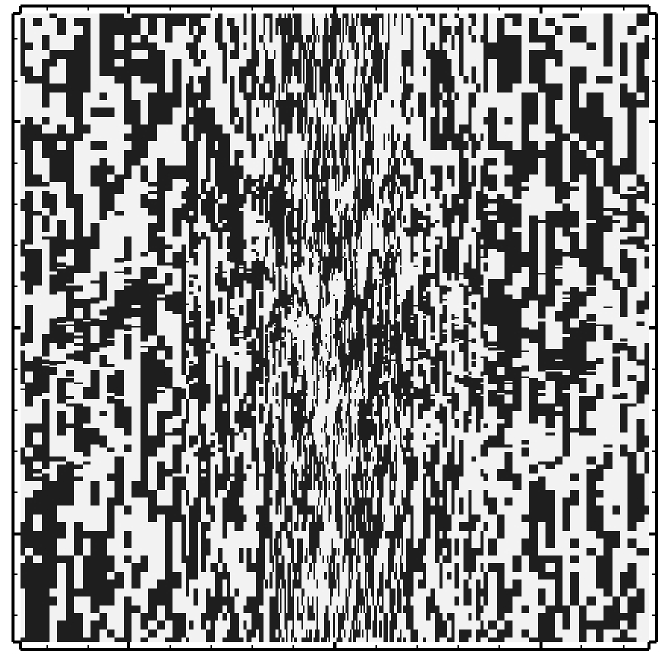
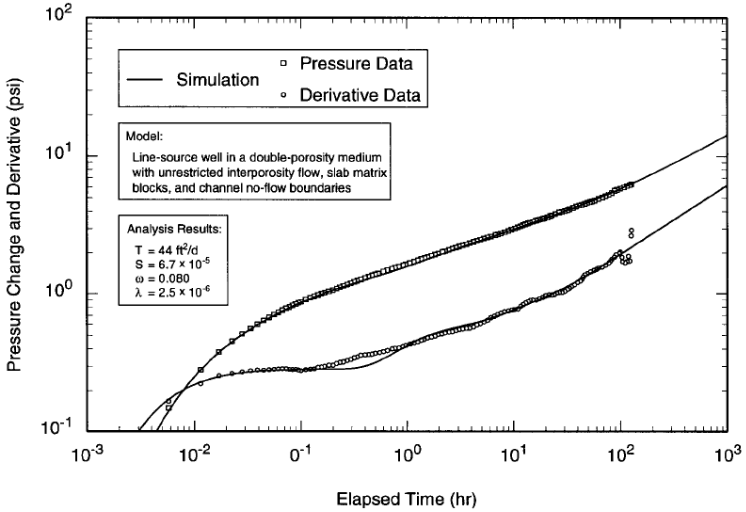
Field Coarsening



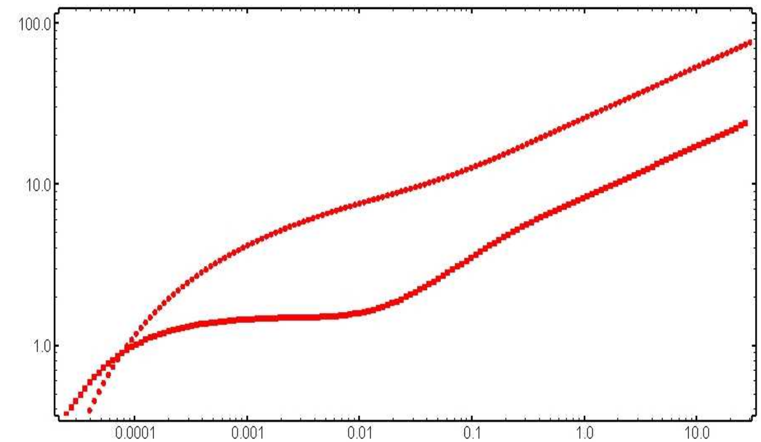


- Original field
  - Correlation length ratio 1:10
- Upward coarsened field
  - Increase in block size from well
  - Point estimates

BRF – Visual Comparison



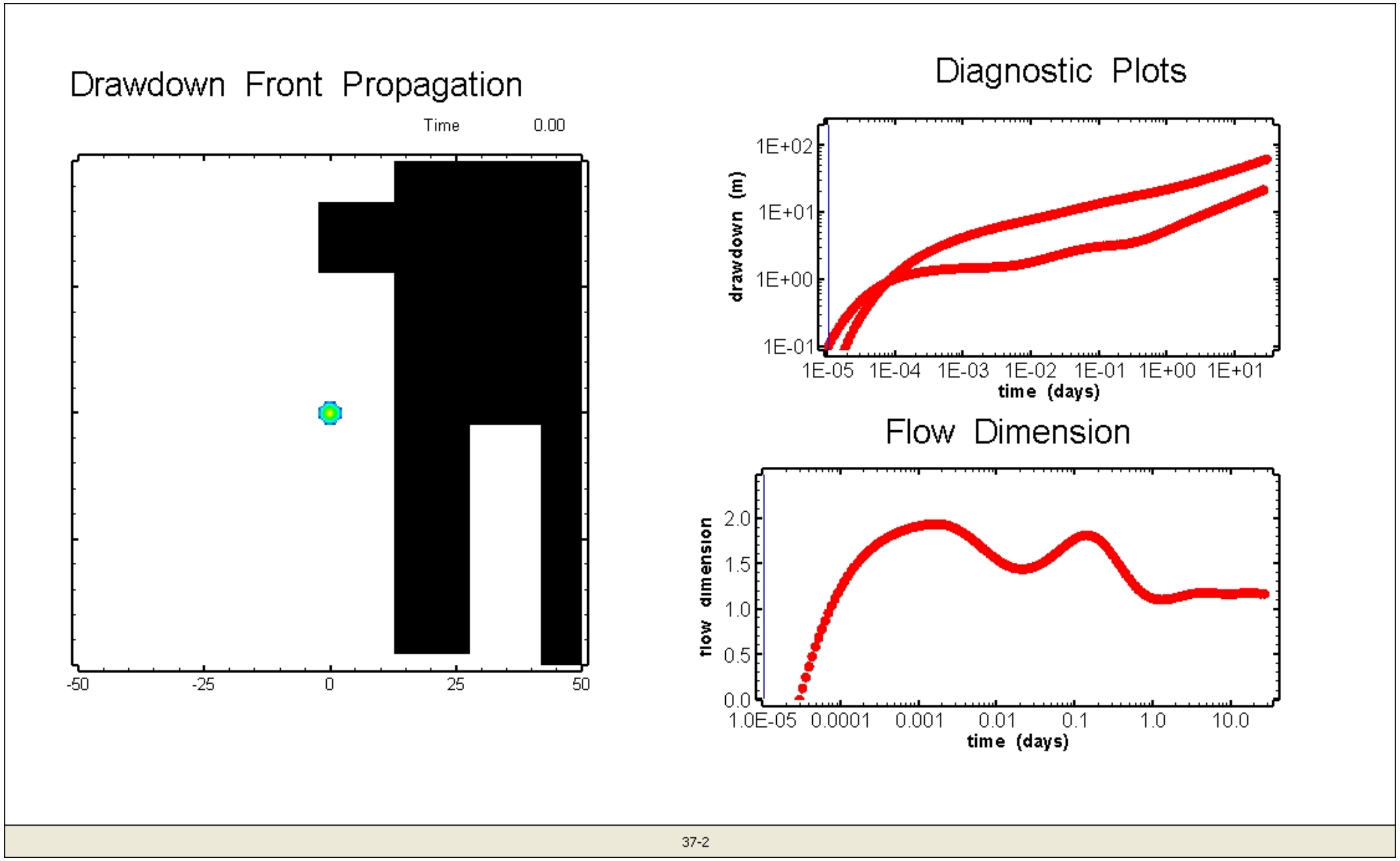
WIPP Field data



Binary Random Field data from our simulations

- BRF similarities to field data
  - Positive slope drawdown
  - Positive slope log-derivative
  - Degrees of variation

BRF - Pumping Test Simulation



Conclusions

- We demonstrate a physical connection of flow dimension to simple field geometries in a two-dimensional finite difference model.
- Using perturbation analysis combined with standard well-test analysis techniques, we are able to accurately estimate T and S values for conduit geometries imbedded in an impermeable host material.

Conclusions (2)

- Using anisotropic, binary random fields, we are able to simulate a realistic representation of heterogeneity in a fractured medium that produces persistent, non-integer flow dimensions and positive-slope diagnostic characteristics commonly seen in WIPP field data.
- The inferred value of transmissivity for the entire system is likely less than the transmissivity of the fracture network.

Future Work

- Investigate the application of conduit geometries to modeling efforts
- Investigation of the effects of differing transmissivity contrasts, *n* values, and parameter estimation for conduit geometries
- Separating positive slope diagnostics
  - Geometry
  - Transmissivity contrast