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Time Dependent Non-Extinction Probability for Fast Burst Reactors

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INTRODUCTION

Fast burst reactors are able to achieve intense neutron pulses for very short durations by assembling the reactor into a super prompt critical state. Sandia has operated the Sandia Pulse Reactor III (SPR-III), a fast burst reactor, over the past few decades in a pulse configuration.[3] As reactivity is being inserted into the reactor, it is possible for the prompt excursion to take place prior to the full reactivity state being reached. When this occurs, the reactor is said to pre-initiate. Without an external source present, the reactor is operating under weak source conditions and the initial buildup of neutrons in the reactor is said to be stochastic.[4] The non-extinction probability that a neutron will exist at a later time from an initial source neutron is the focus of this paper. The results presented below highlight our initial efforts to describe the time dependent behavior of SPR. Others have also presented results for multiplying systems as well as point models.[5]

THEORY

The physics governing the non-extinction probability (frequently call the survival probability), for multiplying systems has been previously derived.[1] The backwards survival probability equation for a one dimensional slab with monoenergetic neutrons is presented with seven term fission multiplicity data. In this equation $p_s(x,\mu,t)$ is the probability that a neutron will still exist at some time t in the past, at some position x, traveling in some direction μ, given that a source neutron was injected at some final time t_f.

$$-\frac{1}{v}\cdot\frac{\partial p_s}{\partial t}-\mu\cdot\frac{\partial p_s}{\partial x}+\sigma_T(x)\cdot p_s(x,\mu,t)=$$
$$\frac{\left(\sigma_S(x)+\bar{v}\cdot\sigma_F(x)\right)}{2}\cdot\int_{-1}^{1}p_s(x,\mu',t)\cdot d\mu'- \qquad (1)$$
$$-\sigma_F(x)\cdot\sum_{i=2}^{7}(-1)^i\cdot\frac{\chi_i}{2^i\cdot i!}\cdot\left(\int_{-1}^{1}p(x,\mu',t)\cdot d\mu'\right)^i$$

with: $-\infty<t\le t_f$, and $0\le x\le L$ subject to vacuum and terminal boundary conditions:

$$p_s(0,\mu,t)=0 \text{ for } \mu<0$$
$$p_s(L,\mu,t)=0 \text{ for } \mu>0$$
$$p_s(x,\mu,t_f)=1.0 \text{ for } -1\le\mu\le 1$$

Using a shifted time variable, the backwards equation can be shifted to a forwards equation. In addition, the angular variable is interchangeable $(\mu=-\mu)$ as the probability profile is symmetric. With the exception of the fifth term being non-linear, the equation is similar to the standard adjoint neutron transport equation. Solution of the survival probability gives a time dependent representation for any surviving neutron in the reactor. In addition, the probability of initiation (POI) can be found by taking the limit as time goes to negative infinity.[1] Bell's quadratic POI approximation is a scalar steady state value which gives the probability that a source neutron will lead to a divergent chain for infinite time. The steady state (SS) POI is itself only a function of constants, namely $\bar{v}$, k, and χ_2. This SS value provides an effective benchmark on the time dependent solution as the final survival probability must asymptote to the POI at long times. For the low reactivity rates of interest, there is minimal difference between the quadratic approximation and the complete fission multiplicity data at long times.

1-D RESULTS

The survival probability equation in 1-D is implicitly solved using a source iteration routine with a time lagged source. For the lagged source, the solution from the previous time step is used as a constant source over the next time interval. Within a given time step both outer and inner iterations are used. Both the linear and non-linear fission terms are held constant over an outer iteration. During the inner iteration the fission source and the time lagged source are held fixed and the scattering term is iterated upon. Upon convergence of the inner iteration a new fission term is computed in the outer iteration. This is then held constant over the next inner iteration. Upon convergence of both the inner and outer iterations using relative error criteria, the simulation is advanced by a timestep. This iteration scheme is shown in the equations below. The standard Sn approximation is

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invoked on the angular domain [-1,1] which is approximated with Gauss-Legendre quadrature ordinates. Both the spatial and time domains utilize diamond differencing. Given equation 1 is non-linear, simplistic iteration and differencing schemes were used until sufficient information was gathered to determine where more refined methods are appropriate.

Time Loop (k iteration)

$$Q(x,\mu)=\frac{P_n^k(x,\mu)}{\Delta t}$$

Outer iteration (s iteration)

$$F=\frac{\bar{\nu}\cdot\sigma_F(x)}{2}\cdot\int_{-1}^{1}p^s(x,\mu')\cdot d\mu'-$$

$$\sigma_F(x)\cdot\sum_{i=2}^{7}(-1)^i\cdot\frac{\chi_i}{2^i\cdot i!}\cdot\left(\int_{-1}^{1}p^s(x,\mu')\cdot d\mu'\right)^i$$

Inner Iteration (l iterations)

$$p^l(x,\mu)=\frac{\sigma_S(x)}{2}\cdot\int_{-1}^{1}p_s(x,\mu',t)\cdot d\mu'+F^s+Q$$

To establish the system reactivity versus slab thickness a standard k-eigenvalue iteration is performed prior to solving the survival probability equation. The non-linear terms are ignored and the fission term is modified by the standard $1/k_{eff}$ and iterated upon until convergence is achieved. With the linear system reactivity established, the survival probability equation is solved forwards in time. As a means to verify the code, reflective boundary conditions were placed on the slab and the time dependent results agree well with 0-D results not presented here. We present results for a slab with fixed system reactivity and vacuum boundary conditions. Monoenergetic cross-sections appropriate for SPR-III were used. Figure 2 plots the time dependent angular integrated solution for various integration time steps. The values plotted in the figure are taken to be the slab mid points. In addition for the 1ns timestep, the time dependent slab edge value is also shown. The lifetime for this system was assumed to be 10ns.

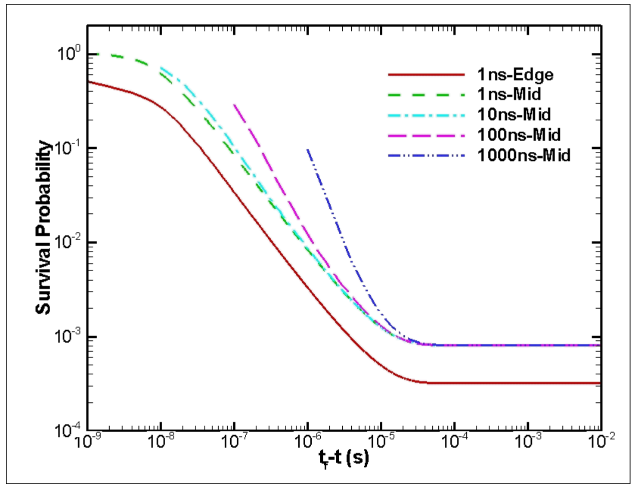

Fig. 2. 1-D slab survival probability v. time.

As the timestep becomes increasingly small, there is negligible change in the time dependent solution. As larger time steps are selected large deviations are seen in the initial time dependent behavior; however, regardless of the timestep selected the steady state solution is achieved. In addition, note that the slab edge and midpoint values follow the same time dependent behavior but converge to different steady state values.

Both the inner and outer iterations are performed until the relative error criteria is reached. As the relative error criteria is relaxed, the total number of inner and outer iterations decreases rapidly. In addition, as the time dependent solution is quickly changing around the insertion time, the total number of inner and outer iterations is relatively high shortly after the insertion time. Table 1 below shows the results for the average number of inner and outer iterations for a simulation time up to 10^{-5} seconds. Beyond this time, the number of inner and outer iterations needed for convergence is on the order of a few as the solution has nearly approached the SS asymptotic value. The results in the table are provided for two different relative errors. As higher relative error criteria are imposed, drastic changes are seen in the number of required iterations. Given the potential for large numbers of inner iterations suggests that techniques such as Diffusion Synthetic Acceleration (DSA) may be beneficial.

TABLE I. Iterations for 10^{-8} / 10^{-5} relative error

Time step (ns)	Average Number of Outer Iterations	Average Number of Inner Iterations
10^0	2.2 / 1.3	6.4 / 1.5
10^1	3.4 / 1.8	34.1/ 3.2
10^2	14.6 / 2.2	242.3 / 14.3
10^3	105.5 / 3.4	1884.7 / 45.0

Since the system reactivities of interest are slightly above prompt critical a large number of inner and outer iterations are required. For system reactivities that are sufficiently high, the total number of inner and outer iterations is on the order of a few across the entire time domain. In addition, the cross-section set used was for a fast system where scattering is the dominant reaction. If thermal data are used rapid convergence for all prompt reactivities are seen. Thus given the reactivity regime of interest as well as the highly scattering media suggests that numerical simulation of the survival probability is particularly taxing for this problem.

At the initial insertion time, there is an equal probability everywhere identical to unity as the source neutron in injected at this time. Figure 3 below shows the survival probability in the slab. At various times from the insertion time the spatial mode quickly develops and then decreases as a function of time. The spatial solution decreases with time until the SS static solution is achieved.

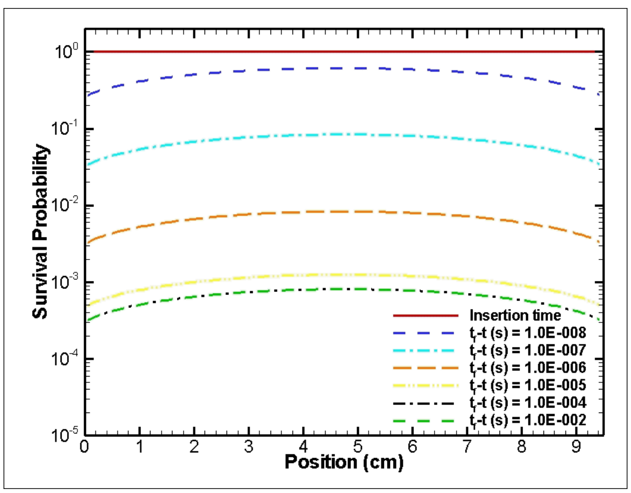

Fig. 3. Spatial probability at fixed times.

The results presented above are for a system that is just slightly above prompt critical (k_p = 1.0007). It is of interest to show the non-extinction probability for different system reactivities. For sub prompt critical systems, the survival probability must go to zero while for super prompt critical systems it has been shown to approach a constant value.[1] For different slab thicknesses, the slab midpoint time dependent solution is shown in Figure 4.

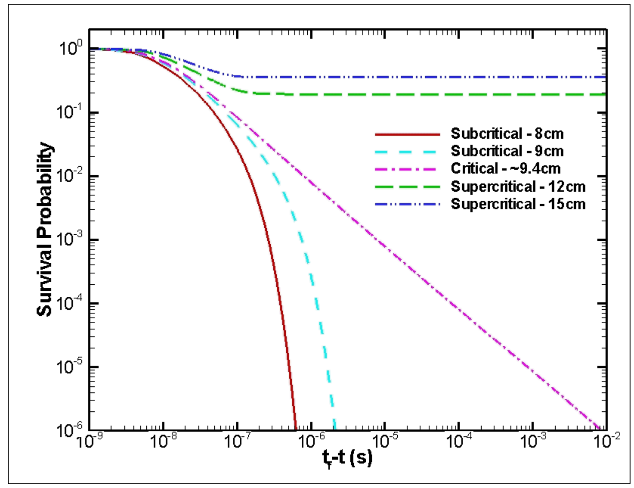

Fig. 4. Time dependent survival probability for different slab thicknesses.

For systems that are highly subcritical, the non-extinction probability drops rapidly suggesting that a given chain will die away quickly. For critical systems the solution behaves according to a decaying exponential. For super prompt critical systems, the solution approaches the SS constant value. The higher the system reactivity, the quicker it approaches the SS value and is of higher magnitude.

CONCLUSION

The time dependent survival probability equation has been solved for both a 0-D and 1-D system. The reflected 1-D solution has excellent agreement with the 0-D time dependent results. The non-extinction probability equation is solved under varying numerical conditions. For large time steps the steady state POI value is obtained; however, the initial time dependent survival probabilities are incorrect. The relative error criteria and simulation timestep can have a drastic impact on the number of simulation iterations. The spatial survival probability at various times is also shown. In addition, the behavior for different slab thicknesses is also presented. In future work, time dependent reactivity will be investigated in the 1-D model in a manner that is appropriate to SPR. In addition, data taken from SPR-III over the operational lifetime will be compared with the theoretical results as a means to benchmark the results.

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