



## Dynamic Analysis of System Behavior

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# Dynamic Analysis of System Behavior

- Analysis capability to complement the national-scale interdependencies models developed as part of the CIPDSS program
- The national-scale model was developed to estimate the consequence of specific disruption scenarios. We recognized an opportunity to leverage this work to build a complementary analysis capability that is intended to address a broader and more fundamental set of questions about the range of possible behaviors that a stressed infrastructure system could exhibit.



# Types of Questions Addressed

- Could a disruption push an infrastructure system past a tipping point into an entirely new behavior regime? If so, where is the tipping point and what is the new behavior?
- Are there specific actions that could be done to prevent or mitigate a catastrophic infrastructure failure?
- How much detail is required in an infrastructure model?



# Objectives

- Comprehensive coverage of range of system behaviors
- High level of confidence in results (including documentation and testing)
- Broad and fast application
- Mathematical foundation well understood and documented
- Not exceeding intuition
- Provides guidance for developing and maintaining the national-scale interdependencies model



# High-Level Steps to Accomplish Objectives

- Start with set of simple, but well understood and fully connected, infrastructure models coupled to a partial equilibrium market model.
- Perform phase-space and bifurcation analysis to investigate system behavior and sensitivity



# Infrastructure Models

- Presently envisioned to be a set of coupled System Dynamic models
- Individual infrastructures would be represented as small (one or two stock) molecules
- We anticipate a small number of fundamental molecule types representing the concepts of storage, transport on networks, process rates, and capacity constraints



# Market Model

- A partial equilibrium model for  $N$  consumers and  $M$  producers
- Accounts for substitution
- Receives as input from the infrastructure model the maximum quantity of each good or service that could be provided
- Calculates the equilibrium price and demand for each good or service



# Phase Space Analysis of System Dynamics Models

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With the help of:

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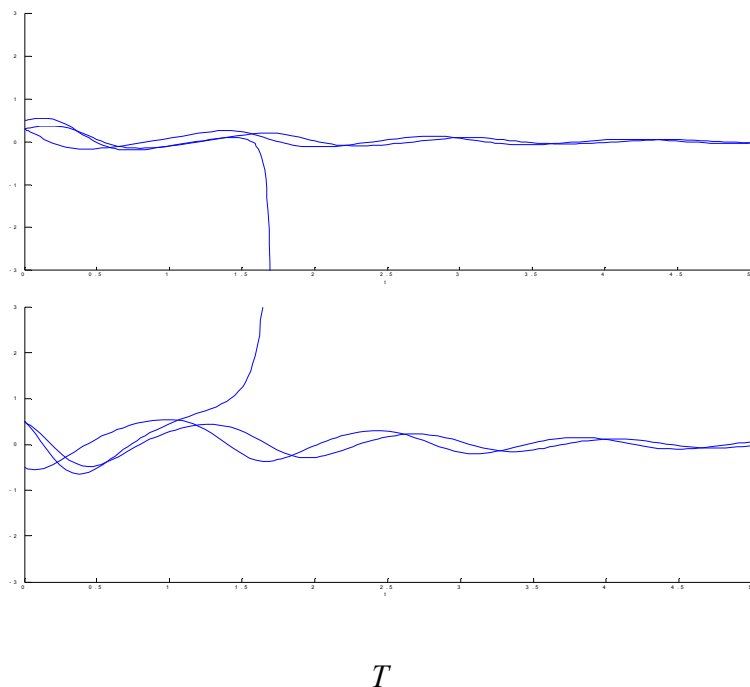




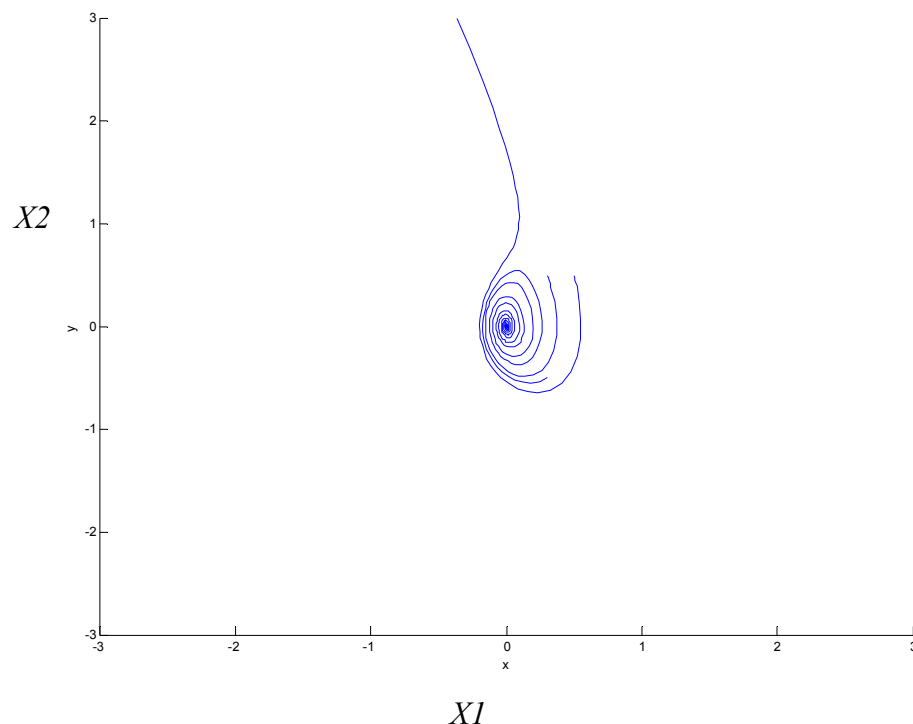
# Phase Diagram Is a Global Behavior Map

## Example System: Faulkner

Time History



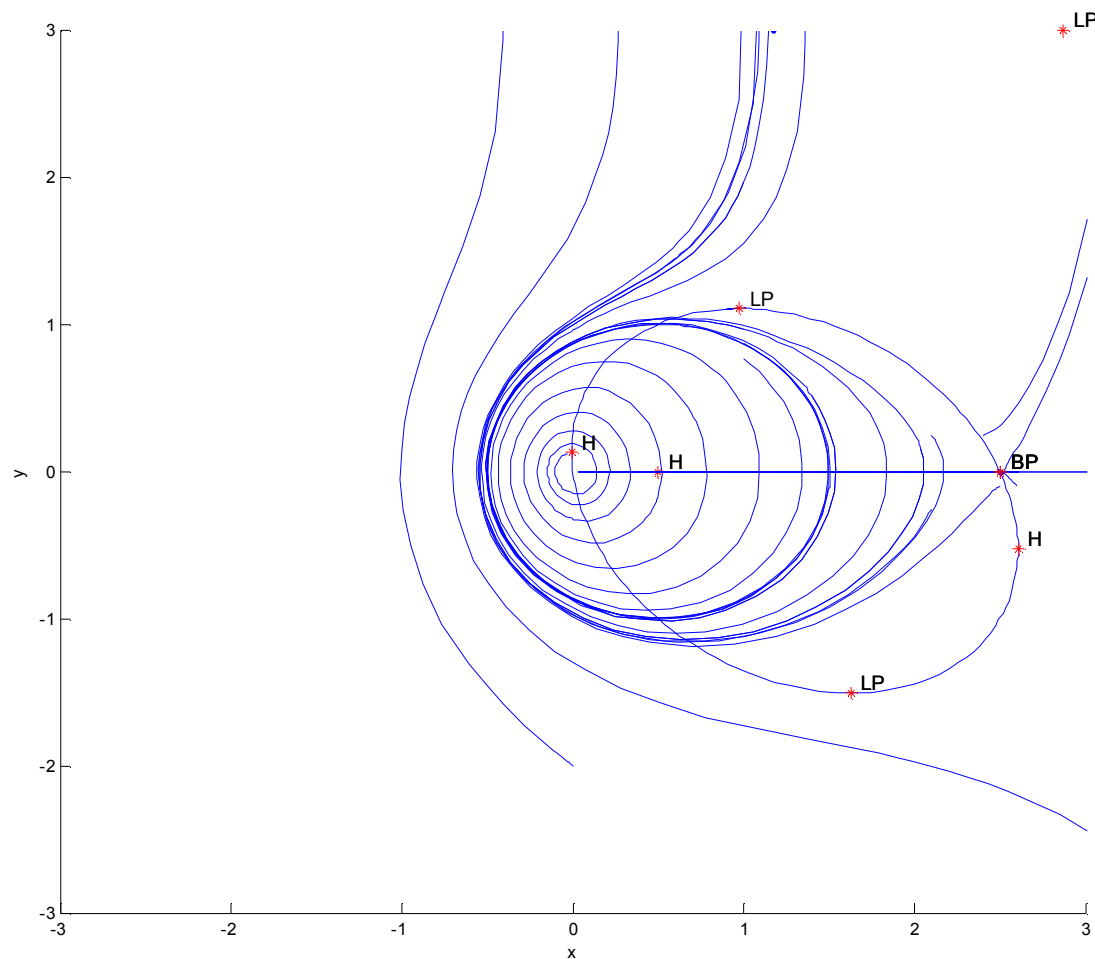
Phase Space Diagram





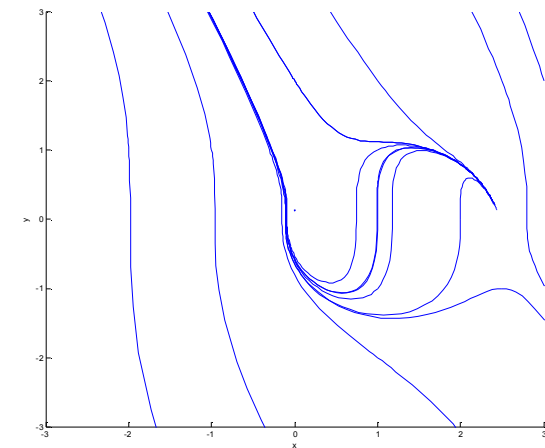
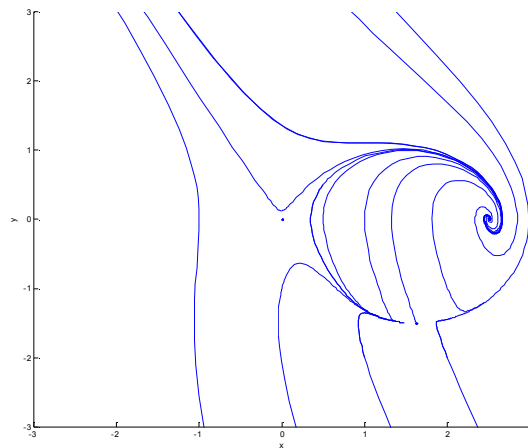
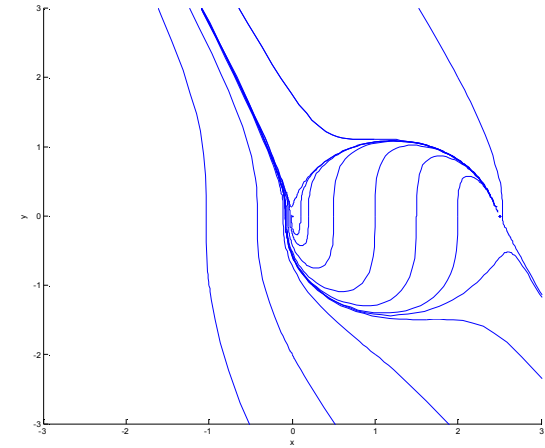
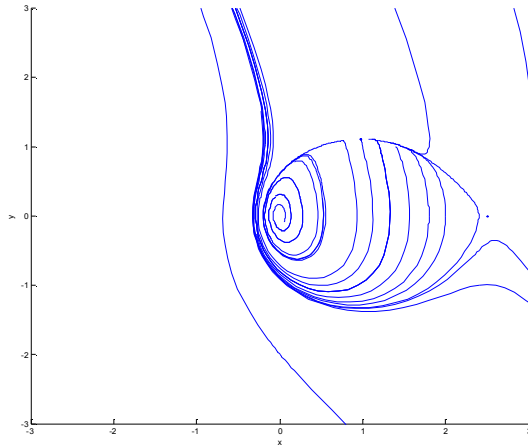
# Bifurcation Analysis Captures Global Behavior Sensitivity

## Example: Faulkner



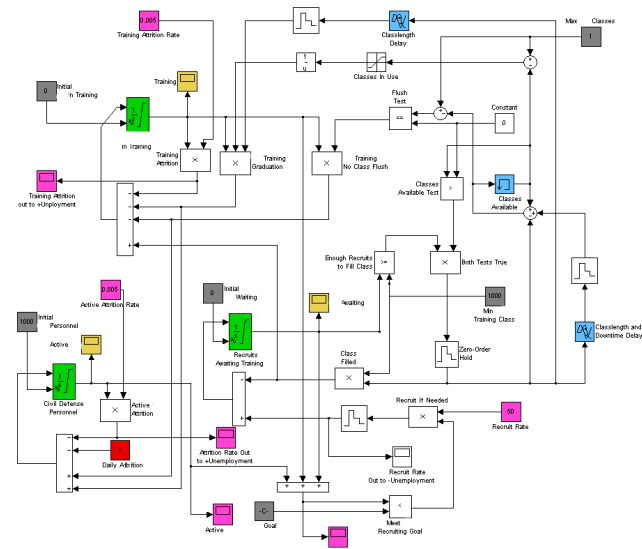
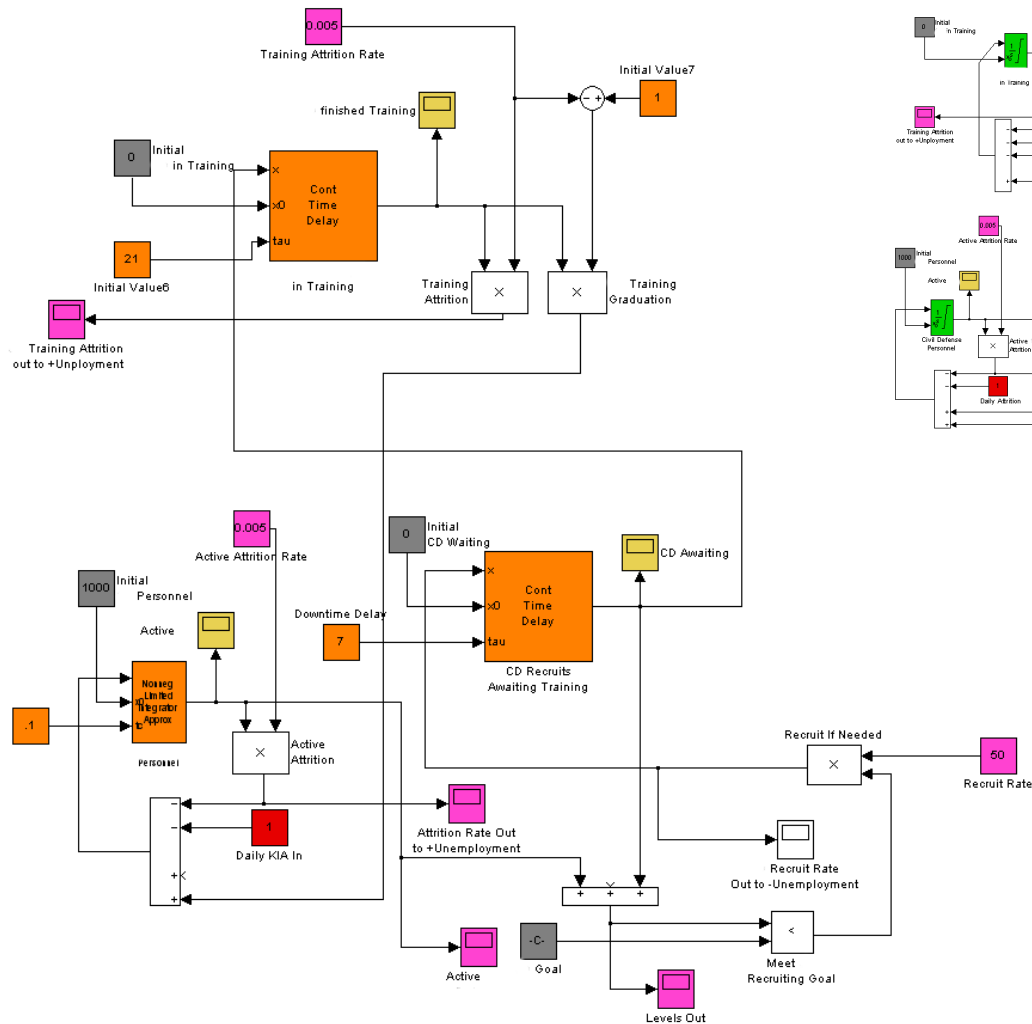


# Some Faulkner Behavior Regimes Identified



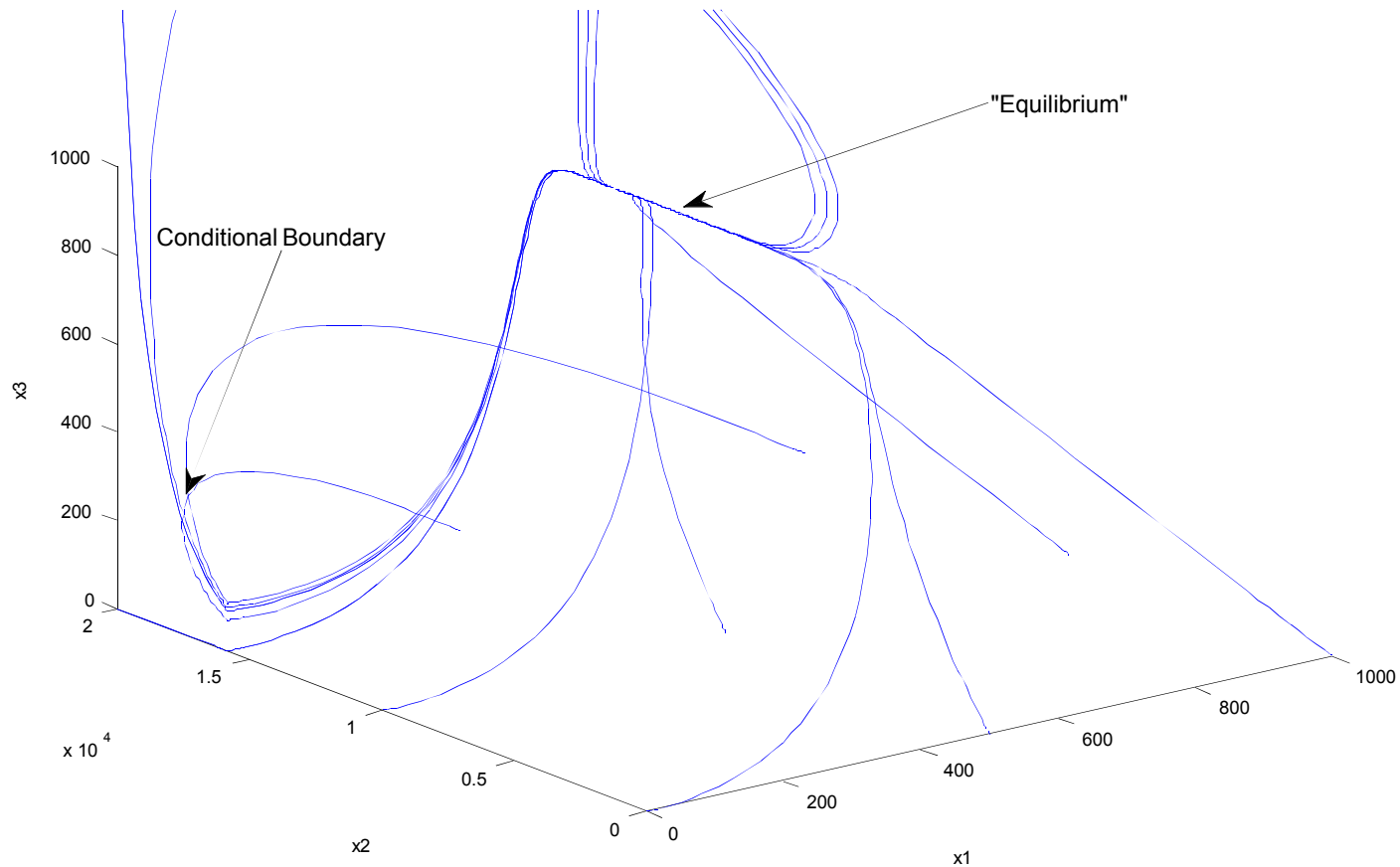


# Example application to SD Model Training





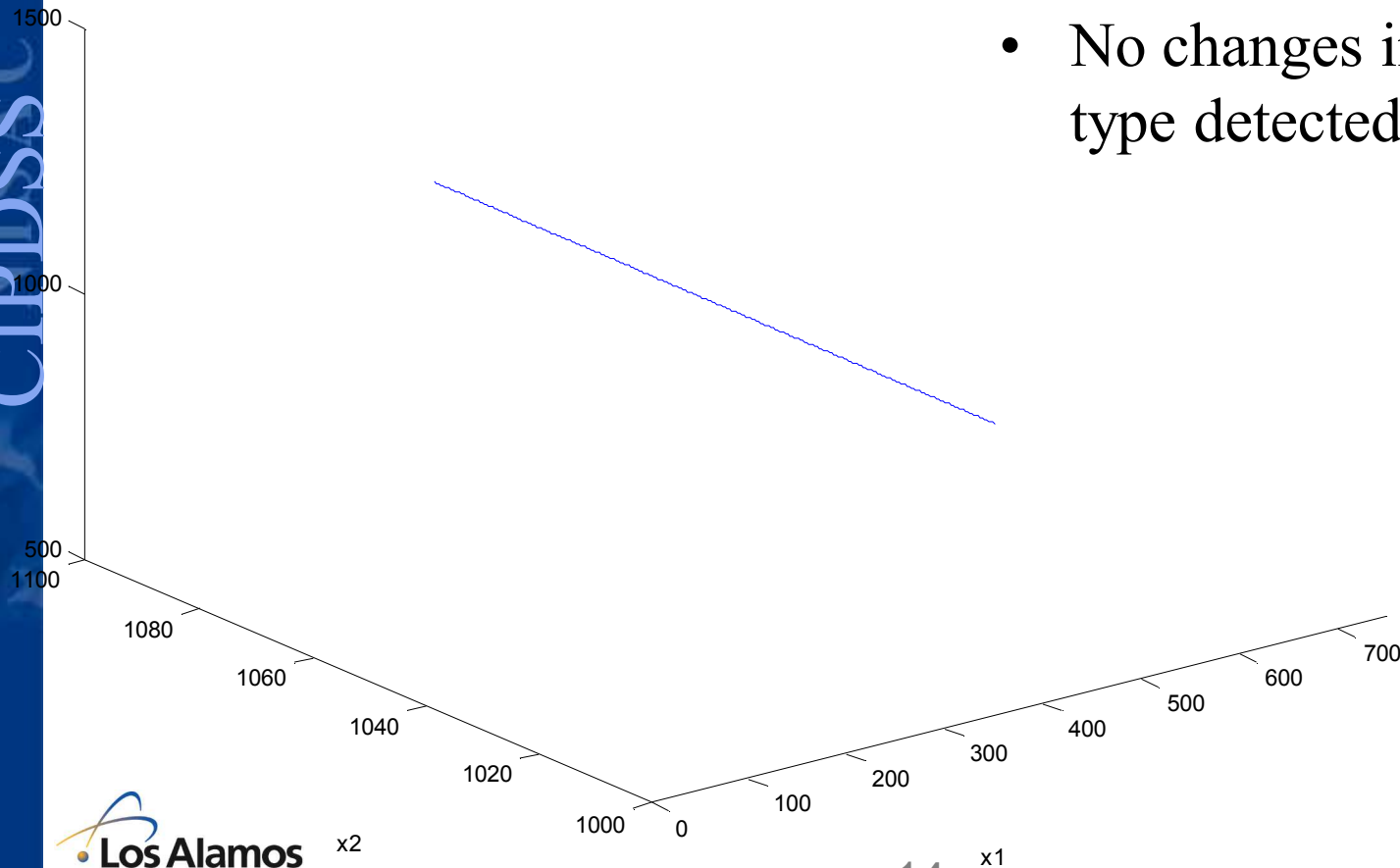
# Phase Portrait of Training Model





# Bifurcation Analysis of Training Model

- 5 parameters produced linear behavior in location of equilibrium
- No changes in equilibrium type detected





# Phase Space Analysis

- Objective: determine the range of potential dynamic behaviors that can be exhibited by a model
- Process:
  1. Extract model and translate to the tool (in our case, a Matlab representation suitable for MatCont). Some modifications are required to enable analysis.
  2. Determine location of equilibria (phase space analysis)
    1. By examination of pictures of vector space or time histories
    2. By search for zeroes of system of equations
  3. Determine how equilibria move and split when model parameters (the constants) change (bifurcation analysis)
    1. Process is automated by MatCont; select a parameter and search begins.
- What the process tells us
  1. Step 1, simplification:
    1. Removal of external drivers and discrete operators focuses attention on simplest model, identifies potentially unimportant structures
  2. Step 2, phase space analysis:
    1. The complexity of the system is captured by the number and kinds of equilibria found.
    2. Sensitivity to initial conditions is captured in the shape of the phase diagram. Within a given basin of attraction, low sensitivity occurs; on opposite sides of a boundary, completely different kinds of behavior occur.
  3. Step 3, bifurcation analysis:
    1. The rate at which equilibria move as a function of model parameters is a measure of the sensitivity of the dynamic behavior to model parameters.
    2. Changes in types of equilibria or splitting/joining of equilibria are fundamental changes in model behavior.