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Model Validation under Both Aleatory and Epistemic Uncertainty

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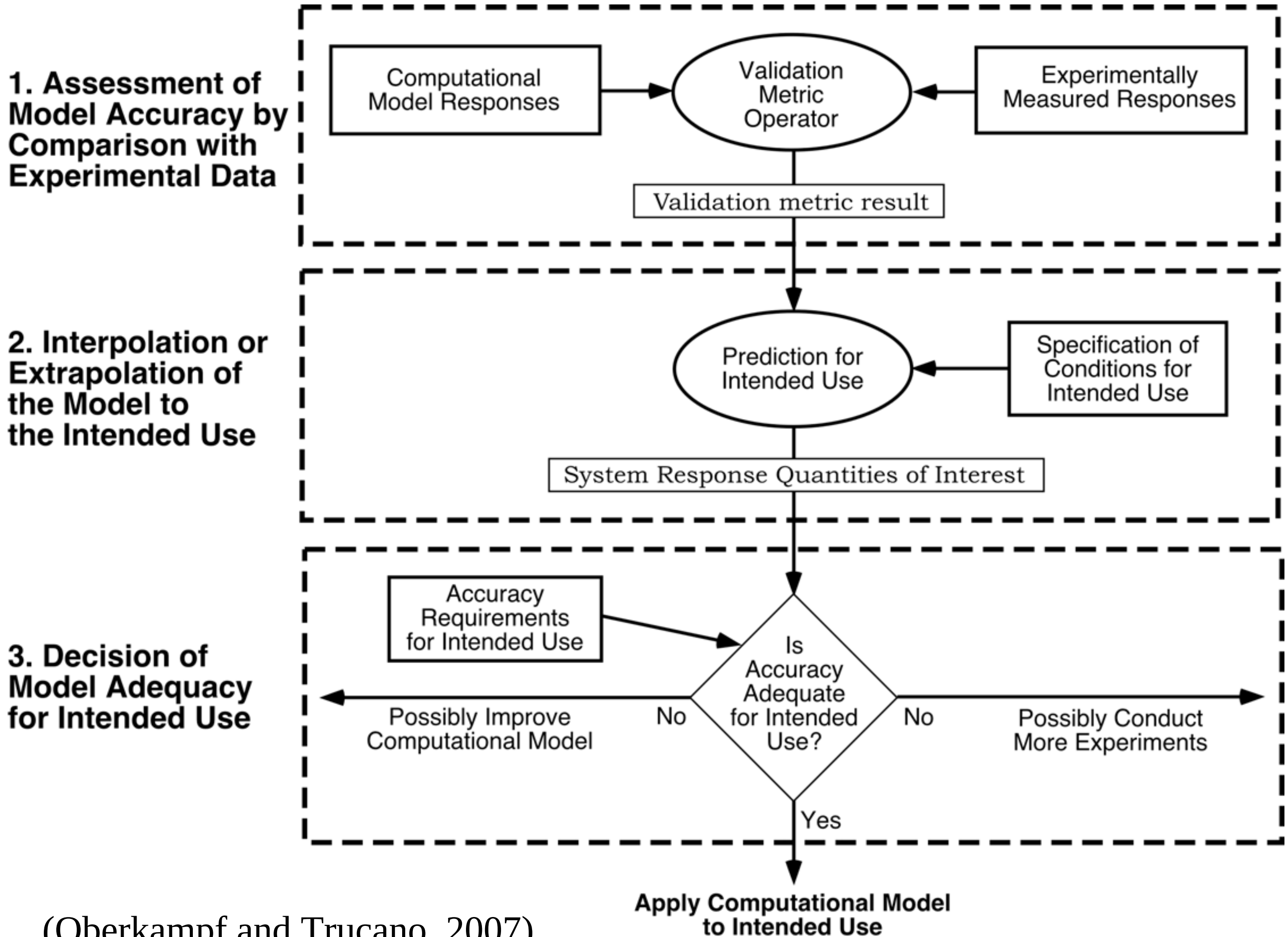
Outline of the Presentation

- Background
- Need for a new approach
- Area validation metric
- Pooling of incomparable comparisons
- Aleatory and epistemic uncertainty
- Closing remarks

Background: Aspects of Validation



(Oberkampf and Trucano, 2007)

Review of Existing Approaches

- **Comparison of mean value from simulation and experiment**
  - Only measures the mismatch between simulation and experiment at the expected value
- **Hypothesis testing**
  - Measures the mismatch between probability distributions from simulation and experiment
  - Mismatch is measured by a probability value
- **Bayesian validation**
  - Focused on evaluating a subjective probability that a simulation is consistent with experiment
  - Emphasis is on updating probability density functions of uncertain parameters to obtain best agreement with experiment
  - Assumes the model form is correct

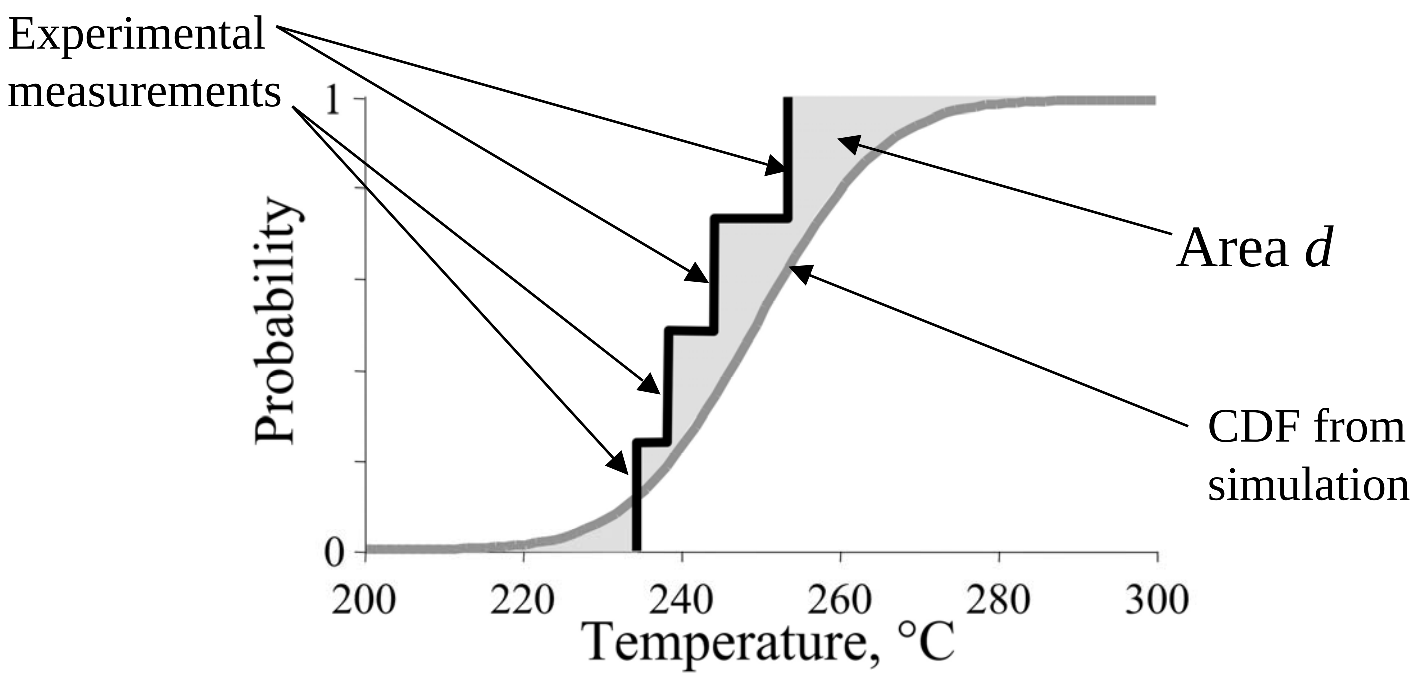
Need for a New Approach

- Our validation metric approach emphasizes:
  - Objectively measuring mismatch between cumulative distribution functions (CDFs) between simulation and experiment
  - Quantifying model form uncertainty, in the spirit of a “blind” comparison with experiment
  - Estimating the mismatch in terms of the units of the system response quantity (SRQ) being compared
- Our approach can address:
  - Comparison between simulation and experiment with as few as one sample each
  - Pooling of comparisons from dissimilar system response quantities
  - Epistemic uncertainty existing either (or both) the simulation and the experimental measurements

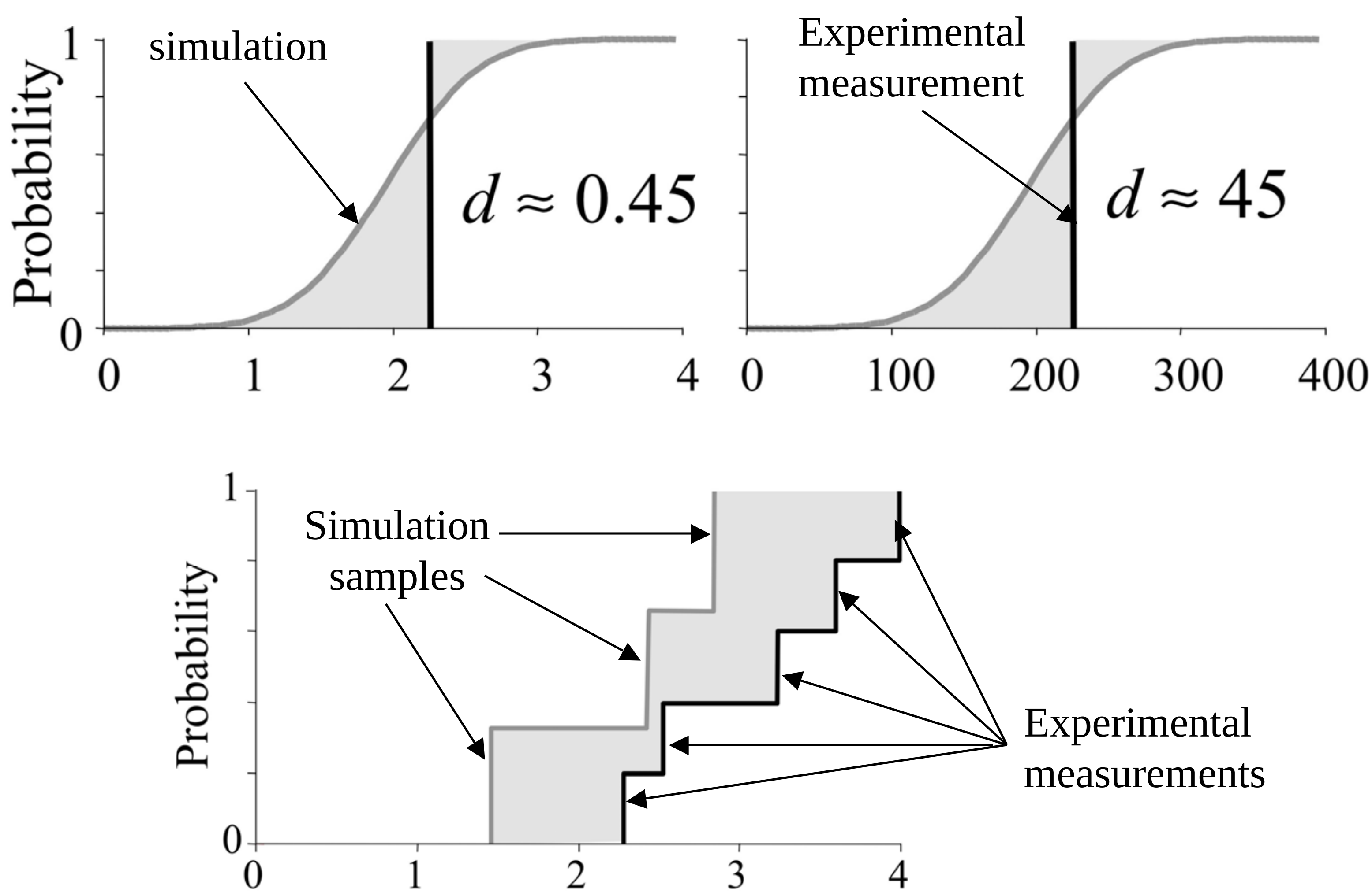
Area Validation Metric

- **The validation metric is defined to be the area between the cumulative distribution function (CDF) from the simulation and the empirical distribution function (EDF) from experiment (Minkowski L_1 metric)**

$$d(F,S_n) = \int_{-\infty}^{\infty} \left| F(x) - S_n(x) \right| \mathrm{d}x$$



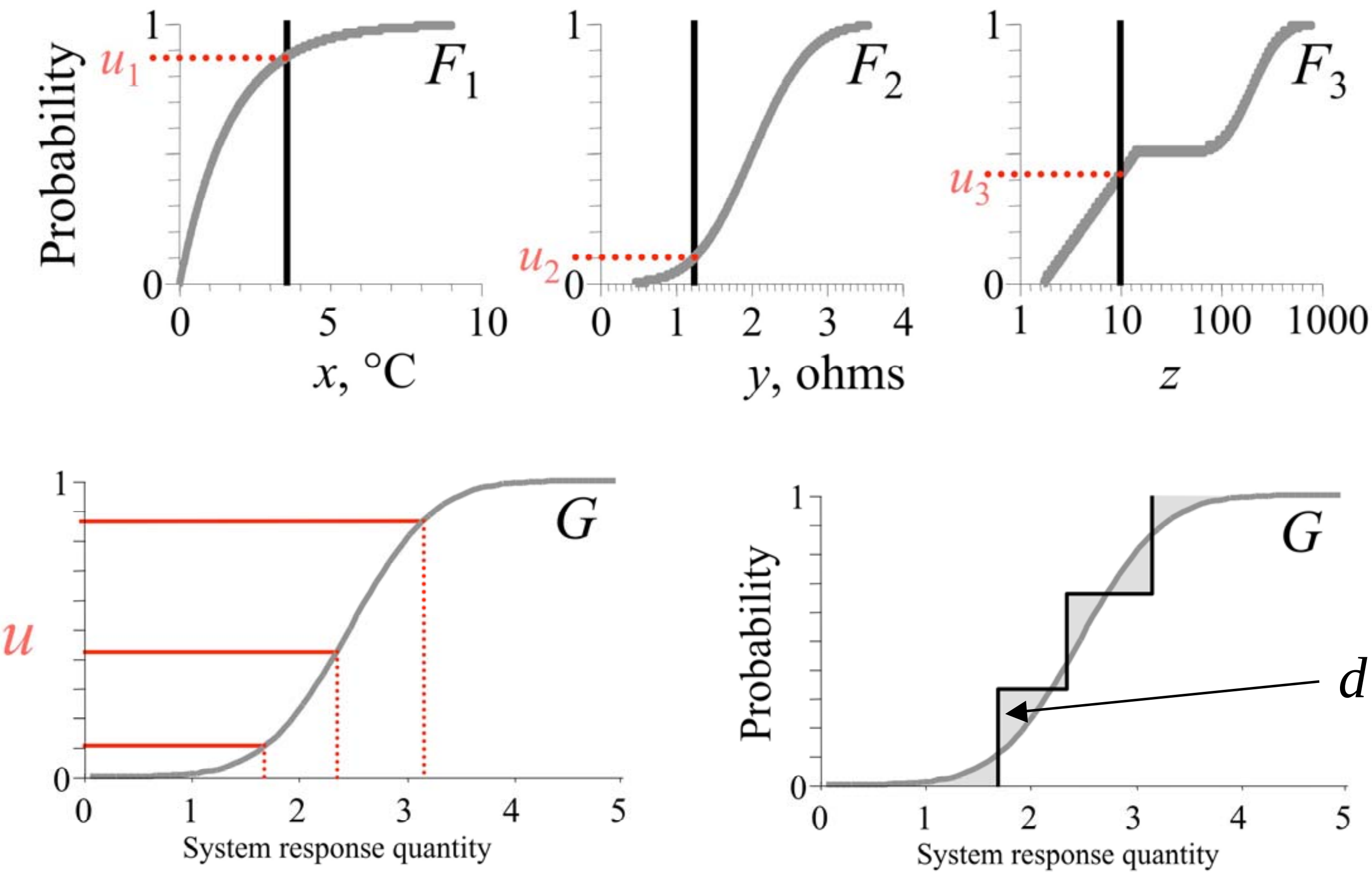
Area Validation Metric (from Ferson et al 2008)



How can Different Validation Metric Results be Combined?

- Two common situations in comparison of simulation and experiment:
  - We have a time dependent SRQ and we have computed individual validation metrics at several instances of time
  - We have computed a validation metric for different SRQs
- Instead of using the probability integral transform theorem (Angus, 1994) in the forward direction
- Given a distribution *F* and a uniform random variable u between zero and one, the value of $F^{-1}(u)$ will be a random variable distributed according to *F*.
- We can:
  - Use it to back-transform from individual SRQs (physical space) to a probability space.
  - Then use an appropriate CDF, for the problem of interest, to transform back into physical space so that a validation metric can be computed.

Pooling of Incomparable Comparisons



Aleatory and Epistemic Uncertainty

- **Aleatory uncertainty is an inherent variation associated with a parameter, physical system, or environment**
  - **Also referred to as variability, stochastic uncertainty, irreducible uncertainty**
- **Examples:**
  - **Variability in geometric parameters due to manufacturing**
  - **Variability in weather conditions**
- **Epistemic uncertainty arises from imperfect knowledge or ignorance**
  - **Also referred to as subjective uncertainty, reducible uncertainty, or model form uncertainty**
- **Examples:**
  - **Insufficient experimental data to precisely characterize a probability distribution**
  - **Poor understanding of physics phenomena or physics coupling**
  - **Poor understanding of failure modes or hostile environments**

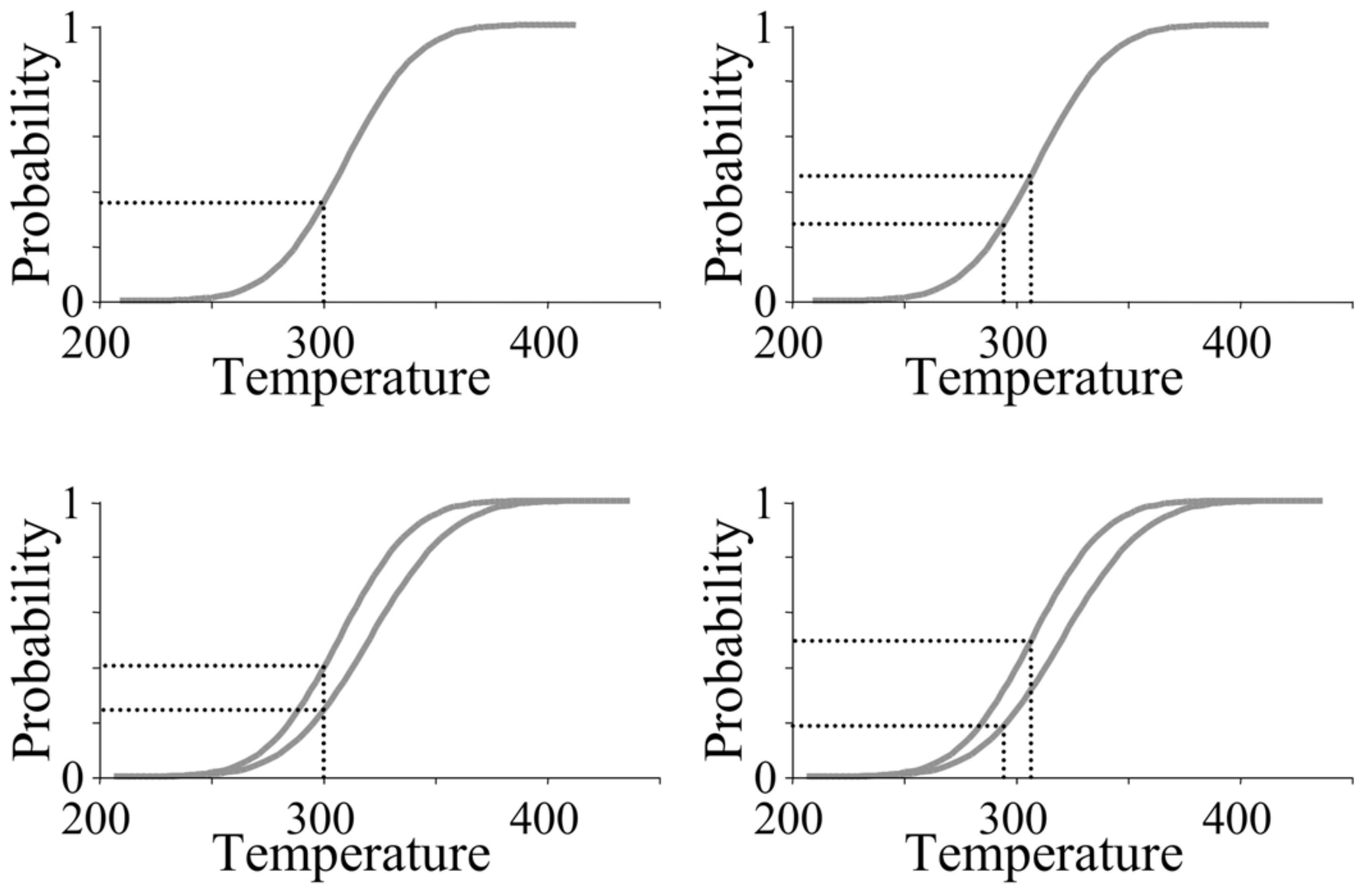
Measurement Uncertainty

- **Traditional ISO/NIST approach is focused on the estimation of random measurement uncertainty of a fixed quantity.**

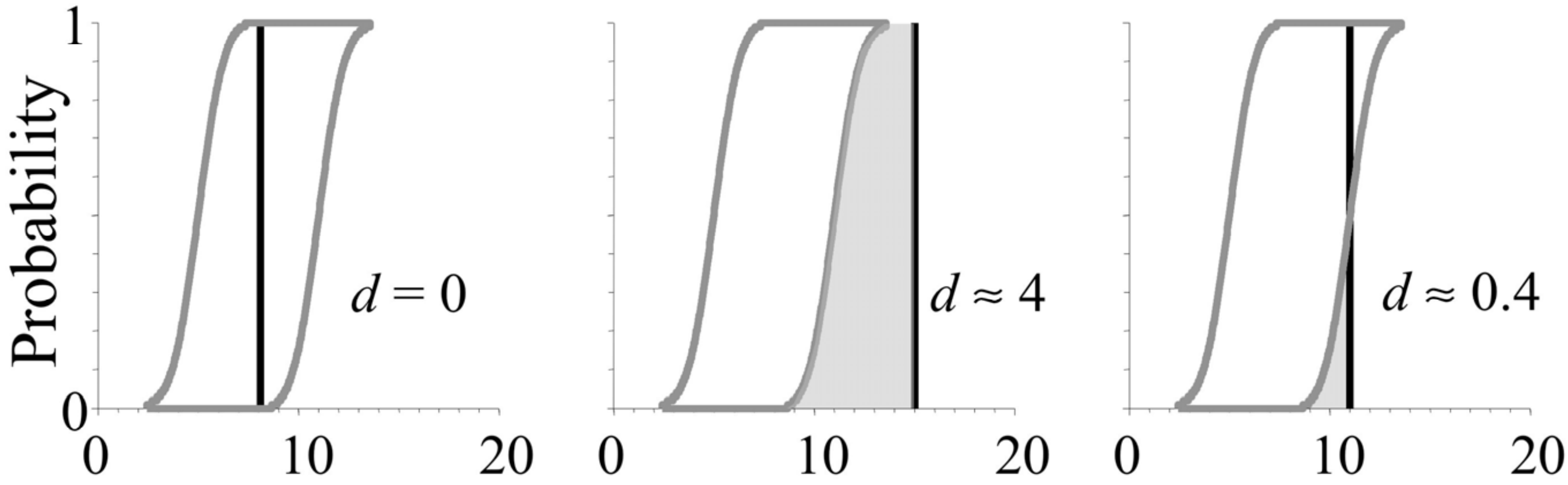
$$Y = X + \varepsilon$$

  - ***X* is the true, but unknown, value of the measurand**
  - **ε is the random measurement error**
  - ***Y* is measured value**
- **Using either higher accuracy measurements or repeated measurements of X, ε is characterized as a parametric probability distribution along with its parameters.**
- **Recent work by Ferson et al (2007) developed methods for estimating epistemic uncertainty in measurements, e.g.,due to:**
  - **Data censoring**
  - **Missing values**
  - **Sampling uncertainty**

Mapping of Epistemic Uncertainty from an SRQ to a Probability



Validation Metric for Aleatory and Epistemic Uncertainty in the Simulation



- **When epistemic uncertainty exist in the simulation, or in the measurements, the validation metric can be zero**
- **This does not mean perfect agreement**
- **It means there is no evidence that the simulation and experiment are in disagreement**

Generalizing the Validation Metric for Aleatory and Epistemic Uncertainty

- **The validation metric is now defined as**

$$\int_{-\infty}^{\infty} \Delta([F_{\mathrm{R}}(x), F_{\mathrm{L}}(x)], [S_{n\mathrm{R}}(x), S_{n\mathrm{L}}(x)])\,\mathrm{d}x$$

  – **where subscripts L and R denote the left and right bounds for any epistemically uncertain CDFs**

  – **and** $\Delta(A, B) = \min_{\substack{a \in A \\ b \in B}} | a - b |$

- **This metric is no longer a true mathematical metric because the measure can attain zero without simulation and experiment being identical**

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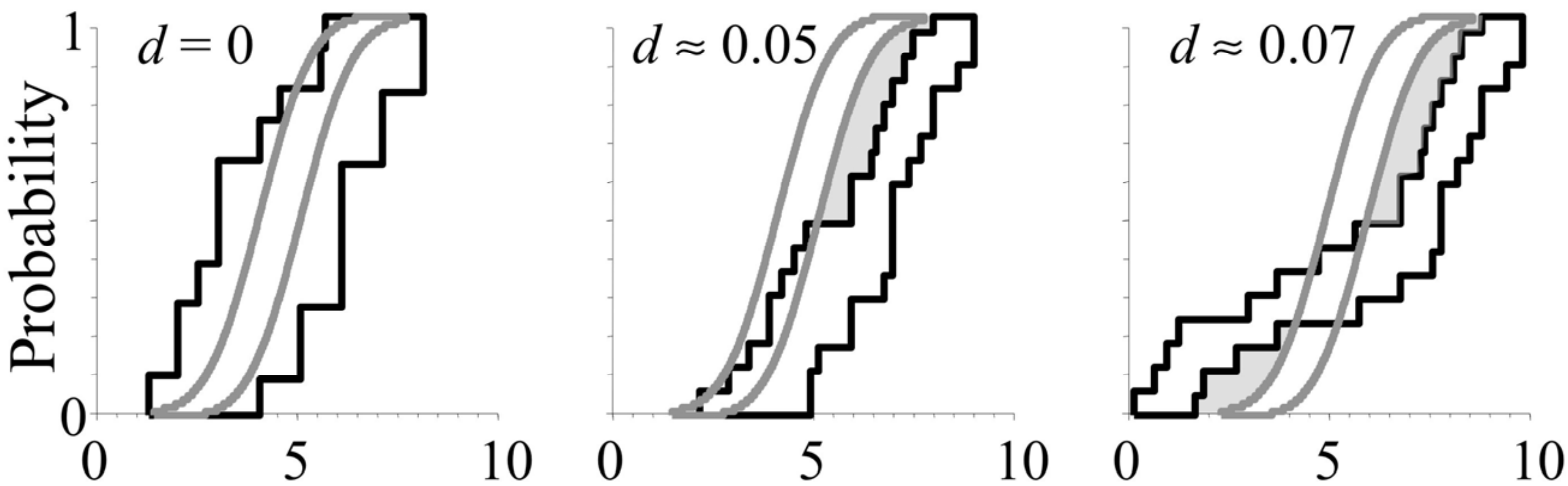
Validation Metric for Aleatory and Epistemic Uncertainty in both the Simulation and Experiment



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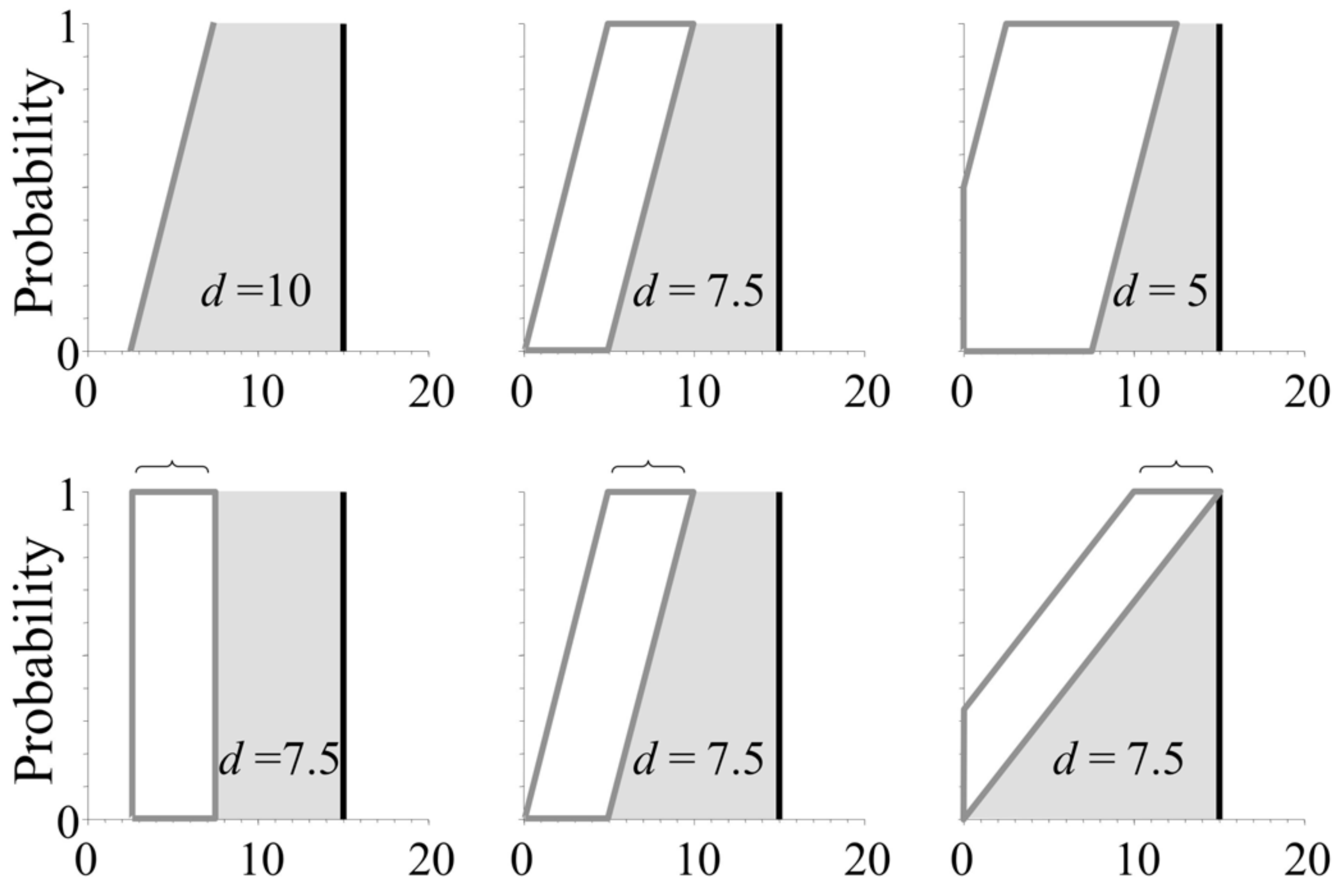
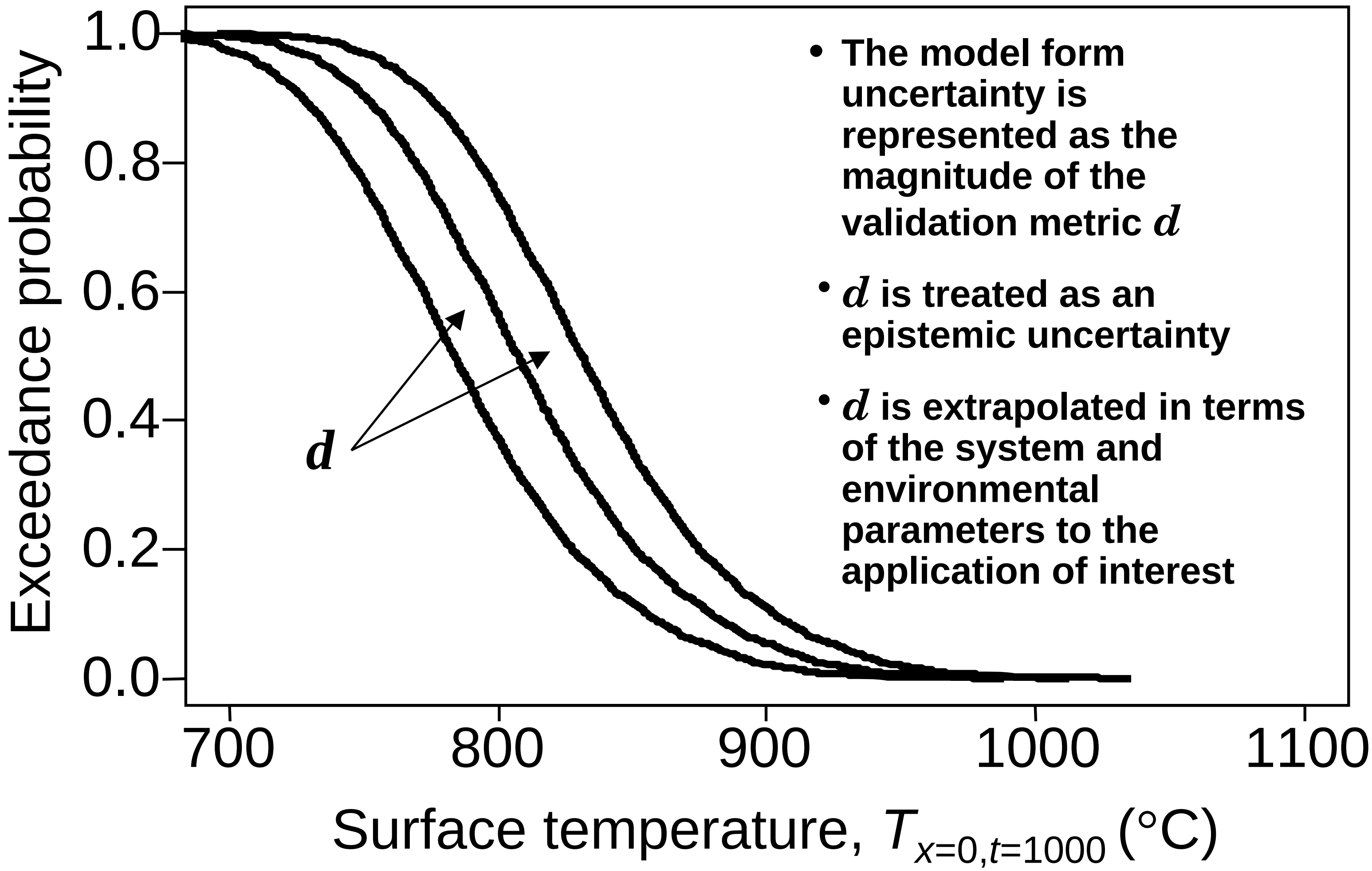
Example of Increasing Epistemic and Aleatory Uncertainty in the Simulation



How is the Validation Metric Result d Used in a Prediction?



- **The model form uncertainty is represented as the magnitude of the validation metric d**
- **d is treated as an epistemic uncertainty**
- **d is extrapolated in terms of the system and environmental parameters to the application of interest**

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Concluding Remarks

- **We believe it is essential to explicitly quantify the mismatch of our models in comparisons with experimental data**
- **Epistemic uncertainty should be characterized using intervals, then use either:**
  - **Second order probability**
  - **Dempster-Shafer theory**
  - **Probability boxes**

  to propagate uncertainty from inputs to SRQs
- **Improve methods are needed to extrapolate the validation metric, d, to the application of interest**
- **We must continue to find ways of testing our predictive capability by "blind" comparisons with experiments**

Goal: Improved Risk-Informed Decision Making

For Engineering Systems