

Effect of Cylindrical Inclusions on the Strength Distributions of Brittle Materials

Rajan Tandon

**Materials Reliability Department
Sandia National Laboratories, Albuquerque, NM 87185**

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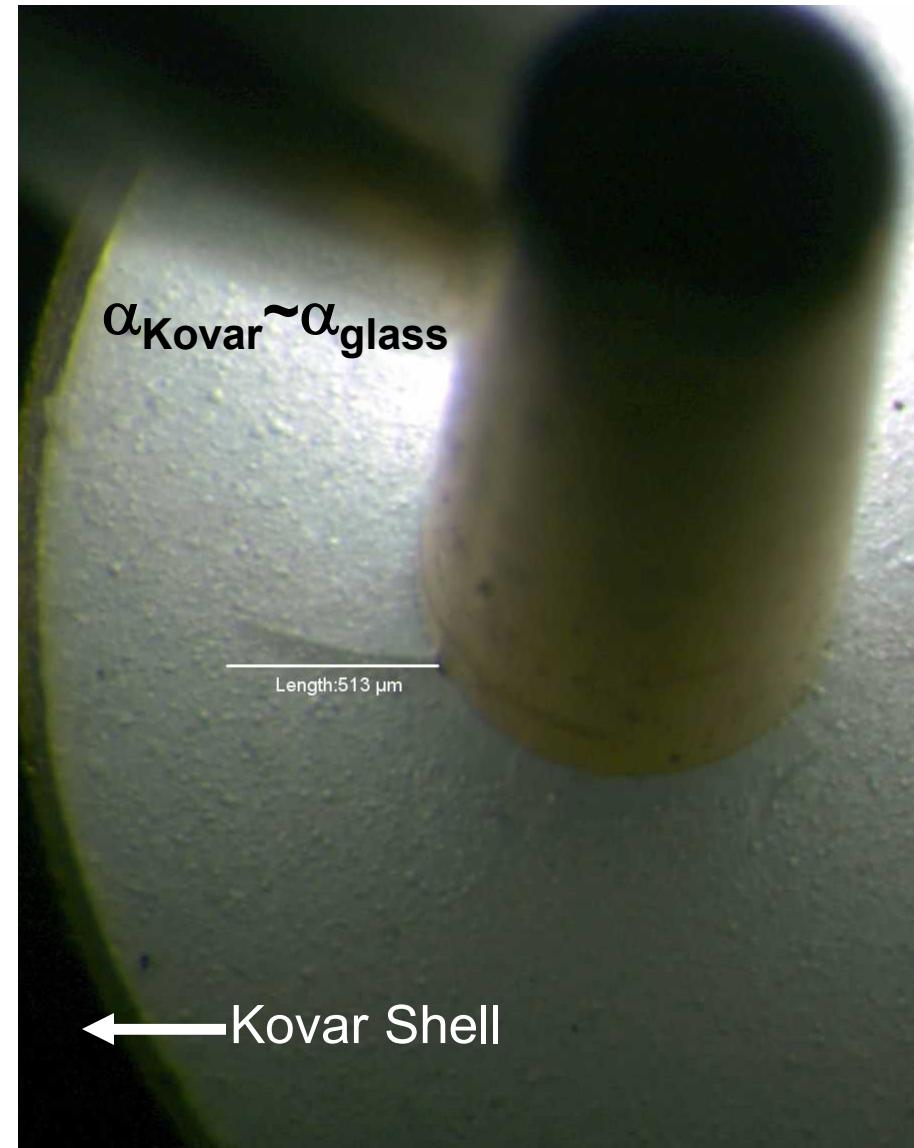
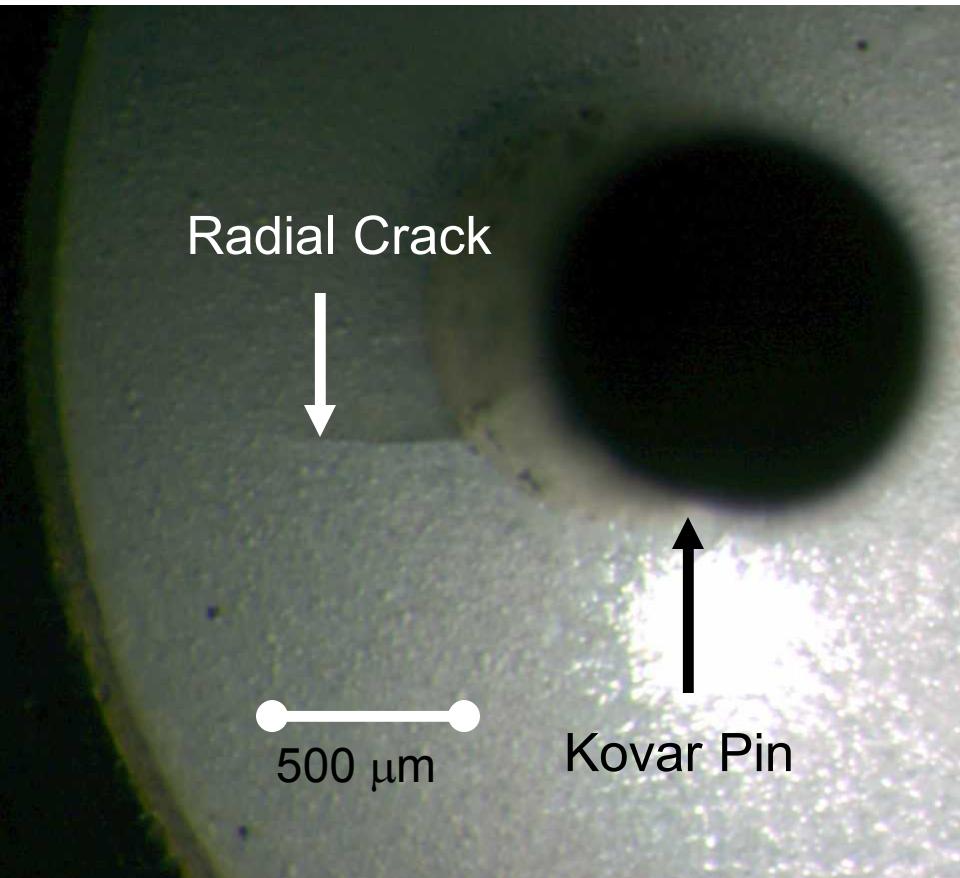


Outline

- **Rationale**
Examples of failures from cylindrical inclusions
 - Impact on SCG ?
- **Fracture Mechanics Analysis**
 - Describe behavior of radial cracks
- **Effect on Strength Variability**
- **Considerations for Design of Material Systems**



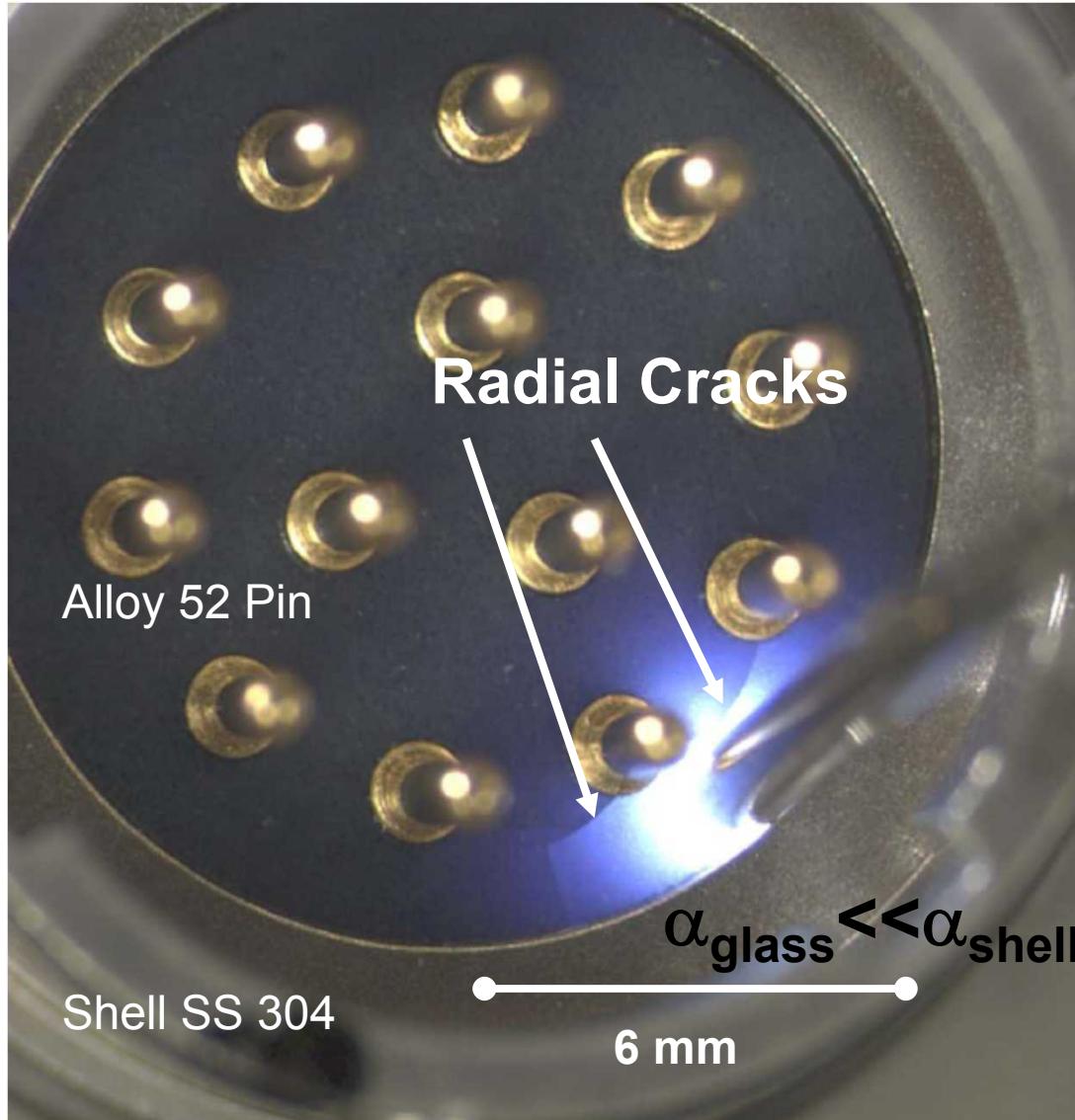
Crack at Pin in a Glass-to-Metal Seal



- Electrical breakdown issue



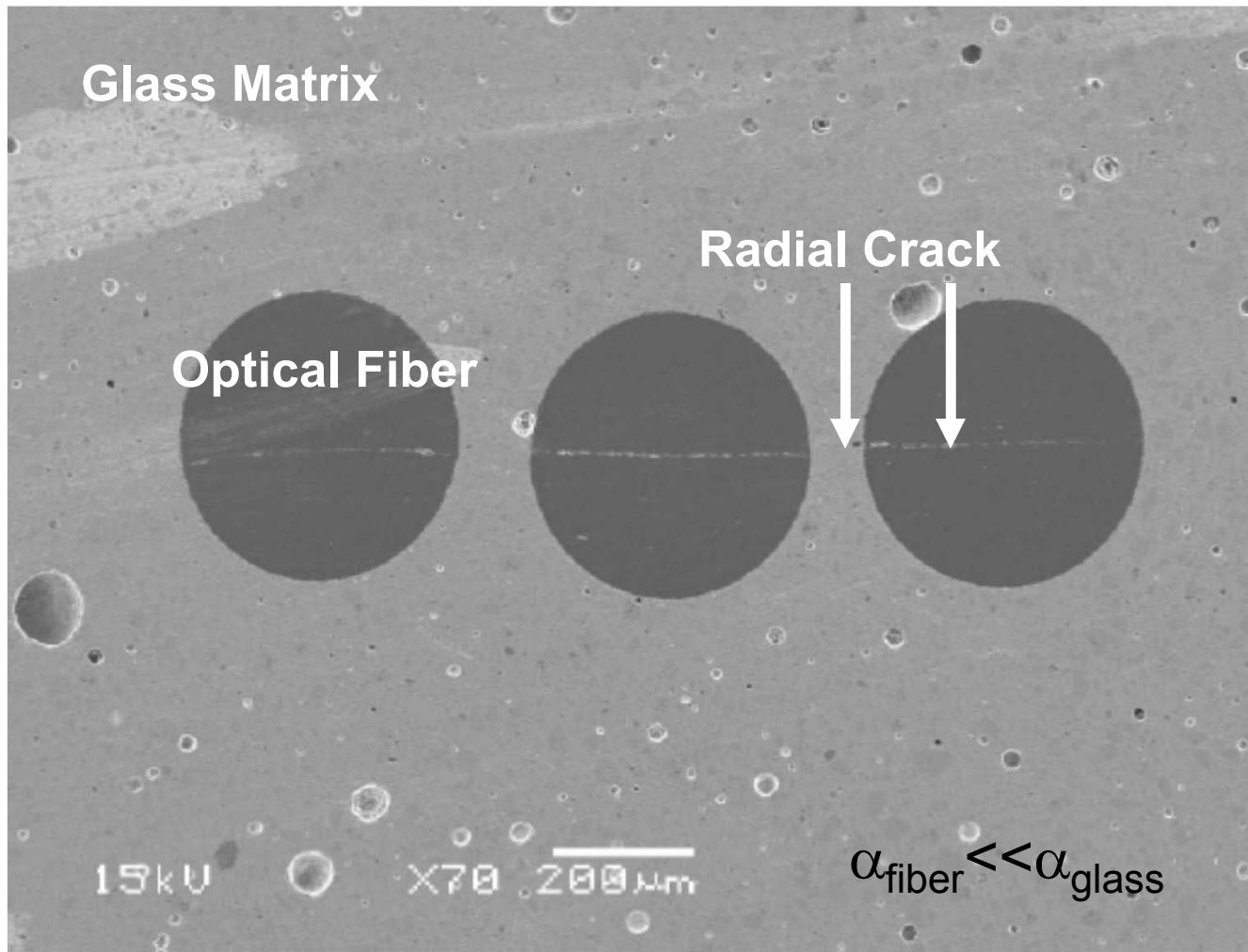
Cracks in Multi-Pin Glass-to-Metal Seal



- Hermeticity and electrical breakdown issues



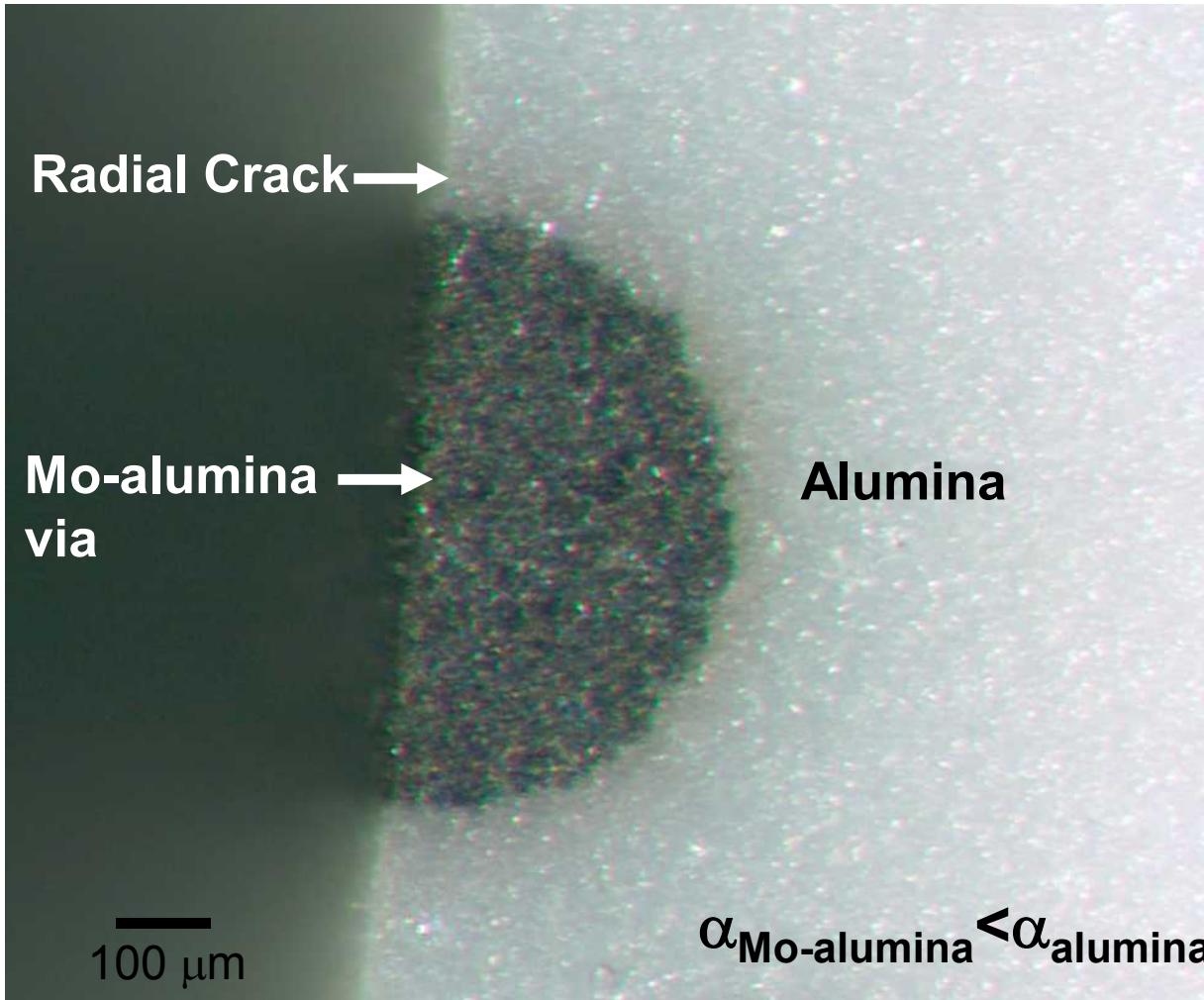
Radial Cracks Within and Outside fiber



- Loss of power-transmission functionality



Radial Crack at Via in Packaging Material

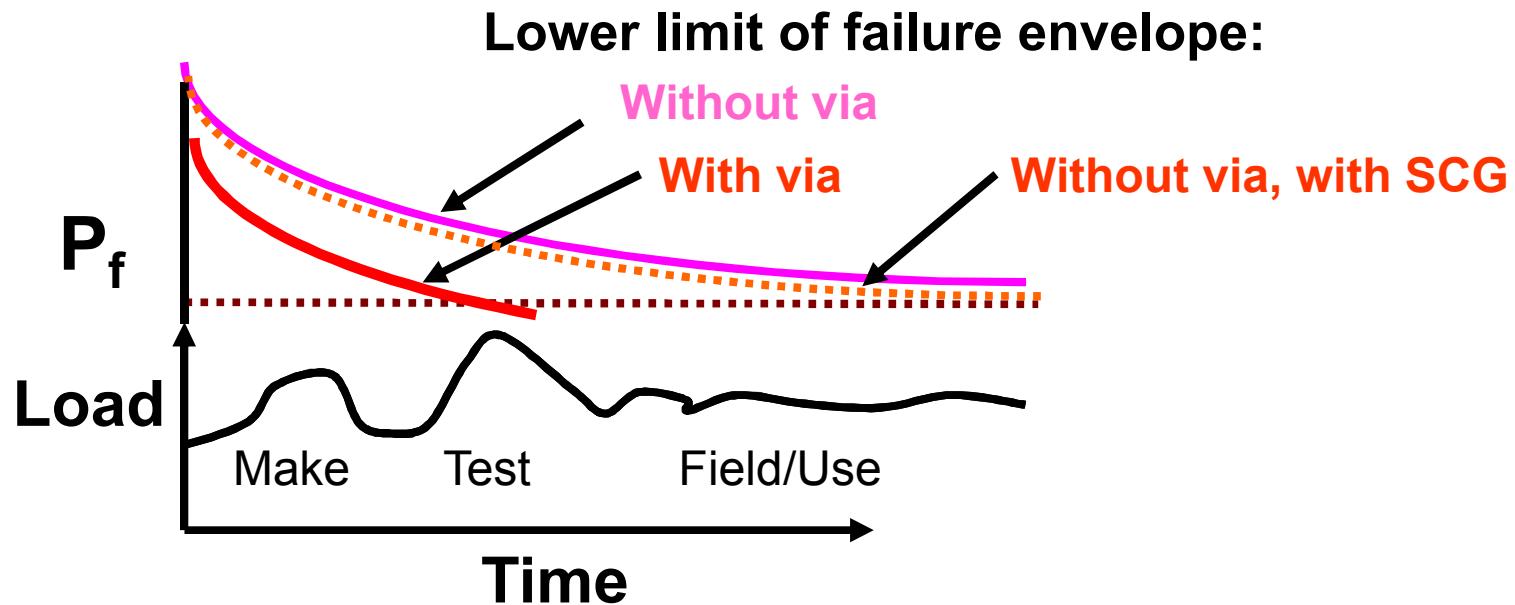


- Strength testing reveals radial cracking at via to be failure mechanism



Stress Fields Around Vias Influence Fracture Behavior

A. Effect on strength variability of components ?

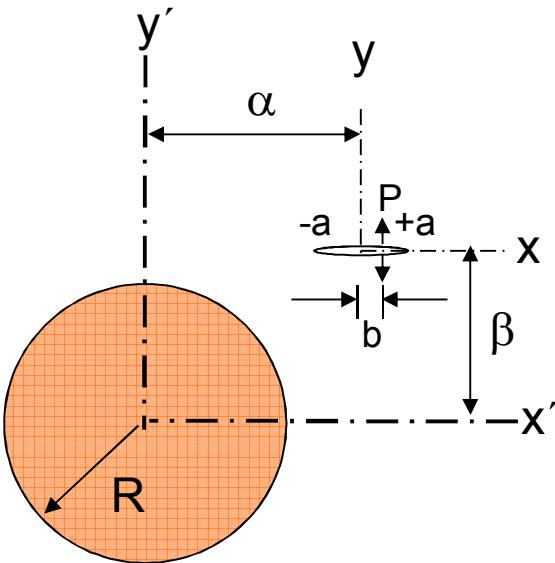


B. Failure envelopes degrade with time due to SCG . How does this stress field impact lifetime predictions of components ?



Fracture Mechanics Description

Stress due to inclusion acting on crack



$$\sigma_{\theta\theta} = -\sigma_{rr} = \Lambda T_{diff} \left(\frac{R}{r} \right)^2$$

where

$$\Lambda = \frac{E_m E_i [\eta_m (1 + \nu_m) - \eta_i (1 + \nu_i)]}{E_i (1 + \nu_m) + E_m (1 - \nu_i - 2\nu_i^2)}, \text{ and } \Lambda = \frac{E_m E_i (\eta_m - \eta_i)}{E_i (1 + \nu_m) + E_m (1 - \nu_i)}$$

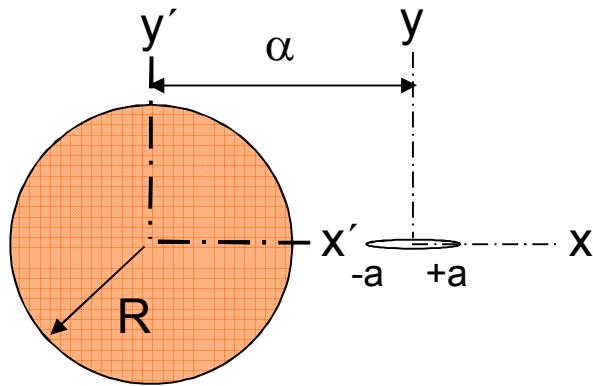
for plane strain, and plane stress respectively.

Point force solution acting on crack

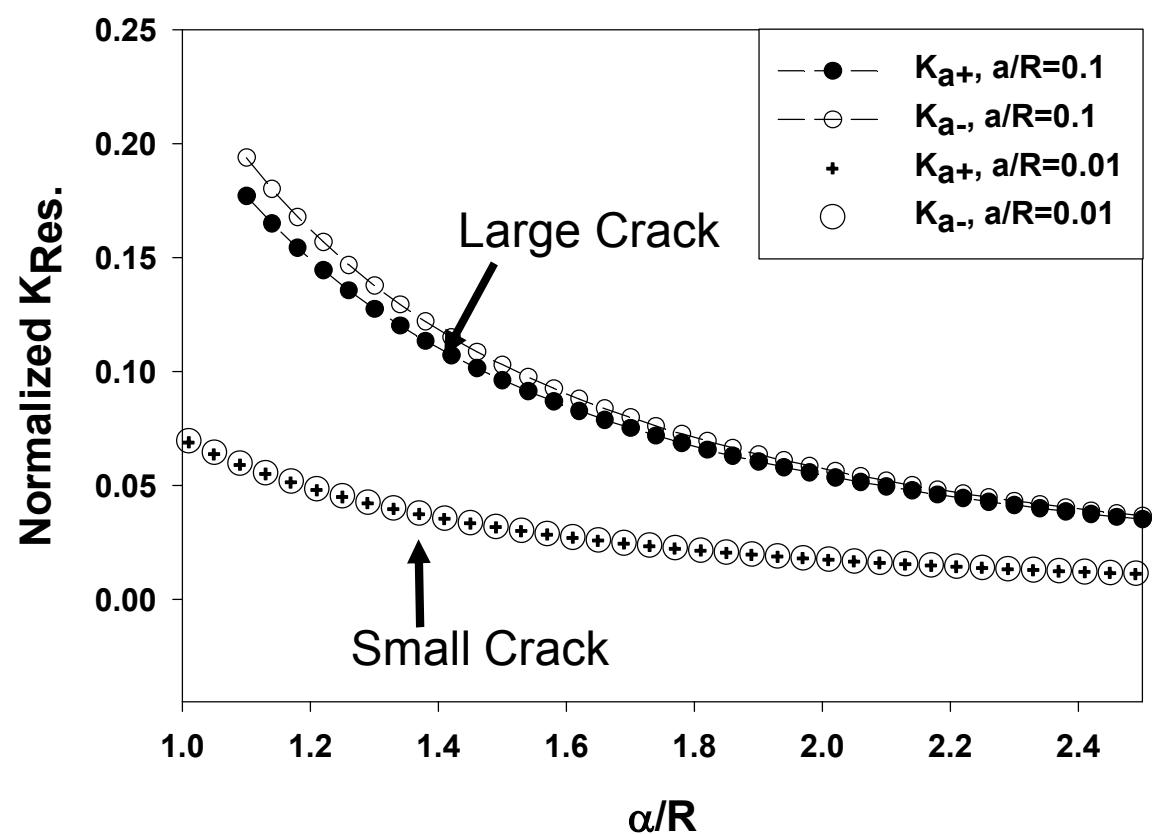
$$K_{\text{Res.}}|_{\pm a} = \frac{1}{\sqrt{\pi a}} \int_{-a}^{+a} \Lambda T_{diff} R^2 \frac{[(x+\alpha)^2 - (y+\beta)^2]}{[(x+\alpha)^2 + (y+\beta)^2]^2} \sqrt{\frac{a \pm x}{a \mp x}} dx$$

Does not include effect of crack on the inclusion
Does not include stresses due to elastic mismatch

Simplification: $\beta=0$



$$K_{\text{Res.}}|_{\pm a} = \frac{\Delta T_{\text{diff}} R^2 \sqrt{\pi a}}{(\alpha \pm a) \sqrt{\alpha^2 - a^2}}$$

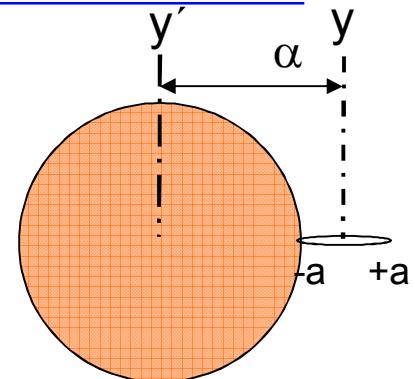


$K_{-a} > K_{+a}$

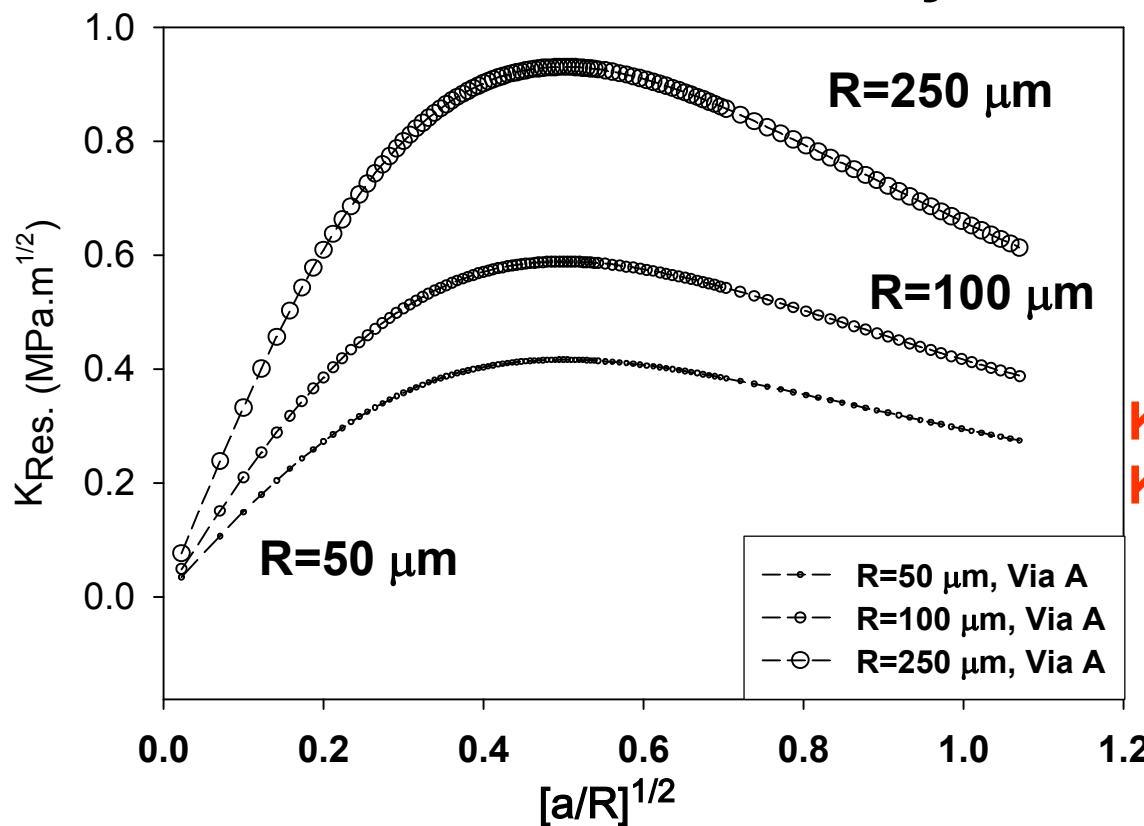
ΔK increases as $\alpha \rightarrow R$

Simplification: $\alpha=R+a$

If $K_{-a} > K_{1c}$, -a tip jumps to inclusion; arrests
Jump likely on external stress application



Results for Materials Systems of Interest



Matrix: Alumina
Via A: Cermet =
Mo + alumina

K_{Res} has a maximum
 K_{Res} increases with radius

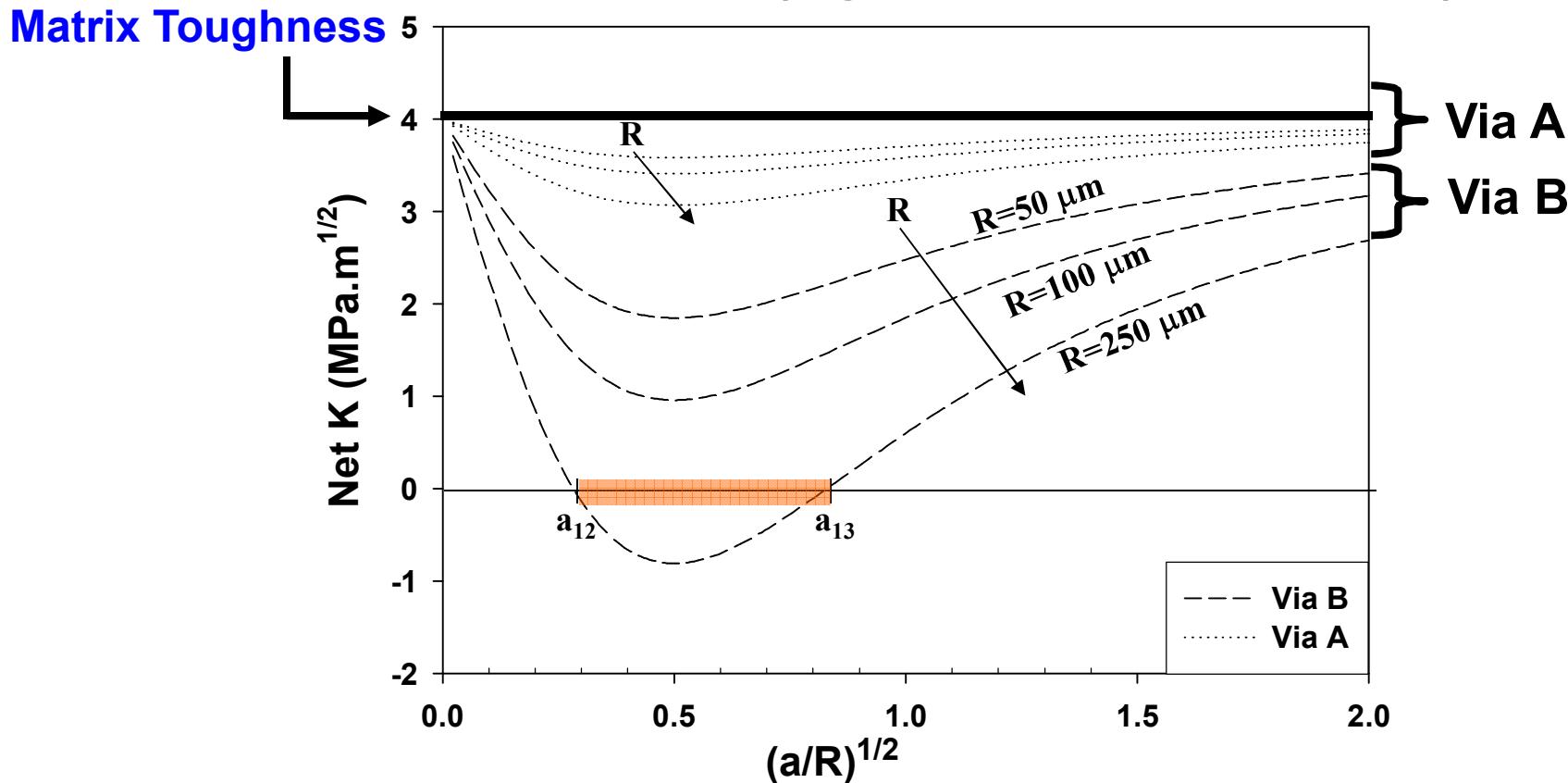
Destabilizing-Stabilizing Fields

Net $K = \text{Matrix Toughness} - K_{\text{Res.}}$ due to inclusion

RESULTS FOR TWO VIA MATERIALS

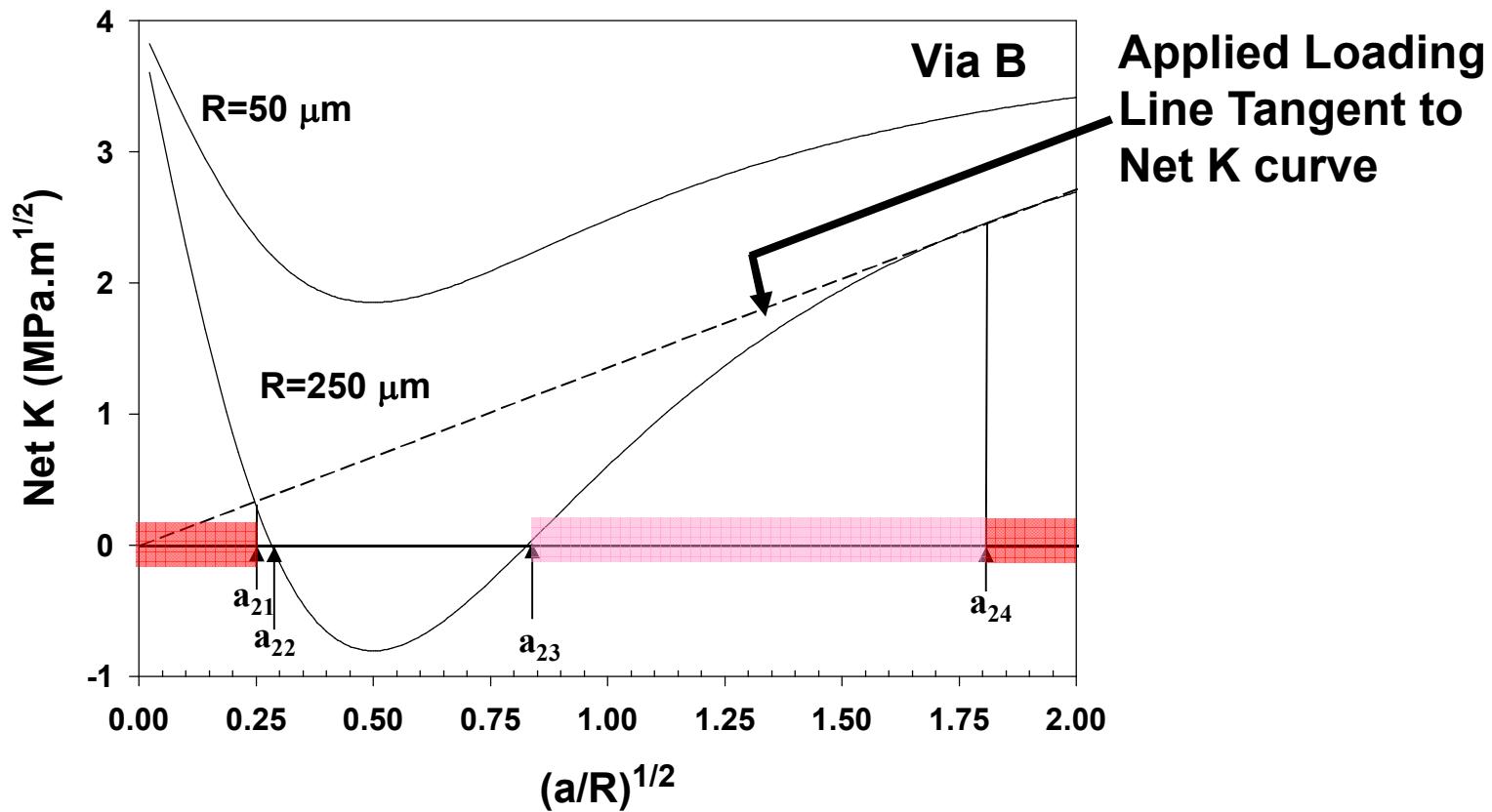
Via A: Mo-alumina

Via B: Pure Mo (higher expansion mismatch)



For largest via, Via B, cracks $a_{12} < a < a_{13}$ grow spontaneously to a_{13}

Behavior of crack at R=250 μm , Via B under Applied Tension



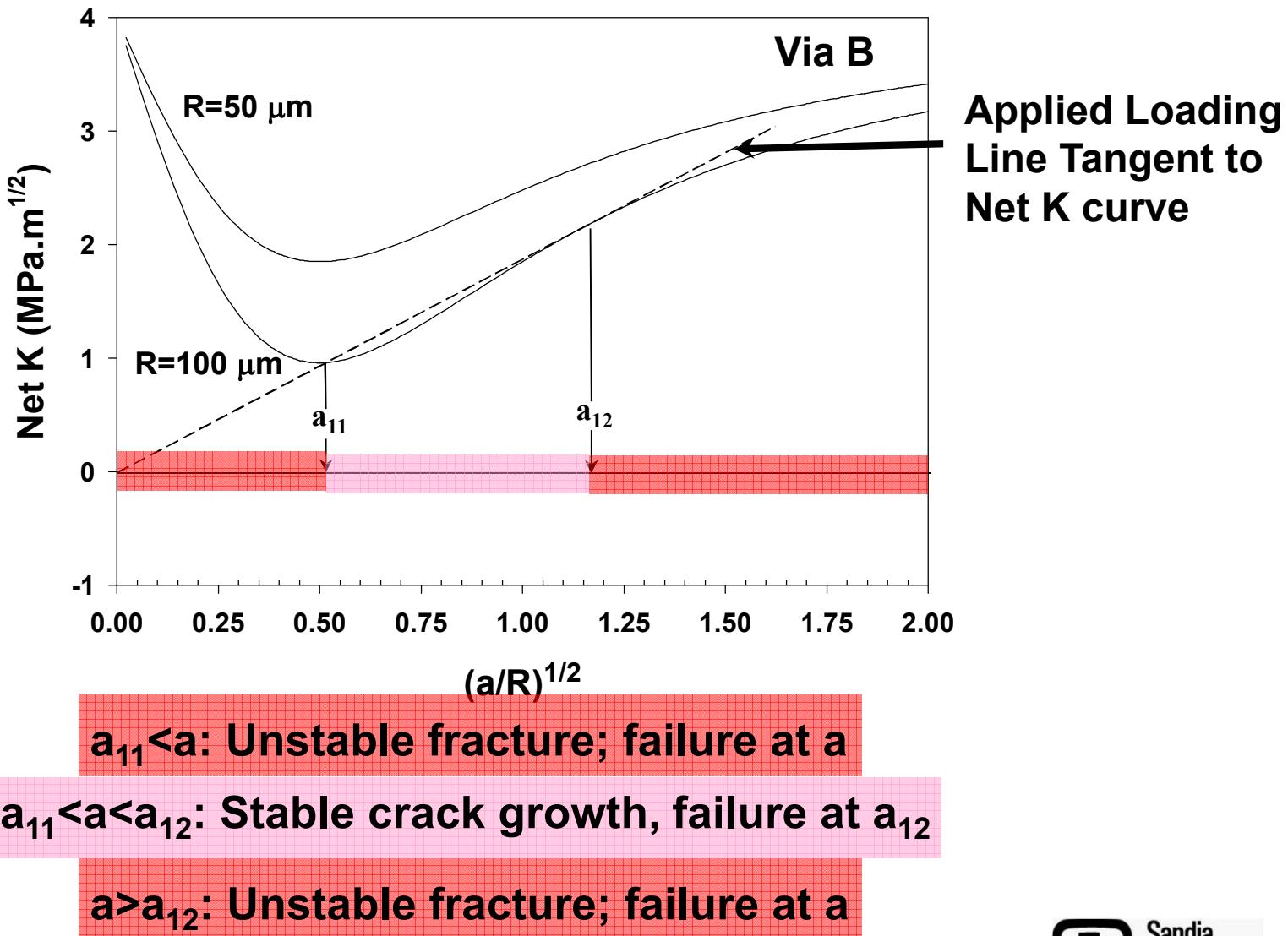
$a_{21} < a < a_{22}$: Unstable fracture; failure at a_{22}

$a_{23} < a < a_{24}$: Stable crack growth, failure at a_{24}

$a > a_{24}$: Unstable fracture; failure at a_{24}

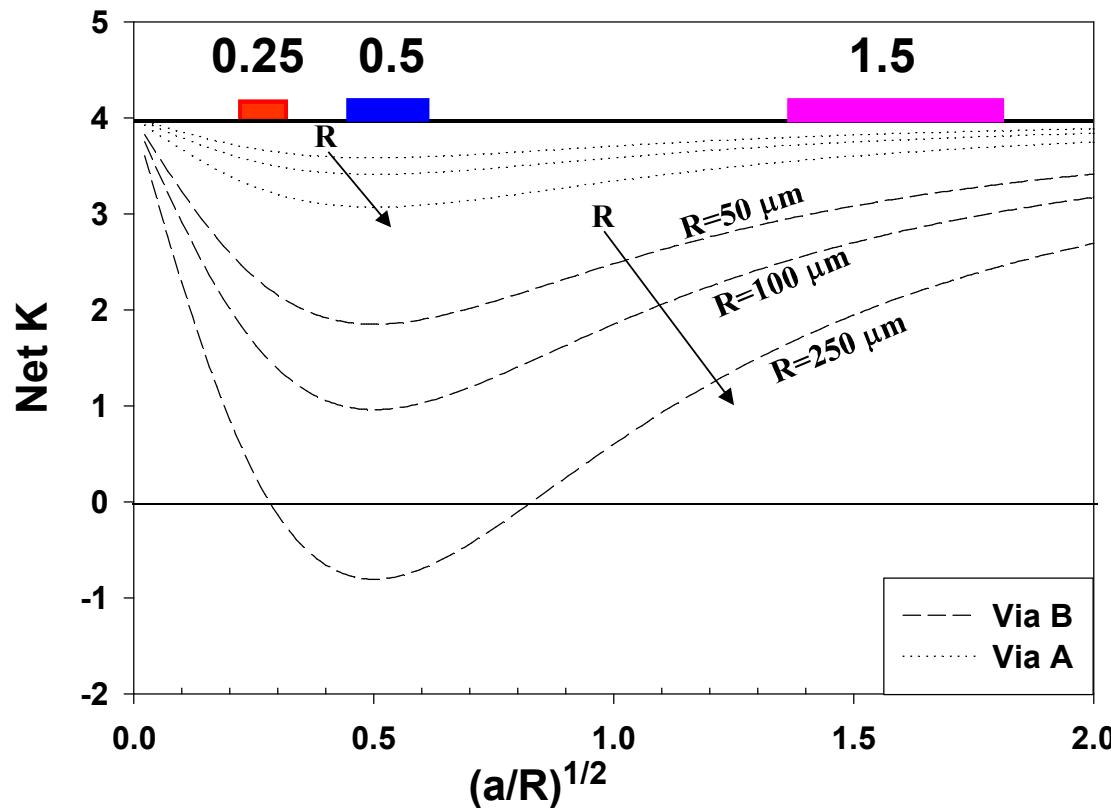


Behavior of crack at R=100 μm , Via B under Applied Tension



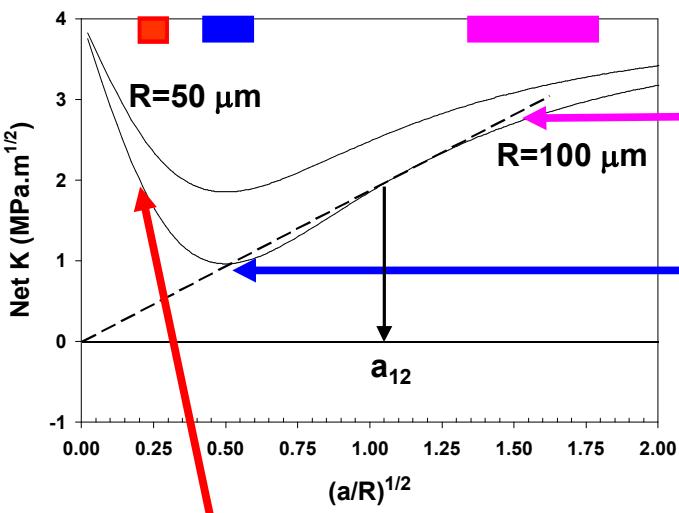


Effect on Strength Variability



Generated crack size distributions centered at various a/R locations such that the Weibull moduli=20 for each distribution on the base material

Strength Variability: R=100 μm

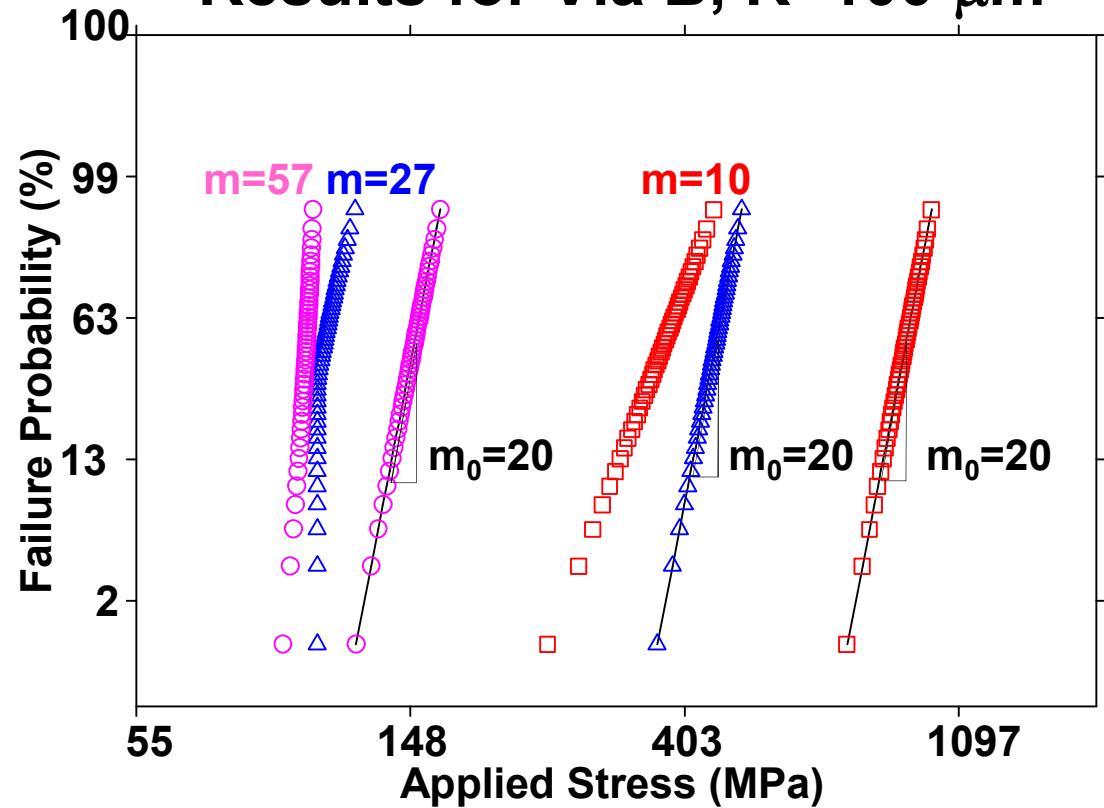


All Cracks in this set: Stabilizing Field

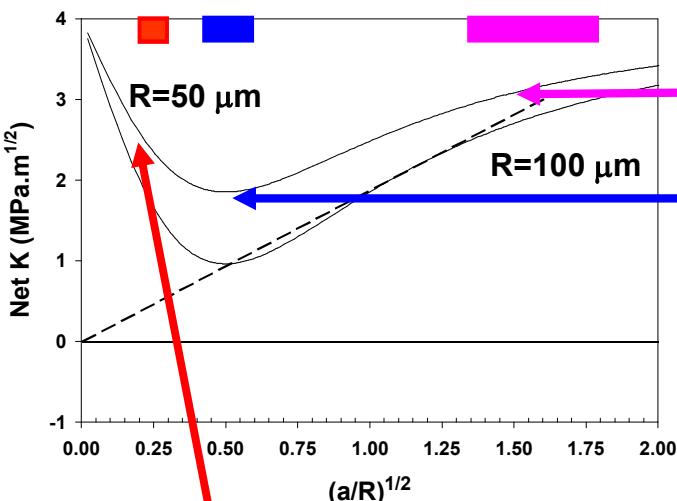
Smaller Cracks in this set: Destabilizing Field
Larger Cracks: Stable growth; failure at a_{12}

All cracks in this set:
Destabilizing Field; no stability

Results for Via B, R=100 μm



Strength Variability: R=50 μm

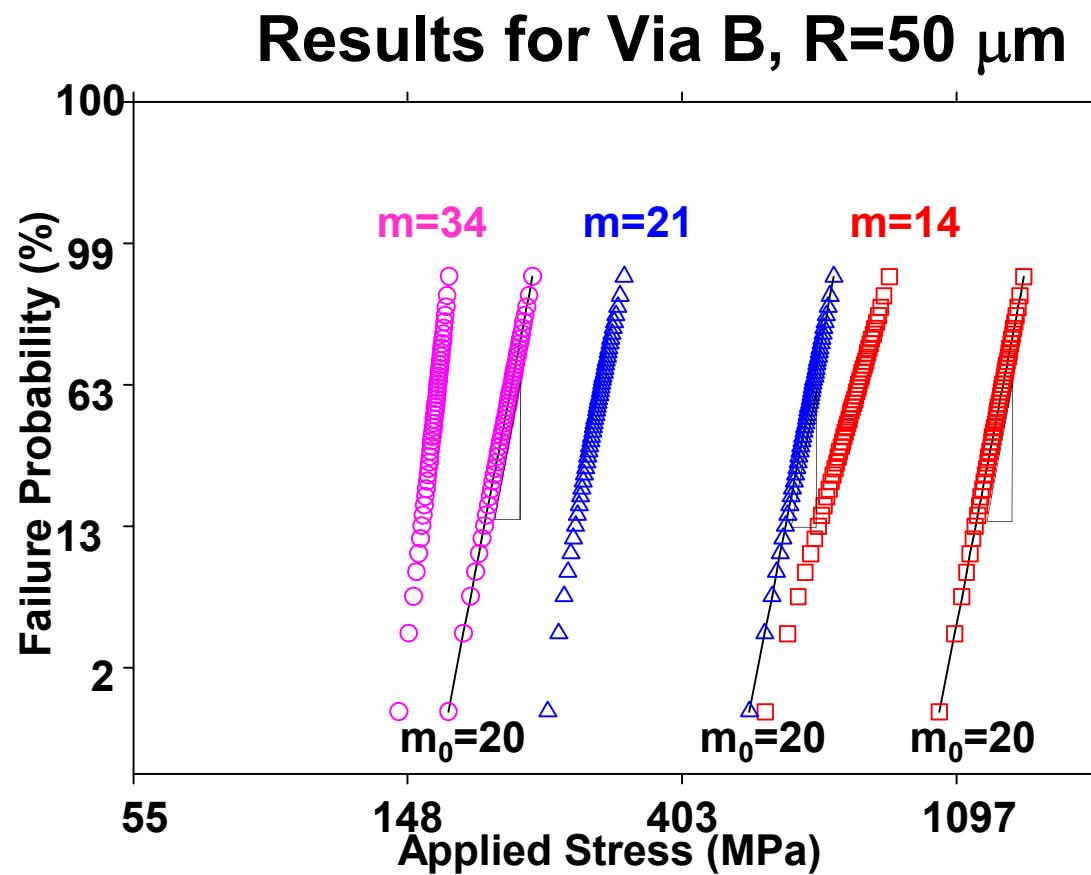


All Cracks in this set: Stabilizing Field

Smaller Cracks: Destabilizing Field

Larger Cracks: Stabilizing Field

All cracks in this set:
Destabilizing Field





Conclusions

Choose smallest inclusion size possible

- Retain most of the strength
- Possibly reduce variability

For High Retained Strength +Reduced Variability

**If crack size is \sim microstructure feature size (g),
then choosing $R < 4a \sim 4g$ ensures that most cracks
lie on the stabilizing branch.**

Fracture Mechanics Vs. Strength of Materials