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STRUCTURE AND RHEOLOGY OF WET FOAMS

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Summary The cell structure and elastic behavior of wet foams under quasi-static conditions is modeled with the Surface Evolver. We consider bulk foams with ordered and random structure, and thin foam layers confined between parallel plates. The liquid is assumed to be located in Plateau borders, which form a continuous network of channels along cell edges. The results for thin foam layers provide a relationship between the 2D structure at the wall and bubble radius, which is relevant to interpreting foam characterization measurements.

BACKGROUND

Gas-liquid foams (and concentrated liquid-liquid emulsions) exhibit elastic behavior because the bubbles (drops) are jammed together and their surface area (strain energy) increases when the material is subjected to quasi-static deformations. Experimental data for the shear modulus of monodisperse foams are well described by $G \sim \sigma/R\ \phi(\phi-\phi^*)$, where σ is the surface tension, R is the equivalent spherical bubble radius, ϕ is the gas volume fraction, and ϕ^* is the volume fraction of randomly packed spheres (~0.64). Above ϕ^* the system becomes jammed; the bubbles are non-spherical; and the foam has a finite shear modulus (and yield stress). Ironically, foam is stiffest in the "dry" limit where it contains mostly gas and can be modeled as two-dimensional surfaces that divide space into polyhedral cells. The structure of wet foams is much more complicated than dry foams because of the Plateau borders that form along cell edges [1]. These characteristic features contain most of the liquid because the thickness of the thin films is determined by colloidal forces that act over distances that are much smaller than the bubble radius R; so small in fact that we will assume that all of the liquid is located in the Plateau borders and the "dry" films that separate neighboring bubbles have zero thickness.

RESULTS

Ordered bulk foams

The Surface Evolver [2] is the standard software for calculating foam structure and rheology under quasi-static conditions. Figure 1 shows the structure of three ordered foams, which all have cubic symmetry and therefore two distinct shear moduli which we find are significantly different for each of these foams. The thickness of the Plateau borders increases with increasing liquid content while the area of the thin films decreases. When $\rho = 0.12$, the area of the smaller films in the Kelvin foam goes to zero along with one of the shear moduli, which indicates that the structure become unstable. The Weaire-Phelan foam becomes unstable when $\rho > 0.15$ for similar reasons. The wet FCC foam is unstable in the dry limit because more than four cell edges meet at cell vertices, which violates Plateau's laws. The shear moduli cannot be calculated over the entire range of ϕ for any of the ordered foam structures; consequently they cannot be used to predict $G(\phi)$ for real foams, which have random structure.

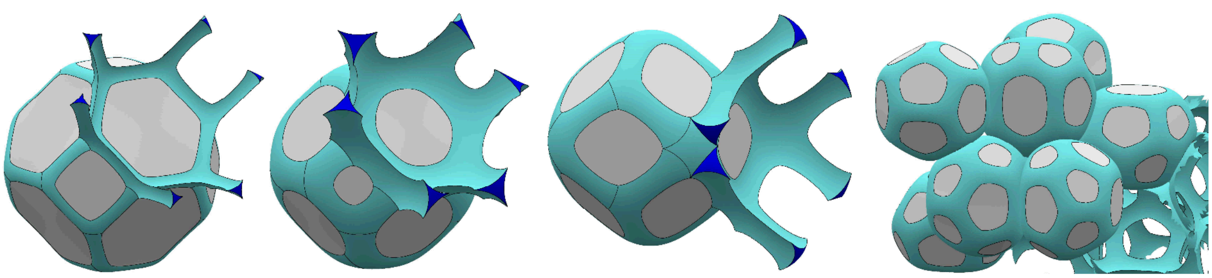

Figure 1. Unit cells of ordered wet foams showing the Plateau borders (cyan) and thin films (white). The first two images show wet Kelvin foams (bcc lattice) with liquid volume fractions $\rho = 1-\phi = 0.01, 0.05$. The last two images show a wet rhombic dodecahedral foam (fcc lattice) and a wet Weaire-Phelan foam (simple cubic), both with $\rho = 0.05$.

Thin layers of ordered foam confined between flat parallel plates

When the bubble size is comparable to the plate spacing, they can form ordered layers. In the "dry" limit, one confined layer is composed of hexagonal cylinders; two layers contain Fejes-Toth cells, which are Kelvin cells modified to pack against a flat surface; and three or more layers contain Kelvin cells sandwiched between Fejes-Toth cells. Two distinct types of Plateau border occur in wet systems: standard Plateau borders that form between three cell edges in the interior of the foam, and wall Plateau borders that form between two cells at the wall, as shown in figure 2. Viewed from "outside," the two-dimensional structure at the wall includes bubbles pressed against the plate, surrounded by the liquid in the wall Plateau borders. In general, the area fraction of liquid at the wall is different from the liquid volume fraction of the foam. A very common method of characterizing the bubble-size distribution of bulk foams uses the area of the wall films to calculate the corresponding bubble size. For example, in the dry limit where this technique is usually applied, the equivalent bubble radius R is assumed to be equal to the wall radius R_W, calculated from the area of the corresponding wall film by assuming circular shape. Our simulations enable us to obtain a correlation between R and R_W and its dependence on the liquid volume fraction ϕ. In general, the macroscopic stress in the foam layer is not isotropic (as in bulk foams), but depends on the ratio R/H where H is the plate spacing. The function R/R_W depends on the foam stress so we adjust R/H until the stress is isotropic and use this condition to obtain correlations that are relevant to experiments; we find that R/R_W is always greater than unity and increases with increasing ϕ. These calculations involve elastic deformations of the foam and provide a measure of the shear modulus of thin foam layers.

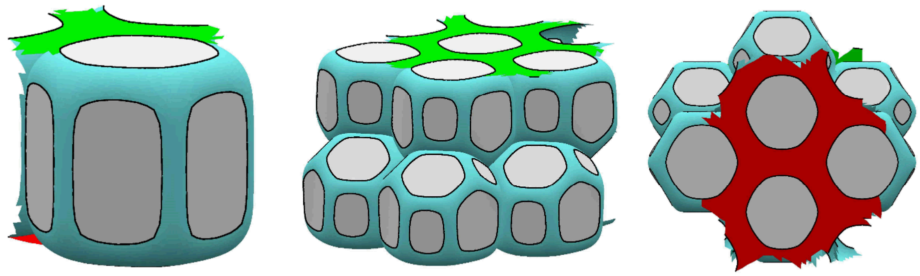

Figure 2. Ordered foams with ρ = 0.05 confined between parallel plates. The wet hexagonal cylinder (left) is oriented to show the (green) surface of the wall Plateau border on top, which would be seen through a glass plate along with the (white) surface of the thin film that is pressed against the plate. Also shown are a side view (center) and top view (right) of two layers of wet Fejes-Toth cells. The red region is the wall Plateau border adjacent to the plate.

Bulk Foams with random structure

Real foams are disordered. Extending previous work on dry random foams [3] to wet random foams is straightforward, in principle, but the demands on computer speed and memory are significantly larger because of the complex geometry of the Plateau borders and their large curvatures. Furthermore, bubbles continually lose neighbors as the liquid content of the foam increases; this requires the development of robust algorithms to deal with topological transitions in wet foams. Figure 3 shows the cell structure of a random monodisperse foam with two different liquid volume fractions.

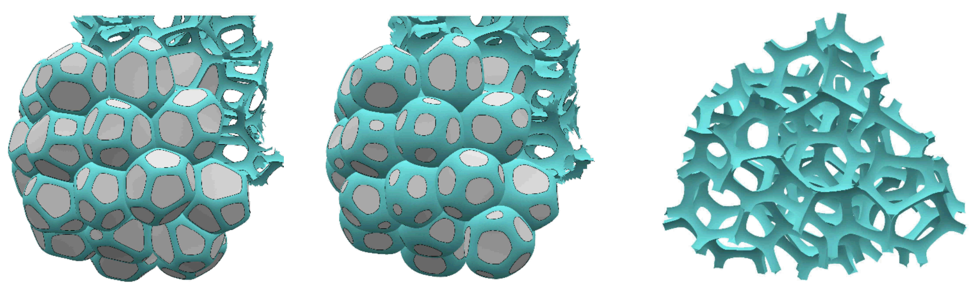

Figure 3. Random monodisperse foams with 27 bubbles and ρ = 0.03 and 0.08. The structure in the center has 19 fewer thin films (bubble contacts) than the foam on the left. The thin films have been removed from the image on the right to reveal the Plateau border structure, which is reminiscent of a microstructure of a solid foam with open cells.

Calculating the shear modulus of random wet foams involves relaxing the lattice to achieve a reference state with isotropic stress. The modulus is then calculated by deforming the foam in different directions to evaluate anisotropic behavior and obtain an effective isotropic modulus. These calculations are currently in progress.

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