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Characteristics of a Spring-Mass System Undergoing Centrifuge Acceleration

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under contract DE-AC04-94AL85000.

Sandia National Laboratories

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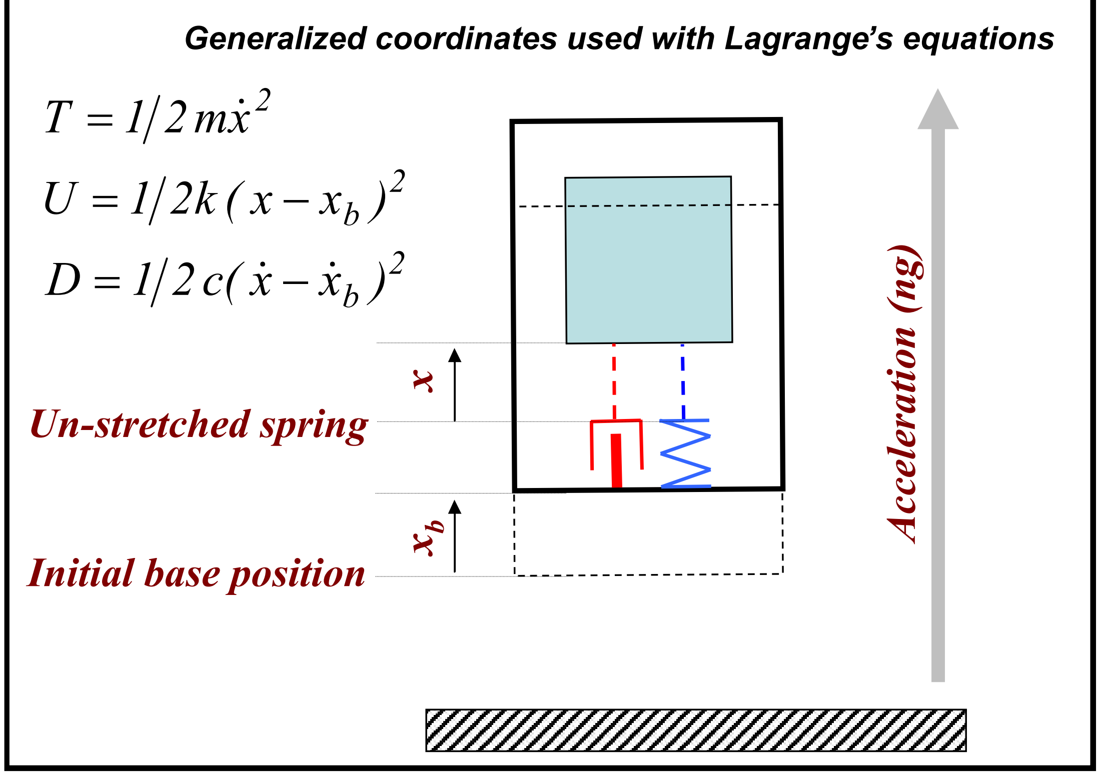
Introduction

- *Environments studied*
- *Analytical solutions*
- *Experimental design and testing*
- *Experimental analysis and results*
- *Implications*
- *Conclusion*

Direct Acceleration



$$m\ddot{x} + c\dot{x} + kx = -mng + c\dot{x}_b + kx_b \qquad [1]$$

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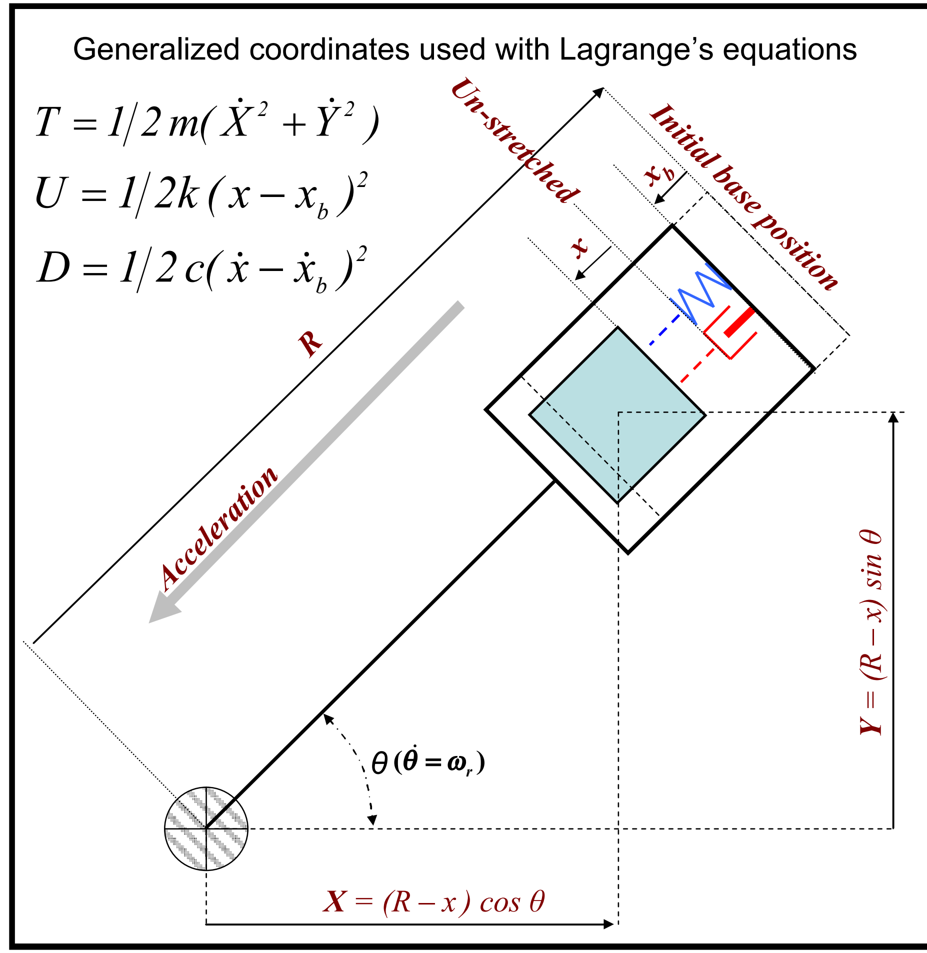
Direct Acceleration

$$\underline{\underline{m\ddot{x} + c\dot{x} + kx = -mng + c\dot{x}_b + kx_b}} \qquad [1]$$

$$\ddot{x} + 2\varsigma\omega_n\dot{x} + \omega_n^2 x = -ng + 2\varsigma\omega_n\dot{x}_b + \omega_n^2 x_b \qquad [1a]$$

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \varsigma = \frac{c}{2\omega_n m}$$

Centrifuge Acceleration



$$m\ddot{x} + c\dot{x} + (k - m\omega_r^{\,2})x = -m\omega_r^{\,2}R + c\dot{x}_b + kx_b \quad [2]$$

Centrifuge Acceleration

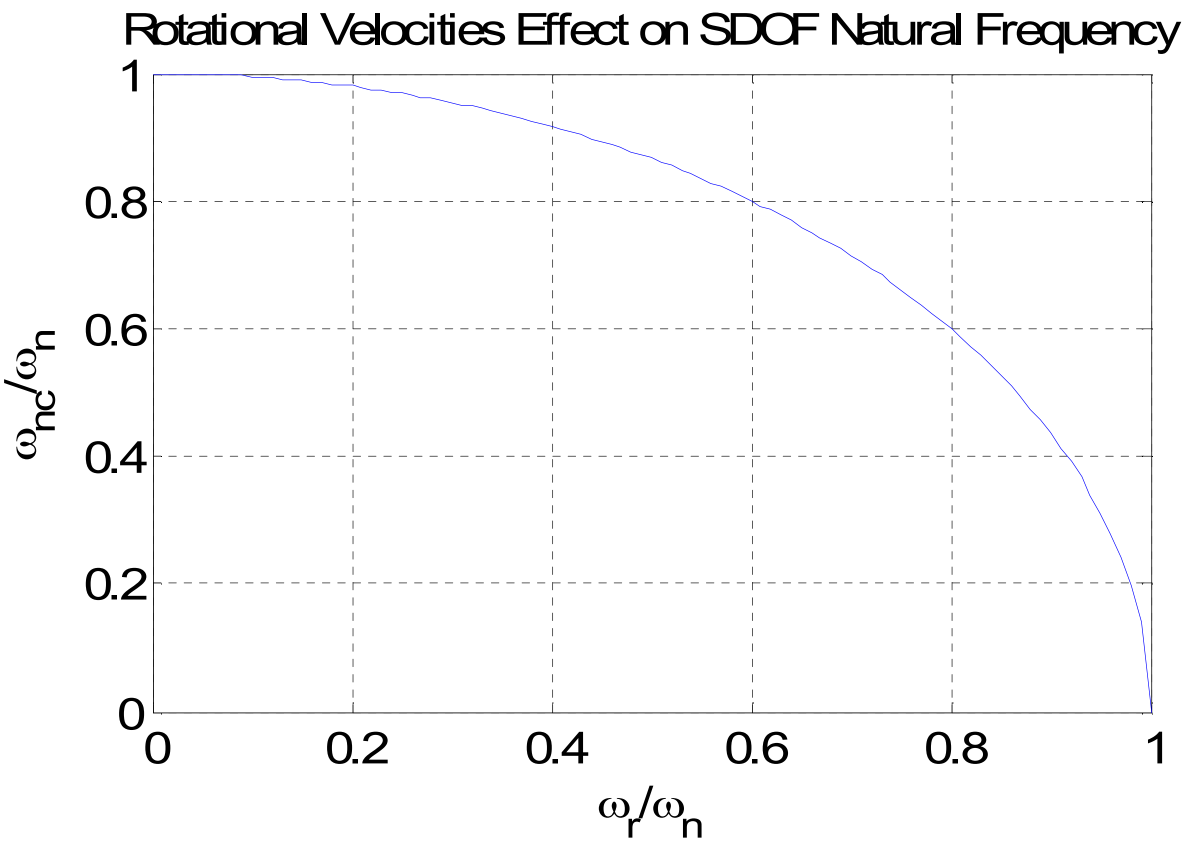
$$\underline{\underline{m\ddot{x} + c\dot{x} + (k - m\omega_r^2)x = -m\omega_r^2 R + c\dot{x}_b + kx_b}} \qquad [2]$$

$$\ddot{x} + 2\varsigma_c\omega_{nc}\dot{x} + \omega_{nc}^2 x = -\omega_r^2 R + 2\varsigma\omega_n\dot{x}_b + \omega_n^2 x_b \qquad [2a]$$

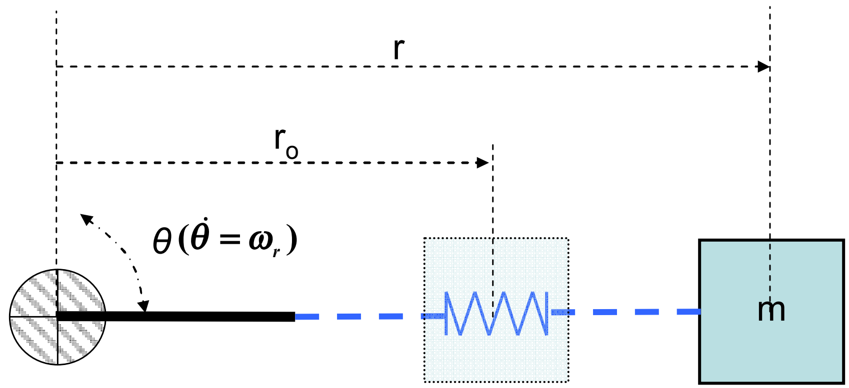
$$\omega_{nc} = \sqrt{\frac{k - m\omega_r^2}{m}}$$

$$\frac{\omega_{nc}}{\omega_n} = \sqrt{1 - (\omega_r/\omega_n)^2}$$

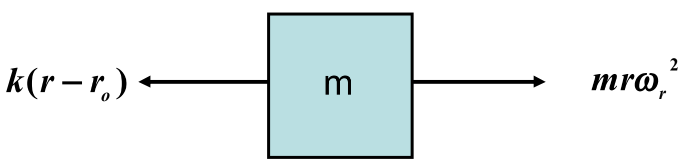
$$\varsigma_c = \sqrt{\frac{\varsigma^2}{1 - (\omega_r/\omega_n)^2}} = \frac{\omega_n}{\omega_{nc}}\varsigma$$



Experimental Design

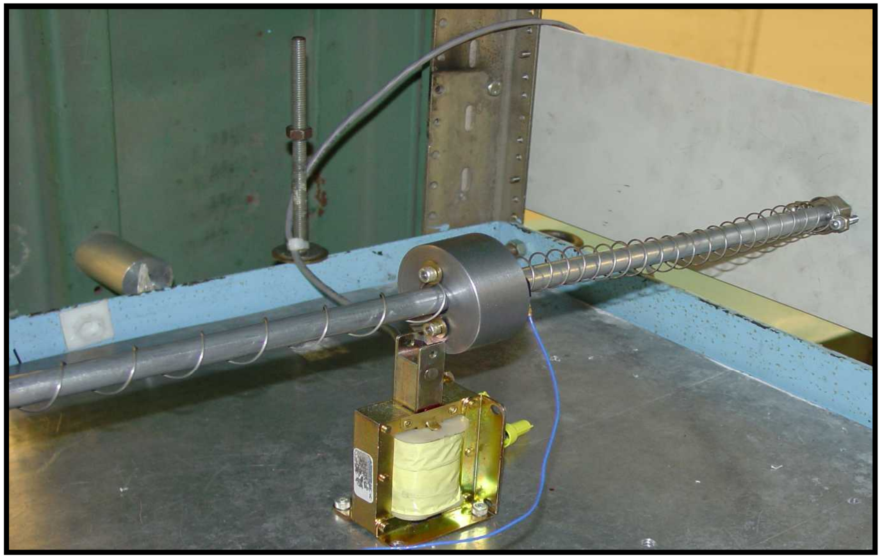


FBD:



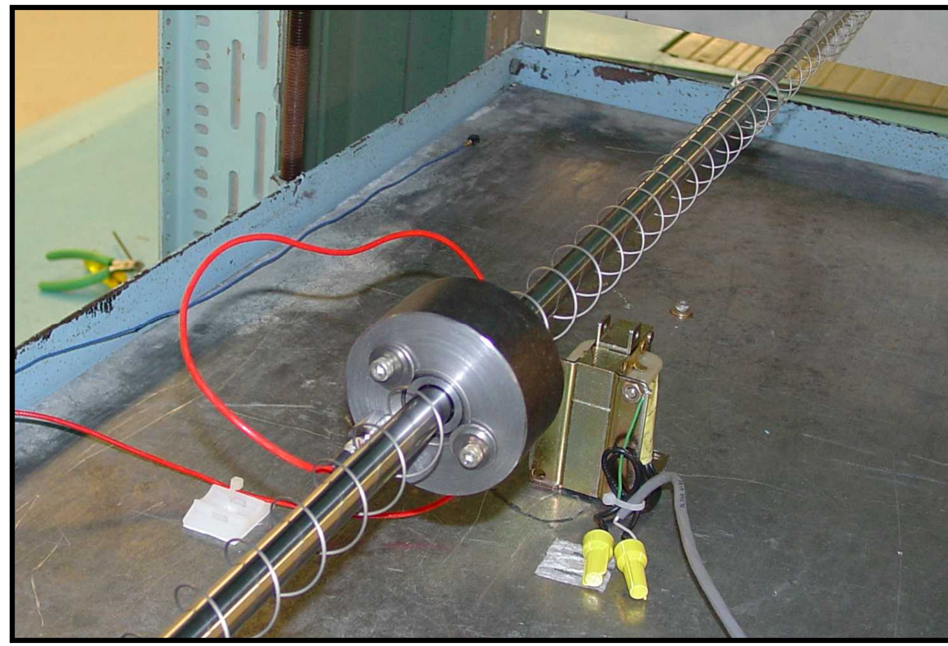
$$\omega_n = \sqrt{\frac{k}{m}}$$

Experimental Design



Design $\omega_n = \sqrt{k/m} = 19.8\ rad/\sec = 3.14Hz$

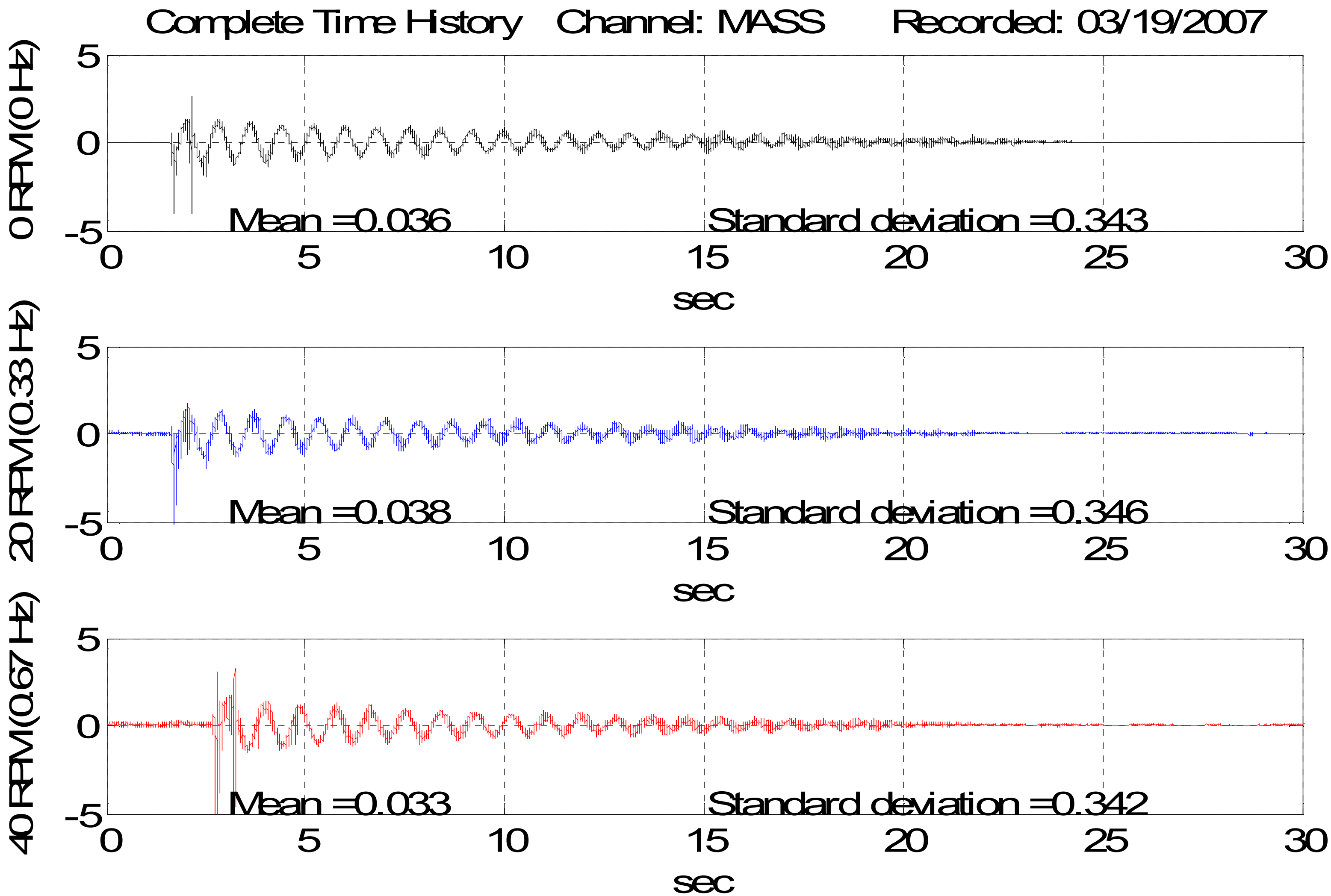
Measured = 3.15 Hz



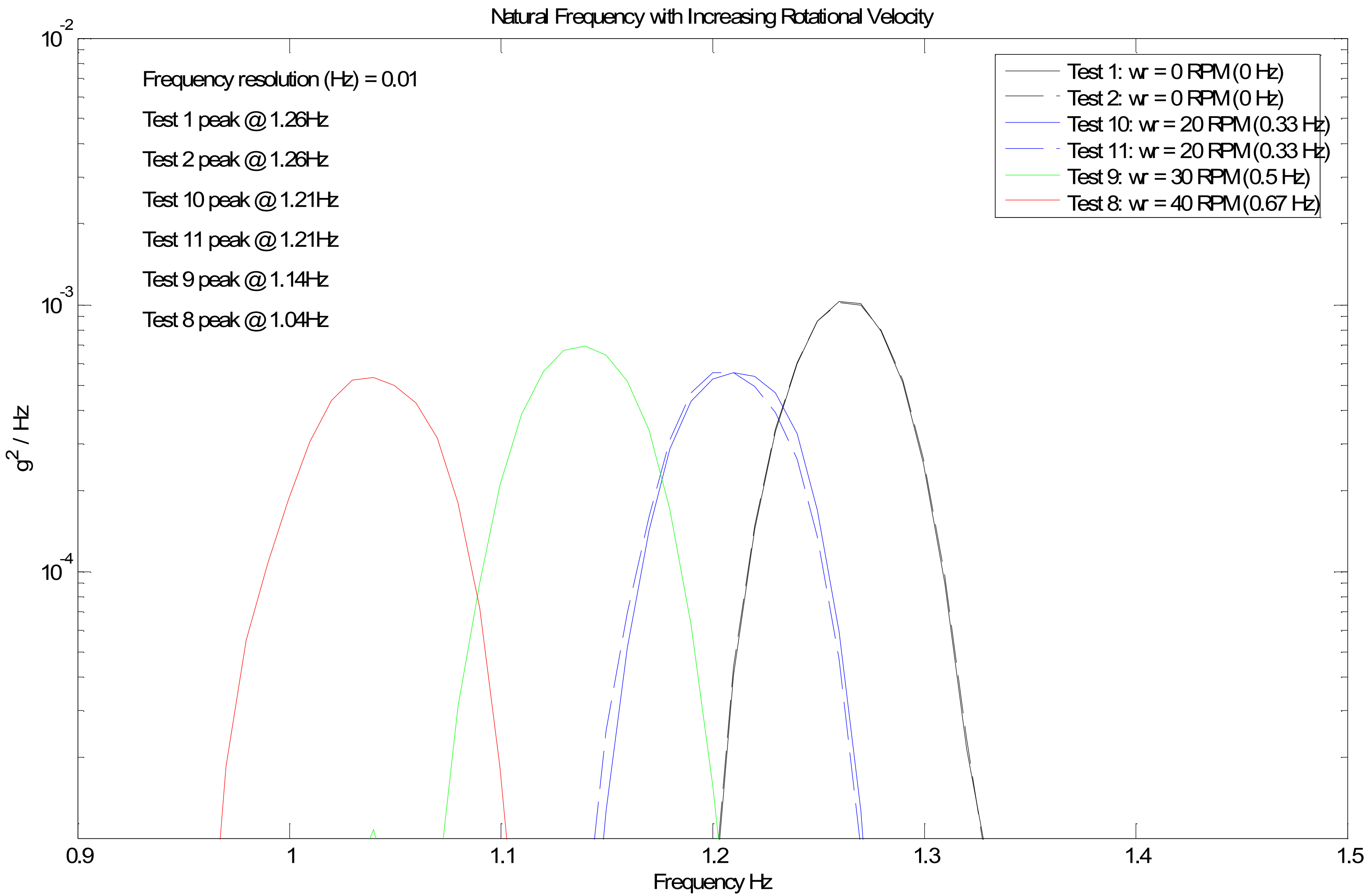
Design $\omega_n = \sqrt{k/m} = 19.8\ \text{rad/sec} = 1.09\ \text{Hz}$

Measured = 1.26 Hz

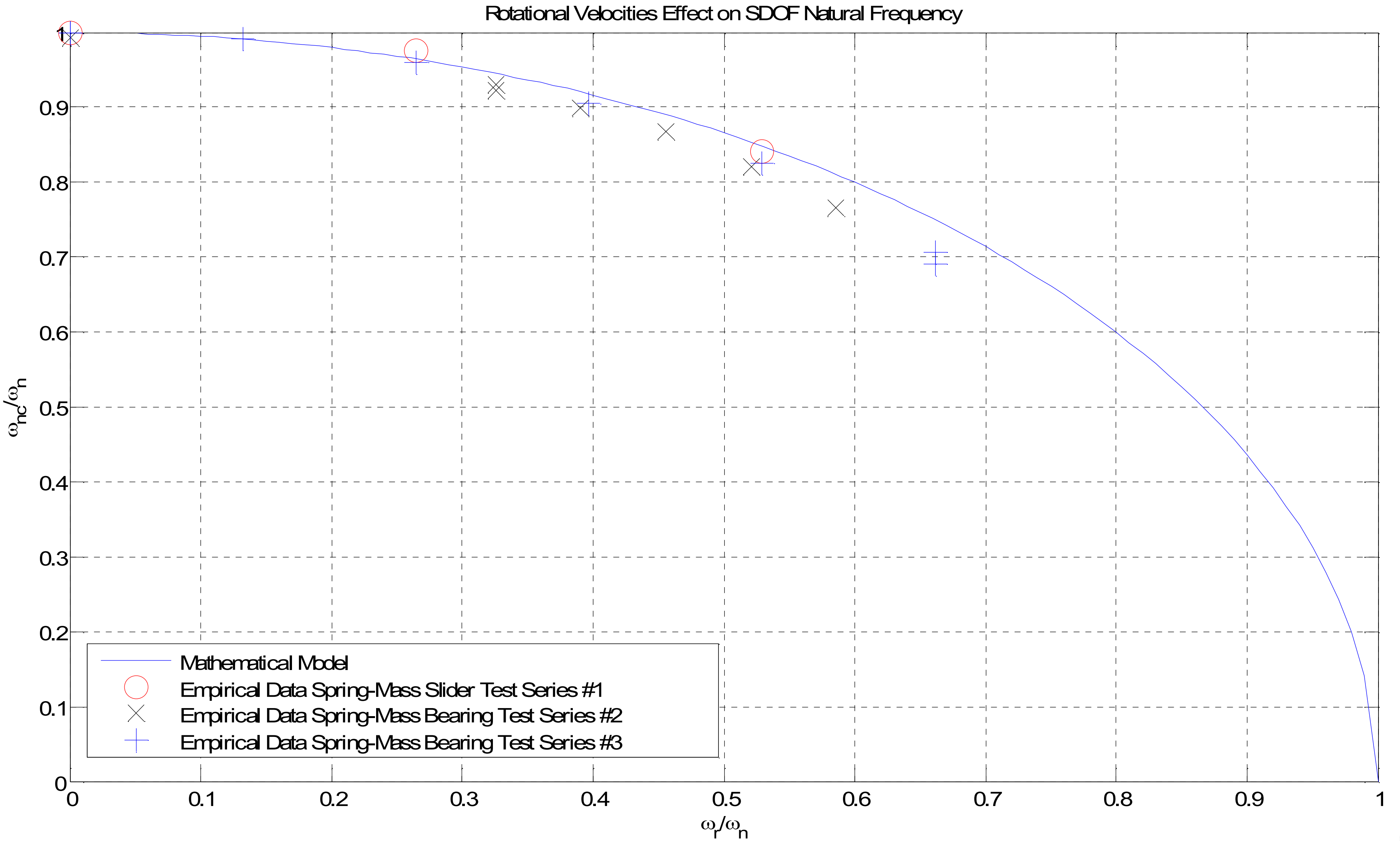
Time Domain Results



Frequency Domain Results



Experimental Results



Centrifuge Survey

SNL: US DOE Sandia National Laboratories, Albuquerque, NM

KCP: US DOE Kansas City Plant, Kansas City, MO

WETL: US DOE Weapons Evaluation Test Laboratory, Amarillo, TX

NASA: US National and Aeronautics Space Administration, Moffett Field, CA

ERDC: US DOD Engineer Research and Development Center, Vicksburg, MS

CIGTF: US DOD Central Inertial Guidance Test Facility, Alamogordo, NM

CU: University of Colorado, Boulder, CO

RPI(NEES): Rensselaer Polytechnic Institute, Troy, NY

UC DAVIS(NEES): University of California, Davis

NEES: Network for Earthquake Engineering Simulation

LARGE	MEDIUM	SMALL	Biochemistry
> 20 ft Radius	5-20 ft Radius	1 - 5 ft Radius	< 1 ft Radius

Facility	*SNL 1*	*SNL 2*	*SNL 3*	*SNL 4*	*KCP 1*	*KCP 2*	*KCP 3*	*KCP 4*	*WETL 1*	*WETL 2*	*NASA*	*ERDC*	*CIGTF*	*CU 1*	*RPI*	*UC DAVIS*
Nominal Radius (ft)	**2.0**	**3.0**	**29.0**	**35.0**	**6.0**	**2.5**	**2.2**	**0.4**	**8.0**	**1.6**	**29.0**	**21.0**	**21.0**	**18.0**	**9.6**	**28.2**
Max Acceleration (g)	400	100	300	270	385	1500	130	1000000	200	205	20	350	100	200	160	76
Max Payload Weight (lb)	50	50	16000	10000	1500	20	100		350	80				4000		4500 kg
Acceleration (ft/s^2)	12880	3220	9660	8694	12397	48300	4186	32200000	6440	6601	644	11270	3220	6440	5152	2447
Angular Velocity (rad/s)	80	33	18	16	45	139	44	9266	28	64	5	23	12	19	23	9
Angular Velocity (Hz)	**13**	**5**	**3**	**3**	**7**	**22**	**7**	**1475**	**5**	**10**	**1**	**4**	**2**	**3**	**4**	**1**
Angualr Velocity (RPM)	766	313	174	151	434	1327	417	88488	271	613	45	221	118	181	221	89

Survey Results

Raw Survey Data Assumed 100000g for Biochemistry Centrifuge (Not Realistic)

	LARGE > 20 ft Radius		MEDIUM 5-20 ft Radius		SMALL 1 - 5 ft Radius		Biochemistry < 1 ft Radius	
Average Radius (ft)	27.2		10.4		2.3		0.4	
Average RPM	133.0		276.7		687.3		88487.9	
Average Angular Velocity (Hz)	2.2		4.6		11.5		1474.8	
	Original (ω_n)	Decreased (ω_{nc})	Original (ω_n)	Decreased (ω_{nc})	Original (ω_n)	Decreased (ω_{nc})	Original (ω_n)	Decreased (ω_{nc})
10% Decrease (Hz)	**5.1**	→ **4.6**	**10.6**	→ **9.5**	**26.3**	→ **23.7**	**3383.4**	→ **3045.1**
50% Decrease (Hz)	**2.6**	→ **1.3**	**5.3**	→ **2.7**	**13.2**	→ **6.6**	**1702.9**	→ **851.5**
80% Decrease (Hz)	**2.3**	→ **0.5**	**4.7**	→ **0.9**	**11.7**	→ **2.3**	**1505.2**	→ **301.0**

Reducing the Maximum Acceleration of Biochemistry Centrifuge

	1000 g on 0.4 ft Centrifuge		***10,000 g on 0.4 ft Centrifuge***		***20,000 g on 0.4 ft Centrifuge***	
	Original (ωn)	Decreased (ωnc)	Original (ωn)	Decreased (ωnc)	Original (ωn)	Decreased (ωnc)
10% Decrease (Hz)	**107.0**	→ **96.3**	**338.3**	→ **304.5**	**478.5**	→ **430.6**
50% Decrease (Hz)	**53.9**	→ **26.9**	**170.3**	→ **85.1**	**240.8**	→ **120.4**
80% Decrease (Hz)	**47.6**	→ **9.5**	**150.5**	→ **30.1**	**212.9**	→ **42.6**

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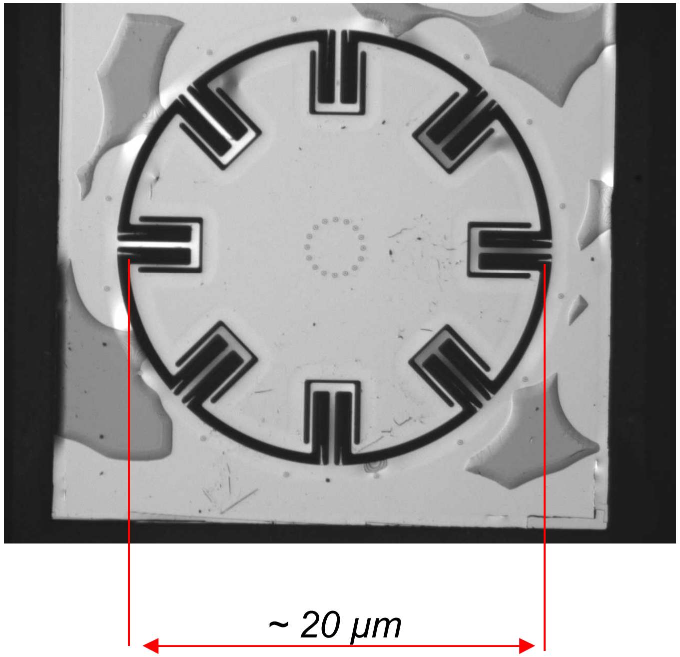
MEMS Example



- *Acceleration switch*
- *Batch of 4 with average 1st frequency of 193 Hz*
- *Small size → Potential to be tested on biochemistry centrifuge*

Conclusion

- *Flight Environment = Centrifuge Environment (With Caveat)*
- *A potential for decrease in natural frequency*
- *Experimental testing demonstrated analytical findings*
- *Survey revealed few cases this characteristic causes concern*

Future Work

- *Refine experiment to obtain more data points*
- *Experiment with two degrees of freedom*
- *Find mechanical systems with low natural frequencies*

Appendix A: *Derivation, single degree of freedom direct acceleration*
(See figure 1 for generalized coordinates)

$$\left.\begin{aligned} T &= \frac{1}{2}m\dot{x}^2 \\ \\ U &= \frac{1}{2}k(x-x_b)^2 \end{aligned}\right\} \rightarrow \quad L = \quad T - U \quad = \quad \frac{1}{2}m\dot{x}^2 - \frac{1}{2}k(x-x_b)^2$$

$$\frac{\partial L}{\partial x} = -\frac{1}{2}k(2)(x-x_b)(1) = \underline{-kx + kx_b}$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{1}{2}m(2)(\dot{x}) = m\dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \underline{m\ddot{x}}$$

$$D = \frac{1}{2}c(\dot{x}-\dot{x}_b)^2$$

$$\frac{\partial D}{\partial \dot{x}} = \frac{1}{2}c(2)(\dot{x}-\dot{x}_b) = \underline{c\dot{x} - c\dot{x}_b}$$

$$\delta W = Q\delta x = -mng\delta x \quad \rightarrow \quad Q = \underline{-mng} \qquad (ng = number\ of\ g's; \quad m = mass)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = Q - \frac{\partial D}{\partial \dot{x}} \quad \leftrightarrow \quad m\ddot{x} - (-kx + kx_b) = -mng - (c\dot{x} - c\dot{x}_b)$$

$$\rightarrow \quad \underline{\underline{m\ddot{x} + c\dot{x} + kx = -mng + c\dot{x}_b + kx_b}}$$

Appendix B: *Derivation, single degree of freedom centrifuge acceleration*

$$X = (R - x)\cos\theta \quad \rightarrow \quad \dot{X} = -(R - x)\dot{\theta}\sin\theta - \dot{x}\cos\theta$$

$$Y = (R - x)\sin\theta \quad \rightarrow \quad \dot{Y} = (R - x)\dot{\theta}\cos\theta - \dot{x}\sin\theta$$

$$T = \frac{1}{2}m(\dot{X}^2 + \dot{Y}^2) \quad \rightarrow \quad \frac{m}{2}\left\{\left[-(R - x)\dot{\theta}\sin\theta - \dot{x}\cos\theta\right]^2 + \left[(R - x)\dot{\theta}\cos\theta - \dot{x}\sin\theta\right]^2\right\}$$

$$U = \frac{1}{2}k(x - x_b)^2$$

$$\rightarrow \quad L = \quad T - U \quad = \quad \frac{m}{2}\left\{\left[-(R - x)\dot{\theta}\sin\theta - \dot{x}\cos\theta\right]^2 + \left[(R - x)\dot{\theta}\cos\theta - \dot{x}\sin\theta\right]^2\right\} - \frac{1}{2}k(x - x_b)^2$$

$$\frac{\partial L}{\partial x} = \frac{m}{2}\left\{2\left[-(R - x)\dot{\theta}\sin\theta - \dot{x}\cos\theta\right]\left(\dot{\theta}\sin\theta\right) + 2\left[(R - x)\dot{\theta}\cos\theta - \dot{x}\sin\theta\right]\left(-\dot{\theta}\cos\theta\right)\right\} - \frac{1}{2}2k(x - x_b)$$

$$\rightarrow \quad \frac{\partial L}{\partial x} = m\left[-(R - x)\dot{\theta}^2\sin^2\theta - \dot{x}\dot{\theta}\sin\theta\cos\theta - (R - x)\dot{\theta}^2\cos^2\theta + \dot{x}\dot{\theta}\sin\theta\cos\theta\right] - kx + kx_b$$

$$\rightarrow \quad \frac{\partial L}{\partial x} = m\left[-(R - x)\dot{\theta}^2\left(\sin^2\theta + \cos^2\theta\right)\right] - kx + kx_b = \underline{-m\dot{\theta}^2(R - x) - kx + kx_b}$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{m}{2}\left\{2\left[-(R - x)\dot{\theta}\sin\theta - \dot{x}\cos\theta\right]\left(-\cos\theta\right) + 2\left[(R - x)\dot{\theta}\cos\theta - \dot{x}\sin\theta\right]\left(-\sin\theta\right)\right\}$$

$$\rightarrow \quad \frac{\partial L}{\partial \dot{x}} = m\left[(R - x)\dot{\theta}\sin\theta\cos\theta + \dot{x}\cos^2\theta - (R - x)\dot{\theta}\sin\theta\cos\theta + \dot{x}\sin^2\theta\right]$$

$$\rightarrow \quad m\left[\dot{x}\left(\sin^2\theta + \cos^2\theta\right)\right] = m\dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \underline{m\ddot{x}}$$

$\rightarrow$

$$D = \frac{1}{2}c(\dot{x} - \dot{x}_b)^2$$

$$\frac{\partial D}{\partial \dot{x}} = \frac{1}{2}c(2)(\dot{x} - \dot{x}_b) = \underline{c\dot{x} - c\dot{x}_b}$$

$$\delta W = Q\delta x = 0 \quad \rightarrow \quad Q = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = Q - \frac{\partial D}{\partial \dot{x}}$$

$$\rightarrow \quad m\ddot{x} - (-m\dot{\theta}^2(R - x) - kx + kx_b) = -(c\dot{x} - c\dot{x}_b)$$

$$\rightarrow \quad m\ddot{x} + m\dot{\theta}^2(R - x) + kx - kx_b = -c\dot{x} + c\dot{x}_b$$

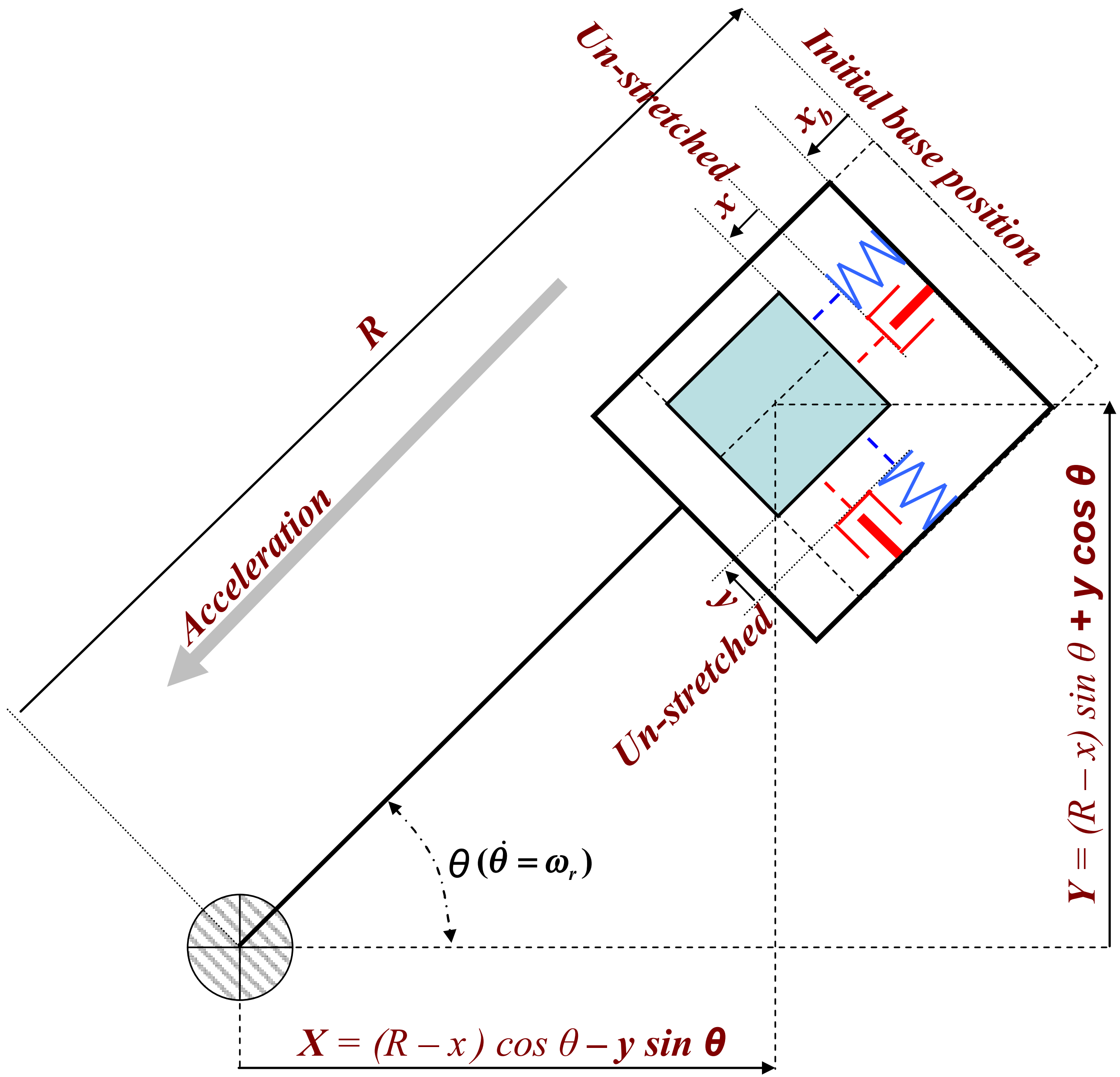
$$\rightarrow \quad m\ddot{x} + m\dot{\theta}^2 R - m\dot{\theta}^2 x + kx - kx_b = -c\dot{x} + c\dot{x}_b$$

$$\rightarrow \quad \underline{\underline{m\ddot{x} + c\dot{x} + (k - m\dot{\theta}^2)x = -m\dot{\theta}^2 R + c\dot{x}_b + kx_b}}$$

Replacing $\dot{\theta}^2$ *with* ω_r^2

$$\underline{\underline{m\ddot{x} + c\dot{x} + (k - m\omega_r^2)x = -m\omega_r^2 R + c\dot{x}_b + kx_b}}$$

Generalized coordinates used with Lagrange's equations



$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} c_x & -2m\omega_r \\ 2m\omega_r & c_y \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} (k_x - m\omega_r^{\,2}) & 0 \\ 0 & (k_y - m\omega_r^{\,2}) \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} -R \\ 0 \end{Bmatrix} m\omega_r^{\,2} + \begin{Bmatrix} c_x \\ 0 \end{Bmatrix}\dot{x}_b + \begin{Bmatrix} k_x \\ 0 \end{Bmatrix} x_b$$

$$\omega_{1,2} = \sqrt{\frac{\left(\omega_{nx}^{\,2} + \omega_{ny}^{\,2} + 2\omega_r^{\,2}\right) \pm \sqrt{\omega_{nx}^{\,4} - 2\omega_{nx}^{\,2}\omega_{ny}^{\,2} + 8\omega_{nx}^{\,2}\omega_r^{\,2} + \omega_{ny}^{\,4} + 8\omega_{ny}^{\,2}\omega_r^{\,2}}}{2}}$$

$$\rightarrow \quad \frac{\omega_{1,2}}{\omega_{nx}} = \sqrt{\frac{\left(1 + \frac{\omega_{ny}^{\,2}}{\omega_{nx}^{\,2}} + \frac{2\omega_r^{\,2}}{\omega_{nx}^{\,2}}\right) \pm \sqrt{1 - \frac{2\omega_{ny}^{\,2}}{\omega_{nx}^{\,2}} + \frac{8\omega_r^{\,2}}{\omega_{nx}^{\,2}} + \frac{\omega_{ny}^{\,4}}{\omega_{nx}^{\,4}} + \frac{8\omega_{ny}^{\,2}\omega_r^{\,2}}{\omega_{nx}^{\,4}}}}{2}}$$