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Validation of Random Vibration Environments

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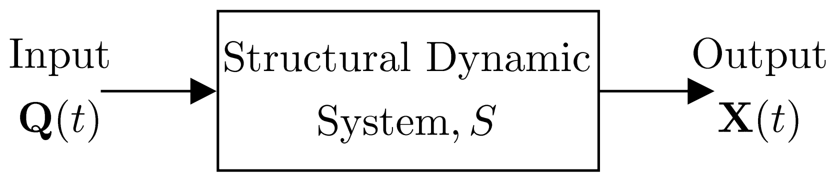
Outline

- Motivation
- Introduction
- A Linear Structure
- Linear Random Vibrations
- A Stationary Random Excitation
- Identification of Random Excitation Parameters
- Validation of Excitation Model

Motivation

- **Many types of structures are subjected to random environments**
  - **Wind loads on buildings, sea loads on off-shore structures, aerodynamic forces on flight vehicles, etc.**
- **Accurate prediction of structural dynamic response requires:**
  - **Validated models for structure (S) AND input (Q)**



- **Most work in model validation is focused on S**
- **Our work is focused on validation of Q**
  - **Many of these environments cannot be measured directly**
  - **Data is always limited**

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Introduction

- Many structures subjected to random vibration environments can be modeled as linear
- Some of those environments are approximately stationary
- We may know something about the general form of the model for random excitation
- We can use this knowledge and the theory of linear random vibrations to identify the parameters of an excitation model
- Validate the environment model following the procedures used for the validation of the structural model

Assumptions

1. Structure is linear and model is accurate
   – Allows us to focus on validation of the environment model; this can be relaxed later

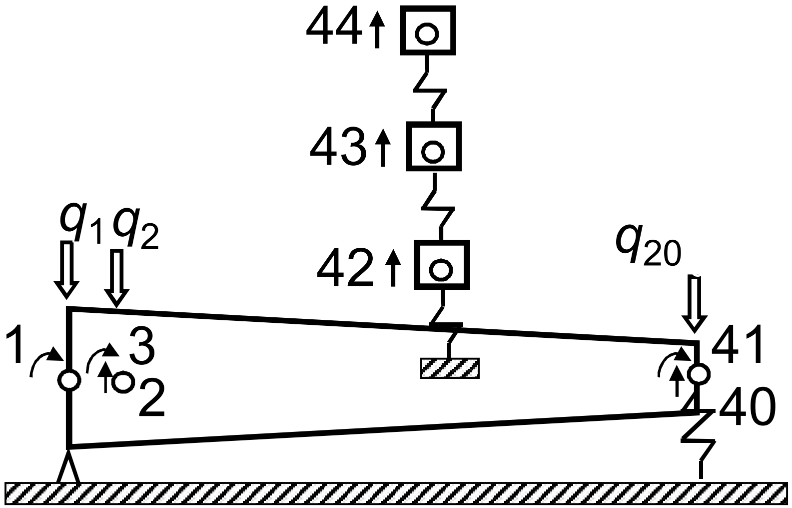
2. Structure is excited by a zero mean, stationary, Gaussian, stochastic process input that cannot be measured directly
   – Because excitation is Gaussian, response is Gaussian, and both can be fully characterized by second-moment properties

3. We have measured structural responses
   – They are free of measurement error
   – They have finite duration

A Linear Structure

For the sake of clarity we consider the random vibration and excitation identification of a specific structure:

- **Tapered-beam part is modeled with beam elements in FE code**
- **Three DOF substructure is modeled with springs / masses / dashpots**
- **Loads applied at transverse beam DOF**



$$\mathbf{m}\,\ddot{\mathbf{X}}(t) + \mathbf{c}\,\dot{\mathbf{X}}(t) + \mathbf{k}\,\mathbf{X}(t) = \mathbf{Q}(t), \quad -\infty < t < \infty$$

$\mathbf{Q}$ represents (random) turbulent flow over the beam

Linear Random Vibrations

- **Stationary, Gaussian stochastic process can be fully specified by its spectral density matrix**
  - **Auto-spectral density characterizes distribution of MS signal content in the frequency domain**
  - **Cross-spectral density characterizes degree of linear relation and average phase between two processes**
- **Two-sided spectral density matrix of excitation is:**

$$\mathbf{S_{QQ}}(\omega) = \begin{bmatrix} S_{Q_1Q_1}(\omega) & S_{Q_1Q_2}(\omega) & \cdots & S_{Q_1Q_N}(\omega) \\ S_{Q_2Q_1}(\omega) & S_{Q_2Q_2}(\omega) & \cdots & S_{Q_2Q_N}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{Q_NQ_1}(\omega) & S_{Q_NQ_2}(\omega) & \cdots & S_{Q_NQ_N}(\omega) \end{bmatrix}, \quad -\infty < \omega < \infty$$

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Linear Random Vibrations

- **Theory of linear random vibration relates spectral density matrix of inputs to spectral density matrix of outputs/responses:**

$$\mathbf{S}_{\ddot{\mathbf{X}}\ddot{\mathbf{X}}}(\omega) = \mathbf{H}_{\ddot{\mathbf{X}}\mathbf{Q}}(\omega)\,\mathbf{S}_{\mathbf{Q}\mathbf{Q}}(\omega)\left[\mathbf{H}^{*}_{\ddot{\mathbf{X}}\mathbf{Q}}(\omega)\right]^{T}$$

where

$$\mathbf{H}_{\ddot{\mathbf{X}}\mathbf{Q}}(\omega) = -\omega^2\left[-\omega^2\mathbf{m} + i\,\omega\,\mathbf{c} + \mathbf{k}\right]^{-1}$$

is a matrix of FRFs

A Stationary Random Excitation

- **Consider a stationary, Gaussian excitation with zero mean and one-sided spectral density matrix of the form**

$$\mathbf{G_{QQ}}(f_k) = G_0(f_k)\,\mathbf{\Lambda}(f_k),\;\; k = 0, \ldots, n_f - 1$$

where $G_0(f_k)$ is the scalar ASD at each input location, and $\mathbf{\Lambda}(f_k)$ is a Hermitian matrix with elements

$$\Lambda_{i,j} = \exp\left(-\alpha|u_i - u_j| + 2\pi\sqrt{-1}\,\frac{\Delta u\, f_k}{v_0}\right),\;\; i, j = 1, \ldots, N$$

Meanings …? What is known and what is unknown?

Two examples follow

- Example #1
  - One output signal available at midpoint of beam
  - Can identify input ASD
  - Identification of environment model only
- Example #2
  - Two output signals available (DOFs 10 and 30)
  - Can identity input ASD and correlation length (α)
  - Identification and validation addressed

$X_{rp} \cong v w^{1/2} U + \bar{x}$

Example #1: Identification

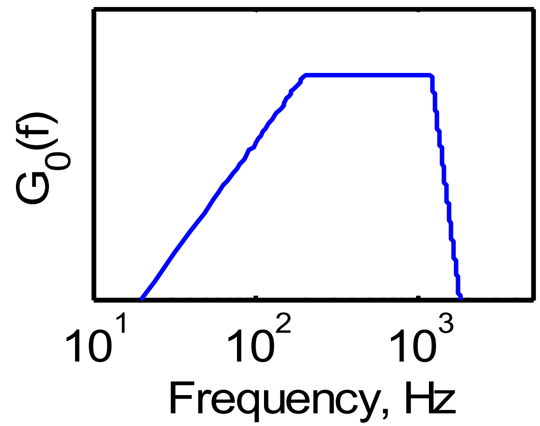
- **One output signal available (mid-point of beam)**
  - **When response is measured at one location and parameter α is known, we can identify ASD of excitation at each input location**

$$G_{\ddot{X}_i \ddot{X}_i}(f_k) = G_0(f_k)\, \mathbf{H}_{\ddot{X}_i \mathbf{Q}}(f_k)\, \mathbf{\Lambda}(f_k) \left[\mathbf{H}^*_{\ddot{X}_i \mathbf{Q}}(f_k) \right]^T,$$
$$i = 1, \ldots, N;\ \ k = 0, \ldots, n_f - 1$$

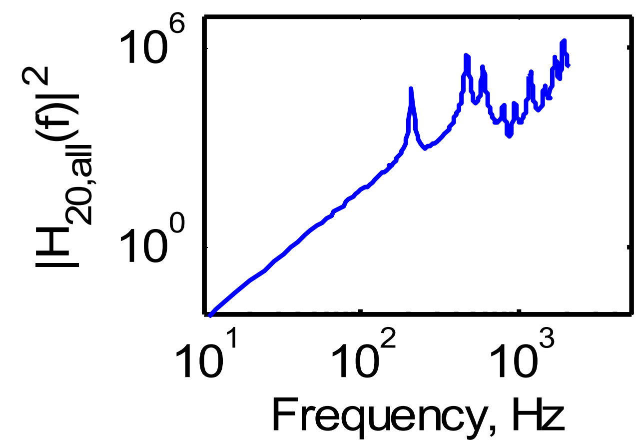
  - **Estimate noise due to finite-length output records**

Example #1: Identification results

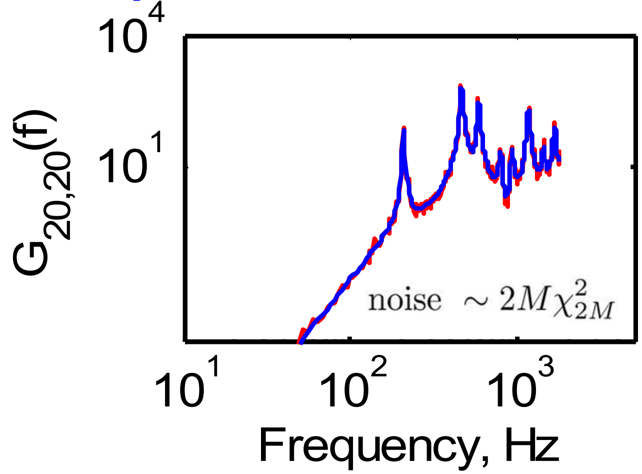
ASD of theoretical input



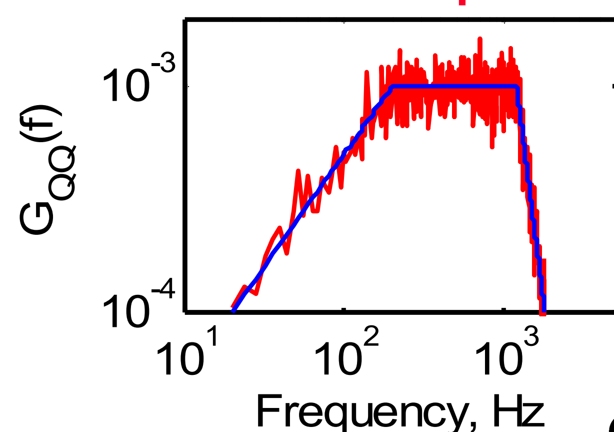
Effective FRF modulus squared



Response ASD w/ and w/o noise



Estimated input ASD



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$X_{rp} \cong v w^{1/2} U + \bar{x}$

Example #2: Identification

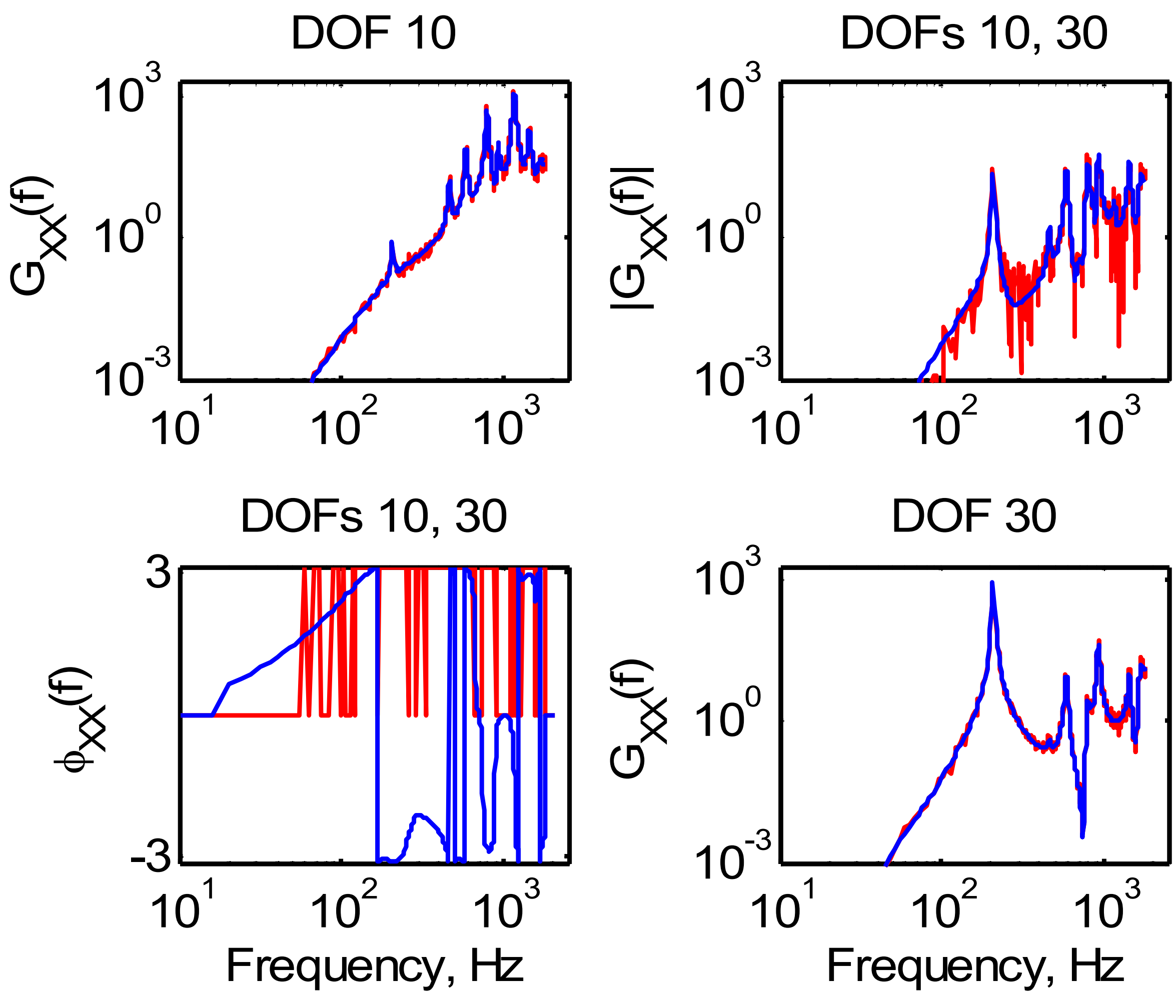
- Two output signals available (DOFs 10 and 30)
  - When response is measured at two locations and α is unknown, we can identify:
    1. ASD of excitation at each input location, and
    2. The value for α (correlation length)

$$\mathbf{G}_{\ddot{\mathbf{X}}_{(i,j)}\ddot{\mathbf{X}}_{(i,j)}}(f_k) = G_0(f_k)\,\mathbf{H}_{\ddot{\mathbf{X}}_{(i,j)}\mathbf{Q}}(f_k)\,\mathbf{\Lambda}(f_k)\left[\mathbf{H}^*_{\ddot{\mathbf{X}}_{(i,j)}\mathbf{Q}}(f_k)\right]^T, \quad i = 1,\ldots,N;\; k = 0,\ldots,n_f - 1$$

  - We do this by minimizing error between estimated response spectral density matrix and spectral density matrix of response excited by input with arbitrary parameters

$\alpha = 0.1 in^{-1}$

Example #2: Identification results



$\alpha = 0.1\ \mathrm{in}^{-1}$

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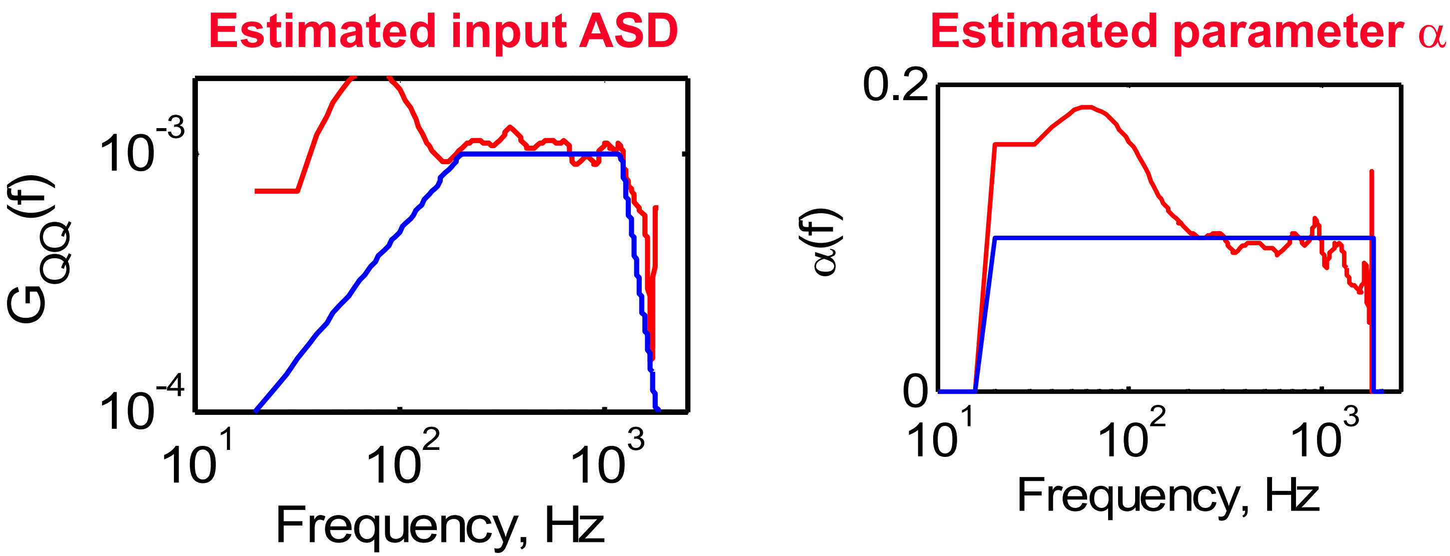
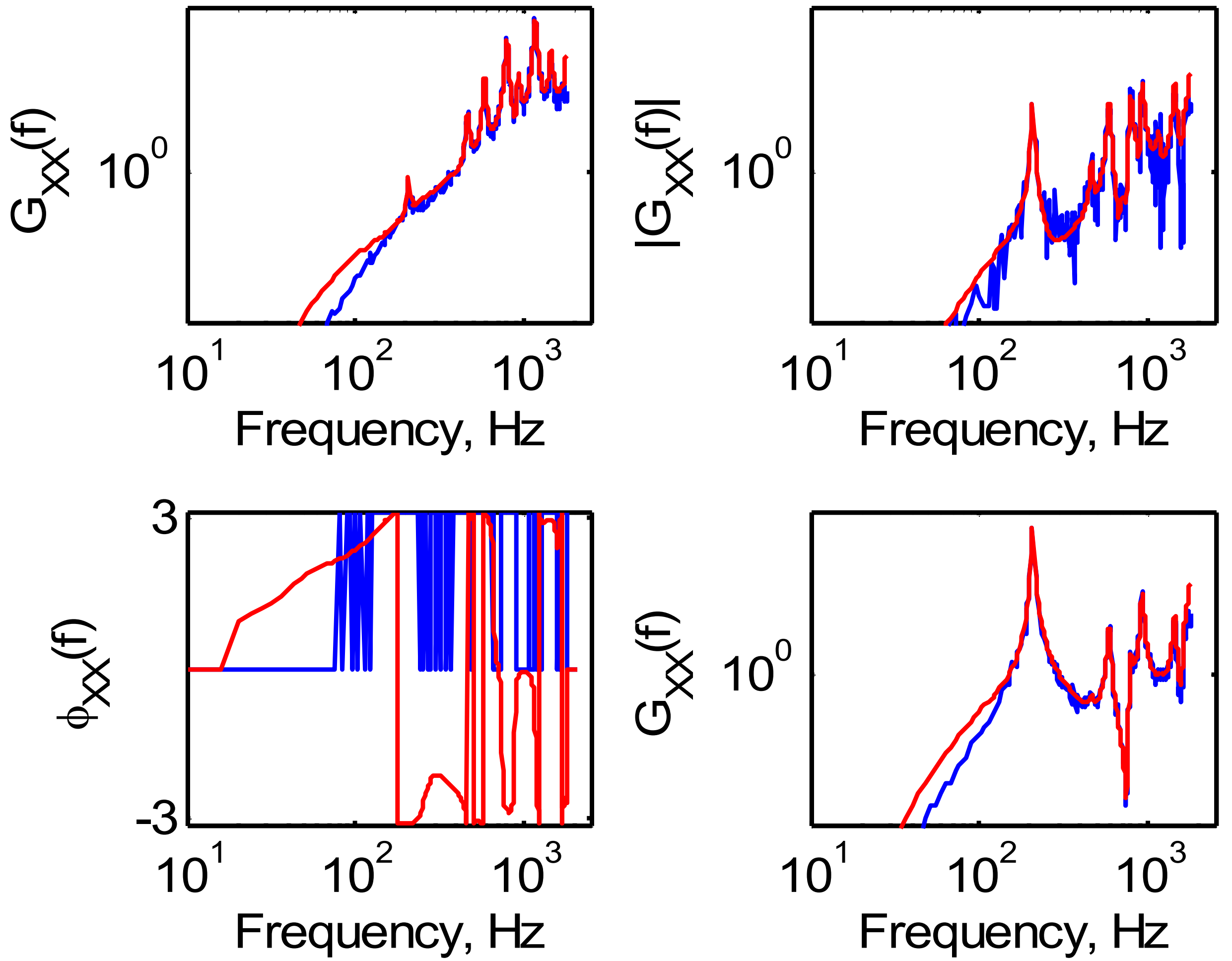
Example #2: Identification results



- **Accurate estimates for f > 100 Hz; inaccurate for f < 100 Hz**
- **System does not respond below 100 Hz (first mode is 315 Hz) so irrelevant (see next slide)**

Example #2: Identification results

- Response spectral densities – estimated from measurements vs. computed from identified input



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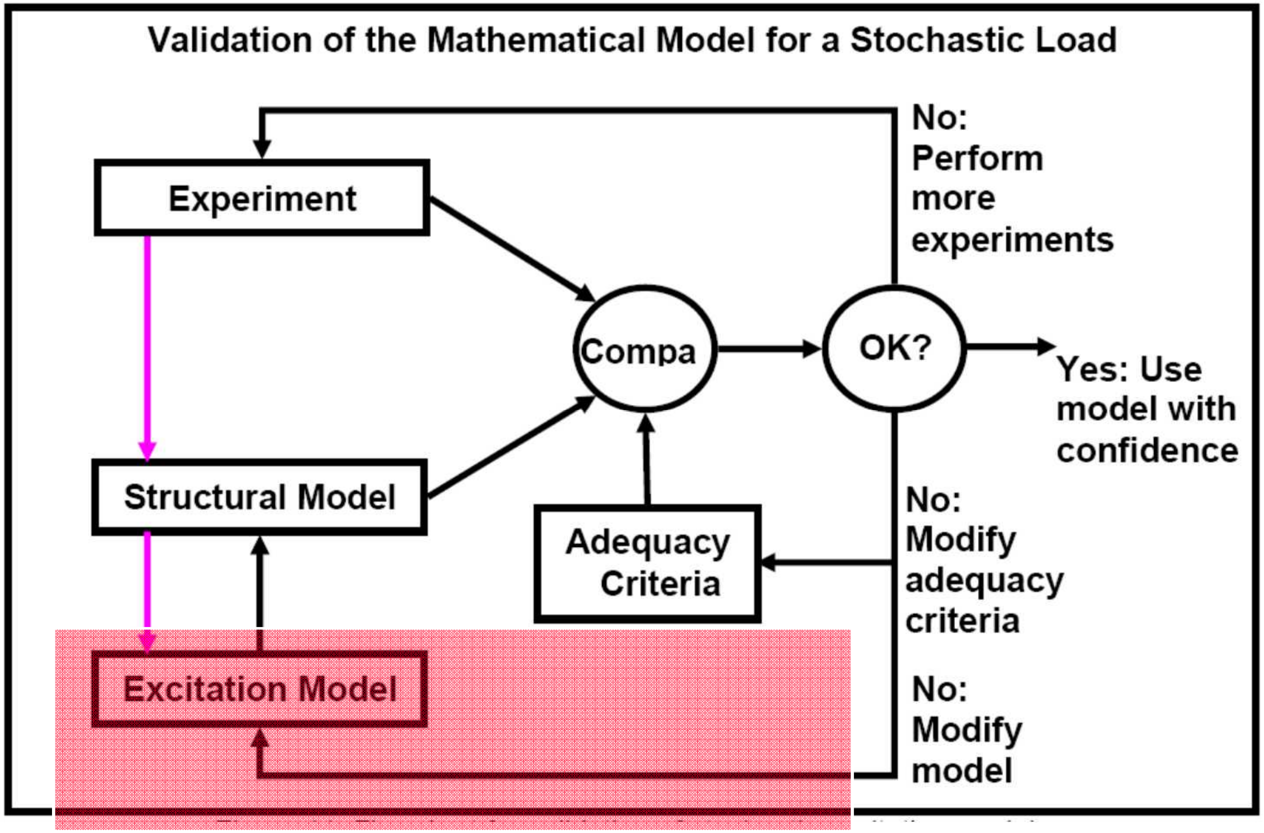
Validation procedure



3 Key Elements

1. Output of interest
2. Metric for comparison
3. Adequacy Criteria

Data for validation cannot be used for identification.

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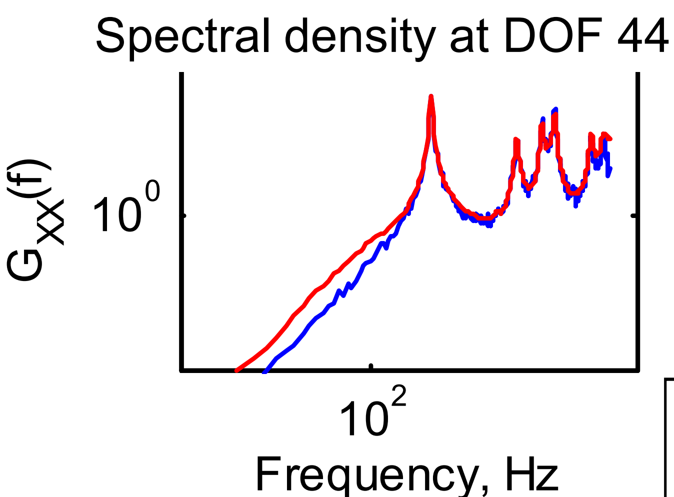
Example #2: Validation

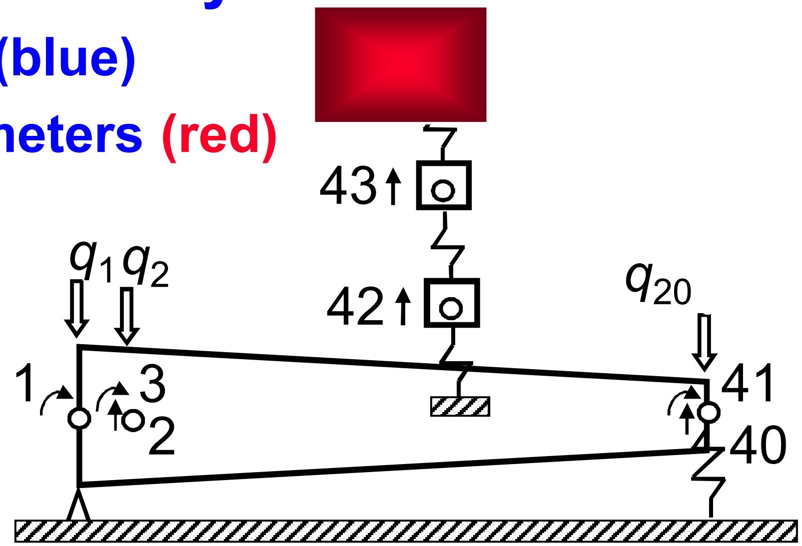
- For Example #2, the following were estimated:
  - Excitation spectral density, $G_0(f_k),\ k = 0, \ldots, n_f - 1$
  - Decay rate parameter, α, from the off-diagonal terms in the excitation spectral density

- Given these quantities, other spectral measures of structural response can estimated. For example:
  - Spectral density of response at any location
  - Cross-spectral densities
  - Measures that are functions of auto- and cross-spectral densities

Example #2: Validation

- Assume that response at DOF 44 is critical
- Estimate its spectral density from
  - Finite measured data (blue)
  - Estimated input parameters (red)





An environment validation might seek to <u>infer</u> validity of the input model by comparing these two curves, or a *measure* of the curves.

Example #2: Validation

- For example, we choose to compare the peak displacement response PDF, a function of the spectral density
- Validation requirement: *Mean and standard deviation of peak response PDF based on identified input parameters must be within ten percent of mean and standard deviation based on measured data*
- Let Z denote peaks in random response; its PDF is

$$f_Z(z) = (1-\beta^2)\,\frac{1}{\sigma_X\sqrt{2\pi(1-\beta^2)}}\,\exp\left(\frac{-z^2}{2\,\sigma_X^2(1-\beta^2)}\right) + \frac{\beta\, z}{\sigma_X^2}\,\Phi\left(\frac{\beta\, z}{\sigma_X(1-\beta^2)}\right)\exp\left(\frac{-z^2}{2\,\sigma_X^2}\right)$$

where Φ is the standard normal CDF and 0 < β < 1 is the "irregularity factor"

$$\beta = \frac{\sigma_{\dot{X}}^2}{\sigma_X\,\sigma_{\ddot{X}}} = \begin{cases} 0 & \text{for "broad-band" process} \\ 1 & \text{for "narrow-band" process} \end{cases}$$

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Example #2: Validation results

- The moments that form β are functions of the spectral density:

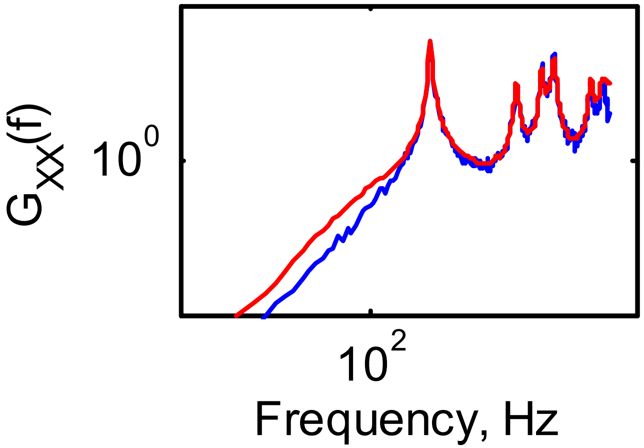
$$\sigma_X = \left[\int_0^\infty G_{XX}(f)\,\mathrm{d}f\right]^{1/2}, \quad \sigma_{\dot{X}} = \left[(2\pi)^2 \int_0^\infty f^2\, G_{XX}(f)\,\mathrm{d}f\right]^{1/2}, \quad \sigma_{\ddot{X}} = \left[(2\pi)^4 \int_0^\infty f^4\, G_{XX}(f)\,\mathrm{d}f\right]^{1/2}$$

- From direct estimates (measured data, blue)

$$\sigma_X = 6.675 \times 10^{-5}\ \text{in}, \quad \sigma_{\dot{X}} = 8.273 \times 10^{-2}\ \text{in/sec}, \quad \sigma_{\ddot{X}} = 254.6\ \text{in/sec}^2 \quad \Longrightarrow \quad \beta = 0.4739$$

- From the identified input model parameters (red)

$$\sigma_X = 5.938 \times 10^{-5}\ \text{in}, \quad \sigma_{\dot{X}} = 8.565 \times 10^{-2}\ \text{in/sec}, \quad \sigma_{\ddot{X}} = 281.2\ \text{in/sec}^2 \quad \Longrightarrow \quad \beta = 0.4394$$



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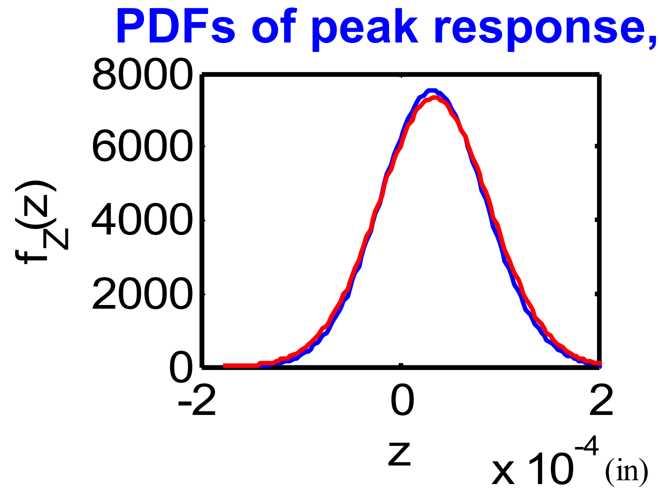
Example #2: Validation results



- **Mean and standard deviation based on direct estimates (measured data)**

$$\mu_Z = 3.369 \times 10^{-5} \text{ in}, \quad \sigma_Z = 5.294 \times 10^{-5} \text{ in}$$

- **Mean and standard deviation based on identified input model parameters**

$$\mu_Z = 3.525 \times 10^{-5} \text{ in}, \quad \sigma_Z = 5.695 \times 10^{-5} \text{ in}$$

- **Latter moments within ten percent of former; therefore *excitation model is valid***

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Conclusions

- Developed techniques for estimating model parameters of excitation spectral density
  - Guidelines sought
- Extended current validation procedures to consider models for random vibration environments
- Assumptions
  - Structure is linear with known parameters
  - Functional form of input spectral density known
  - If assumptions are inaccurate – estimate of input spectral density inaccurate
- Future work
  - Nonlinear system with non-Gaussian input
  - Joint validation of system AND excitation models

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