SAND2008-3678C

Computational Modeling Sciences Department

Sheet-Based Hexahedral Mesh Generation

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May 2008

LOCKHEED MARTIN

Sandia National Laboratories

Outline

- Research Motivation
- Hexahedral Meshing Definitions
- Methods
- Research Definitions
- Hexahedral Isosurfacing
- Multi-surface Hexahedral Mesh Generation

Mesh Generation Needs

"Ironically, as numerical analysis is applied to larger and more complex problems, non-numerical issues play a larger role. Mesh generation is an excellent example of this phenomenon. Solving current problems in structural mechanics or fluid dynamics with finite difference of finite element methods *depends on the construction of high-quality meshes of surfaces and volumes*. *Geometric design and construction of these meshes are typically much more time-consuming than the simulations that are performed with them*."

- **John Guckenheimer, "Numerical Computation in the Information Age" in June 1998 issue of SIAM News.**

Setup Time: A Key Problem

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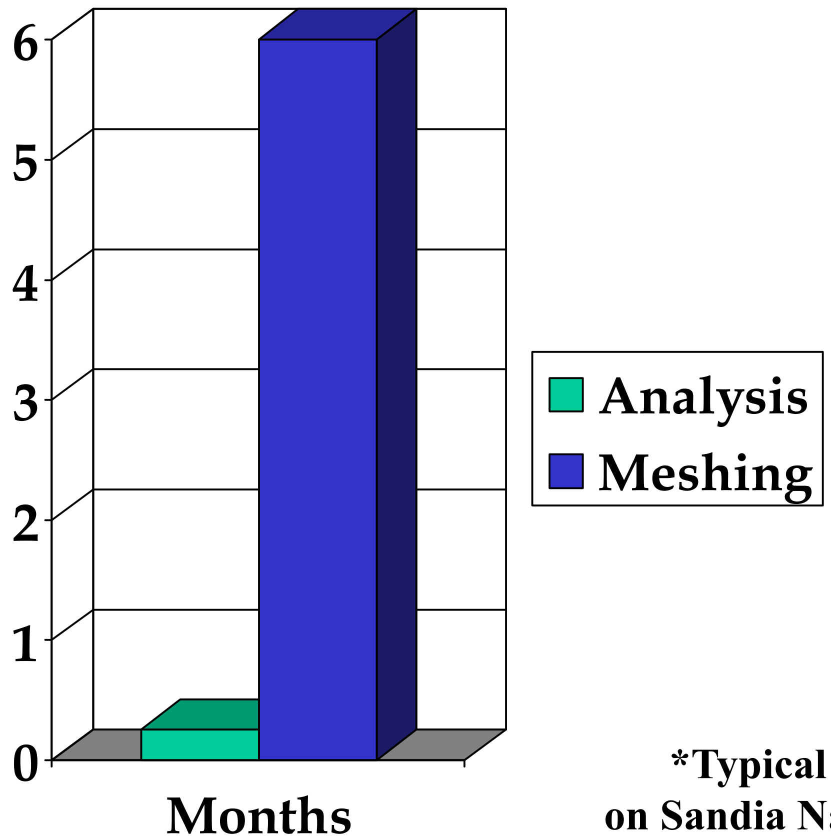

***Typical case for hexahedral meshes on Sandia National Laboratories geometries**

DTA Process Map

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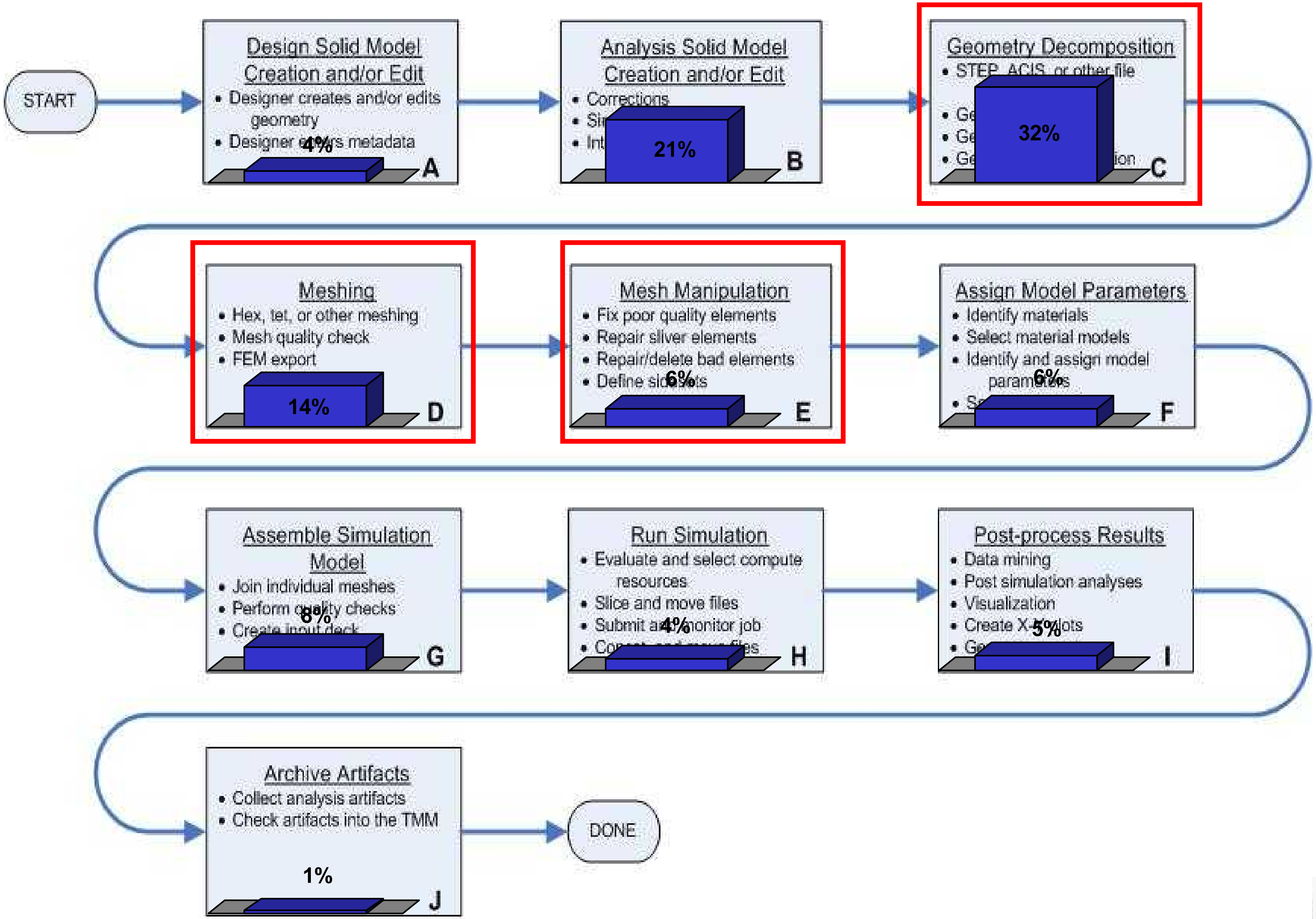
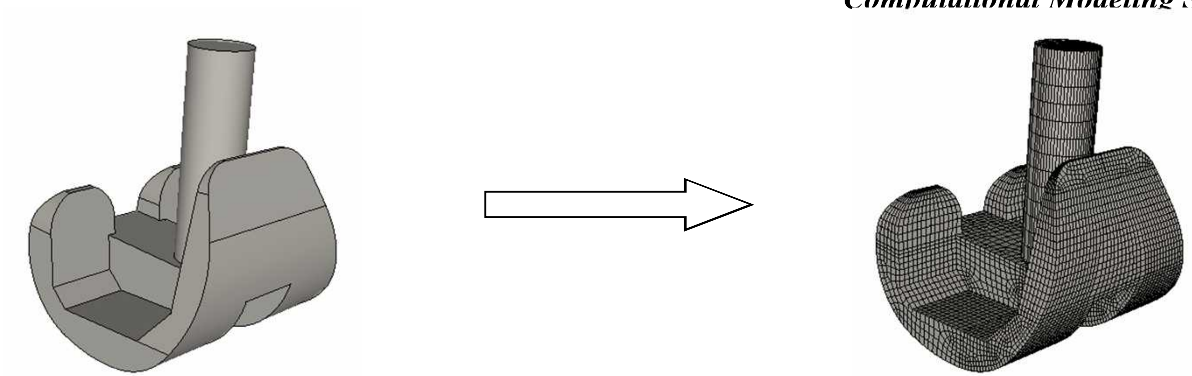


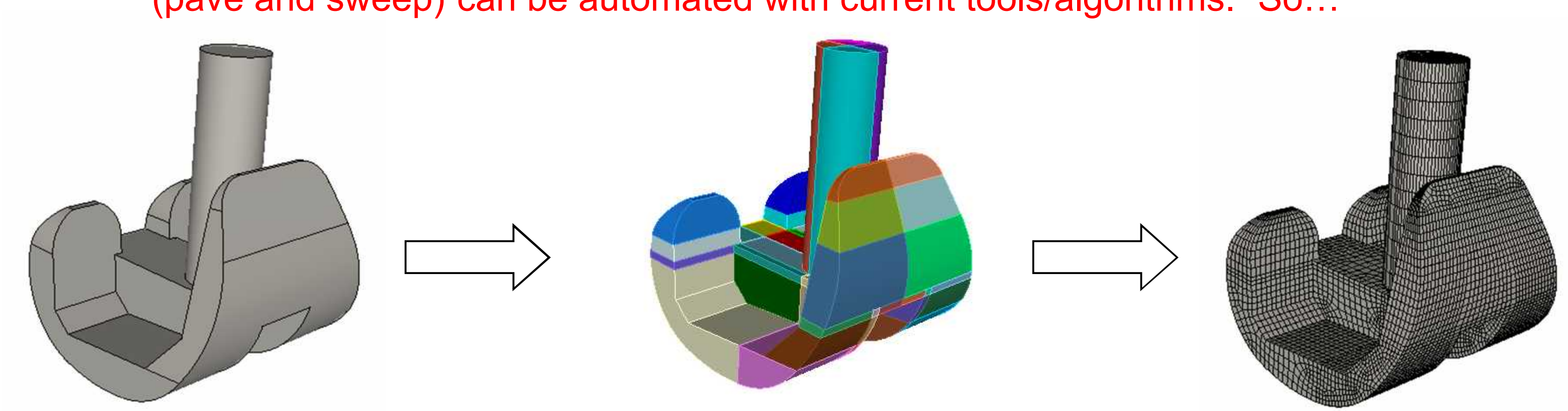
Hexahedral Meshing



Given a geometric representation of an object, G,…

… create an alternate geometric representation consisting of hexahedral elements…

Unfortunately, there is only a limited class of geometries for which hexahedral meshing (pave and sweep) can be automated with current tools/algorithms. So…



Model Creation
25% of DTA time*

Decomposition for Hex Meshing
32% of DTA time*

Hex Meshing
14% of DTA time*

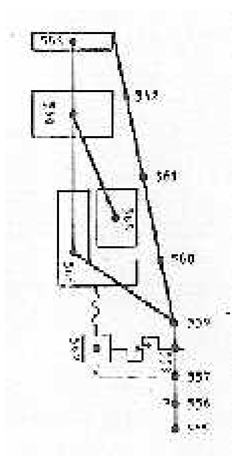
However, decomposition for pave-sweep is more art than science…

***timings based on a Design-thru-Analysis study conducted by the DART project**

Models get more complex over time…

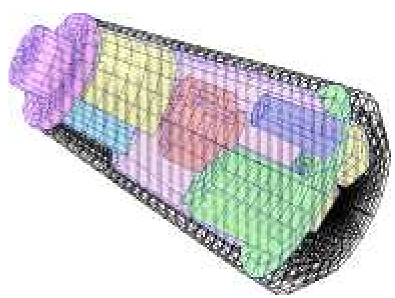
As model complexity increases, decomposition for pave-sweep quickly becomes intractable…

ca. 1988



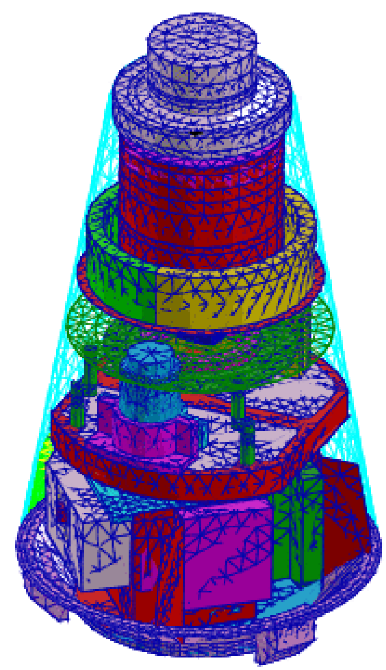
200 dof
Shellshock 2D

ca. 1995



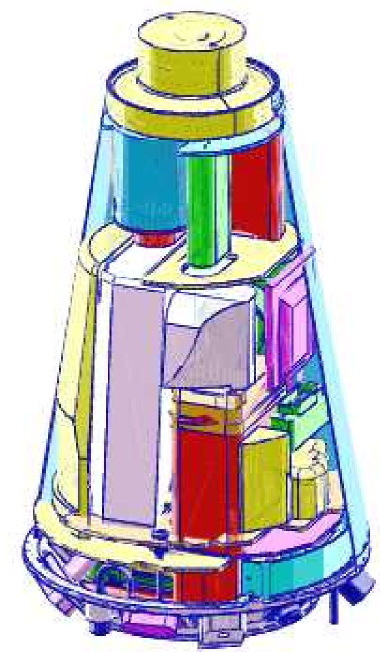
40,000 dof
NASTRAN

ca. 1998



800K dof
MP Salinas

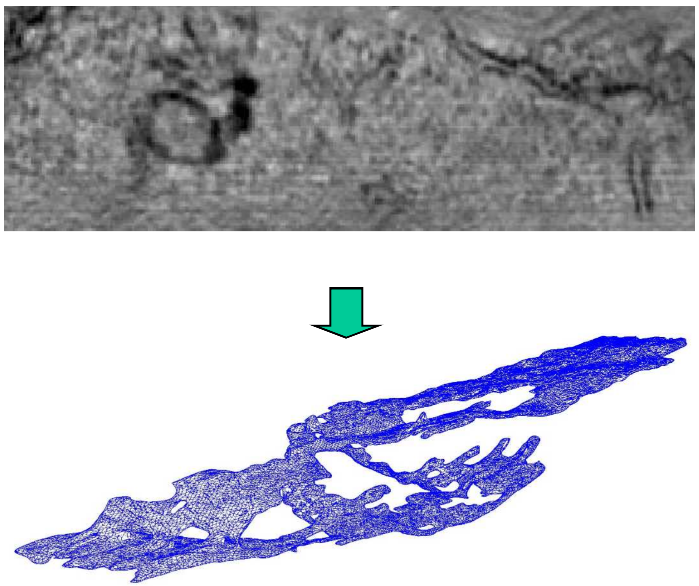
ca. 2000



>10M dof
MP Salinas

Capacity and Resolution

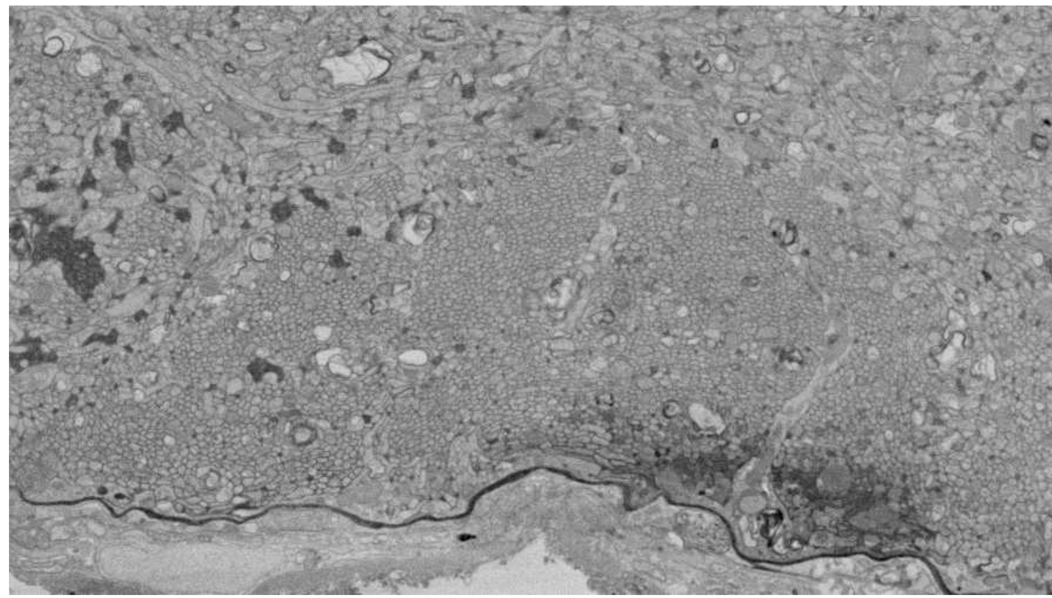
ca. 2002



Endoplasmic Reticulum

(courtesy of Bridget Wilson, et al. University of New Mexico)

2007-2008?



Neural Fiber Bundles (Zebra Fish)

(courtesy of Liz Jurrus & Chi-Bin Chien, University of Utah and Winfried Denk, Max Planck Institute for Medical Research)

Definitions

Definitions

A hexahedral mesh can be defined as a geometric cell complex composed of 0-dimensional nodes, 1-dimensional edges, 2-dimensional quadrilaterals (residing in $\Re^3$), and 3-dimensional hexahedra, such that:

- Topologic Constraints:
  - **Each node is contained by at least three edges**
  - **Each edge contains two distinct nodes.**
  - **If two edges contain the same nodes, the edges are identical.**
  - **Each quadrilateral is bounded by a cycle of four distinct edges.**
  - **Two quadrilaterals share at most one edge.**
  - **If two quadrilaterals share four edges, they are identical.**
  - **Each hexahedra is bounded by six distinct quadrilaterals.**
  - **Every quadrilateral is contained by at least one hexahedra and no more than 2.**
  - **Two hexahedra share at most one quadrilateral.**
  - …

Definitions

- Boundary Constraints:
  - **Every quadrilateral on the boundary of the hexahedral mesh must correspond to a surface on the geometric boundary.**
  - **Every surface on the geometric boundary must have a collection of simply connected quadrilaterals on the boundary of the hexahedral mesh that approximates the surface.**
  - **Every curve on the geometric boundary must have a collection of simply connected edges on the boundary of the hexahedral mesh that approximates the curve.**
  - **Every vertex on the geometric boundary must be associated with a node in the hexahedral mesh.**
- Geometric or Quality Constraints:
  - **All quadrilaterals and hexahedra should be convex and have positive volume.**
    - In Cubit, we typically require that min($|J_i|$) > 0. (scaled) for i = 1 to 8, and strive for min($|J_i|$) > 0.2 (scaled)

Hexahedral Layers

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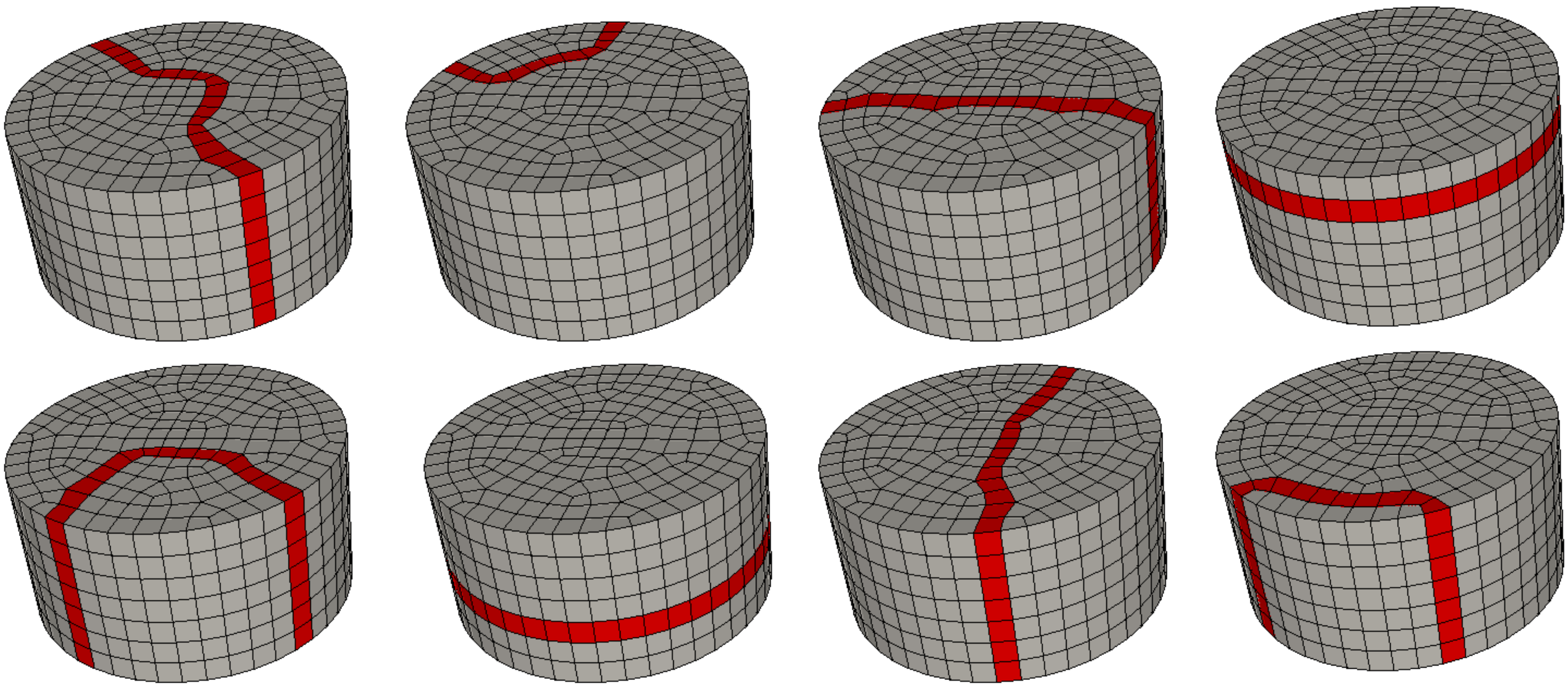
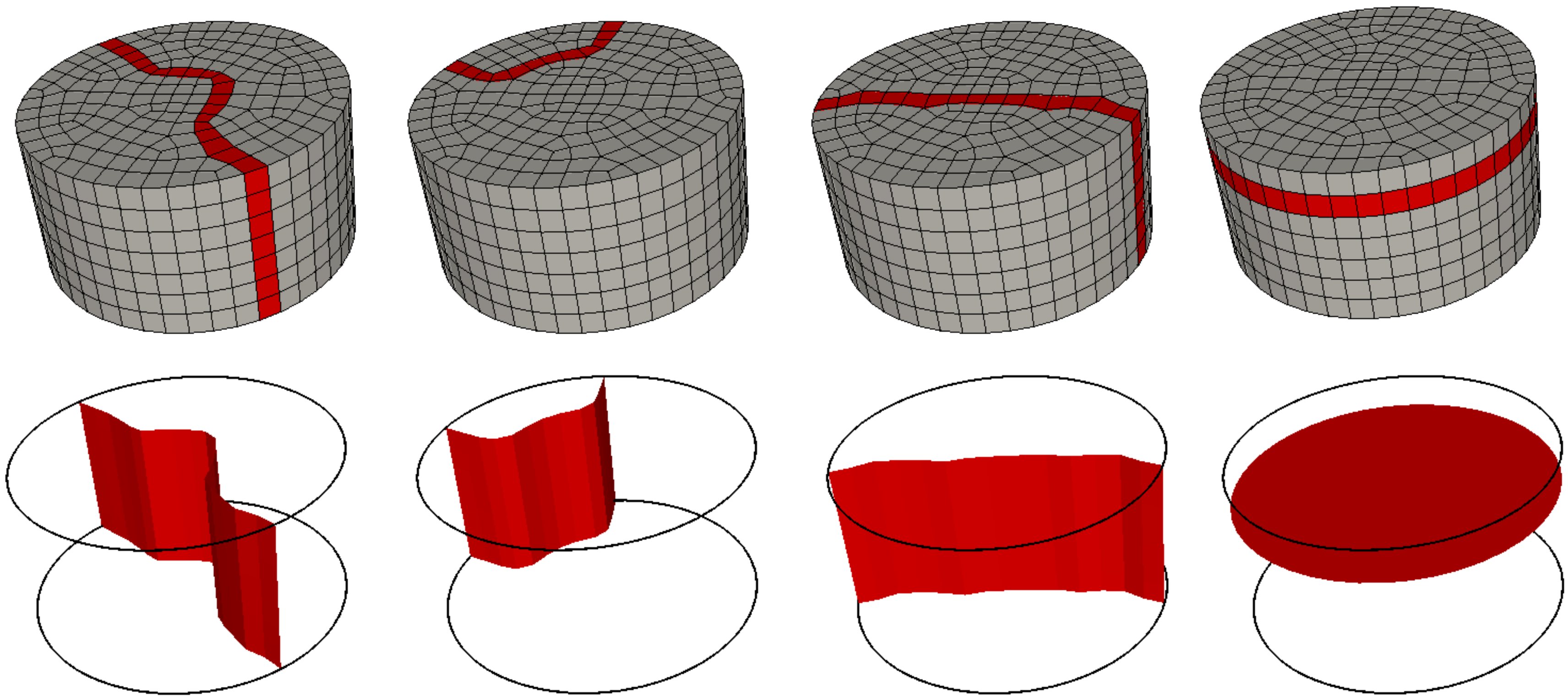
- **Due to symmetry within a hexahedral element, all hexahedral meshes are created as layers of hexahedral elements.**



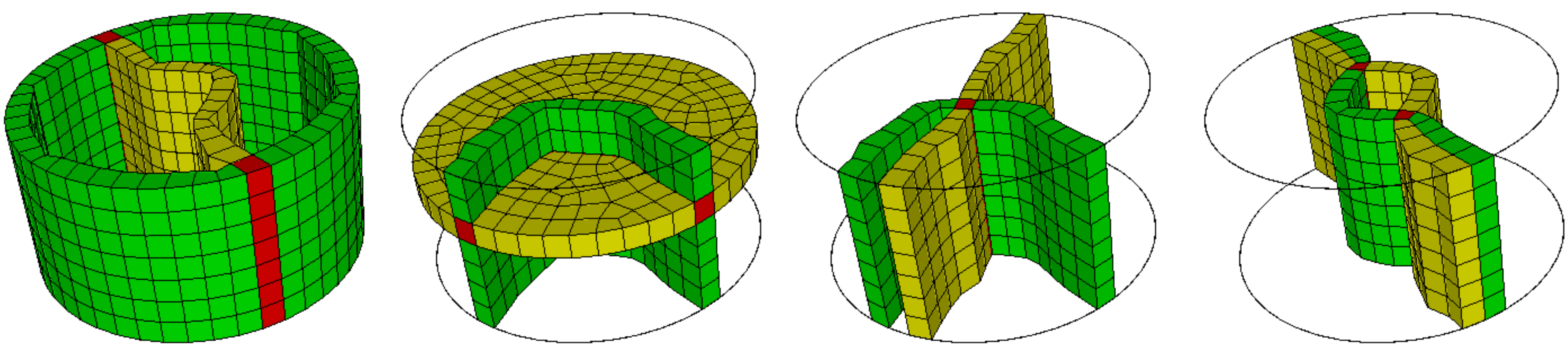
Hexahedral Layers

- **Due to symmetry within a hexahedral element, all hexahedral meshes are created as layers of hexahedral elements.**
- **Hexahedral layers can also be visualized as manifold surfaces (also known as ‘sheets’)**



Hexahedral Columns

- **The intersection of one or more sheets of hexahedra form 'columns' of hexahedra (also known as a chords).**
- **Sheets and chords are elements within the 'dual' description of a hexahedral mesh.**



Topologic Constraints in the Dual of a Hexahedral Mesh

- The dual of the mesh M is the set of intersecting 2-manifolds (sheets) M* such that:
  - **At any point there can be at most three intersecting 2-manifolds.**
  - **A set of 2-manifolds cannot meet at a tangency**
  - **A single intersection of any 2-manifold (including self-intersection) must result in a 1-manifold.**
  - **The intersection of a 1-manifold and a 2-manifold result in a triple-point intersection.**
  - **Every 2-manifold must contain at least one triple-point intersection.**
  - **Every compact 2-manifold must contain more than one triple point intersection.**
  - **…**

Boundary Constraints in the Dual of a Hexahedral Mesh

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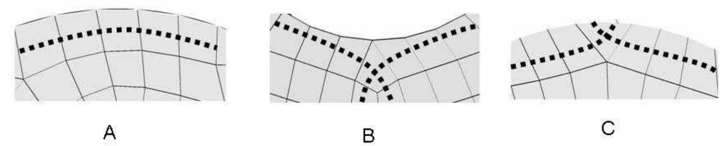
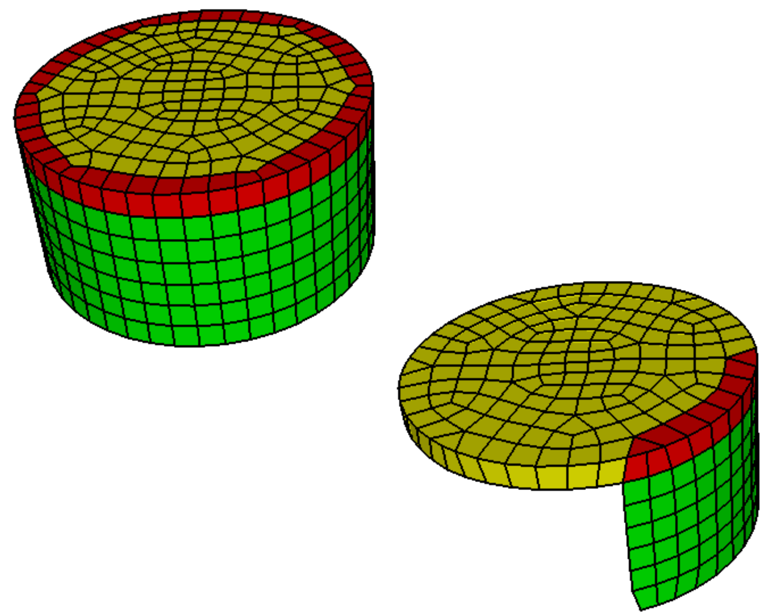
- Every geometric surface in the model must have at least one layer (sheet) of hexes beneath it (but may have more than one).
- Every geometric curve in the model must have at least one column (chord) of hexes adjacent to it (but may have more than one).
- Every geometric vertex in the model must map to at least c intersections of three sheets, where c is equal to:

$$\max(v - 2, 1)$$

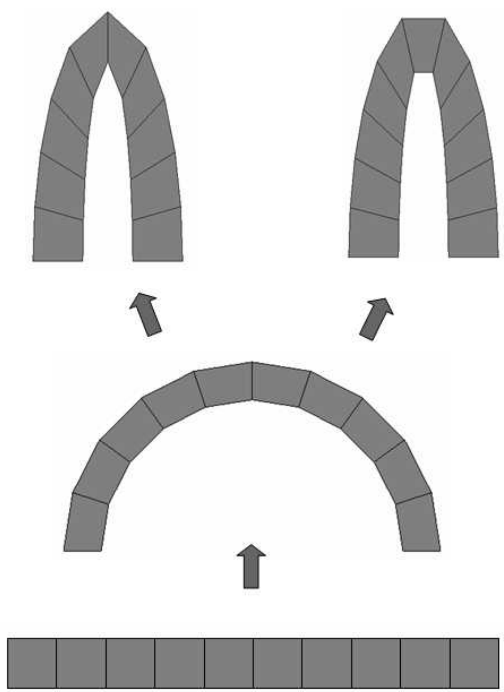
where v is the vertex valence. (Vertex valence is the number of geometric curves connected to the vertex. A vertex with 3 curves has a valence of 3.)

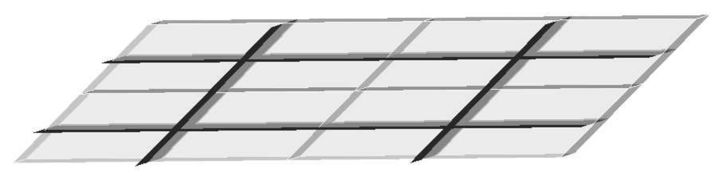




Geometric Constraints in the Dual of a Hexahedral Mesh

- Minimize sheet curvature.
- Maximize sheet orthogonality.





Methods

Methods –
Sheet Insertion and Extraction

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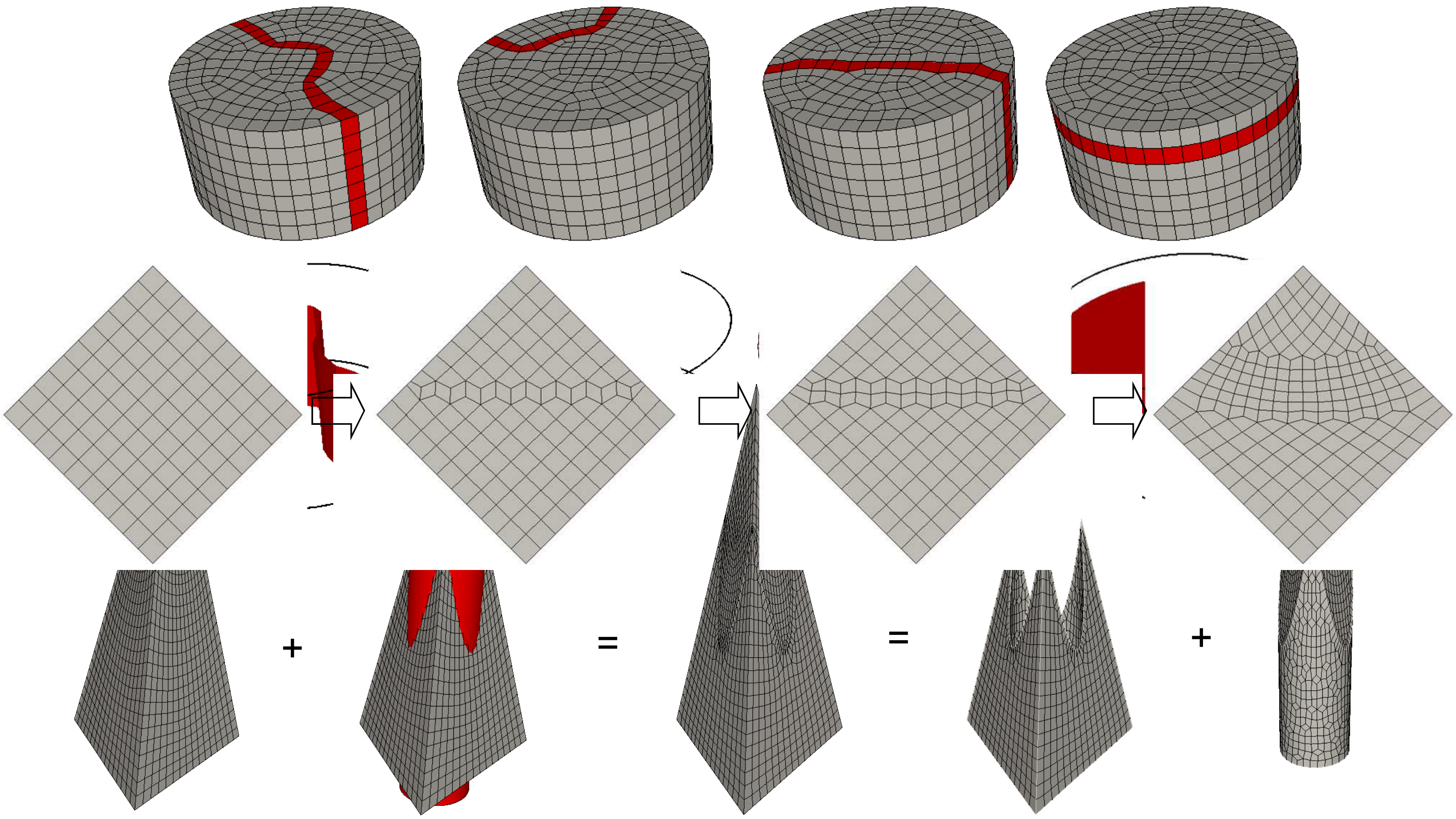
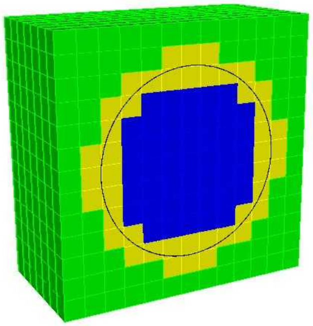
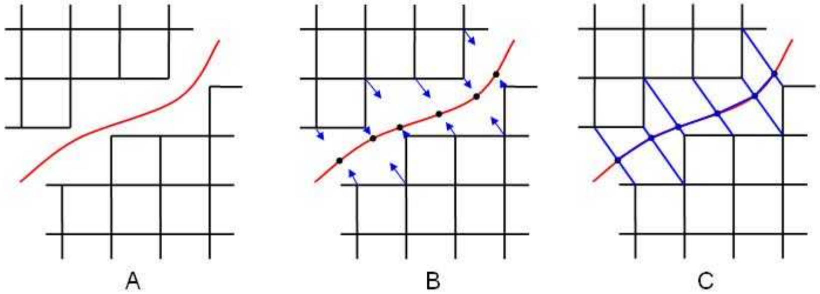


Methods – Sheet Insertion (Pillowing)

Given a hexahedral mesh (not necessarily octree) and a triangle mesh on a manifold

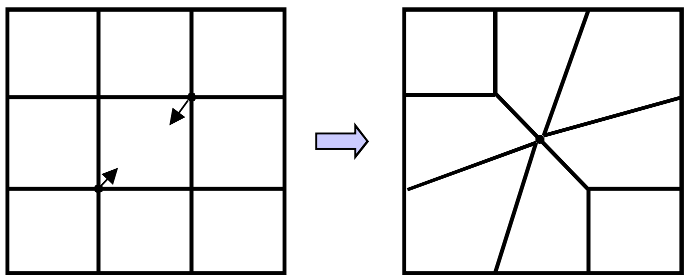


1. **Separate the hexahedra into three groups**
    1. Hexes intersected by the triangle mesh
    2. Hexes to one side of the triangle mesh (Side1), and
    3. Hexes on the opposite side of the triangle mesh (Side2).
2. **Placing the intersected hexes with one of the two sides, insert two sheets of hexahedra between the resulting groups projecting the new nodes to the original triangle mesh.**

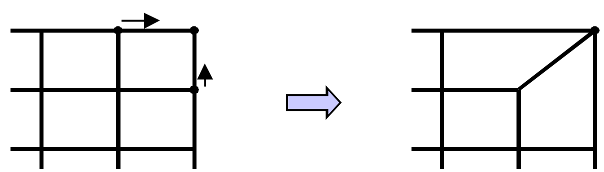


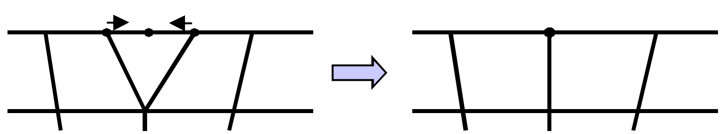
Methods – Hexahedral Flipping

- **Two examples of hexahedral 'flips'-**
  - Face collapse:



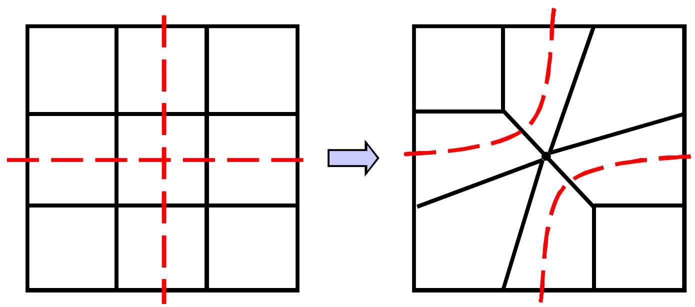
  - Boundary Face collapse:



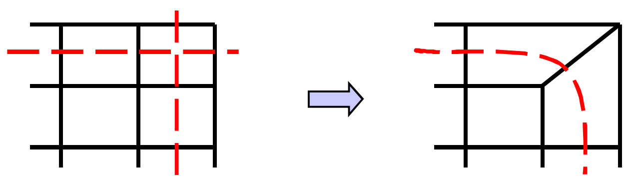


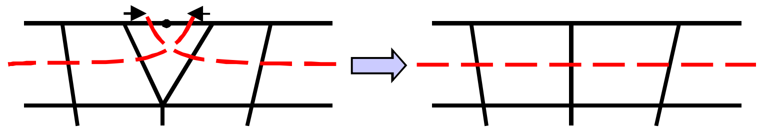
Methods – Hexahedral Flipping

- **Dual changes due to 'flip' operations-**
  - Face collapse:



  - Boundary Face collapse:

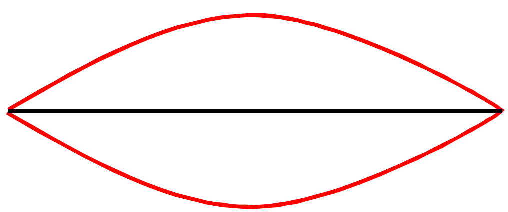




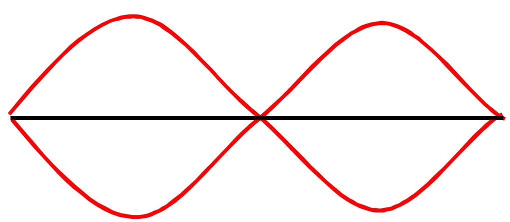
Research Definitions

Waves on a String

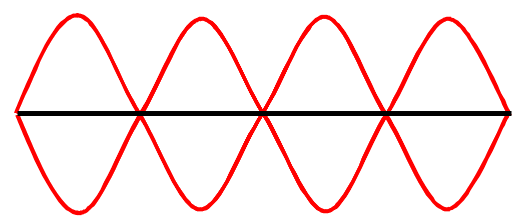
In wave theory, a string with the two endpoints fixed at opposite ends vibrates with several frequencies. The lowest frequency wave that can be formed on the string has a wavelength that is twice the length of the string, and is known as the first harmonic, or the ***fundamental*** frequency, of the string.



First Harmonic, or **'Fundamental'** Frequency



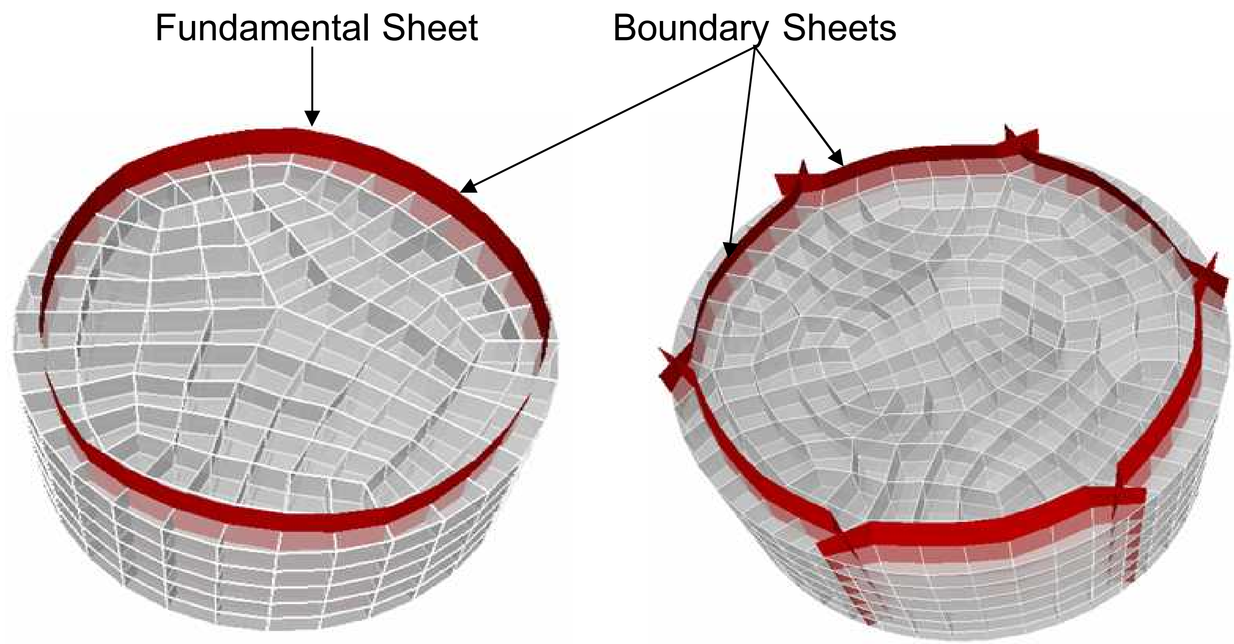
2nd Harmonic



4th Harmonic

Fundamental Hexahedral Meshes

- **Definition:** A ***fundamental mesh*** is a hexahedral mesh that contains one sheet for every surface, at least one continuous two-sheet intersection (chord) for every curve, and (vertex valence - 2) triple-point intersections (centroids) for every geometric vertex.



Fundamental Mesh

Non-Fundamental Mesh

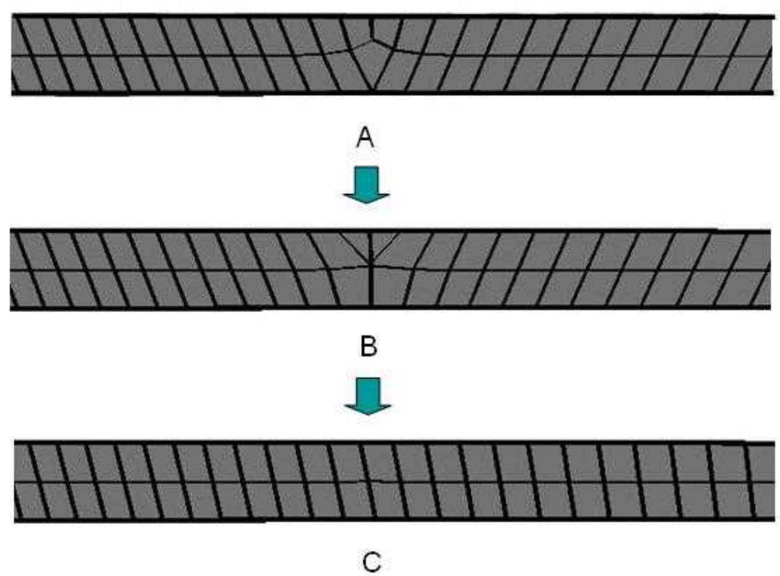
Converting to Fundamental Hexahedral Meshes

- **Assertion 1:** For any given hexahedral mesh of a geometric object, there exists a set of transformations that converts the set of boundary sheets into a set of fundamental sheets for the geometry.

- **A proof of existence for this transformation is demonstrated fairly easily by inserting a single sheet of elements interior to the boundary of the geometric object, and then doing a similar operation on each of the surfaces (if fundamental curves don't exist).**

Converting to Fundamental Hexahedral Meshes

- **Hexahedral Flipping operations can also be used to convert hexahedral meshes to fundamental meshes.**
  - Example 1:



Converting to Fundamental Hexahedral Meshes

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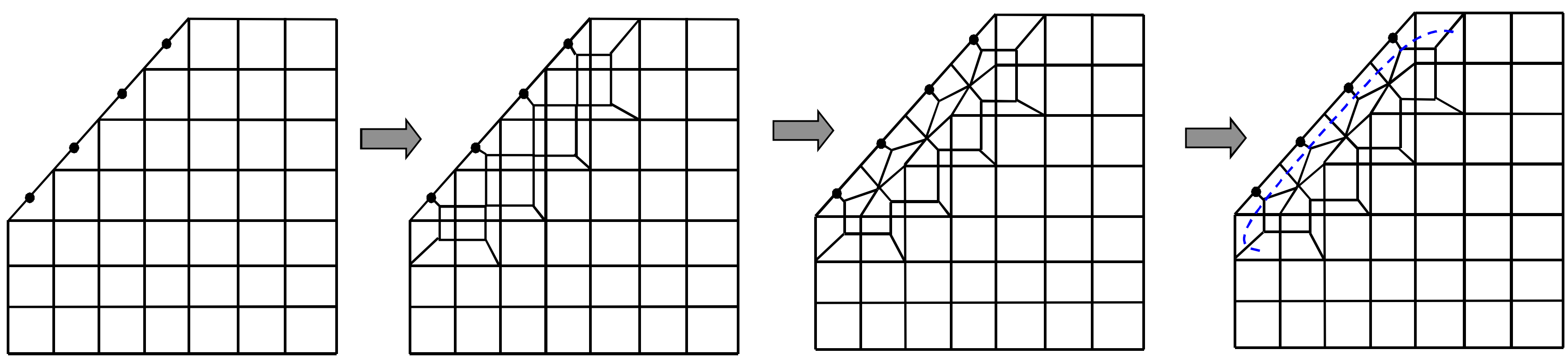
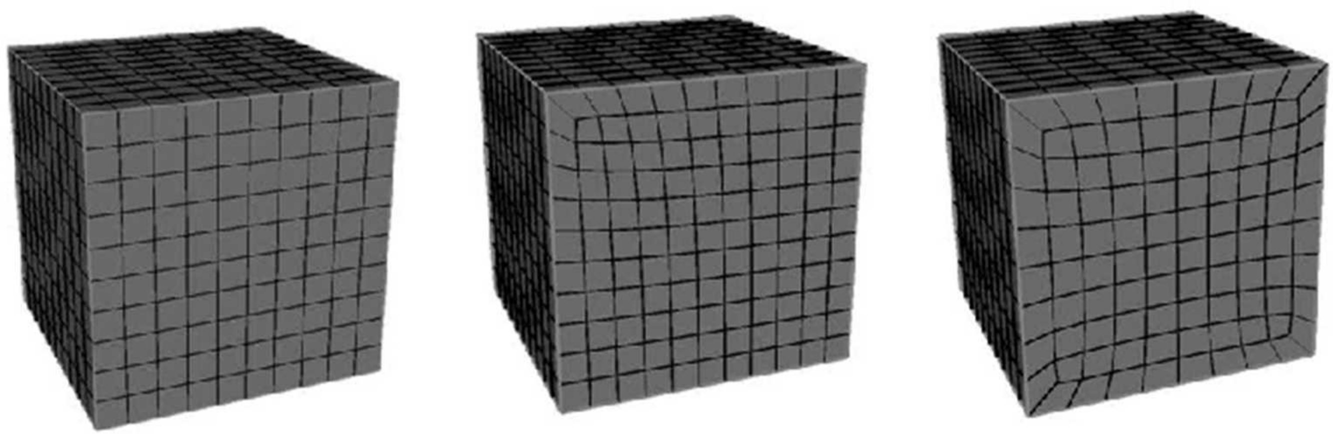
- **Hexahedral Flipping operations can also be used to convert hexahedral meshes to fundamental meshes.**
  - Example 2:



Non-fundamental Mesh — Pillow nodes — Face Collapse operations — Fundamental Mesh

The Set of Fundamental Hexahedral Meshes

- **A given geometric object may have several sets of fundamental sheets which satisfy the definition of a fundamental mesh.**



The Set of Fundamental Hexahedral Meshes

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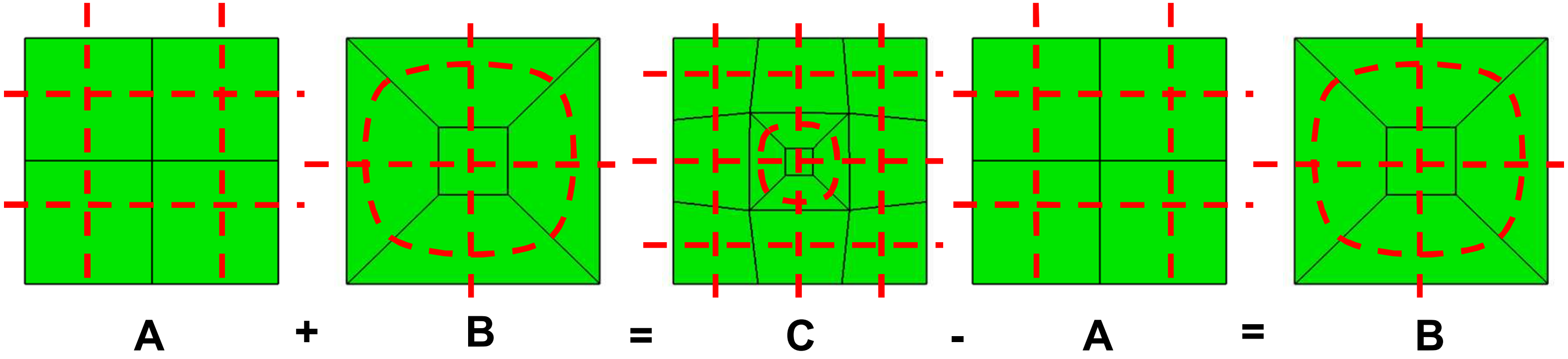
- **Assertion 2:** There exists a set of operations that will convert one mesh into another mesh.

- Specifically, we want a set of operations to convert one fundamental mesh in G into an alternate fundamental mesh in G
- A working proof of this concept can be given as follows:
  - **Given two different, fundamental meshes for a given geometric object, (the set of sheets from these two meshes will be designated A & B), then using sheet insertion (+) and sheet extraction (-) the following statement will be true:**

A + B – A = B



Another view of Assertion 2

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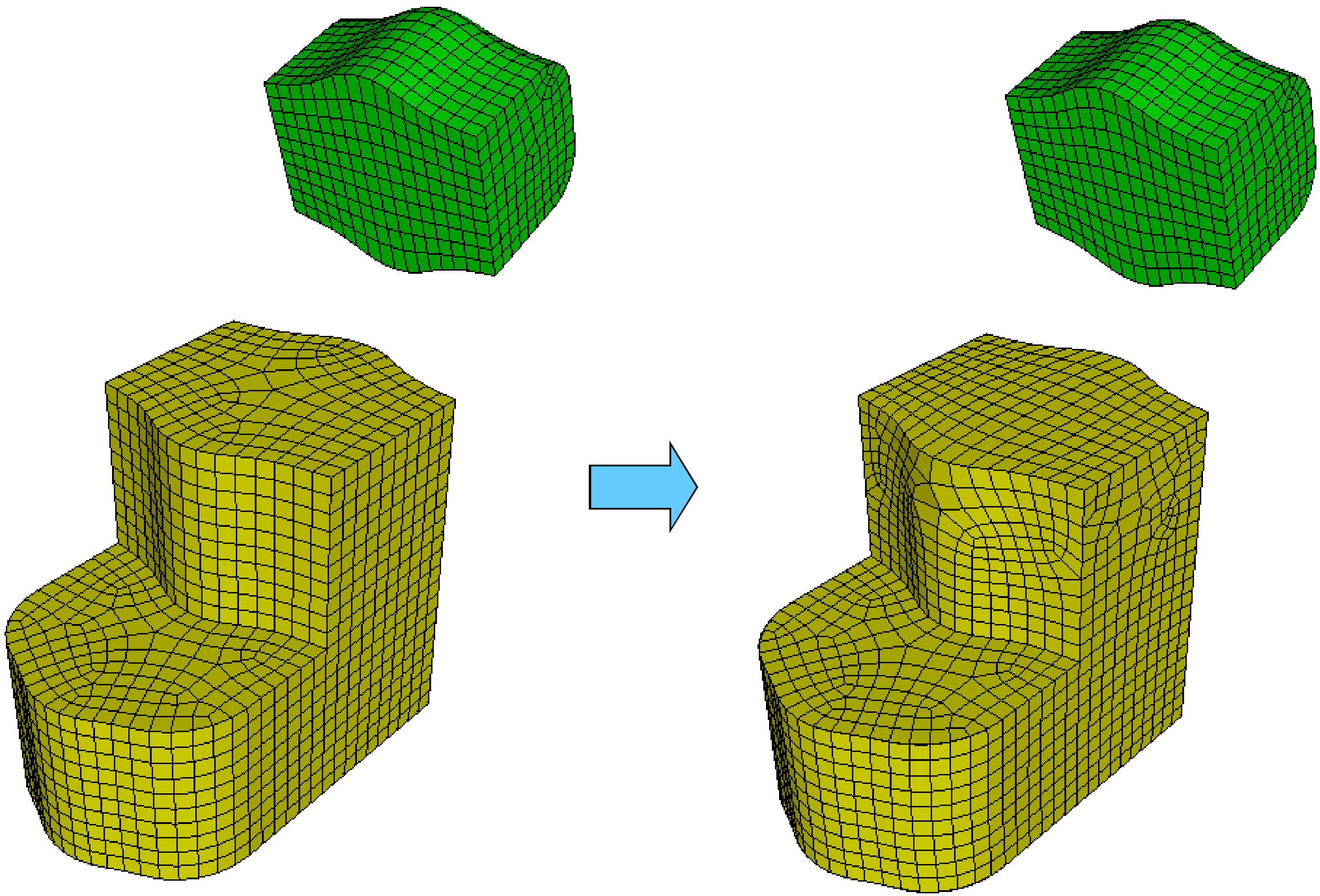

Examples from Matt Staten, et al., Poster at the 16th International Meshing Roundtable, Seattle, WA.

The Set of Fundamental Hexahedral Meshes

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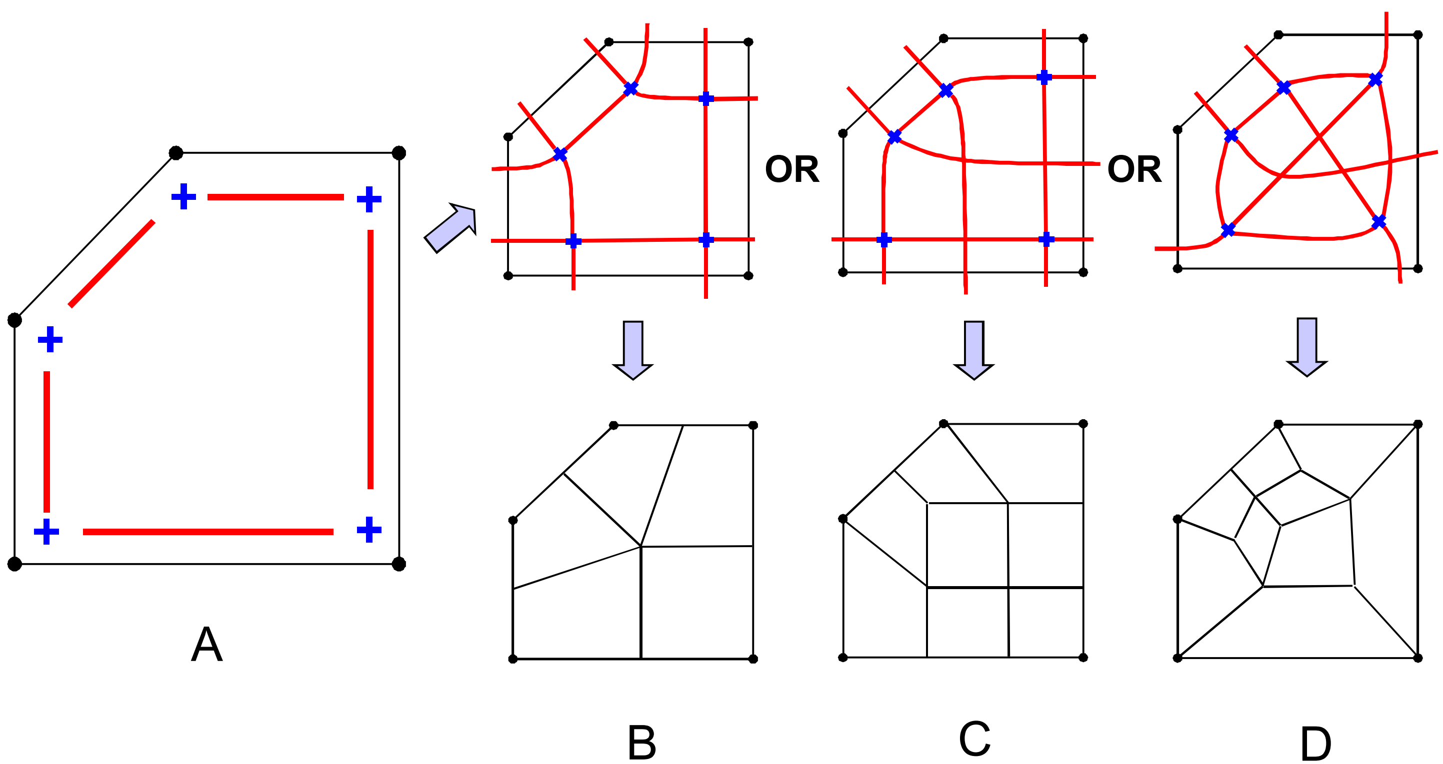
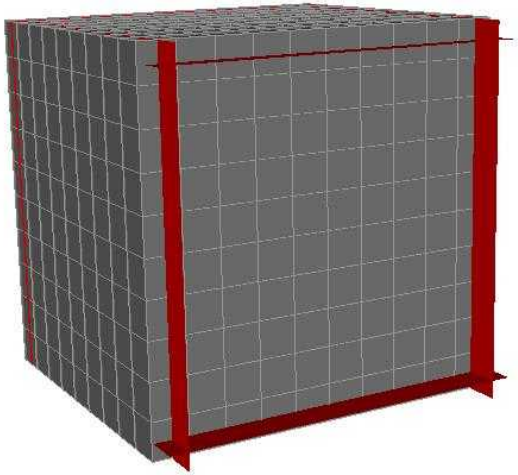
- **An alternative view of Assertion 2: For any geometry, there exists a set of sheets, chords and centroids necessary for the mesh to be fundamental with the geometric object. How these pieces are connected determines the various fundamental meshes possible in the geometric object.**

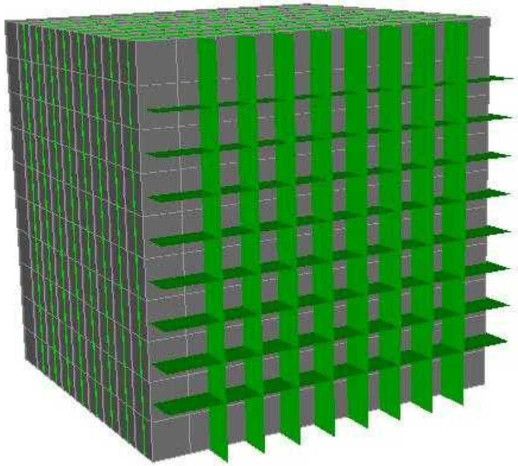


Minimizing Hexahedral Meshes

- **Definition:** A ***secondary sheet*** is a sheet in a mesh that is not a boundary sheet or a fundamental sheet in that mesh. Secondary sheets are typically utilized to meet shape/size requirements within the final mesh.



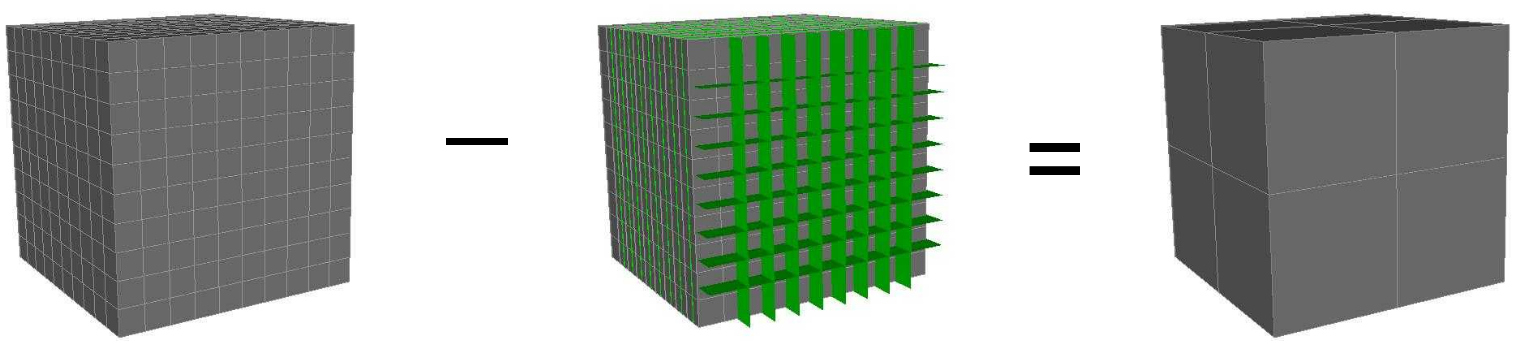
Fundamental Sheets



Secondary Sheets

Minimizing Hexahedral Meshes

- **The fundamental sheets are necessary for maintaining the geometric fidelity of the hexahedral mesh with the original solid geometry. Removing the secondary sheets from the mesh results in a mesh which is coarse, but maintains the geometric fidelity to the original geometry.**
  – Example 1:

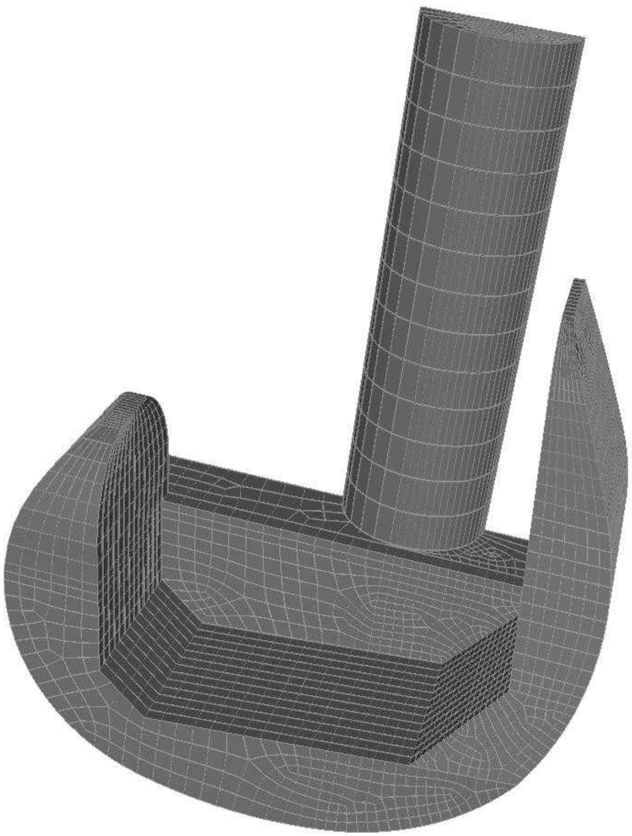


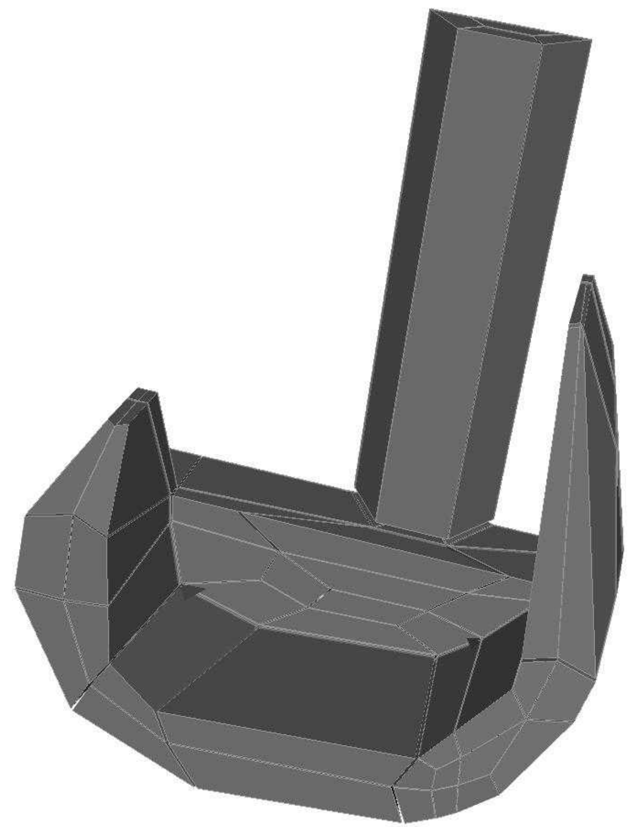
1000 hexes

8 hexes

Minimizing Hexahedral Meshes

– Example 2:





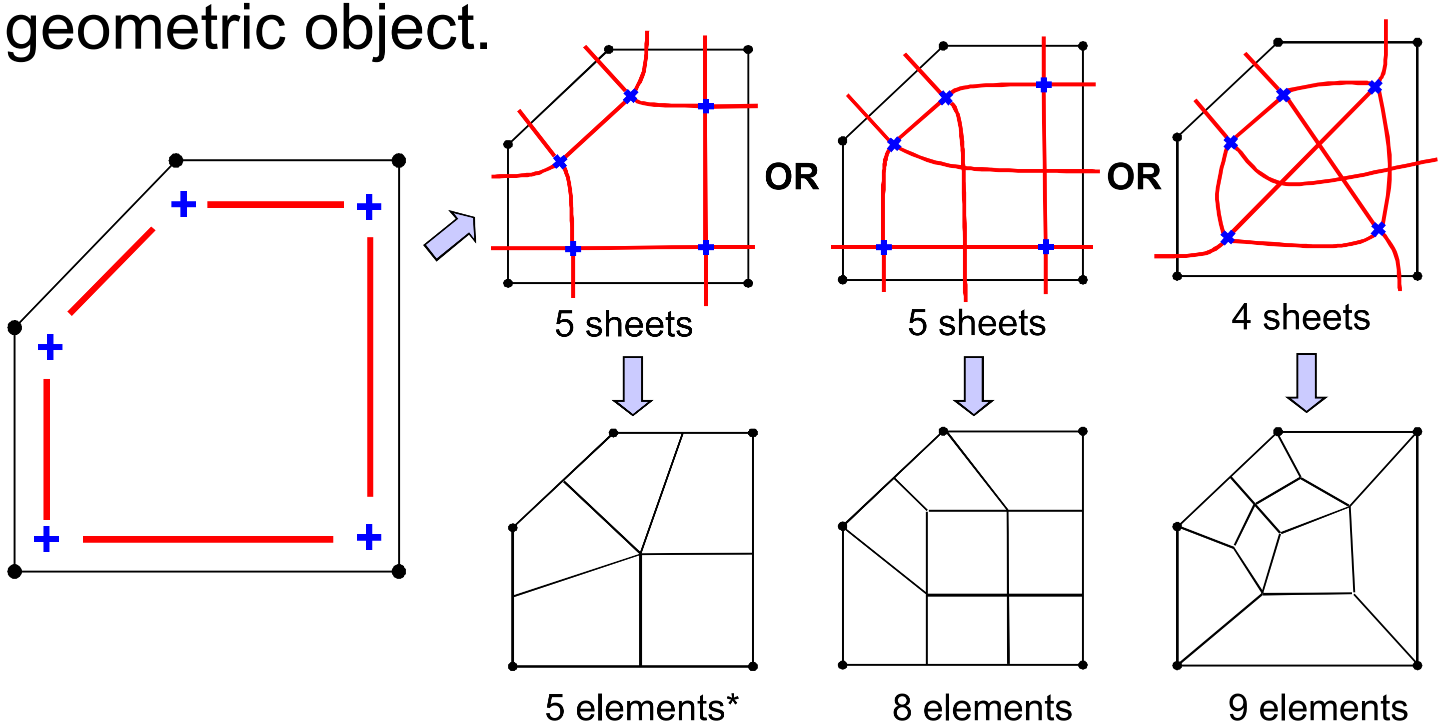
10,534 hexes - Secondary sheets = 77 hexes

Minimal Hexahedral Mesh

- **Definition:**
  - A hexahedral mesh is minimal within a geometric object if:
    - **1. it contains the fewest number of hexahedra for all sets of possible hexahedral meshes for a given object**
    - **2. The mesh does not contain any doublets**
    - **3. The mesh does not contain any 'geometric' doublets**
      - (i.e. two adjacent faces on a hex cannot belong to a single surface, and two adjacent edges of a hex cannot belong to a single curve.)
- **Definition:**
  - A thin-region exists within a mesh when a single sheet is fundamental to two opposing surfaces within the mesh (i.e. there is only a single layer of hexahedra within this portion of the geometry.)

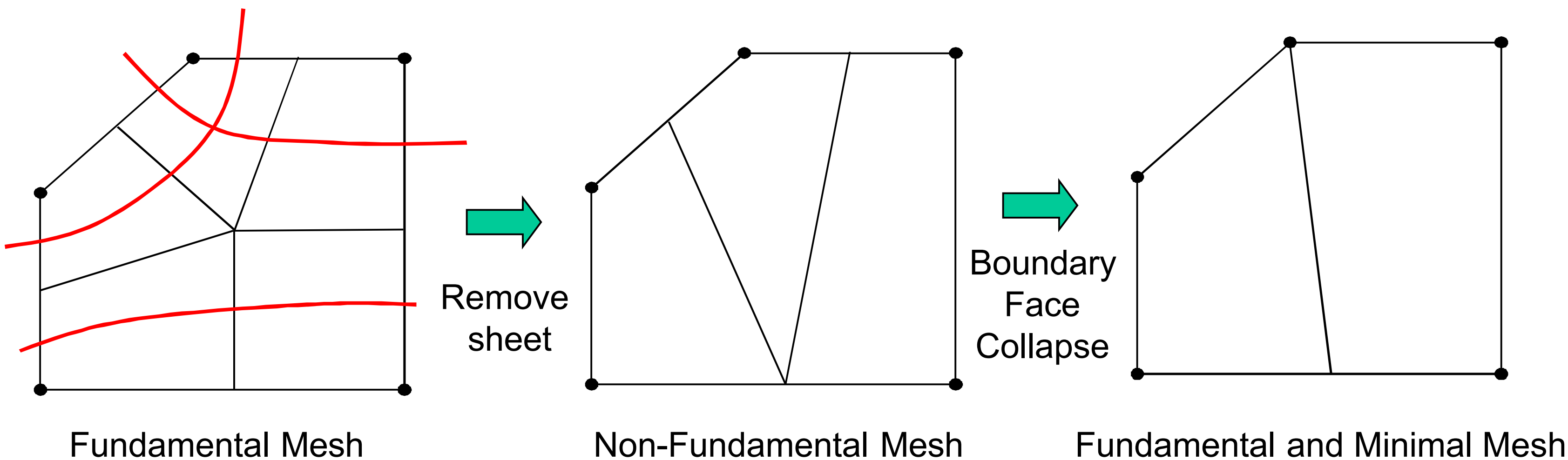
Minimal Hexahedral Meshes

- **Conjecture 1**: The ***minimal hexahedral mesh*** for a geometric object *without thin regions* is defined by one of the possible sets of fundamental sheets for a geometric object.



Minimal Hexahedral Meshes

- **Conjecture 2**: The ***minimal hexahedral mesh*** for a geometric object *with thin regions* will be fundamental with respect to at least one of side of the thin region in the geometric object.



Minimal Hexahedral Meshes

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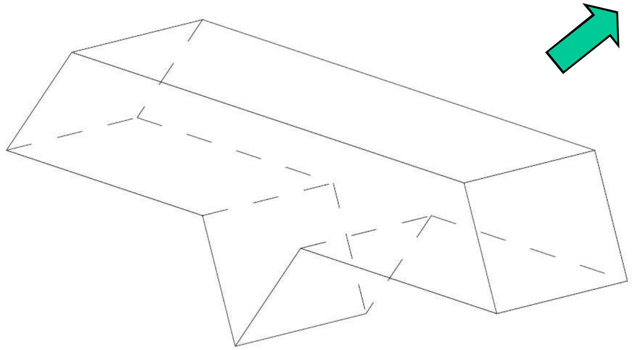
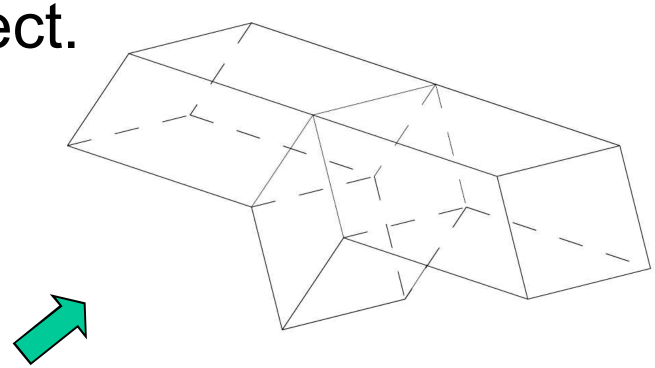
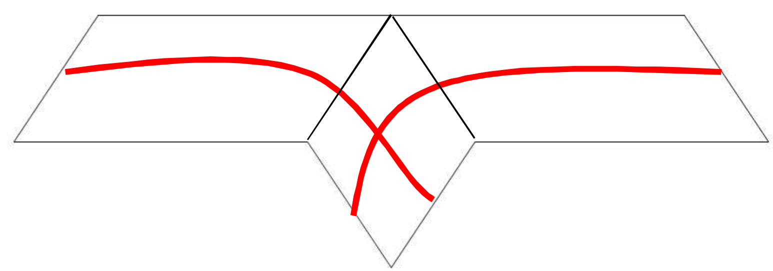


Original Geometry



Minimal Mesh (3 elements)

Not fundamental with respect to this side of the thin region



Fundamental with respect to this side of the thin region

Why is this important?

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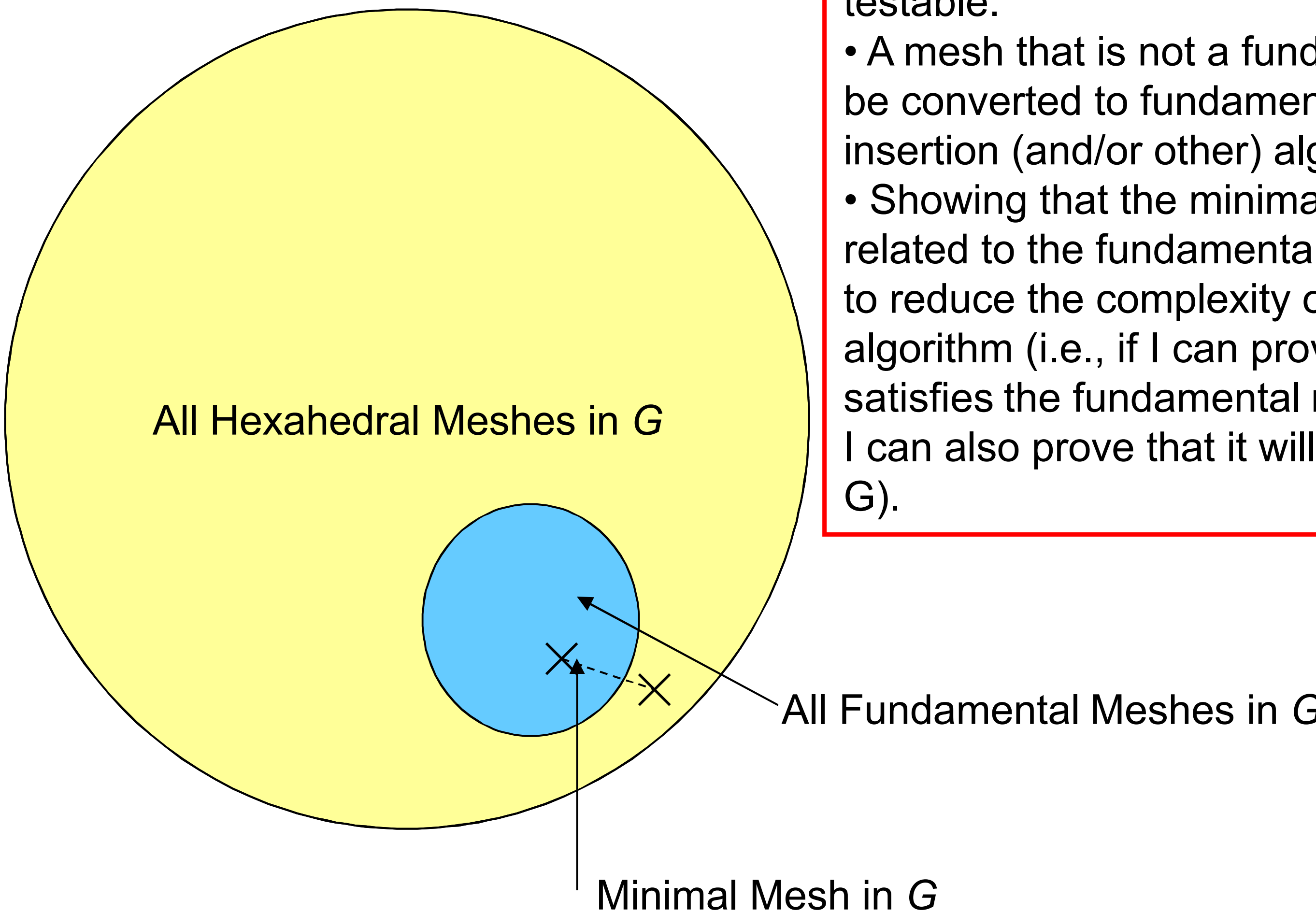
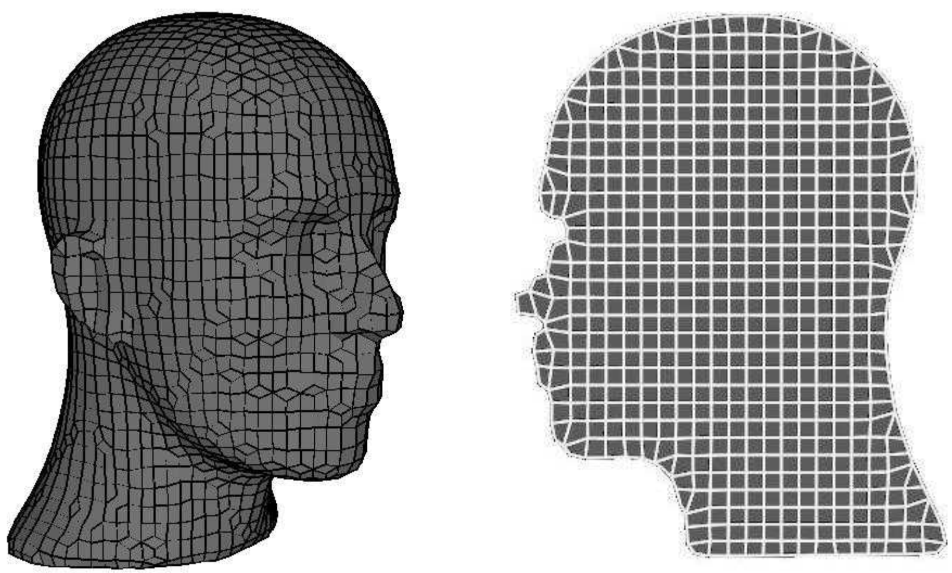


- Characterizing a mesh as fundamental is testable.
- A mesh that is not a fundamental mesh can be converted to fundamental using sheet insertion (and/or other) algorithms.
- Showing that the minimal mesh is also related to the fundamental mesh can be used to reduce the complexity of an all-hex algorithm (i.e., if I can prove that my algorithm satisfies the fundamental mesh requirements, I can also prove that it will generate a mesh in G).

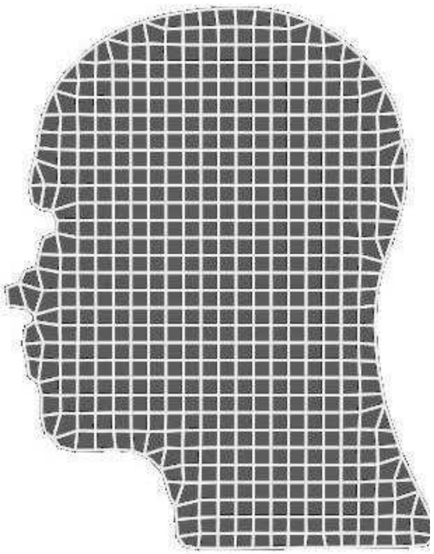
Hexahedral Isosurfacing

- **In 2005, Zhang et al. introduced an algorithm for generating hexahedral topologies from volumetric data in a process similar to Marching Cubes and dual contouring methods of generating triangle isosurfaces.**
  - Similar results to other hexahedral octree methods (i.e. poor and inverted hexahedra found at the boundary of the resulting mesh).
  - Zhang et al. worked to improve these meshes by smoothing the boundary elements. However, many elements still retained poor or inverted shapes.

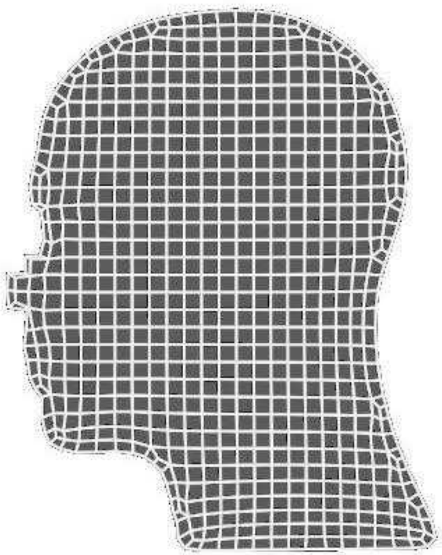


Hexahedral Isosurfacing

- **By introducing a fundamental sheet for the isosurface into the mesh, the quality of these meshes can be dramatically improved without altering the original quadrilateral boundary mesh.**
  - Example:



Before



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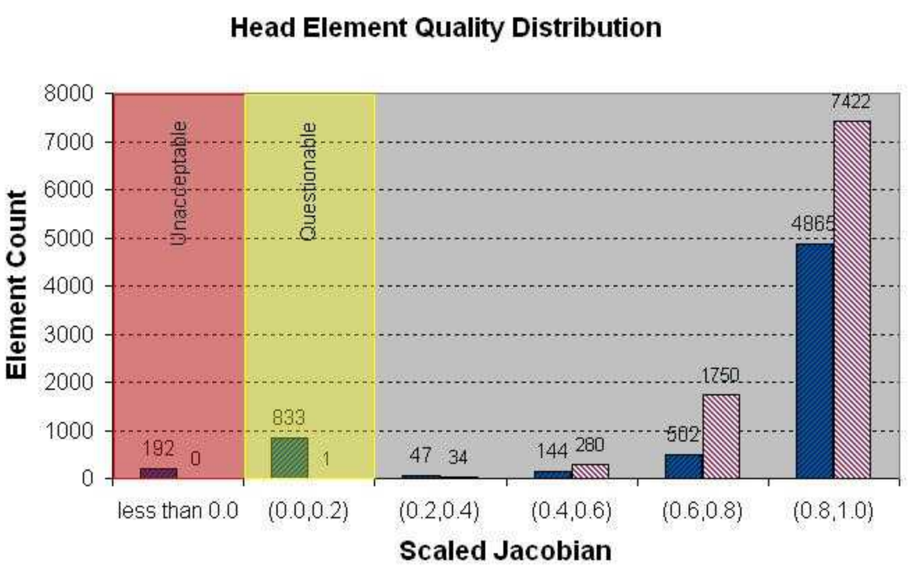

Hexahedral Isosurfacing

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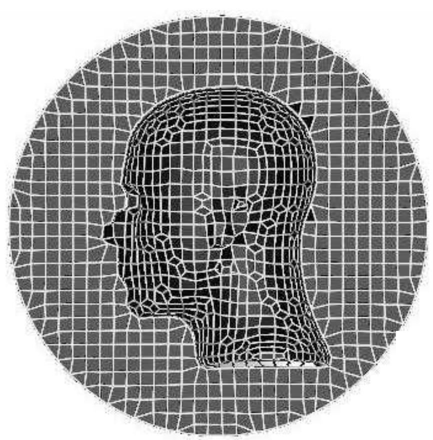
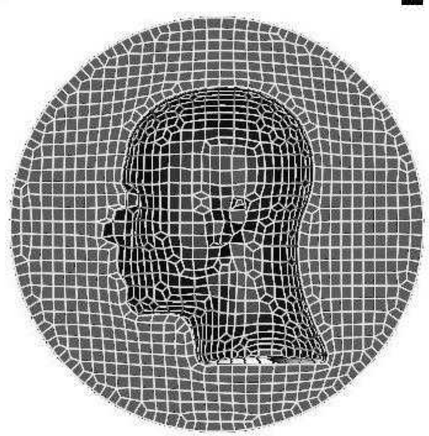
- **Additionally, because the original method does not alter the original octree topology, it is possible to generate conformal meshes between interior and exterior sets of elements.**
  - Example:



Before



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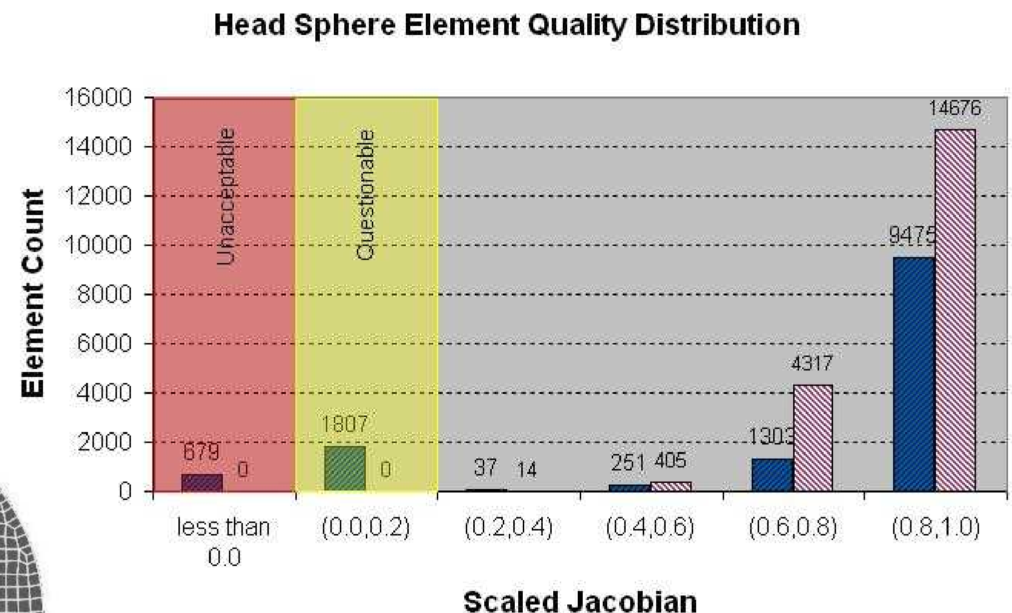

Hexahedral Isosurfacing

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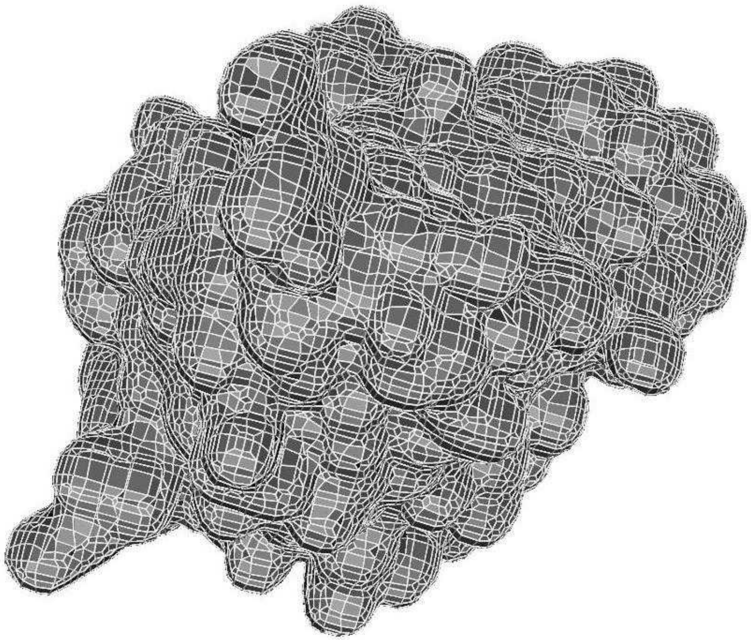
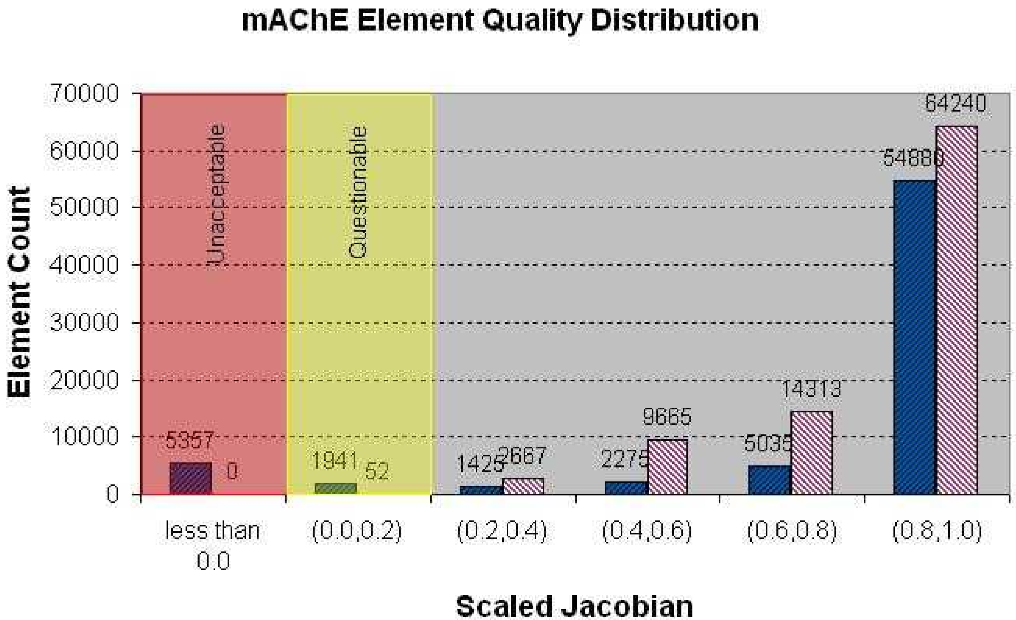
– Example:



mAChE biomolecule



Hexahedral Isosurfacing

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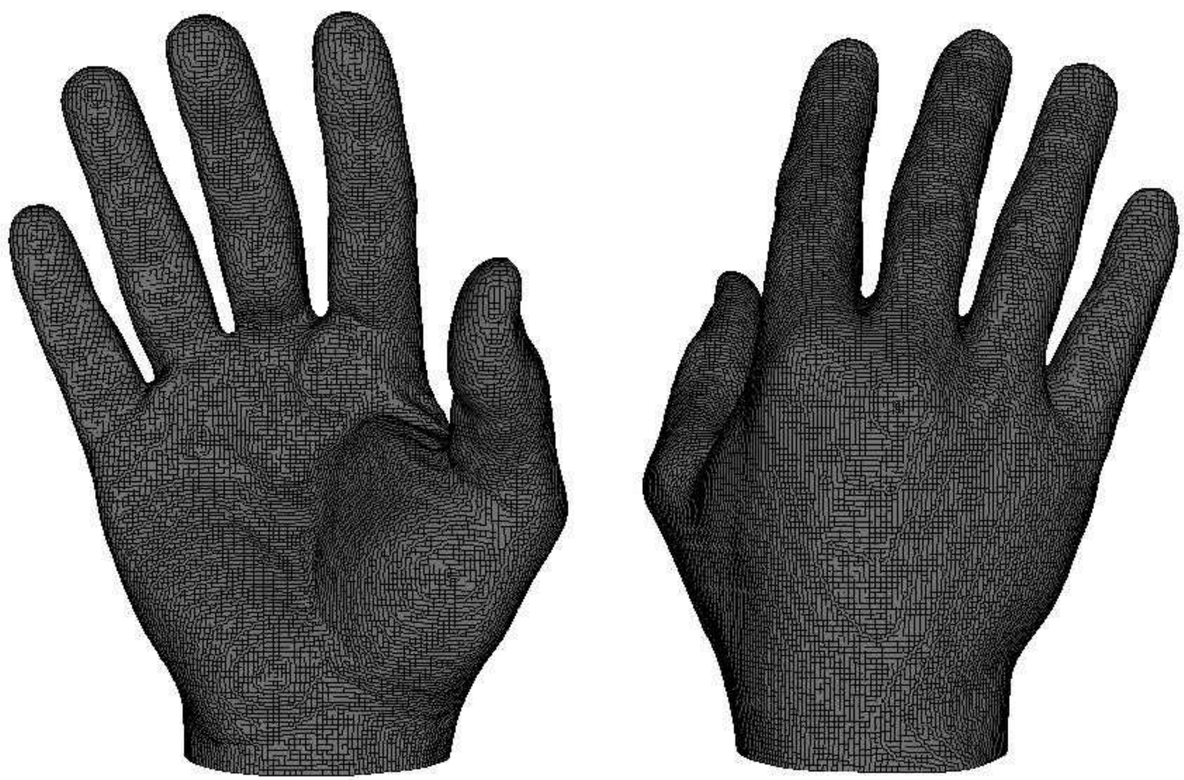
– Example 1 (Hand)- 202,974 hexahedra



Hexahedral Isosurfacing

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– Example 1 (Hand)- Geometric Fidelity to original Triangle Mesh

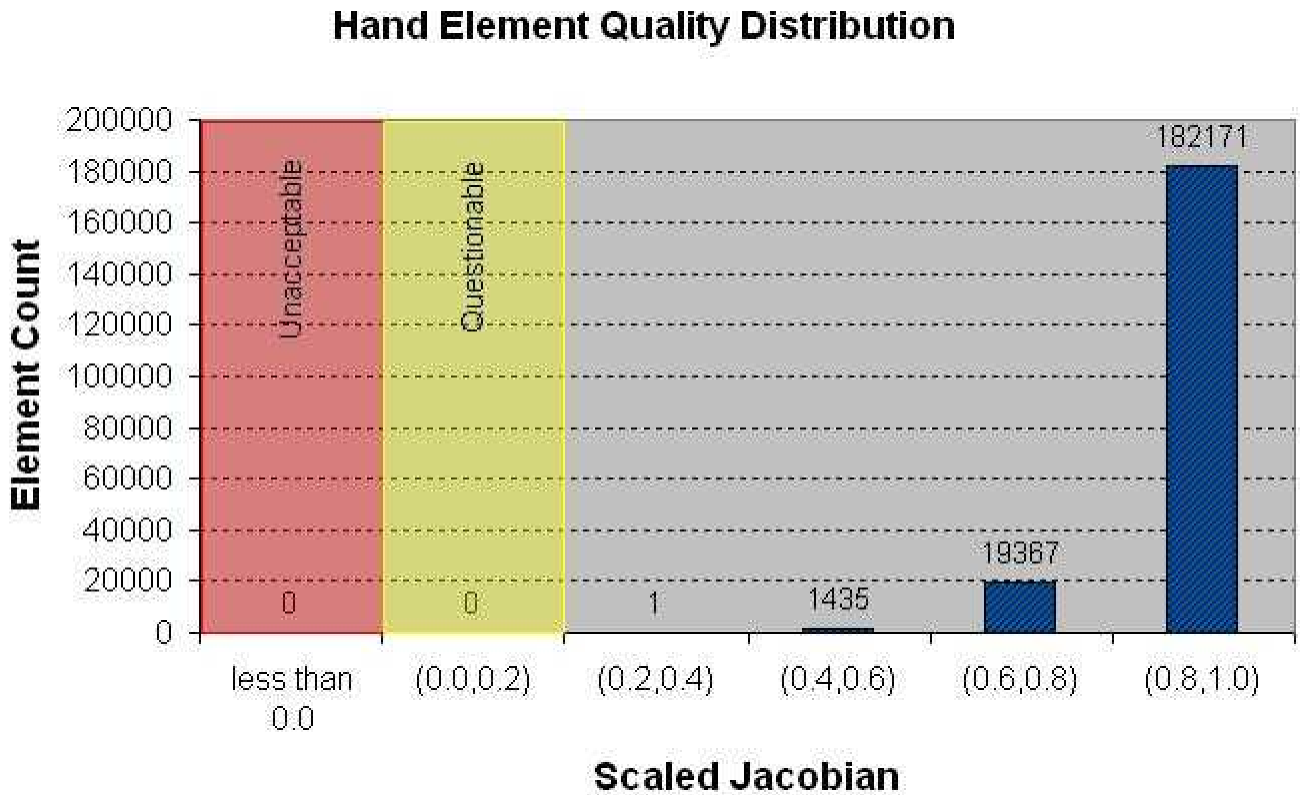
Original Triangle Mesh

Hexahedral Facets

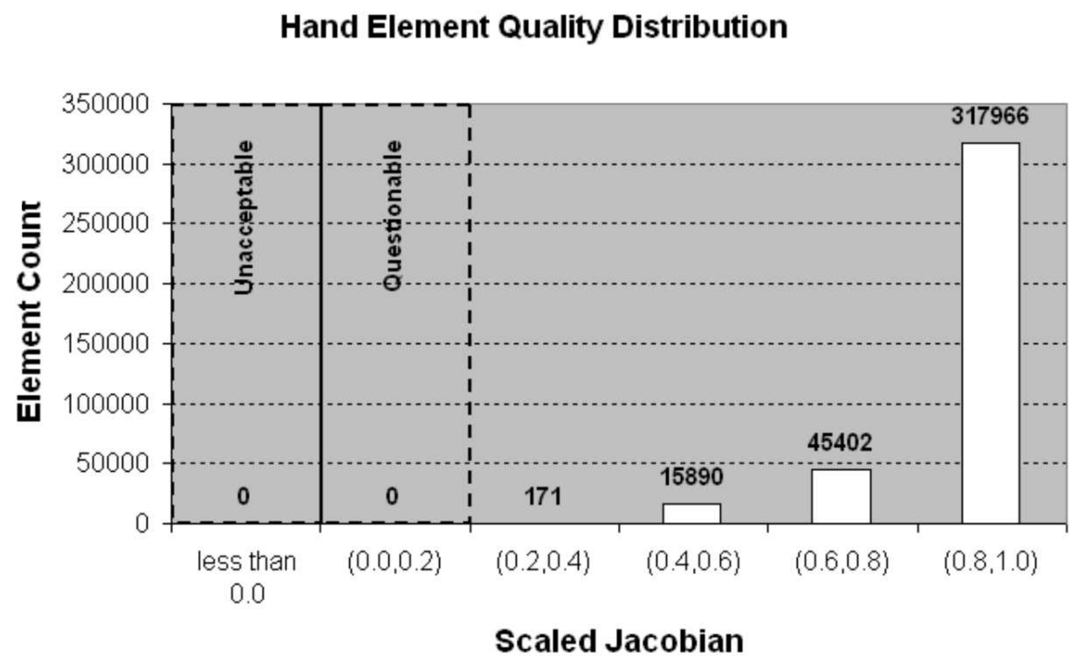
Composite facets (triangle facets in red, hexahedral facets in green)

Hexahedral Isosurfacing

– Example 1 (Hand)-

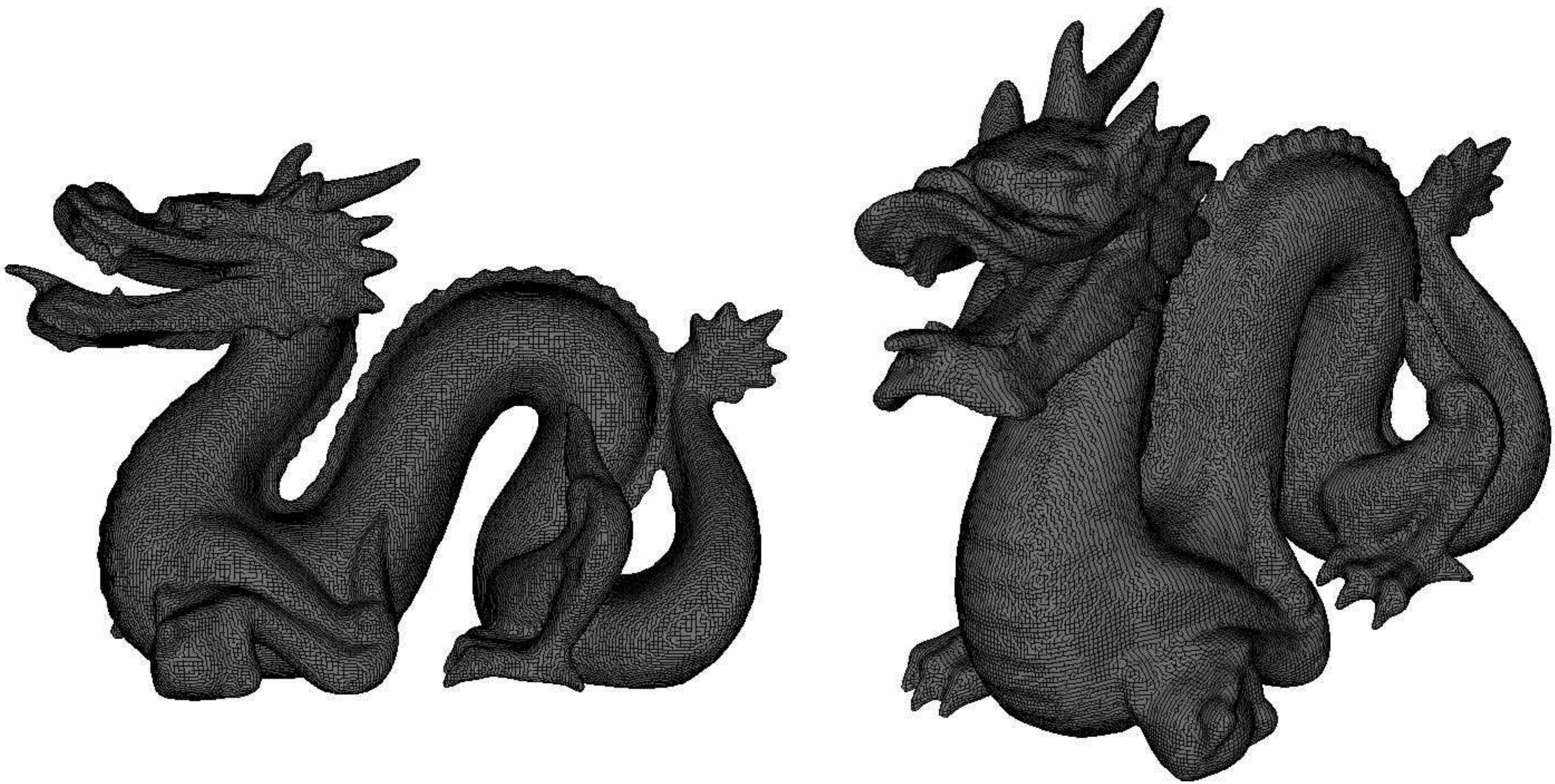


Sheet Insertion



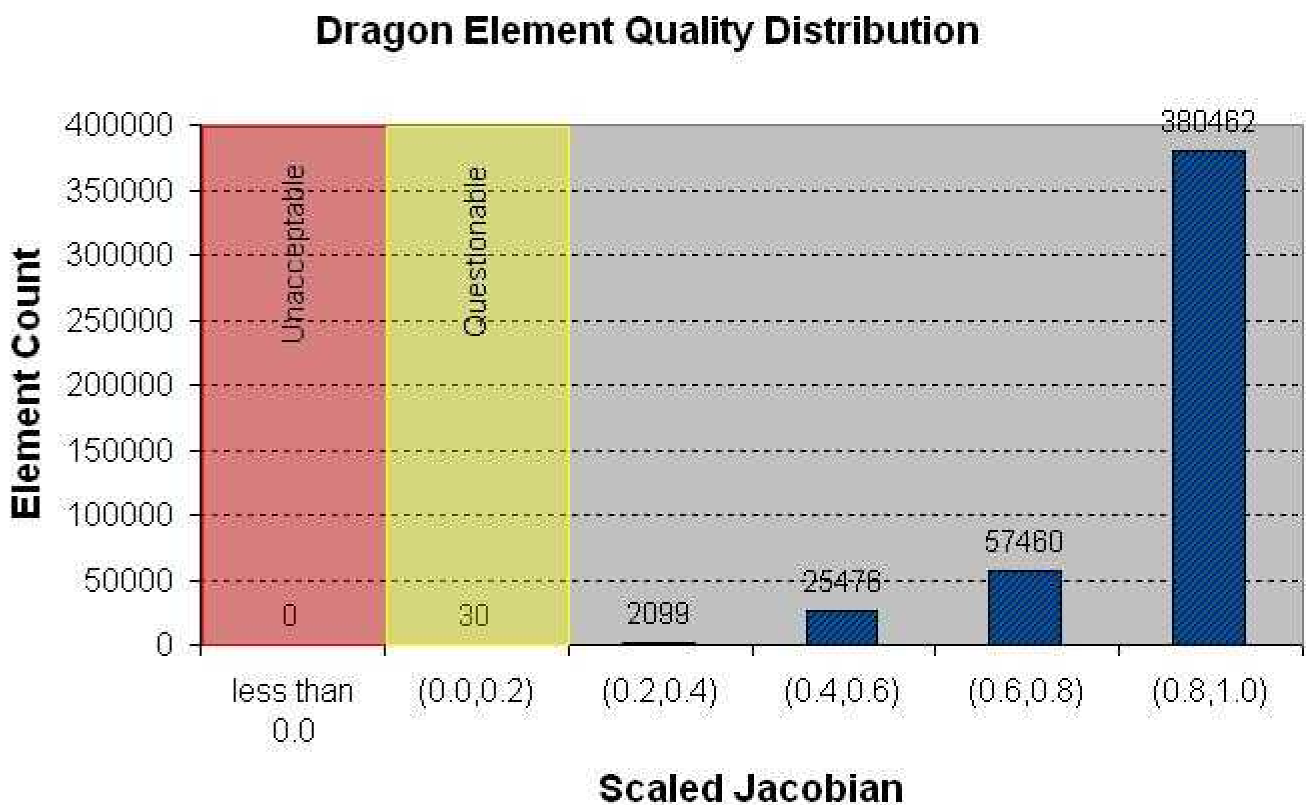
Hexahedral Isosurfacing

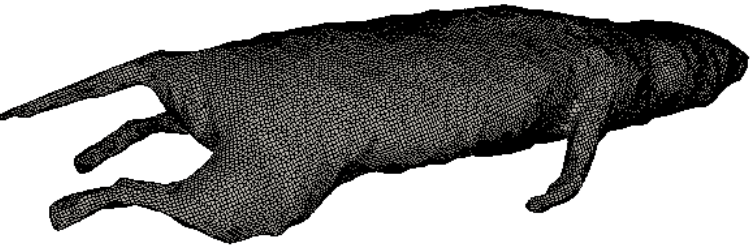
– Example 1 (Dragon)- 465,527 hexahedra

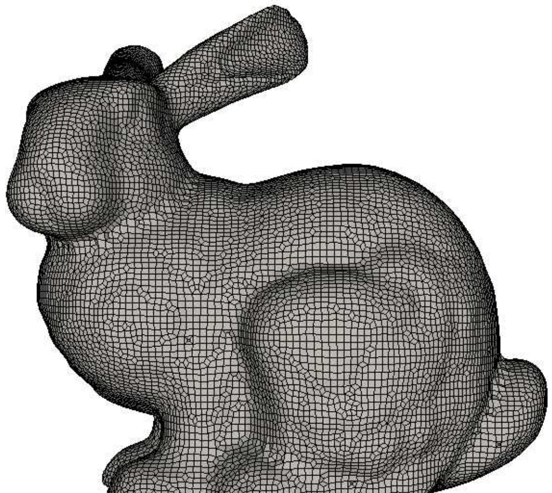


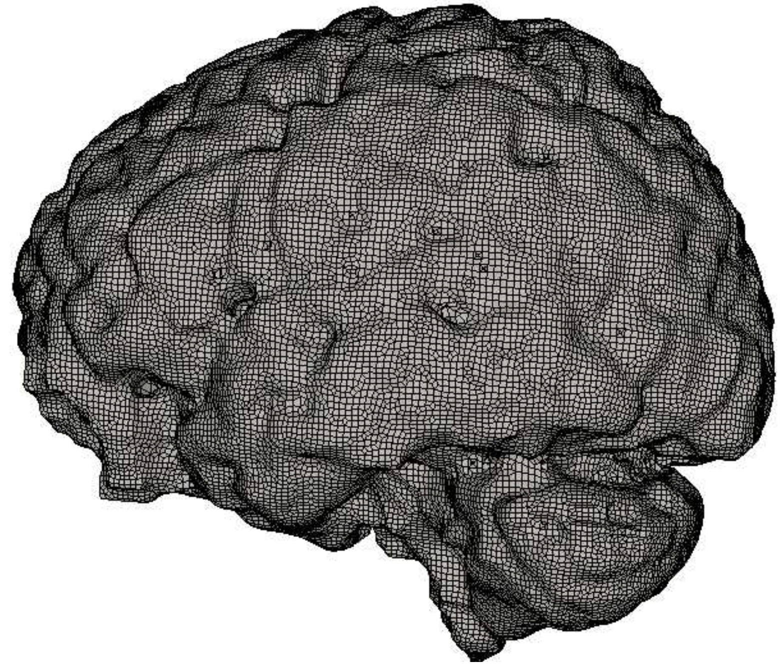
Hexahedral Isosurfacing

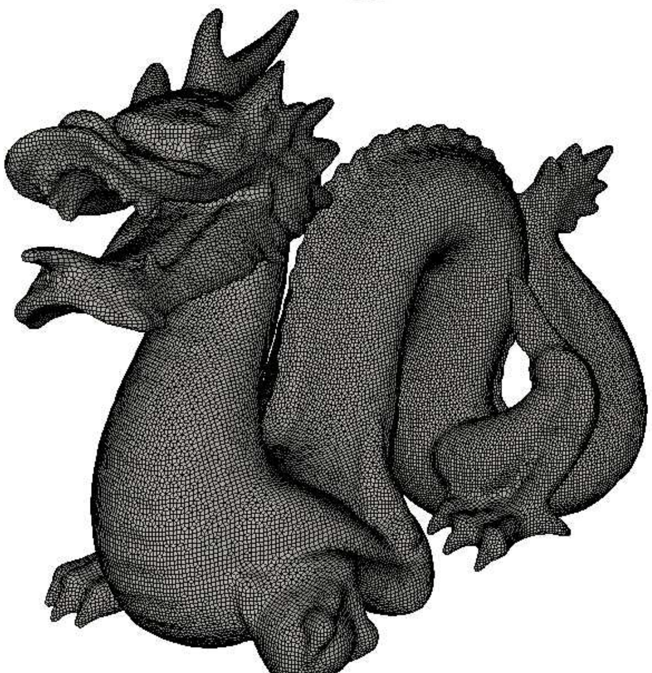
– Example 1 (Dragon)-

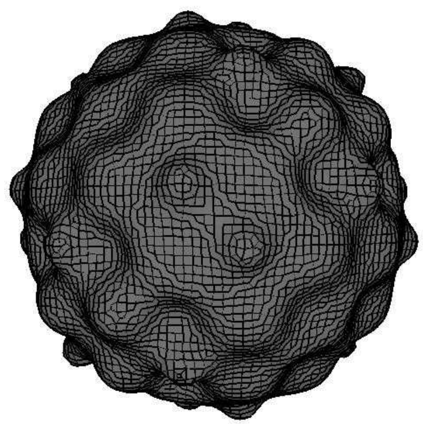


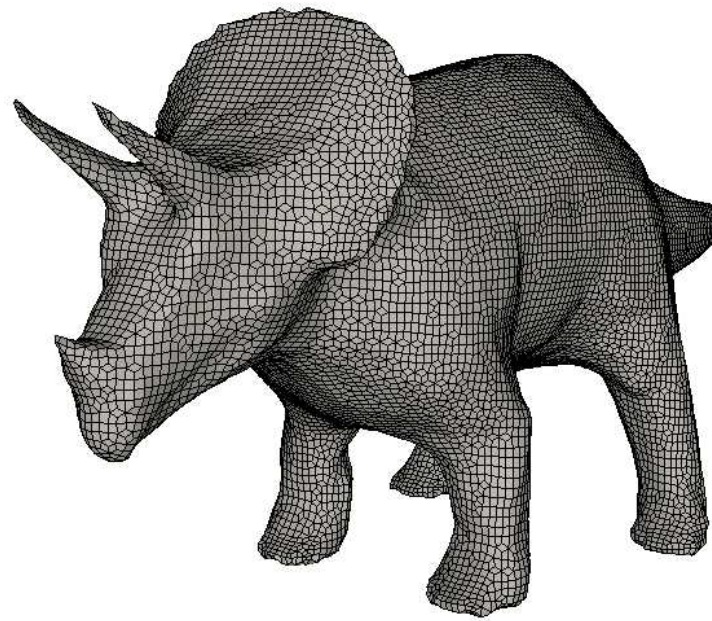












-Mouse model is courtesy of Jeroen Stinstra of the SCI Institute at the University of Utah
-Bumpy Sphere model is provided courtesy of mpii by the AIM@SHAPE Shape Repository.
-Brain and Hand Models are provided courtesy of INRIA by the AIM@SHAPE Shape Repository.

Multi-surface Hexahedral Mesh Generation

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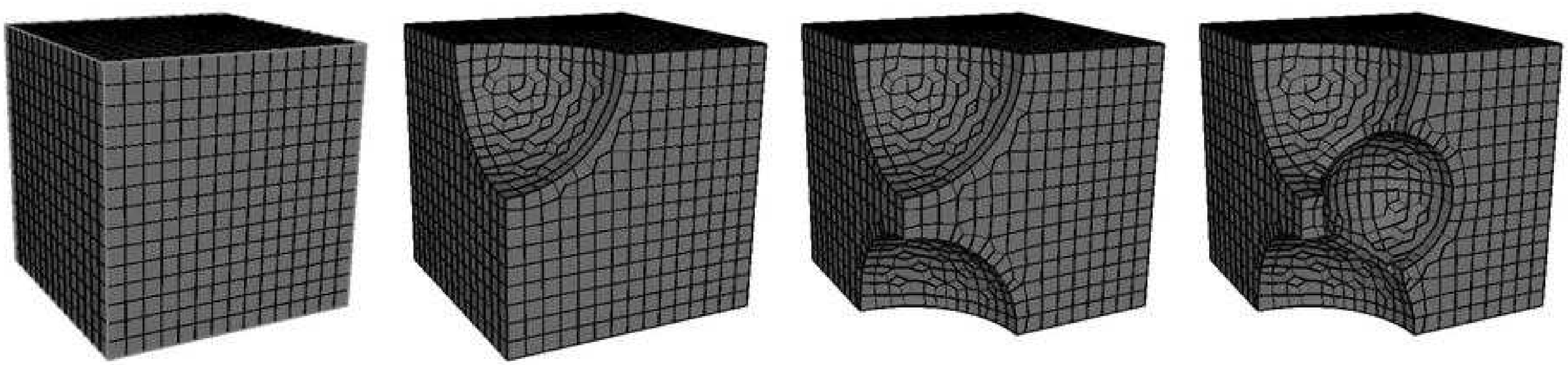
- **Using the same algorithm developed for isosurfaces, we can insert multiple sheets whenever it is desirable to capture a hard curve in the hexahedral mesh. Coupling this algorithm with geometric Boolean operations enables hexahedral mesh generation of increasingly complex geometric solids.**



Multi-surface Hexahedral Mesh Generation

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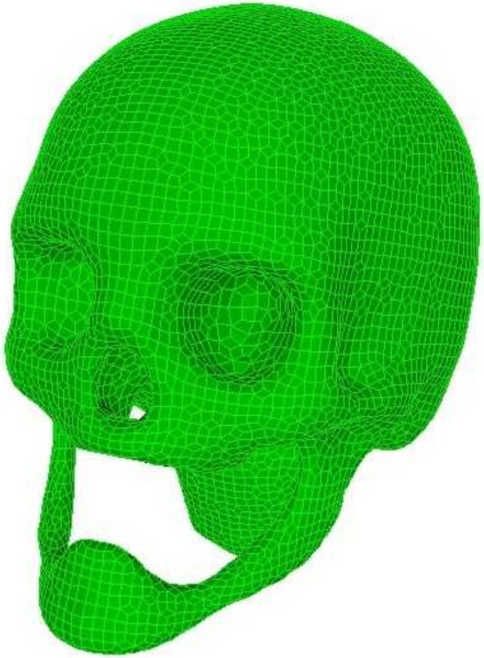
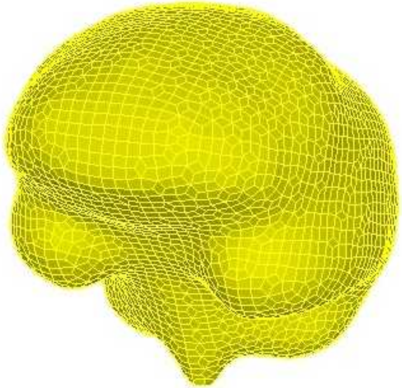
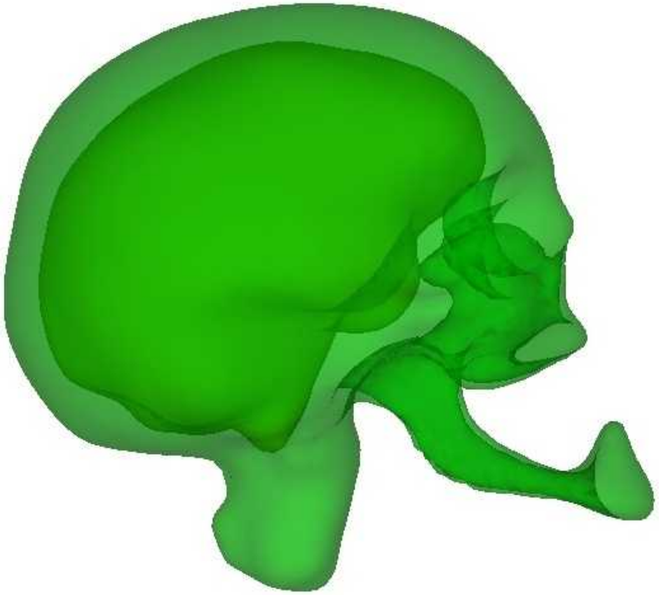
- **Example (skull)**



Skull mesh:
Contains 19,330 hexahedra



Cranial Mesh:
Contains 34,815 hexahedra



Composite facets
(transparent view)

Multi-surface
Hexahedral Mesh Generation

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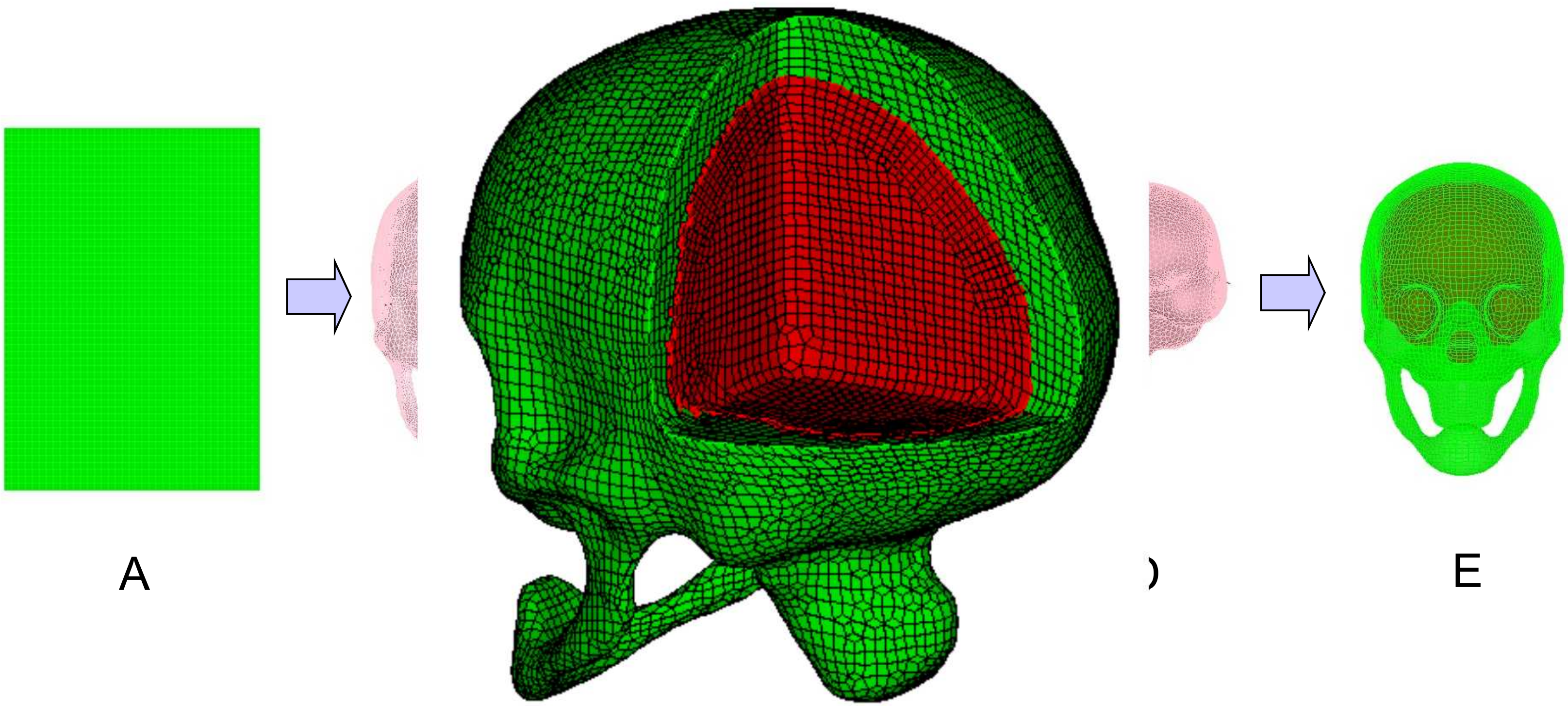

Multi-surface
Hexahedral Mesh Generation

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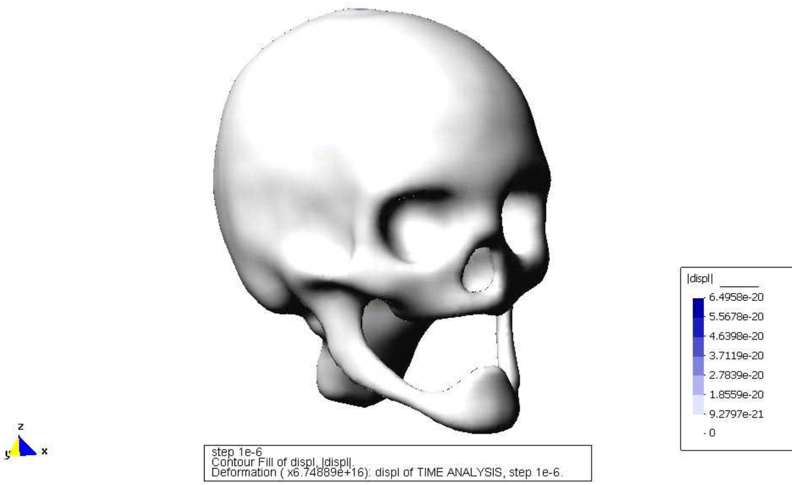
- **Example (skull) -**

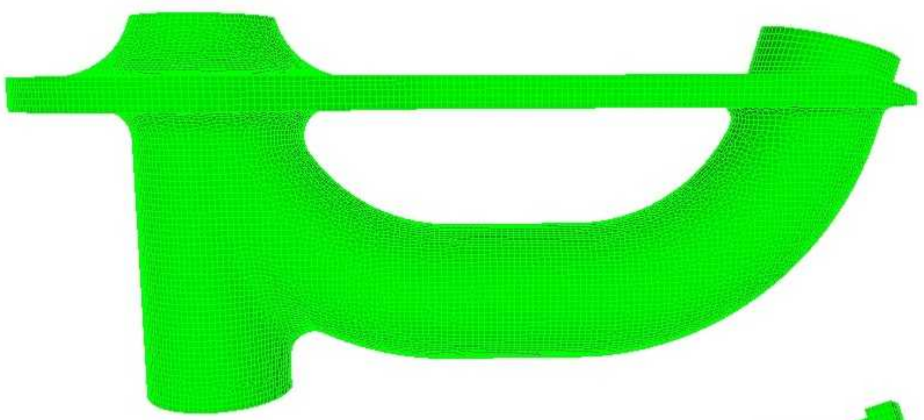


Skull bone shown in blue
Cranial cavity shown in magenta

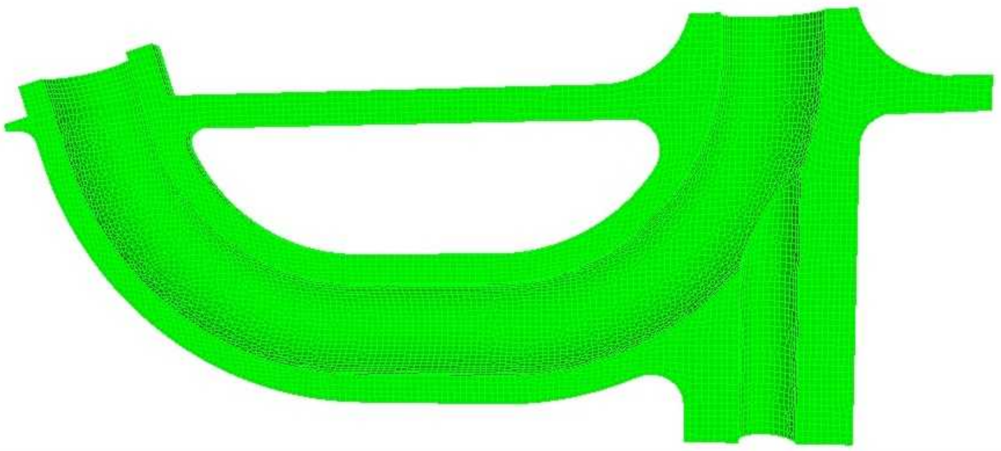
Impact analysis courtesy of Dr. Marco Stupazzini, Department fuer Geo- und Umweltwissenschaften Sektion Geophysik Ludwig-Maximilians-Universitaet Theresienstrasse 41

Multi-surface Hexahedral Mesh Generation

- **Example (goose16) – contains 57,114 hexahedra**



Front view



Back View

Multi-surface Hexahedral Mesh Generation

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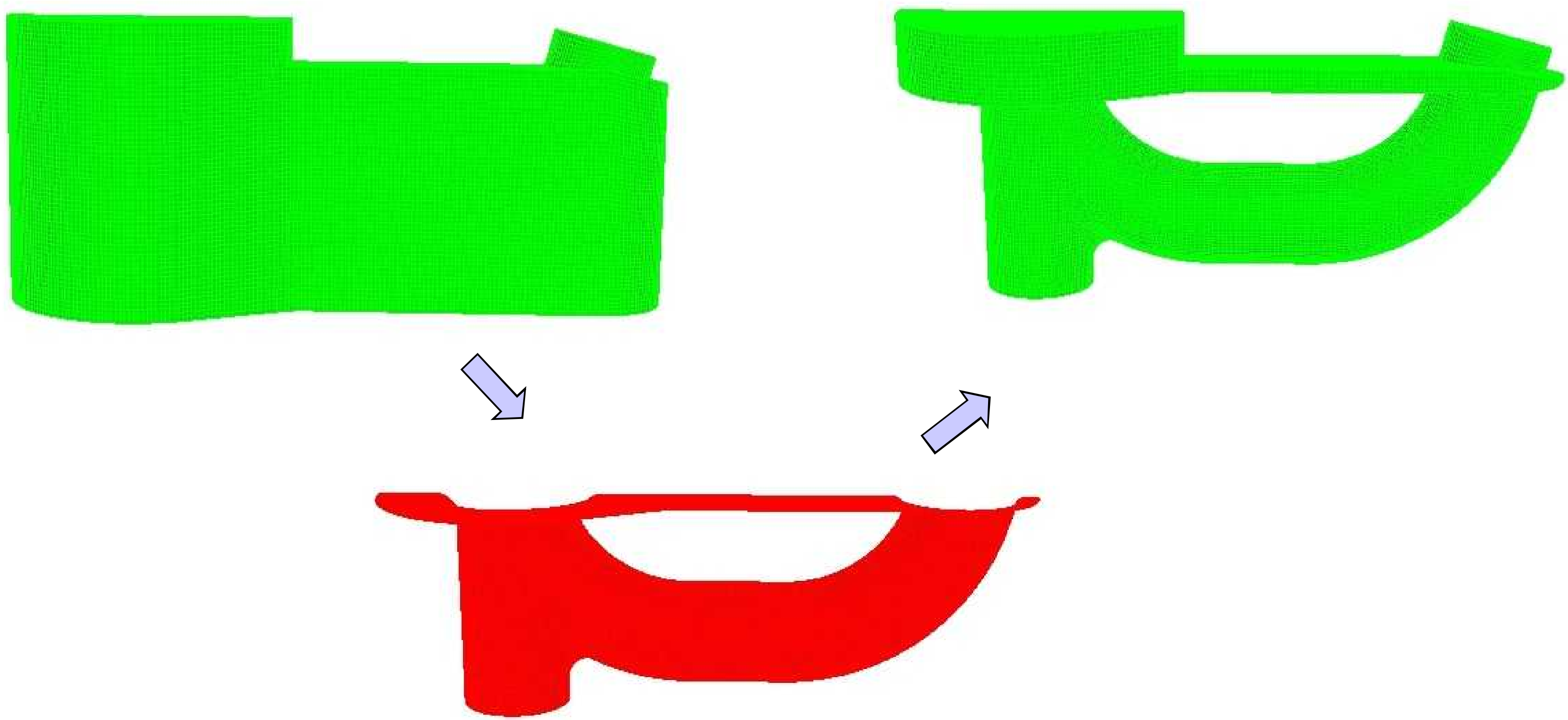
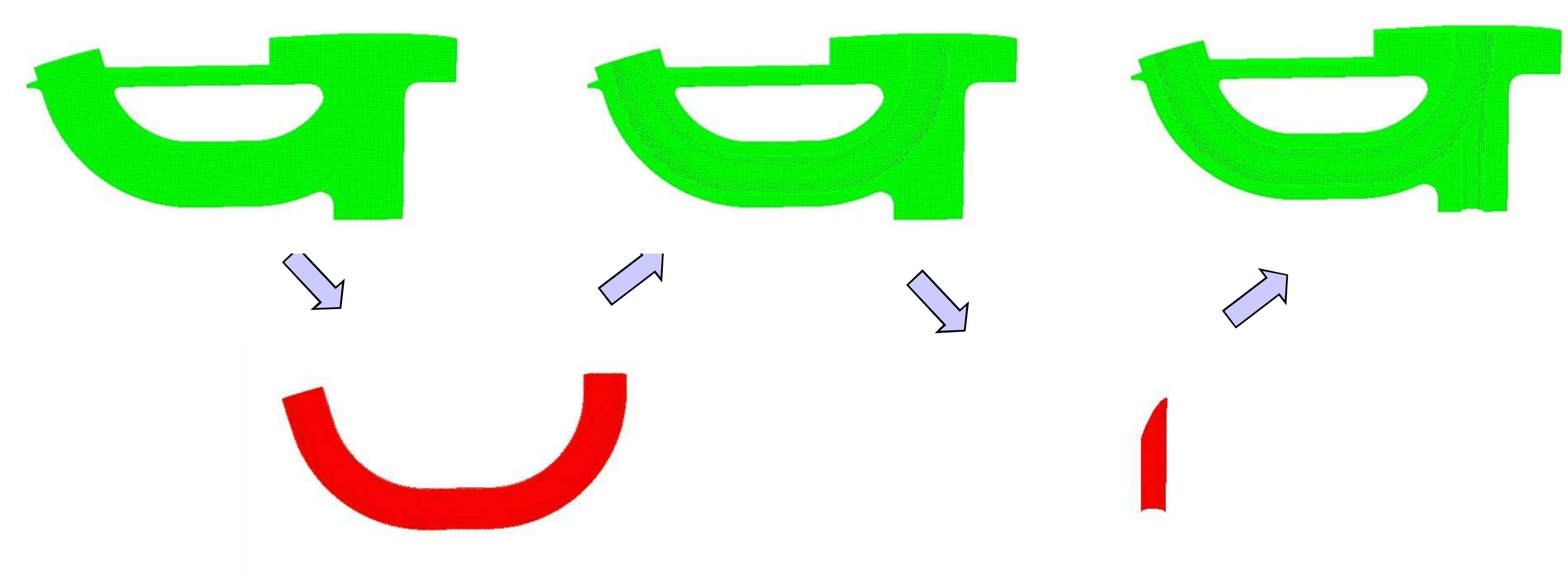
- **Example (goose16) - process**



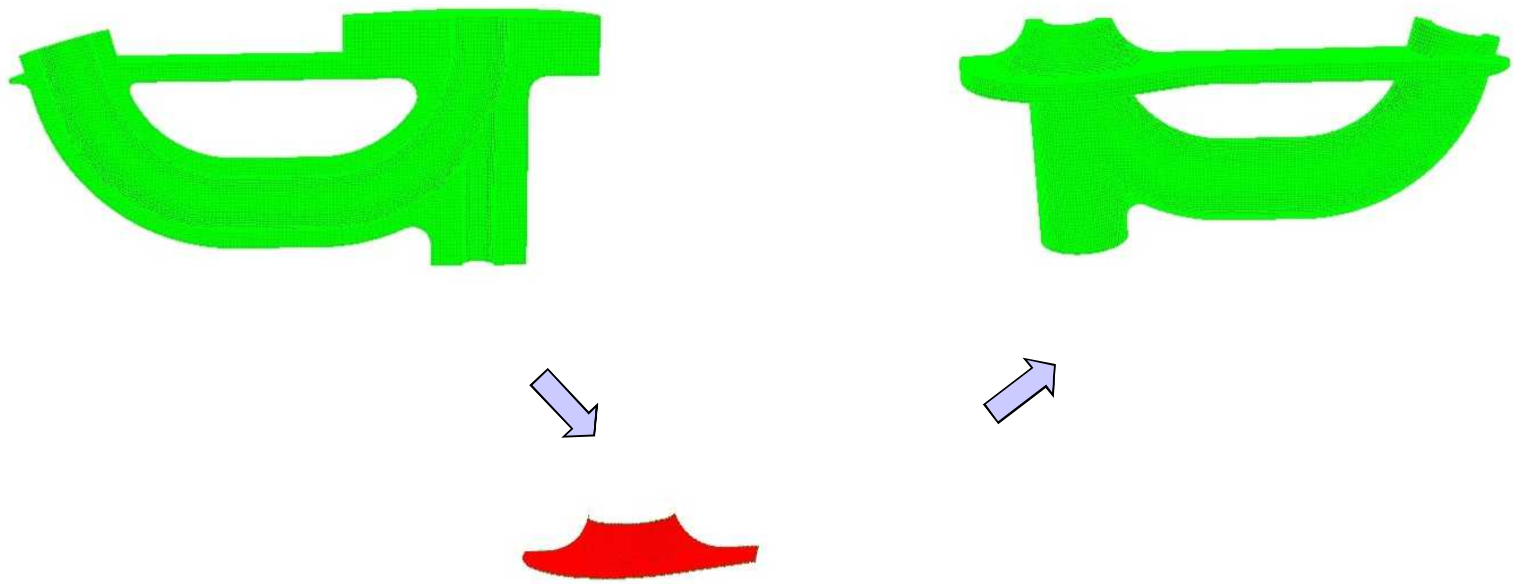
Multi-surface
Hexahedral Mesh Generation

- **Example (goose16) – process (cont'd.)**

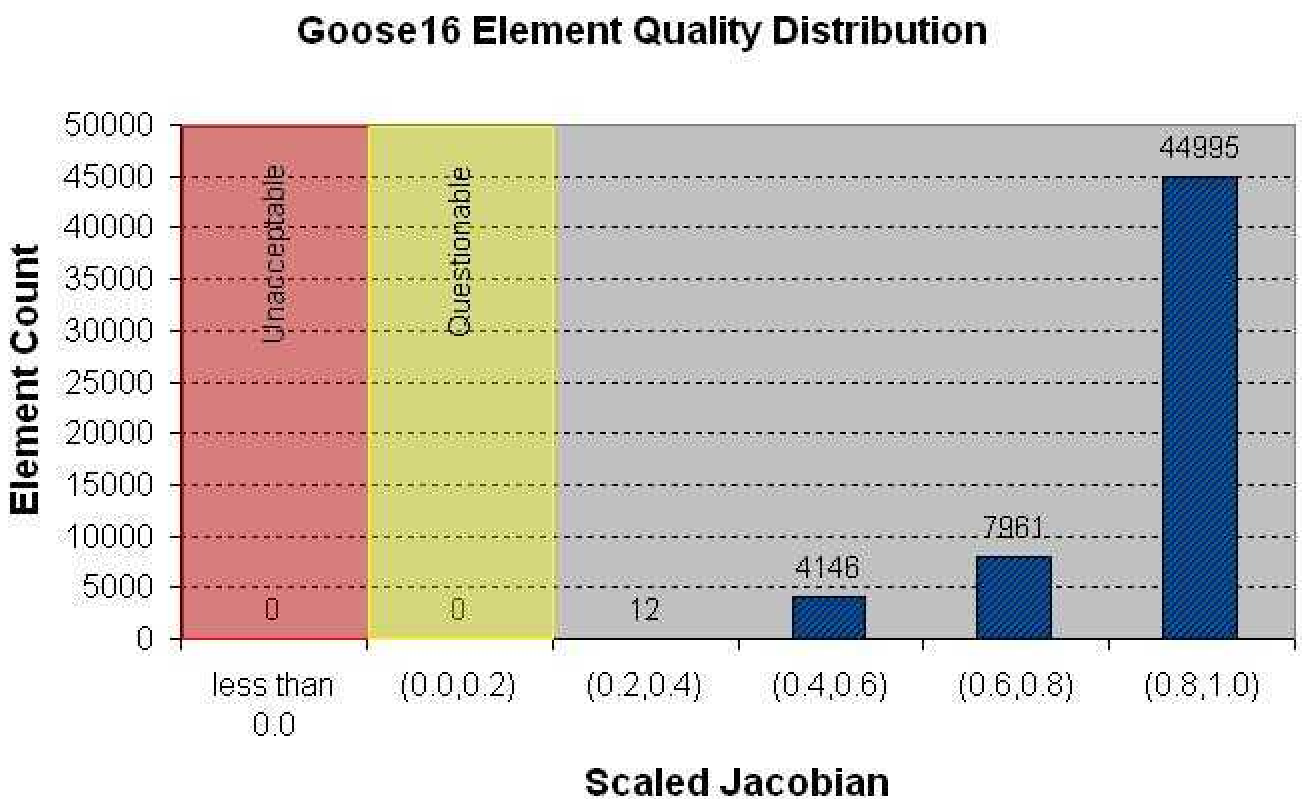


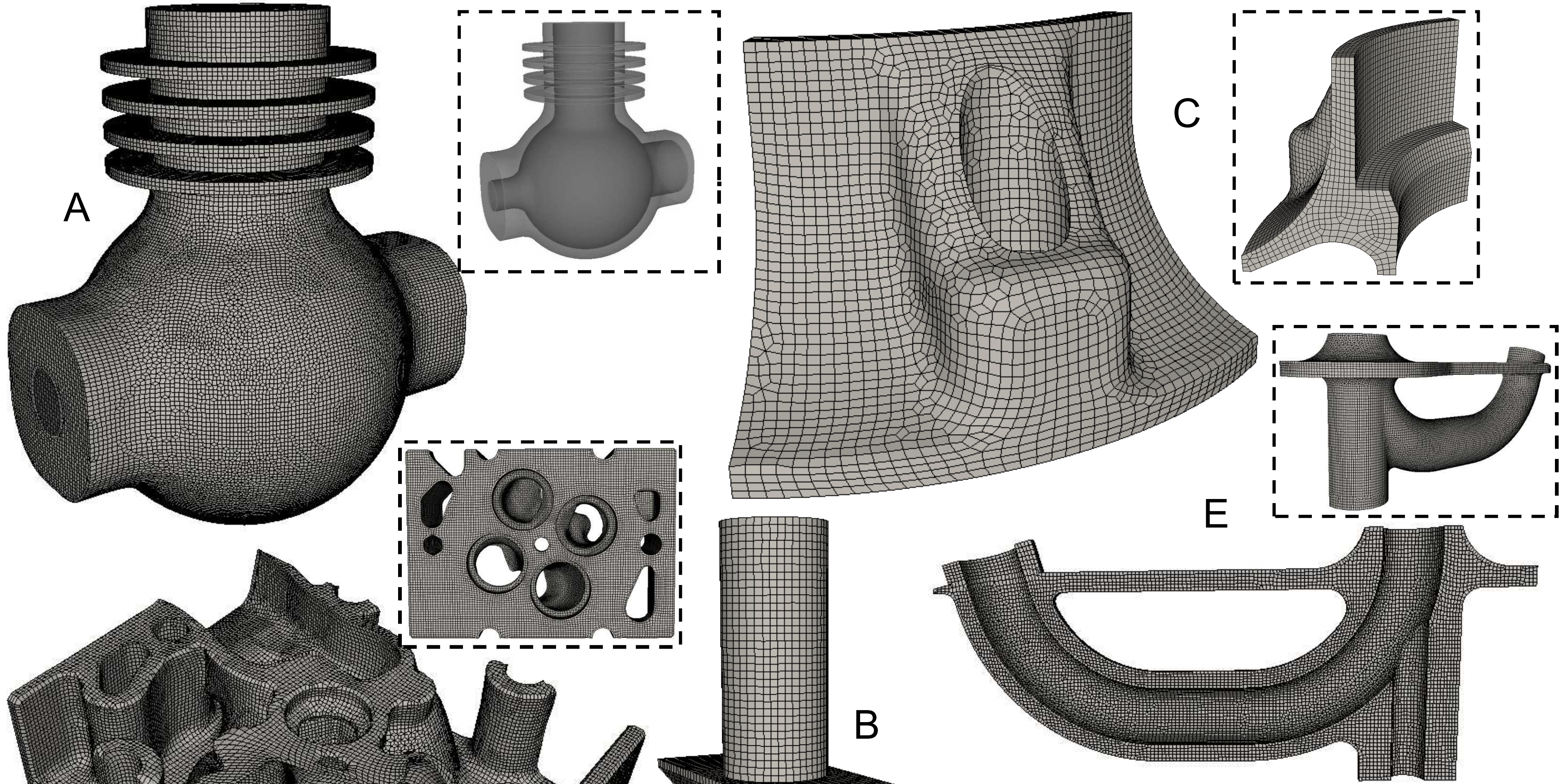
Multi-surface
Hexahedral Mesh Generation

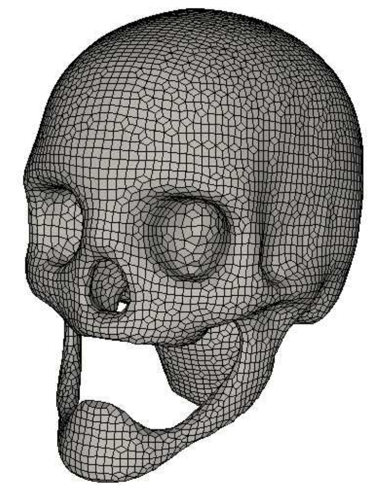
- **Example (goose16) – process (cont'd.)**



- **Example (goose16) -**



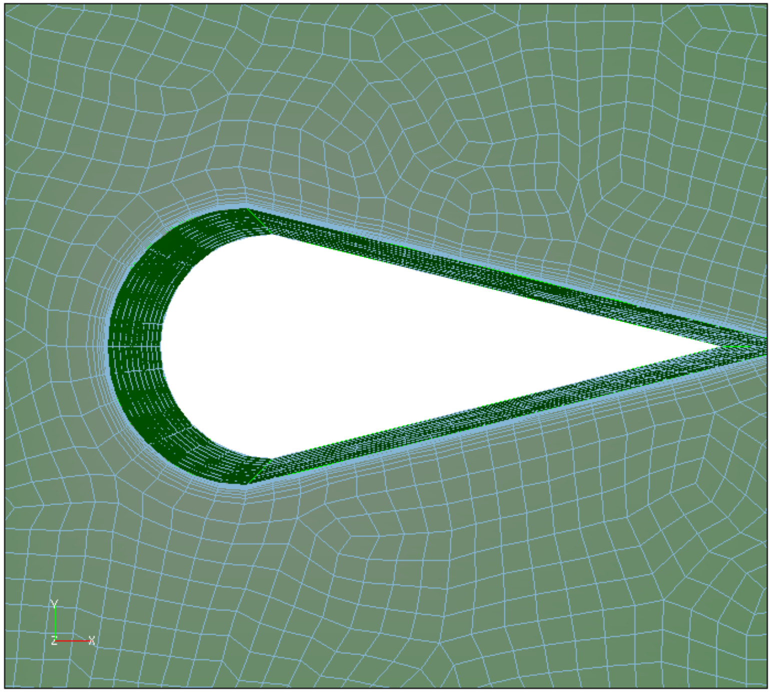


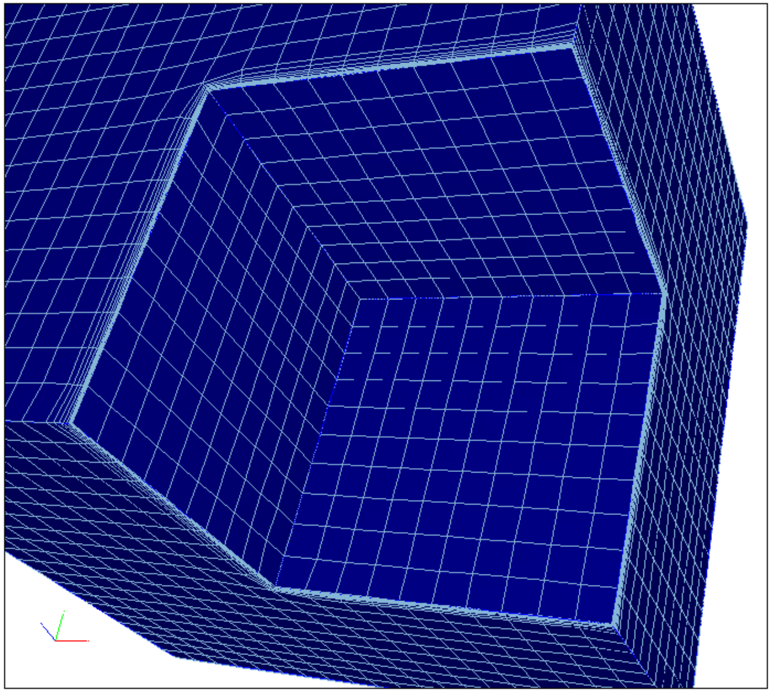


-Models A, C, D, E are provided courtesy of ANSYS
-Model B is provided courtesy of Tim Tautges by the AIM@SHAPE Shape Repository
-Model F is provided courtesy of Inria by the AIM@SHAPE Shape Repository

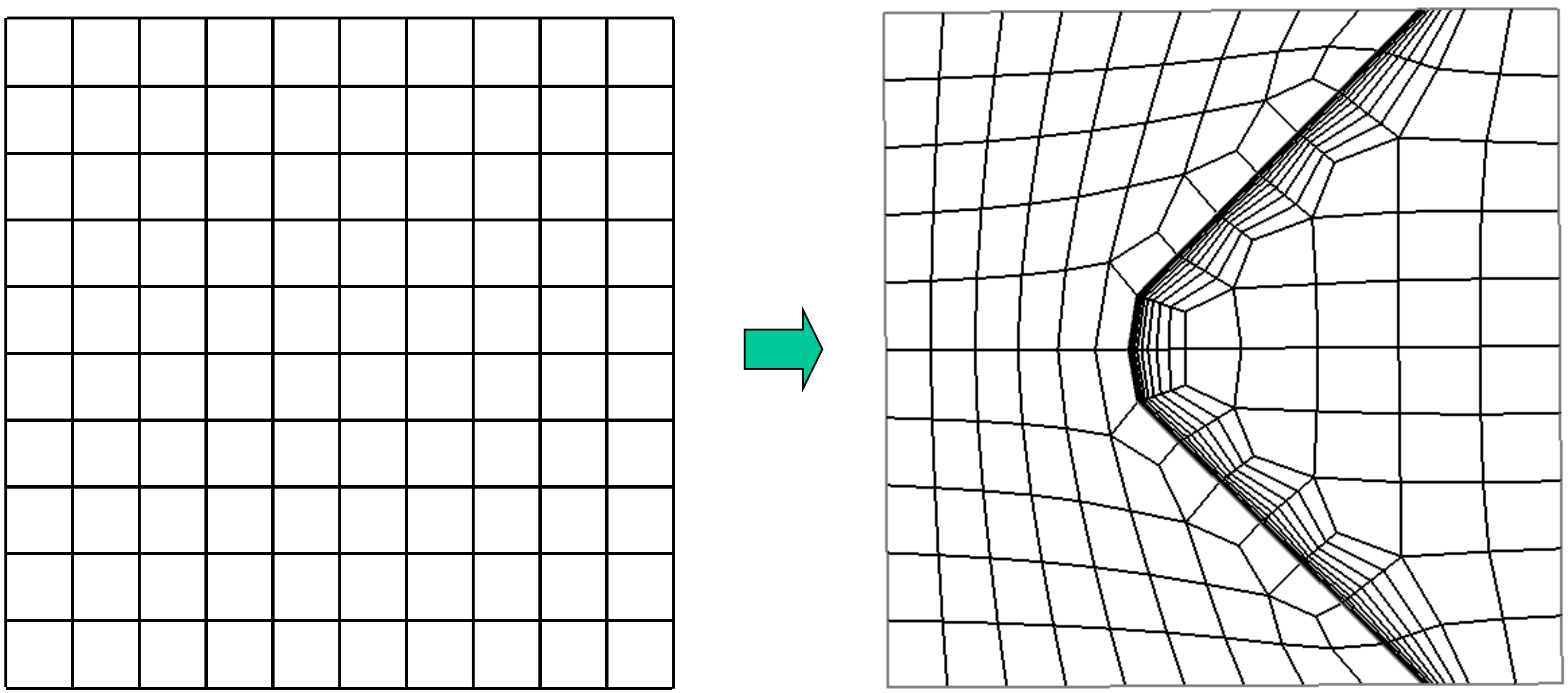
R-Adaptive Refinement

- **R-Adaptive Refinement**





R-Adaptive Refinement



Conclusion

- Decomposition time for hexahedral mesh generation is significant, but required for traditional approaches. New approaches to hexahedral mesh generation can reduce this overhead.
- Mesh transformation operations exist that will allow us to convert one mesh to an alternate mesh without destroying geometric integrity.
- The fundamental mesh is related to the minimal mesh in a geometric object.
- The fundamental mesh gives a quantifiable set of structures for determining geometric integrity of a mesh to a given geometry.
- Introduction of fundamental sheets/chords can be used in place of decomposition to build up new meshes which conform to difficult geometries.

Publications

Dissertation:

- J.F. Shepherd, "Topologic and Geometric Constraint-Based Hexahedral Mesh Generation," Doctoral Dissertation, University of Utah, 2007. (Dr. Chris R. Johnson, advisor)

Papers:

- S.J. Owen, M.L. Staten, M.J. Borden, J.F. Shepherd, "New Strategies for Unstructured All-Hexahedral Mesh Generation," submitted to APCOM'07 in conjunction with EPMESC X, October 2007.
- S.J. Owen, B. Clark, D.J. Melander, M. Brewer, J.F. Shepherd, K. Merkley, C.D. Ernst, R.G. Morris, "An Immersive Topology Engine for Meshing," Proceedings, 16th International Meshing Roundtable, October 2007.
- K. Merkley, C. D. Ernst, J.F. Shepherd, M.J. Borden, "Methods and Applications of Generalized Sheet Insertion for Hexahedral Meshing," Proceedings, 16th International Meshing Roundtable, October 2007.
- J.F. Shepherd, C. R. Johnson, "Hexahedral Mesh Generation for Biomedical Models in SCIRun," accepted to Engineering with Computers, September 2007.
- M. Callahan, M. J. Cole, J.F. Shepherd, J. Stinstra, C. R. Johnson, "BioMesh3D: A Meshing Pipeline for Biomedical Models," submitted to Engineering with Computers, June 2007.
- P. P. Pebay, D. C. Thompson, J.F. Shepherd, P. Knupp, "New Applications of the Verdict Library for Standardized Mesh Verification: Pre, Post, and End-to-End Processing," Proceedings, 16th International Meshing Roundtable, October 2007.
- F. Stenger, B. Baker, C. Brewer, G. Hunter, S. Kaputerko, J.F. Shepherd, "Periodic Approximations Based on Sinc," submitted to Sampling Theory in Signal and Image Processing, November 2006.
- C.D. Carbonera , J.F. Shepherd "On the Existence of a Perfect Matching for 4-Regular Graphs derived from Quadrilateral Meshes," submitted to the Journal of Graph Theory, November, 2006.
- J.F. Shepherd, C. Tuttle, C. Silva, X. Zhang, "Quality Improvement and Feature Capture in Hexahedral Meshes," Proceedings, International Society of Grid Generation, September 2007.
- C.D. Carbonera, J.F. Shepherd, "A Constructive Approach to Constrained Hexahedral Mesh Generation," Proceedings, 15th International Meshing Roundtable, pp. 435-452, Birmingham, AL, September 2006.
- J.F. Shepherd, C. R. Johnson, "Hexahedral Mesh Generation Constraints," invited to a special issue of Engineering with Computers, August 2006.
- Suzuki, T., Yamakawa, S., Shepherd, J.F., "An Interior Surface Generation Method for All-Hexahedral Meshing", accepted to a special issue of Engineering with Computers, September 2007.
- Shawn Means, Alexander J. Smith, Jason Shepherd, John Shadid, John Fowler, Richard Wojcikiewicz, Tomas Mazel, Gregory D. Smith, and Bridget S. Wilson, "Reaction Modeling of Calcium Dynamics with Realistic ER Geometry", Biophysical Journal 91(2), pp. 537-557, July 15, 2006.

Impact

Computational Modeling Sciences Department

Patents (granted):

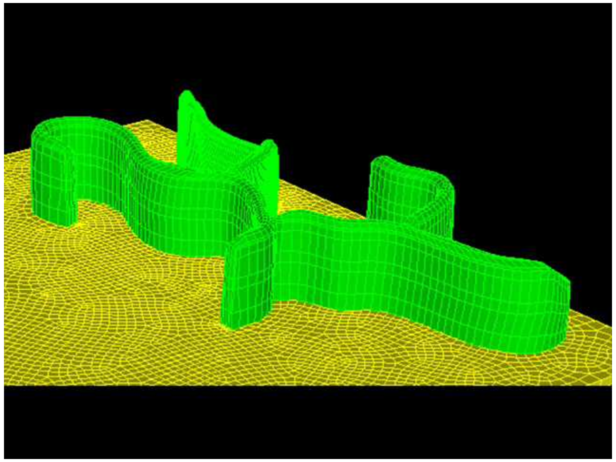
- **•U.S. Patent No. 7098912 - J.F. Shepherd, M.J. Borden, "Method of Modifying a Volume Mesh using Sheet Insertion", Granted Aug. 29, 2006.**
- **•U.S. Patent No. 7181377 – M.J. Borden, J.F. Shepherd, "Method of Modifying a Volume Mesh using Sheet Extraction," Granted Feb. 20, 2007.**
- **•U.S. Patent No. 7219039 – J.F. Shepherd, S.R. Jankovich, S.E. Benzley, S.A. Mitchell, "Method for Generating a Mesh Representation of a Region Characterized by a Trunk and a Branch Thereon", Granted March 2007.**

Patents (pending):

- **J.F. Shepherd, B.J. Grover, S.E. Benzley, "Quadrilateral mesh generation and modification for surfaces by dual creation and manipulation," U.S. Patent Pending, 2002.**
- **M.L. Staten, A.C. Woodbury, S.E. Benzley, J. F. Shepherd, "Finite Element Mesh Coarsening Using Pillowing Technique," U.S. Patent Pending, 2007.**

External Collaborations:

- **Collaborator, UNM's NIH center proposal, 2007.**
  - Dynamic, adaptive meshing for cellular models



- **Consultant, UCSD's NIH NCMIR center proposal, 2007.**
  - Automated, qualitative mesh generation for cellular modeling
- **Collaborator, UofU's NIH CIBC center proposal, 2006.**
  - Mesh generation tool suite for biomedical modeling
- **CEA-DAM, Franck Ledoux, Jean-Christophe Weill.**
  - NNSA collaboration on advanced hexahedral whisker weaving.
- **Brigham Young University contract, Dr. Steven Benzley + 3 students.**
  - Hexahedral coarsening and refinement

Other:

- **Winner, "Best Technical Poster," – M.L. Staten, R.L. Kerr, J.F. Shepherd, A.C. Woodbury, S.J. Owen, "Advanced Hexahedral Mesh Generation and Modification," 16th International Meshing Roundtable, Seattle, Washington, October 2007.**